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Student Number

SCEGGS Darlinghurst

2019

**Year 11 Preliminary Course
Semester 2 Examination**

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Answer Section I on the multiple-choice answer sheet provided
- Answer Section II questions in the writing spaces provided and show relevant mathematical reasoning and/or calculations
- Marks will be deducted for careless or badly arranged work
- Extra space is available at the end of the examination
- Calculators approved by NESA may be used
- A Reference Sheet is provided on a separate sheet

Total marks:
70

Section I - 8 marks

- Attempt Questions 1-8
- Allow about 12 minutes for this section

Section II - 62 marks

- Attempt Questions 9-28
- Allow about 1 hour 48 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ A. 2 B. 6 C. 8 D. 9

 A. ☐ B. ☒ C. ☐ D. ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A. ☒ B. ☒ C. ☐ D. ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A. ☒ B. ☒ C. ☐ D. ☐

correct

Section I – Multiple Choice

8 marks

Attempt Questions 1–8

Allow about 12 minutes for this section

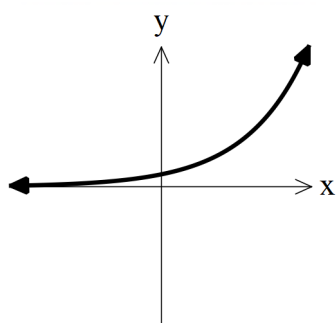
Use the multiple-choice answer sheet for Questions 1-8.

1. Which of the following is an example of a discrete random variable?

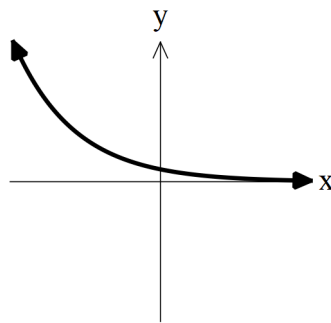
- A. The total number of tails showing when 5 coins are tossed
- B. The hair colour of students in a class
- C. The heights of players on a soccer team
- D. The first initial of people on the electoral roll

2. Which of the following curves best represents $y = -3^x$?

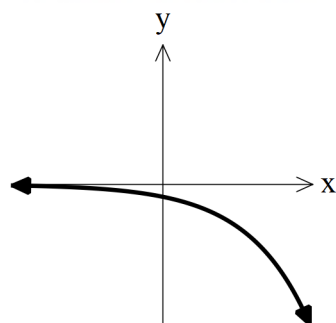
A.



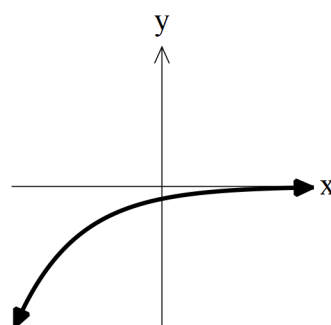
B.



C.



D.



3. Which of the following is equivalent to $2\log_3 8 - \log_3 2$?

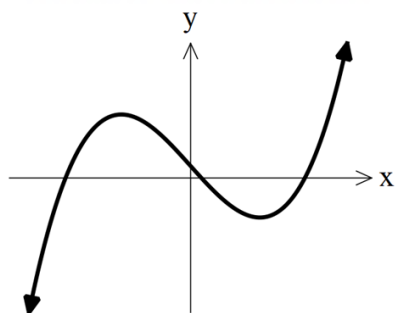
- A. $2\log_3 4$
- B. $\log_3 8$
- C. $\log_3 32$
- D. $\log_3 62$

4. When $x^2 - 8x + 5$ is expressed in the form $(x - a)^2 + b$, what is the value of b ?

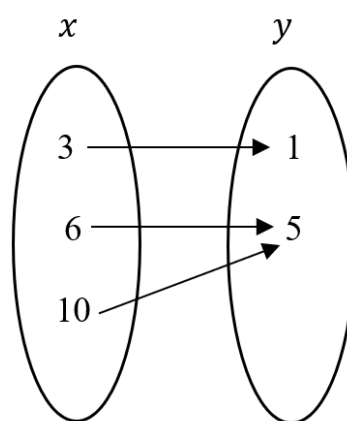
- A. -11
- B. 5
- C. 16
- D. 21

5. Which one of the following relations is **not** many-to-one?

A.



B.



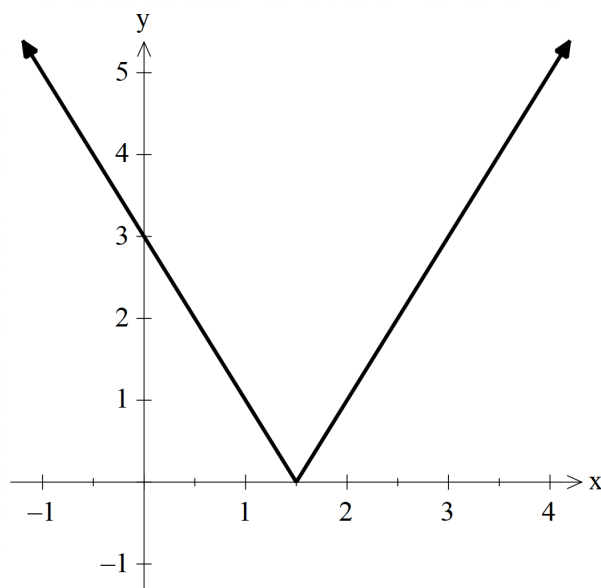
C. $(-1,6) (-1,4) (2,10) (3,5)$

D. A relation which maps 100 students to the month in which they were born.

6. What is the value of $\cot \frac{\pi}{2}$?

- A. -1
- B. 0
- C. 1
- D. undefined

7. What is the equation of the graph below?



- A. $y = |2x + 3|$
- B. $y = |2x - 3|$
- C. $y = |x - 1.5|$
- D. $y = |x + 1.5|$

8. The discrete random variable X has the following probability distribution.

x	0	1	2	3	4
$P(X = x)$	$\frac{5}{20}$	$\frac{9}{20}$	a	$\frac{1}{20}$	$\frac{3}{20}$

Let μ be the mean of X .

What is $P(X < \mu)$?

- A. $\frac{1}{10}$
- B. $\frac{7}{5}$
- C. $\frac{1}{4}$
- D. $\frac{7}{10}$

End of Section I

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Mathematics Advanced Section II

Student Number

62 marks

Attempt Questions 9-28

Allow about 1 hour and 48 minutes for this section

Write your student number at the top of this page.

Answer the questions in the spaces provided. There is extra writing space at the back of the booklet.
If you use this space, clearly label each answer.

Please turn over

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Question 9 (3 marks)

Let $f(x) = x^2 + 5$ and $g(x) = -4x$.

- (a) Find $g(x) - f(x)$. 1

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- (b) What is the degree of the polynomial $f(x) \times g(x)$? 1

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- (c) Find $f(g(x))$. 1

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Question 10 (2 marks)

Simplify $\sec \theta \times \cot \theta$. 2

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Question 11 (1 mark)

Simplify $\frac{5\sqrt{21}}{20\sqrt{7}}$. 1

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Question 12 (3 marks)

Simplify $\frac{x^2-16}{4x^2-12x-16}$.

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Question 13 (2 marks)

Find the exact value of $\sec \frac{7\pi}{6}$.

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Question 14 (1 mark)

Find the value of $\log_a \sqrt[3]{a}$.

1

Question 15 (3 marks)

Let $f(x) = 3x^2$.

3

Using first principles, show that $f'(x) = 6x$.

Question 16 (2 marks)

Solve $6^x = 12$, giving your answer to 3 significant figures.

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Question 17 (2 marks)

Solve $|3x + 1| = 2$

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Question 18 (4 marks)

Solve the following trigonometric equations:

(a) $\cos \theta = 0$ for $0 \leq \theta \leq 360^\circ$.

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(b) $4 \sin^2 \theta = 1$ for $-\pi \leq \theta \leq \pi$.

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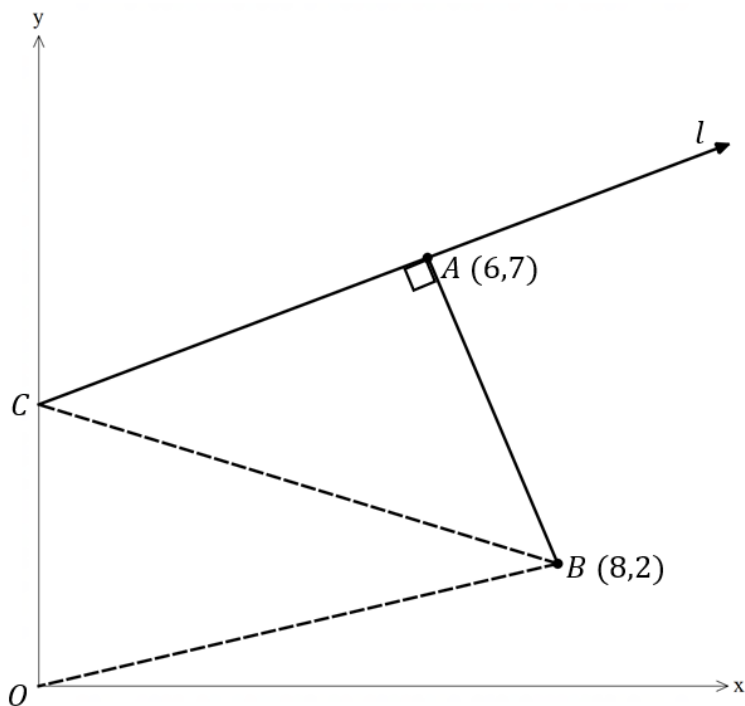
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Question 19 (5 marks)

The points A and B have coordinates $(6,7)$ and $(8,2)$ respectively.

The line l passes through the point A and is perpendicular to the line AB , as shown in the diagram below. Line l intersects the y -axis at the point C .



- (a) Find the gradient of AB .

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Question 19 (continued)

- (b) Find the equation of line l in general form.

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- (c) Find the coordinates of C .

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- (d) The area of $\triangle OCB$, where O is the origin.

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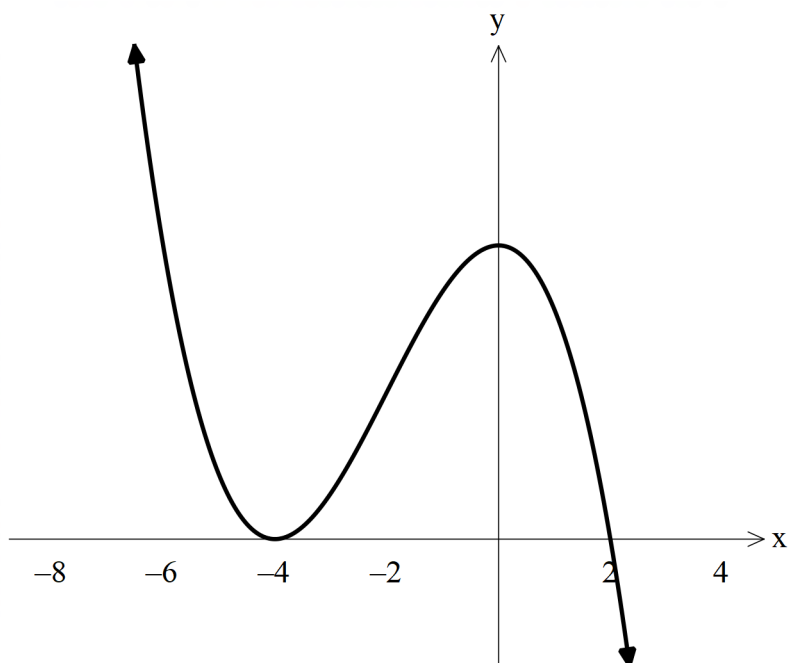
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End of Question 19

Question 20 (2 marks)

A function, $f(x)$, has been sketched below.



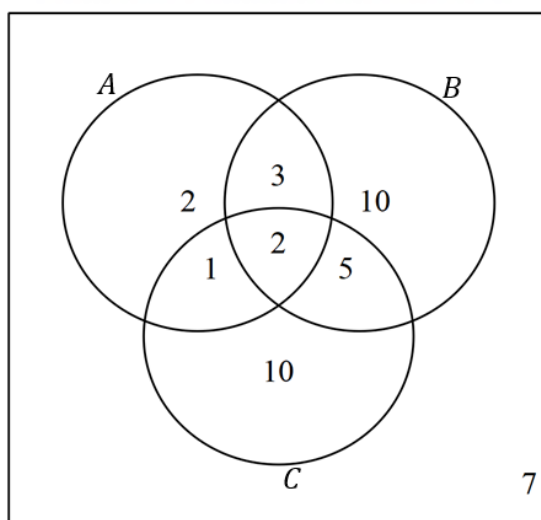
Sketch the gradient function, $f'(x)$.

2

Question 21 (4 marks)

Teatree College is a school with 40 teachers.

The Venn diagram below shows the number of teachers at Teatree College who visit daily any of 3 popular news websites A , B and C .



- (a) How many teachers visit websites A or B ? 1

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- (b) Calculate $P(B|C)$. 1

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- (c) With suitable calculations, determine whether visiting website A and visiting website B are independent events. 2

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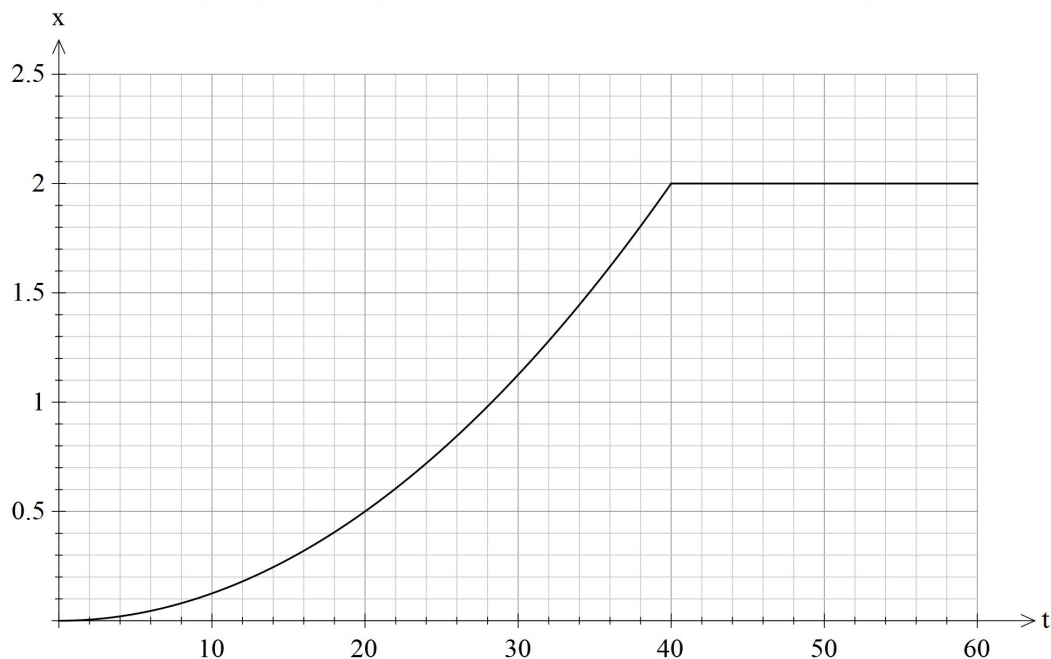
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Question 22 (4 marks)

The graph below, $f(t)$, displays a student's journey to school, walking x km away from home over t minutes.



- (a) Find $f(20)$ and explain its meaning in the context of this question. 2

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- (b) What is the average rate of change from $t = 20$ to $t = 40$ minutes. 1

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- (c) What does $f'(20)$ represent in the context of this question? 1

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Question 23 (5 marks)

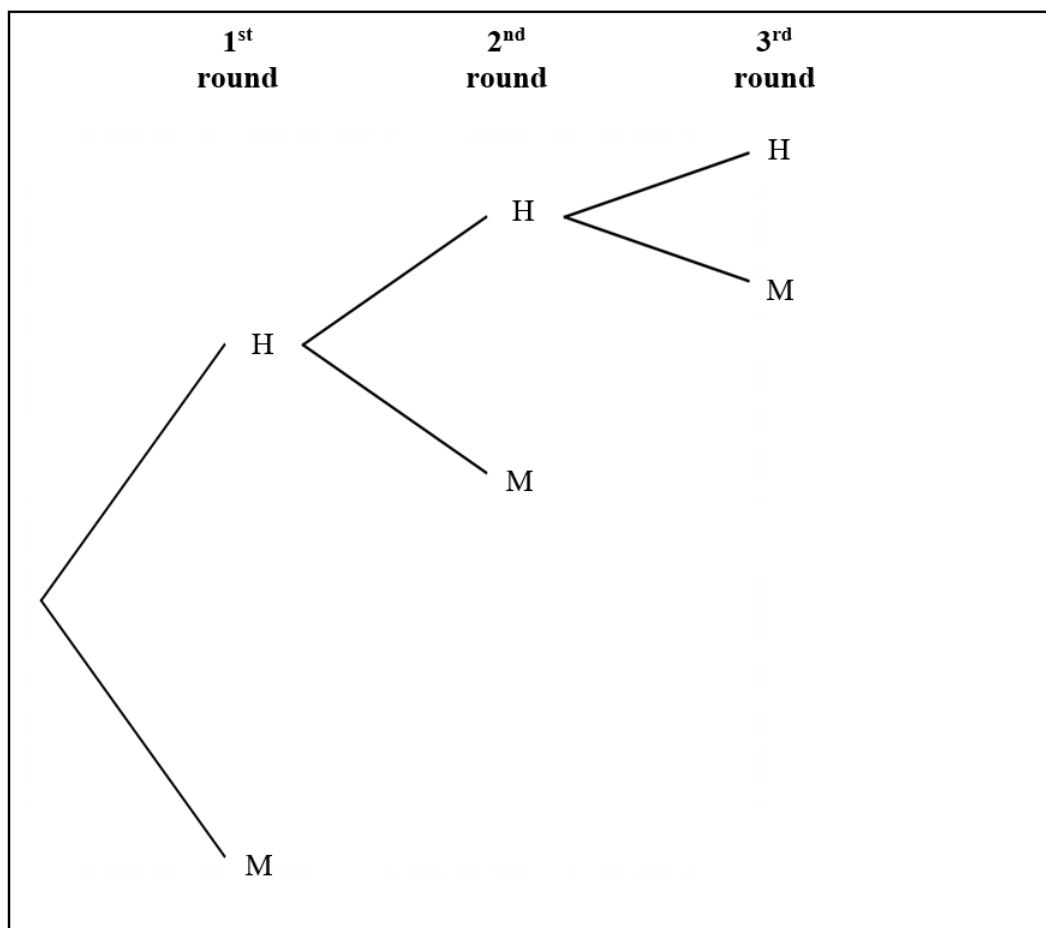
A fairground game involves trying to hit a moving target with a gunshot. A round consists of up to three shots. Ten points are scored if a player hits the target, but the round is over if the player misses.

Linda has a probability of 0.6 of hitting the target and shots are independent of one another.

The random variable X is the number of points that Linda scores in a round.

- (a) By completing the tree diagram, and showing clear calculations and working, show that the probability distribution of X is as shown in the table below. 2

x	0	10	20	30
$P(X = x)$	0.4	0.24	0.144	0.216



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Question 23 (continued)

- (b) Find the expected value of the distribution.

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- (c) Show that the variance of X is approximately 137.7

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End of Question 23

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Question 24 (5 marks)

- (a) Find the points of intersection of $x - y = 2$ and $xy = 15$.

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- (b) Sketch the two functions, clearly showing the points of intersection and all important features.

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Question 25 (3 marks)

The Richter scale measures the magnitude of the seismic waves of an earthquake. An instrument called a seismograph detects movement in the Earth, with the smallest movement that can be detected shown as a wave with amplitude A_0 .

The Richter magnitude, R , is given by the equation

$$R = \log_{10} \frac{A}{A_0}$$

where A is the amplitude of the earthquake wave and A_0 is the amplitude of the smallest detectable wave.

- (a) An earthquake is measured with an amplitude 485 times as great as A_0 . What is the magnitude of this earthquake, using the Richter scale, to 1 decimal place? **1**

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- (b) How many times greater is the amplitude of an earthquake with magnitude 3.5 than an earthquake with magnitude 2? **2**

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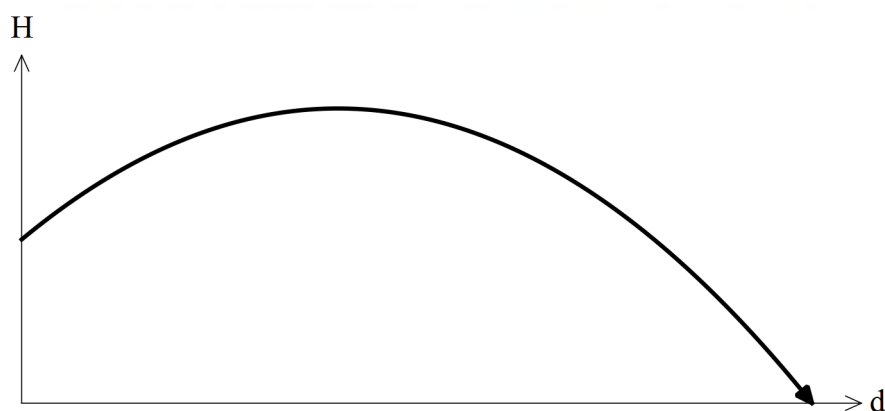
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Question 26 (5 marks)

An archer shoots an arrow, as shown in the picture below.



The height, H metres, of the arrow above the ground is modelled by the formula

$$H = -0.1d^2 + 0.8d + 2 \quad \text{where } d \geq 0, H \geq 0.$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

- (a) Find the horizontal distance travelled by the arrow.

2

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- (b) With reference to the context, interpret the significance of the constant, 2, in the formula.

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Question 26 (continued)

Marks

- (c) Find the maximum height of the arrow above the ground.

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End of Question 26

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Question 27 (3 marks)

Prove that $\frac{\cos \theta}{1+\sin \theta} - \frac{\cos \theta}{1-\sin \theta} = -2 \tan \theta$.

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A diagram showing a sector of a circle with radius a cm and angle $\frac{\pi}{6}$. The sector is bisected by a vertical dashed line, which represents the plane of symmetry. The sector is divided into two regions by this line, with the left region being a smaller sector and the right region being a larger sector.

3

Year 11 Preliminary Course Semester 2 Examination, 2019
Mathematics Advanced

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8



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Student Number

SCEGGS Darlinghurst

2019

**Year 11 Preliminary Course
Semester 2 Examination**

SOLUTIONS

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours 10 minutes
- Write using black pen
- Answer Section I on the multiple-choice answer sheet provided
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Total marks:

70

Section I - 8 marks

- Attempt Questions 1-8
- Allow about 12 minutes for this section

Section II - 62 marks

- Attempt Questions 9-28
- Allow about 1 hour 48 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$

A. 2 B. 6 C. 8 D. 9

A. ☐ B. ☒ C. ☐ D. ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A. ☒ B. ☒ C. ☐ D. ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A. ☒ B. ☒ C. ☐ D. ☐

correct

↓

Section I – Multiple Choice

8 marks

Attempt Questions 1–8

Allow about 12 minutes for this section

Use the multiple-choice answer sheet for Questions 1-8.

1. Which of the following is an example of a discrete random variable?

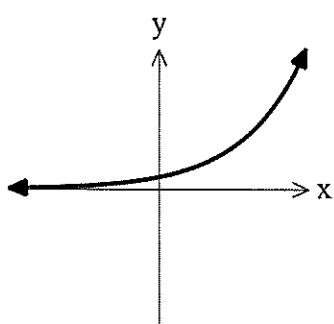
S

- ☒ A. The total number of tails showing when 5 coins are tossed
- B. The hair colour of students in a class
- C. The heights of players on a soccer team
- D. The first initial of people on the electoral roll

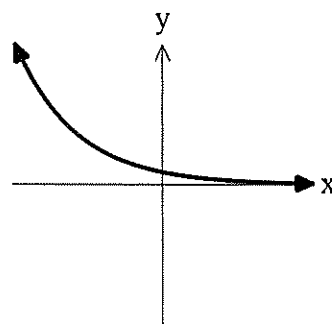
2. Which of the following curves best represents $y = -3^x$?

E

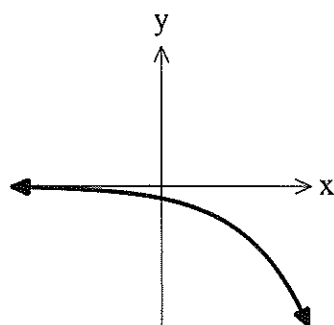
A.



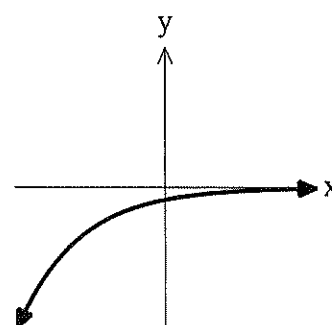
B.



☒ C.



D.



3. Which of the following is equivalent to $2\log_3 8 - \log_3 2$? E

- A. $2\log_3 4$
- B. $\log_3 8$
- ☒ C. $\log_3 32$
- D. $\log_3 62$

$$\begin{aligned} & 2\log_3 8 - \log_3 2 \\ &= \log_3 8^2 - \log_3 2 \\ &= \log_3 64 - \log_3 2 \\ &= \log_3 \frac{64}{2} \\ &= \log_3 32 \end{aligned}$$

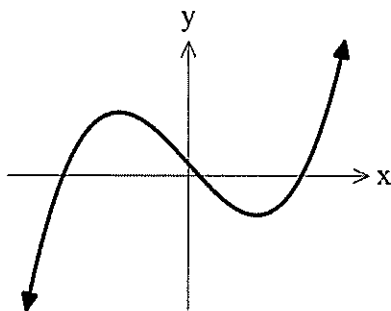
4. When $x^2 - 8x + 5$ is expressed in the form $(x - a)^2 + b$, what is the value of b ? F

- ☒ A. -11
- B. 5
- C. 16
- D. 21

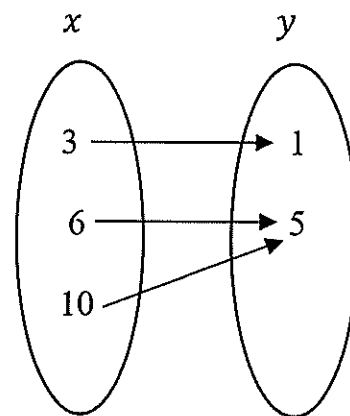
$$\begin{aligned} & x^2 - 8x + 5 \\ &= x^2 - 8x + 16 - 16 + 5 \\ &= (x - 4)^2 - 11 \end{aligned}$$

5. Which one of the following relations is **not** many-to-one? F

A.



B.



- ☒ C. $(-1,6) (-1,4) (2,10) (3,5)$

D. A relation which maps 100 students to the month in which they were born.

6. What is the value of $\cot \frac{\pi}{2}$?

T

A. -1

☒ B. 0

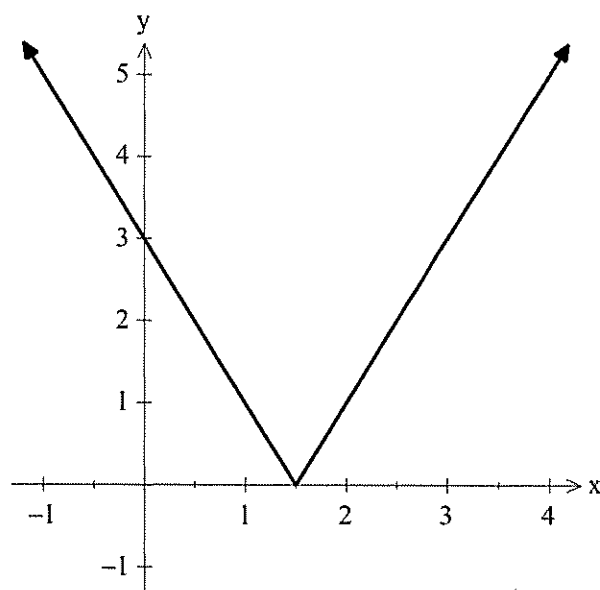
C. 1

D. undefined

$$\begin{aligned}\cot \frac{\pi}{2} &= \frac{\cos \frac{\pi}{2}}{\sin \frac{\pi}{2}} \\ &= \frac{0}{1} \\ &= 0\end{aligned}$$

7. What is the equation of the graph below?

F



A. $y = |2x + 3|$

☒ B. $y = |2x - 3|$

C. $y = |x - 1.5|$

D. $y = |x + 1.5|$

x -intercept = 1.5
gradient = ± 2

8. The discrete random variable X has the following probability distribution.

x	0	1	2	3	4
$P(X = x)$	$\frac{5}{20}$	$\frac{9}{20}$	a	$\frac{1}{20}$	$\frac{3}{20}$

Let μ be the mean of X .

What is $P(X < \mu)$?

A. $\frac{1}{10}$

B. $\frac{7}{5}$

(C) $\frac{1}{4}$

(D) $\frac{7}{10}$

$$\begin{aligned} \mu = E(X) &= 0 \times \frac{5}{20} + 1 \times \frac{9}{20} + 2 \times \frac{2}{20} + 3 \times \frac{1}{20} + 4 \times \frac{3}{20} \\ &= \frac{7}{5} \\ &= 1.4 \end{aligned}$$

$$\begin{aligned} P(X < 1.4) &= \frac{5}{20} + \frac{9}{20} \\ &= \frac{14}{20} \\ &= \frac{7}{10} \end{aligned}$$

End of Section I

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Mathematics Advanced Section II

Student Number

62 marks

Attempt Questions 9-28

Allow about 1 hour and 30 minutes for this section

Write your student number at the top of this page.

Answer the questions in the spaces provided. There is extra writing space at the back of the booklet.
If you use this space, clearly label each answer.

Please turn over

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Question 9 (3 marks)

Let $f(x) = x^2 + 5$ and $g(x) = -4x$.

- (a) Find $g(x) - f(x)$.

1 F

$$-4x - (x^2 + 5) = -4x - x^2 - 5$$

- (b) What is the degree of the polynomial $f(x) \times g(x)$?

1 F

degree 3

- (c) Find $f(g(x))$.

1 F

$$\begin{aligned} f(-4x) &= (-4x)^2 + 5 \\ &= 16x^2 + 5 \end{aligned}$$

Question 10 (2 marks)

Simplify $\sec \theta \times \cot \theta$.

2 T

$$\begin{aligned} \frac{1}{\cos \theta} \times \frac{\cos \theta}{\sin \theta} &= \frac{\cos \theta}{\cos \theta \sin \theta} \\ &= \frac{1}{\sin \theta} \\ &= \operatorname{cosec} \theta \end{aligned}$$

Question 11 (1 mark)

Simplify $\frac{5\sqrt{21}}{20\sqrt{7}}$.

1 F

$$= \frac{\sqrt{3}}{4}$$

Question 12 (3 marks)

Simplify $\frac{x^2-16}{4x^2-12x-16}$.

3 F

$$= \frac{(x+4)(x-4)}{4(x^2-3x-4)}$$

$$= \frac{(x+4)(x-4)}{4(x-4)(x+1)}$$

$$= \frac{(x+4)}{4(x+1)}$$

$$= \frac{(x+4)}{4(x+1)}$$

$$= \frac{(x+4)}{4(x+1)}$$

$$= \frac{(x+4)}{4(x+1)}$$

Question 13 (2 marks)

Find the exact value of $\sec \frac{7\pi}{6}$.

2 T

$$\sec \frac{7\pi}{6} = \frac{1}{\cos \frac{7\pi}{6}}$$

$$= \frac{1}{-\sqrt{3}/2}$$

$$= \frac{-2}{\sqrt{3}}$$

$$\cos \frac{7\pi}{6} = -\cos \frac{\pi}{6}$$

$$= -\frac{\sqrt{3}}{2}$$

Question 14 (1 mark)

Find the value of $\log_a \sqrt[3]{a}$.

1 E

$$\begin{aligned} & \log_a a^{1/3} \\ &= \frac{1}{3} \log_a a \\ &= \frac{1}{3} \end{aligned}$$

Question 15 (3 marks)

Let $f(x) = 3x^2$.

3 C

Using first principles, show that $f'(x) = 6x$.

$$\begin{aligned} f(x) &= 3x^2 \\ f(x+h) &= 3(x+h)^2 \\ &= 3(x^2 + 2xh + h^2) \\ &= 3x^2 + 6xh + 3h^2 \\ f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(6x + 3h)}{h} \\ &= 6x + 3 \times 0 \\ &= 6x \end{aligned}$$

Question 16 (2 marks)

Solve $6^x = 12$, giving your answer to 3 significant figures.

2 E

$$\log_{10} 6^x = \log_{10} 12$$

$$x \log_{10} 6 = \log_{10} 12$$

$$x = \frac{\log_{10} 12}{\log_{10} 6}$$

$$= 1.3868$$

$$\approx 1.39$$

Question 17 (2 marks)

Solve $|3x + 1| = 2$

2 F

$$3x + 1 = 2$$

$$-(3x + 1) = 2$$

$$3x = 1$$

$$-3x - 1 = 2$$

$$x = \frac{1}{3}$$

$$-3x = 3$$

$$x = -1$$

$$\therefore x = -1, x = \frac{1}{3}$$

Question 18 (4 marks)

Solve the following trigonometric equations:

(a) $\cos \theta = 0$ for $0 \leq \theta \leq 360^\circ$.

1 T

From the graph

$$\theta = 90^\circ, 270^\circ$$

(b) $4 \sin^2 \theta = 1$ for $-\pi \leq \theta \leq \pi$.

3 T

$$\sin^2 \theta = \frac{1}{4}$$

$$\sin \theta = \pm \frac{1}{2}$$

$$\sin \theta = \frac{1}{2}$$

$$\sin \theta = -\frac{1}{2}$$

(quad 1 and 2)

(quad 3 and 4)

Related acute angle = $\frac{\pi}{6}$

$$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$$

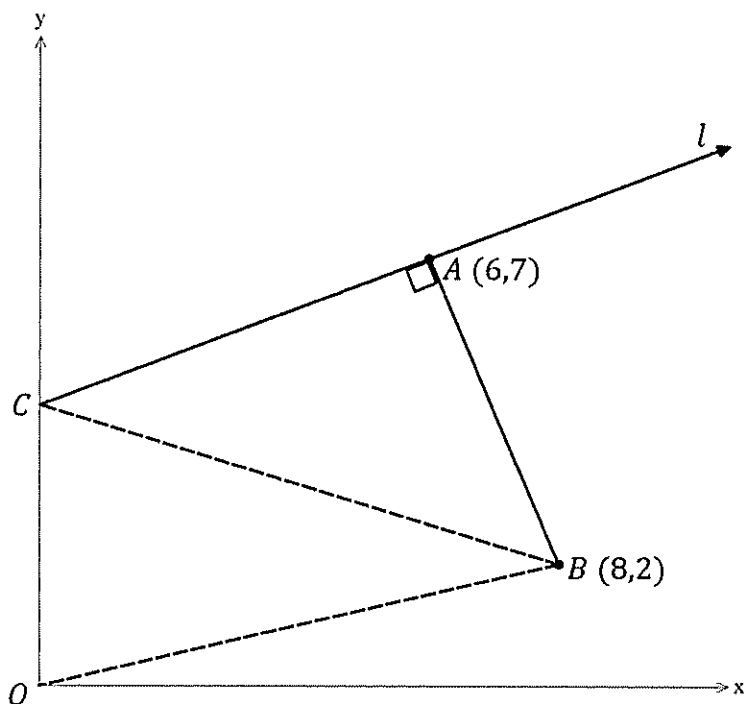
$$\theta = -\frac{\pi}{6}, -\frac{5\pi}{6}$$

$$= \frac{\pi}{6}, \frac{5\pi}{6}$$

Question 19 (5 marks)

The points A and B have coordinates $(6,7)$ and $(8,2)$ respectively.

The line l passes through the point A and is perpendicular to the line AB , as shown in the diagram below. Line l intersects the y -axis at the point C .



- (a) Find the gradient of AB .

1 F

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{7 - 2}{6 - 8} \\
 &= -\frac{5}{2}
 \end{aligned}$$

- (b) Find the equation of line l in general form.

2 F

$$\begin{aligned}
 &\text{Gradient of perpendicular line} = \frac{2}{5} \\
 &y - 7 = \frac{2}{5}(x - 6) \\
 &5y - 35 = 2x - 12 \\
 &2x - 5y + 23 = 0
 \end{aligned}$$

Question 19 (continued)

- (c) Find the coordinates of C .

1 F

$$\begin{aligned} \text{y-intercept: let } x &= 0 \\ -5y + 23 &= 0 \\ 5y &= 23 \\ y &= 4.6 \\ \therefore C(0, 4.6) \end{aligned}$$

- (d) The area of $\triangle OCB$, where O is the origin.

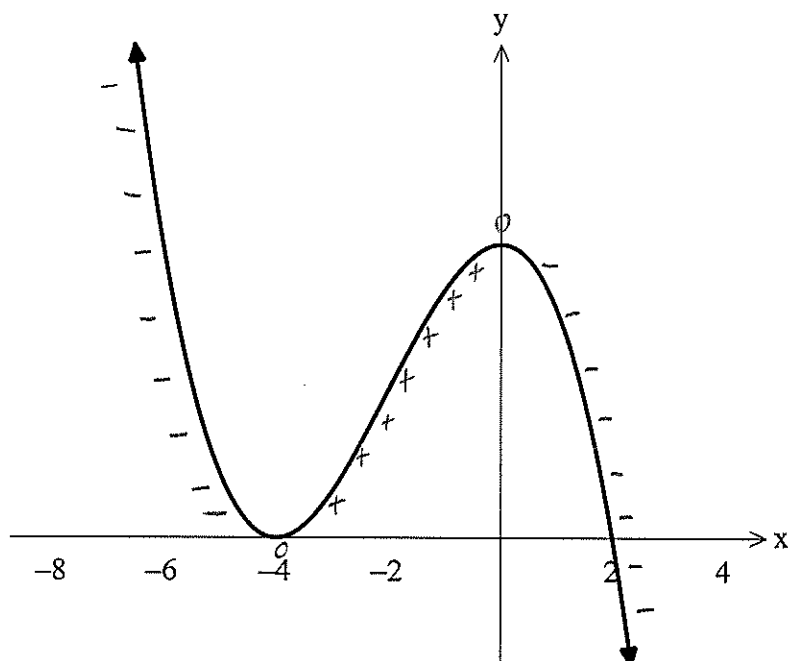
1 F

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 4.6 \times 8 \\ &= 18.4 \text{ units}^2 \end{aligned}$$

End of Question 19

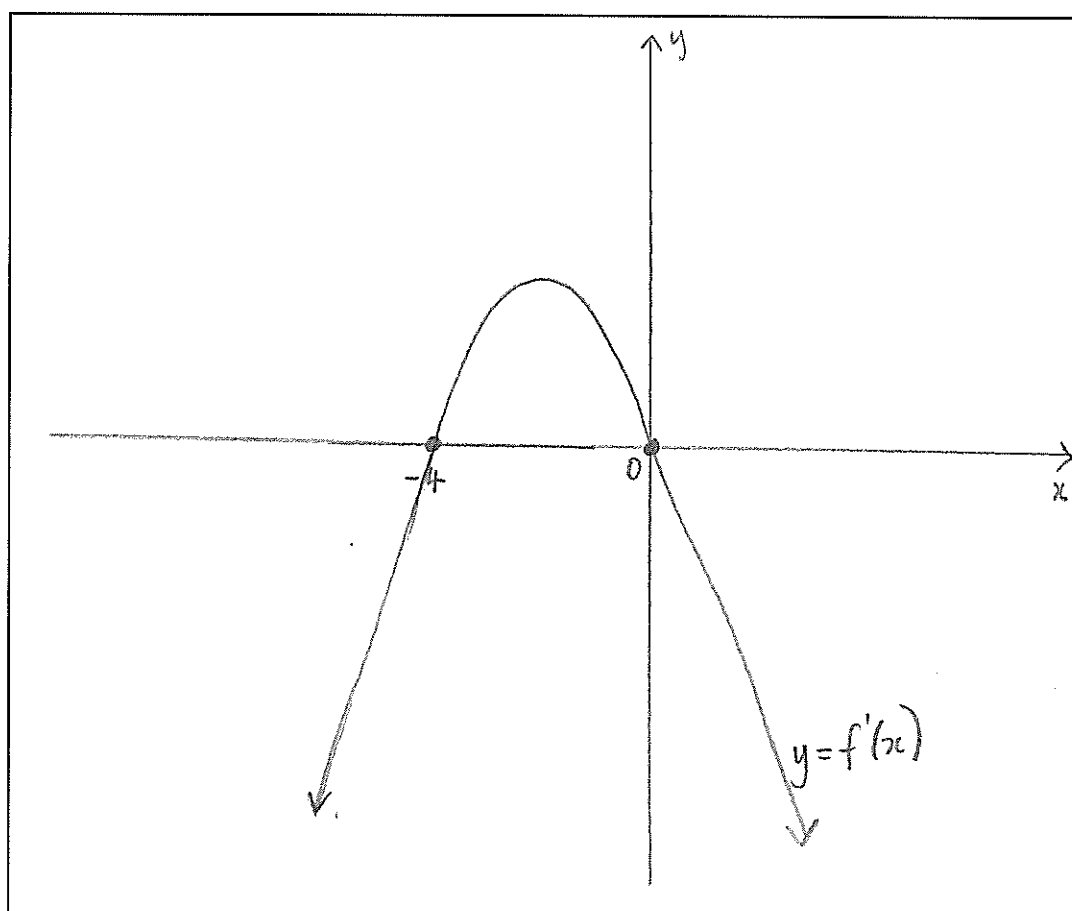
Question 20 (2 marks)

A function, $f(x)$, has been sketched below.



Sketch the gradient function, $f'(x)$.

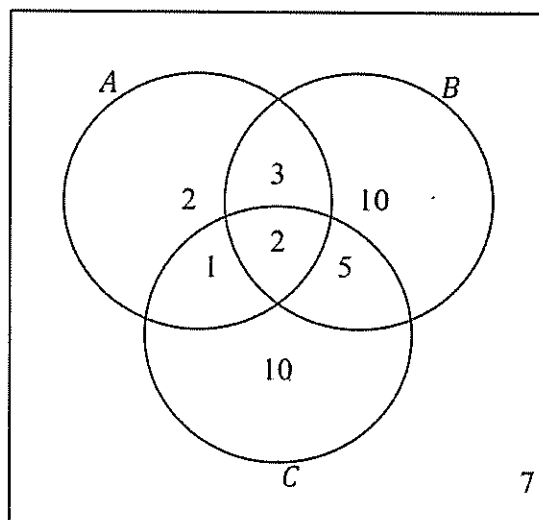
2 C



Question 21 (4 marks)

Teatree College is a school with 40 teachers.

The Venn diagram below shows the number of teachers at Teatree College who visit daily any of 3 popular news websites A , B and C .



- (a) How many teachers visit websites A or B ?

1 5

..... 23

- (b) Calculate $P(B|C)$.

1 5

..... $P(B|C) = \frac{7}{18}$

- (c) With suitable calculations, determine whether visiting website A and visiting website B are independent events.

2 5

..... $P(A) = \frac{8}{40}$ $P(B) = \frac{20}{40}$ $P(A \cap B) = \frac{5}{40}$

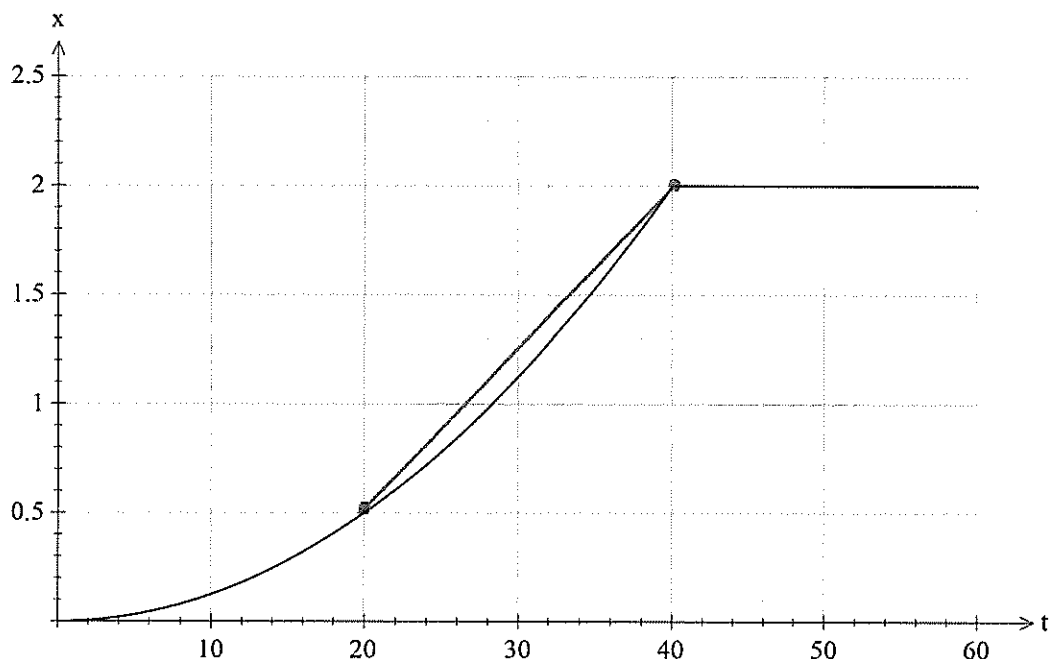
..... $P(A) \times P(B) = \frac{8}{40} \times \frac{20}{40}$

..... $= \frac{4}{40}$ Since $P(A) \times P(B) \neq P(A \cap B)$,
the events are not independent.

..... $\neq P(A \cap B)$

Question 22 (4 marks)

The graph below, $f(t)$, displays a student's journey to school, walking x km away from home over t minutes.



- (a) Find $f(20)$ and explain its meaning in the context of this question.

2 C

$f(20) = 0.5$
This means that the student is 0.5 km after walking for 20 minutes.

- (b) What is the average rate of change from $t = 20$ to $t = 40$ minutes.

1 C

Average rate of change = $\frac{f(40) - f(20)}{40 - 20}$
 $= \frac{2 - 0.5}{20}$
 $= 0.075 \text{ km/min}$

- (c) What does $f'(20)$ represent in the context of this question?

1 C

$f'(20)$ represents the speed that the student is walking at exactly 20 minutes after leaving home.

Question 23 (5 marks)

A fairground game involves trying to hit a moving target with a gunshot. A round consists of up to three shots. Ten points are scored if a player hits the target, but the round is over if the player misses.

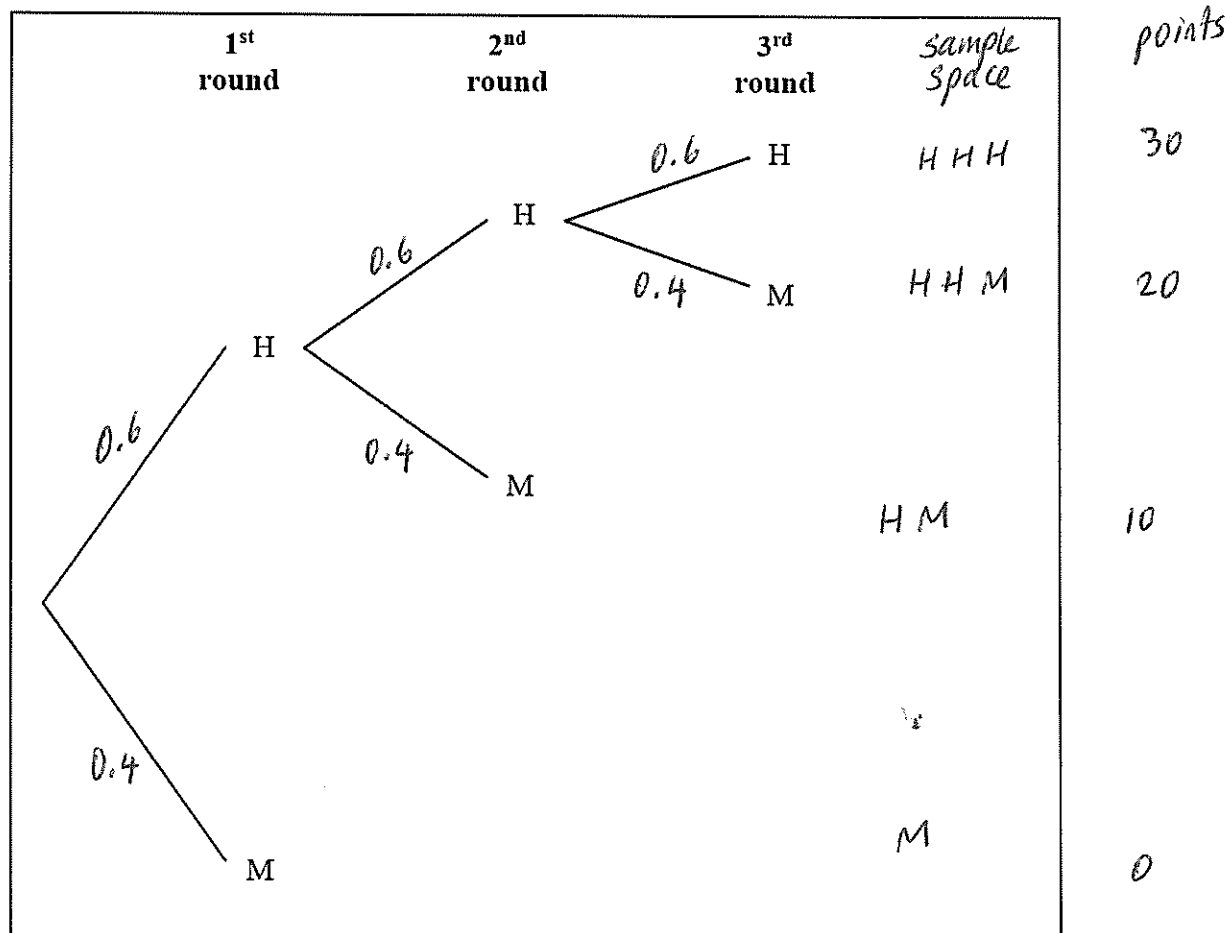
Linda has a probability of 0.6 of hitting the target and shots are independent of one another.

The random variable X is the number of points that Linda scores in a round.

- (a) By completing the tree diagram, and showing clear calculations and working, show that the probability distribution of X is as shown in the table below.

2 5

x	0	10	20	30
$P(X = x)$	0.4	0.24	0.144	0.216



$$P(\text{scoring } 0) = 0.4$$

$$P(\text{scoring } 10) = 0.6 \times 0.4 = 0.24$$

$$P(\text{scoring } 20) = 0.6 \times 0.6 \times 0.4 = 0.144$$

$$P(\text{scoring } 30) = 0.6 \times 0.6 \times 0.6 = 0.216$$

Question 23 (continued)

- (b) Find the expected value of the distribution.

1 S

$$E(X) = 0 \times 0.4 + 10 \times 0.24 + 20 \times 0.144 + 30 \times 0.216$$

$$= 11.76$$

- (c) Show that the variance of X is approximately 137.7

2 S

$$\text{Var}(X) = E(X^2) - \mu^2$$

$$= 276 - 11.76^2$$

$$= 137.7024$$

$$\approx 137.7$$

End of Question 23

x	0	10	20	30	
$p(X=x)$	0.4	0.24	0.144	0.216	
$x p(x)$	0	2.4	2.88	6.48	11.76
x^2	0	100	400	900	
$x^2 p(x)$	0	24	57.6	194.4	276

Question 24 (5 marks)

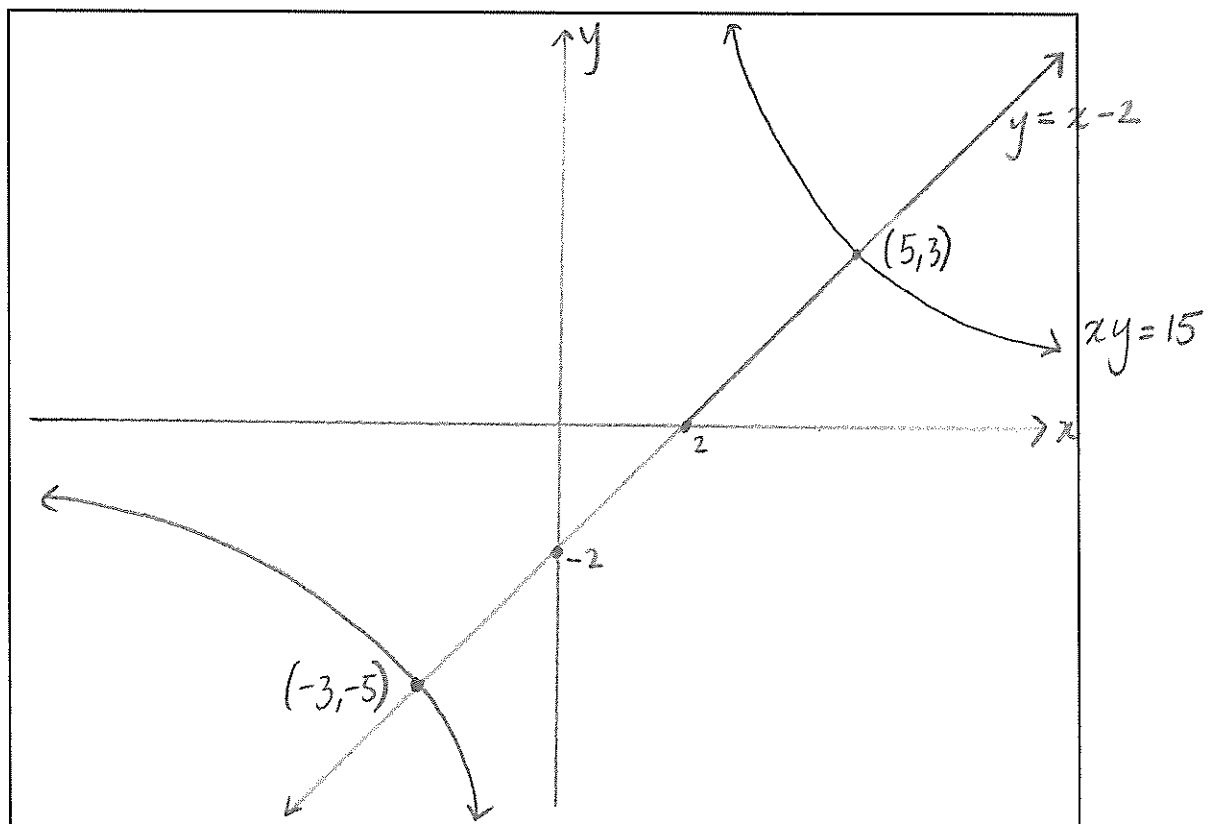
- (a) Find the points of intersection of $x - y = 2$ and $xy = 15$.

3 F

$$\begin{aligned}
 x &= y + 2 & (1) \\
 xy &= 15 & (2) \\
 \text{Sub (1) into (2)} & \quad (y+2)y = 15 \\
 & \quad y^2 + 2y = 15 \\
 & \quad y^2 + 2y - 15 = 0 \\
 & \quad (y+5)(y-3) = 0 \\
 \text{When } y &= -5 & \text{When } y = 3 \\
 x &= -5 + 2 & x = 3 + 2 \\
 &= -3 & x = 5 \\
 \therefore & \quad (-3, -5) \text{ and } (5, 3) \text{ are the points} \\
 & \quad \text{of intersection.}
 \end{aligned}$$

- (b) Sketch the two functions, clearly showing the points of intersection.

2 F



Question 25 (3 marks)

The Richter scale measures the magnitude of the seismic waves of an earthquake. An instrument called a seismograph detects movement in the Earth, with the smallest movement that can be detected shown as a wave with amplitude A_0 .

The Richter magnitude, R , is given by the equation

$$R = \log_{10} \frac{A}{A_0}$$

where A is the amplitude of the earthquake wave and A_0 is the amplitude of the smallest detectable wave.

- (a) An earthquake is measured with an amplitude 485 times as great as A_0 . What is the magnitude of this earthquake, using the Richter scale, to 1 decimal place? 1 E

$$R = \log_{10} \frac{485A_0}{A_0}$$

$$= \log_{10} 485$$

$$= 2.6857...$$

$$\approx 2.7$$

- (b) How many times greater is the amplitude of an earthquake with magnitude 3.5 than an earthquake with magnitude 2? 2 E

$$3.5 = \log_{10} \frac{A}{A_0}$$

$$2 = \log_{10} \frac{A}{A_0}$$

$$\frac{A}{A_0} = 10^{3.5}$$

$$\frac{A}{A_0} = 10^2$$

$$A_0$$

$$A_0$$

$$= 3162.27766$$

$$= 100$$

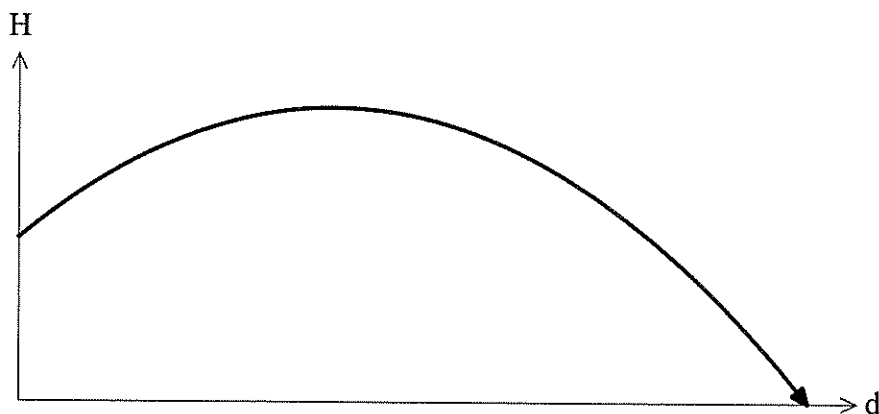
$$\frac{3162.27766}{100} \approx 31.6$$

$$100$$

A 3.5 magnitude earthquake is approximately 31.6 times bigger than a magnitude 2 earthquake.

Question 26 (5 marks)

An archer shoots an arrow, as shown in the picture below.



The height, H metres, of the arrow above the ground is modelled by the formula

$$H = -0.1d^2 + 0.8d + 2 \quad \text{where } d \geq 0, H \geq 0.$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

- (a) Find the horizontal distance travelled by the arrow.

2 F

Find when $H=0$.

$$-0.1d^2 + 0.8d + 2 = 0$$

$$-d^2 + 8d + 20 = 0$$

$$d^2 - 8d - 20 = 0$$

$$(d - 10)(d + 2) = 0$$

$$d = 10 \quad d = -2$$

$\therefore$ The arrow has travelled 10 m.

- (b) With reference to the context, interpret the significance of the constant, 2, in the formula.

1 F

2m is the initial height that the arrow was shot from. This could be the height of the archer.

- (c) Find the maximum height of the arrow above the ground.

2 $\frac{F}{7}$

The maximum height of the arrow occurs at the vertex of the parabola. This will be halfway between the d intercepts, at $d=4$.

When $d=4$

$$H = -0.1 \times (4)^2 + 0.8 \times 4 + 2$$
$$= 3.6$$

$\therefore$ The maximum height of the arrow is 3.6m.

End of Question 26

Question 27 (3 marks)

Prove that $\frac{\cos \theta}{1+\sin \theta} - \frac{\cos \theta}{1-\sin \theta} = -2 \tan \theta$.

3 T

$$\text{LHS} = \frac{\cos \theta}{1+\sin \theta} - \frac{\cos \theta}{1-\sin \theta}$$

$$= \frac{\cos \theta (1-\sin \theta) - \cos \theta (1+\sin \theta)}{(1+\sin \theta)(1-\sin \theta)}$$

$$= \frac{\cos \theta - \cos \theta \sin \theta - \cos \theta - \cos \theta \sin \theta}{1 - \sin^2 \theta}$$

$$= \frac{-2 \cos \theta \sin \theta}{\cos^2 \theta}$$

$$= \frac{-2 \sin \theta}{\cos \theta} \quad \leftarrow \text{since } \sin^2 \theta + \cos^2 \theta = 1 \quad \cos^2 \theta = 1 - \sin^2 \theta$$

$$= -2 \tan \theta$$

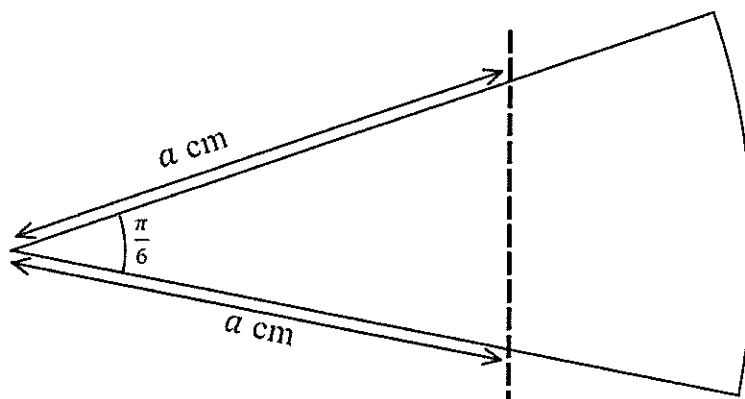
$$= \text{RHS}$$

Question 28 (3 marks)

Sandra buys a slice of cake in the shape of a sector.

The radius is r cm and the sector angle is $\frac{\pi}{6}$ radians.

She wants to give half of the slice to her friend Dee. She makes a cut across the sector as shown, a cm away from the centre.



Show that when they each have half the slice, $a = r\sqrt{\frac{\pi}{6}}$.

3 T

Area of triangle

$$A = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times a^2 \times \sin \frac{\pi}{6}$$

$$= \frac{1}{2} a^2 \times \frac{1}{2}$$

$$= \frac{a^2}{4}$$

Area of sector

$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} r^2 \times \frac{\pi}{6}$$

$$= \frac{\pi r^2}{12}$$

When is area of triangle equal to $\frac{1}{2}$ area of sector

$$\frac{a^2}{4} = \frac{\pi r^2}{24} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{half area of sector}$$

$$a^2 = \frac{\pi r^2}{6}$$

$$a = \pm \sqrt{\frac{\pi r^2}{6}}$$

Since a is a length, $a > 0$.

$$\therefore a = r\sqrt{\frac{\pi}{6}}$$

End of Paper