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Student Number

SCEGGS Darlinghurst

2022 YEAR 11 PRELIMINARY EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Answer Section I on the multiple-choice answer sheet provided
- Answer Section II questions in the writing spaces provided and show relevant mathematical reasoning and/or calculations
- Marks will be deducted for careless or badly arranged work
- Extra space is available at the end of the examination
- Calculators approved by NESA may be used
- A Reference Sheet is provided on a separate sheet

Total marks:

71

Section I - 7 marks

- Attempt Questions 1-7
- Allow about 12 minutes for this section

Section II - 64 marks

- Attempt Questions 8-28
- Allow about 1 hour 48 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ A. 2 B. 6 C. 8 D. 9

 A. ☐ B. ☒ C. ☐ D. ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A. ☒ B. ☒ C. ☐ D. ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A. ☒ B. ☒ C. ☐ D. ☐

correct

Section I – Multiple Choice

10 marks

Attempt Questions 1–10

Allow about 12 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1 What is the exact value of $(1 + \sqrt{3})(2 - \sqrt{3})$?

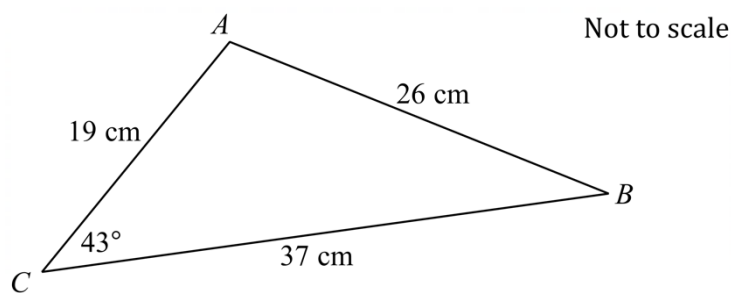
A. $-1 + \sqrt{3}$

B. $-1 + 3\sqrt{3}$

C. $5 + \sqrt{3}$

D. $5 + 3\sqrt{3}$

2 Which of the following is the correct expression involving $\angle ABC$ in $\triangle ABC$?



A. $\frac{19}{\sin \angle ABC} = \frac{26}{\sin 43^\circ}$

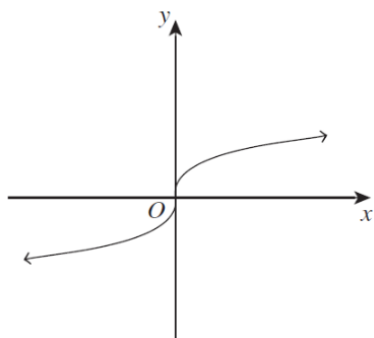
B. $\frac{\sin \angle ABC}{26} = \frac{\sin 43^\circ}{19}$

C. $19^2 = 26^2 + 37^2 + 2 \times 26 \times 37 \cos \angle ABC$

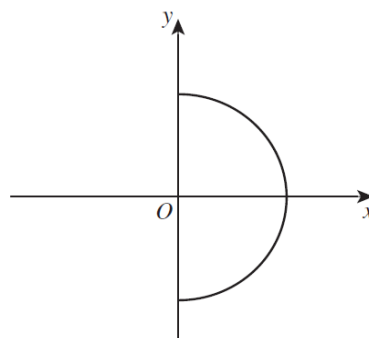
D. $\cos \angle ABC = \frac{26^2 + 37^2 - 19^2}{2 \times 37 \times 19}$

3 Which of the following graphs represent a one-to-many relationship?

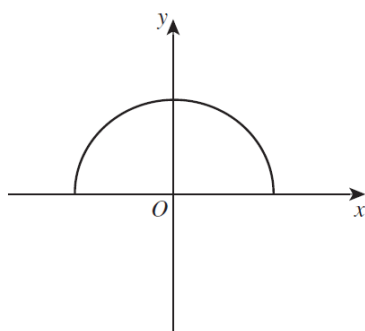
A.



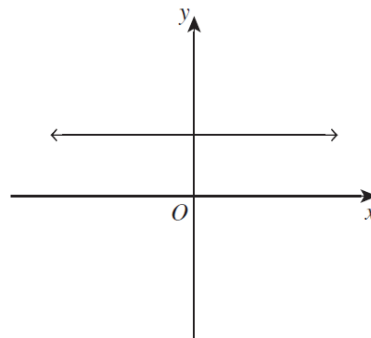
B.



C.



D.



4 If $y = \frac{-3}{x^2}$ then what is $\frac{dy}{dx}$?

A. $\frac{-6}{x^3}$

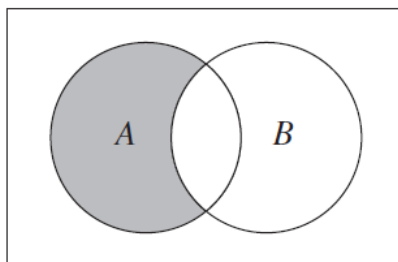
B. $\frac{-3}{x}$

C. $\frac{3}{x}$

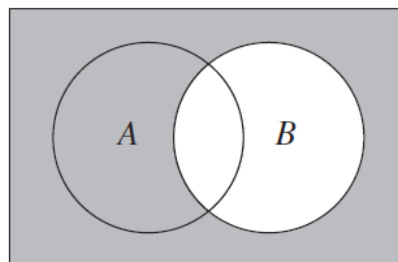
D. $\frac{6}{x^3}$

5 Which of the following Venn diagrams represents $A \cup \bar{B}$?

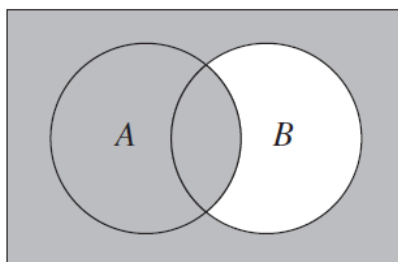
A.



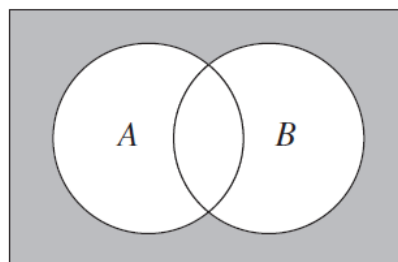
B.



C.



D.



6 How many solutions of the equation $5 \sin^2 \theta + 2 \sin \theta - 3 = 0$ lie between 0° and 360° ?

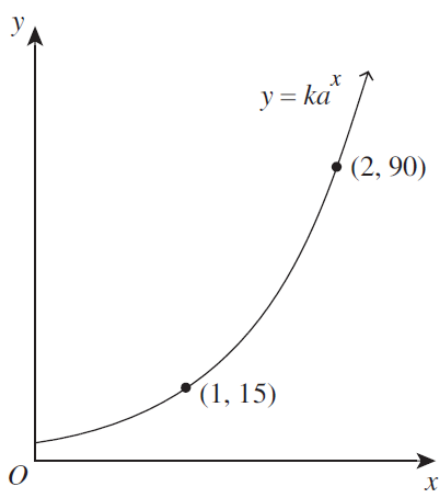
A. 2

B. 3

C. 4

D. 5

- 7 The graph shows $y = ka^x$.



What are the values of a and k ?

- A. $a = 15, k = 6$
- B. $a = 6, k = 15$
- C. $a = 2.5, k = 6$
- D. $a = 6, k = 2.5$

End Of Section I

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Student Number

Mathematics Advanced Section II

64 marks

Attempt Questions 8-28

Allow about 1 hour and 48 minutes for this section

Write your student number at the top of this page.

Answer the questions in the spaces provided. There is extra writing space at the back of the booklet. If you use this space, clearly label each answer.

Please turn over

Question 8 (2 marks)

Solve: $|3x + 1| = 8$

2

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Question 9 (2 marks)

Simplify: $\frac{2x^2}{y^3} \times (4x^2y)^2$

2

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Question 10 (3 marks)

Find the equation of the tangent to the curve $y = 2x^2 - 4x + 3$ at the point $(2, 3)$

3

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Question 11 (2 marks)

Given that $\tan \theta = -\frac{8}{5}$ and $90^\circ \leq \theta \leq 180^\circ$, find the exact value of $\sin \theta$. **2**

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Question 12 (2 marks)

State the domain and range for the function $y = 2^x - 3$ **2**

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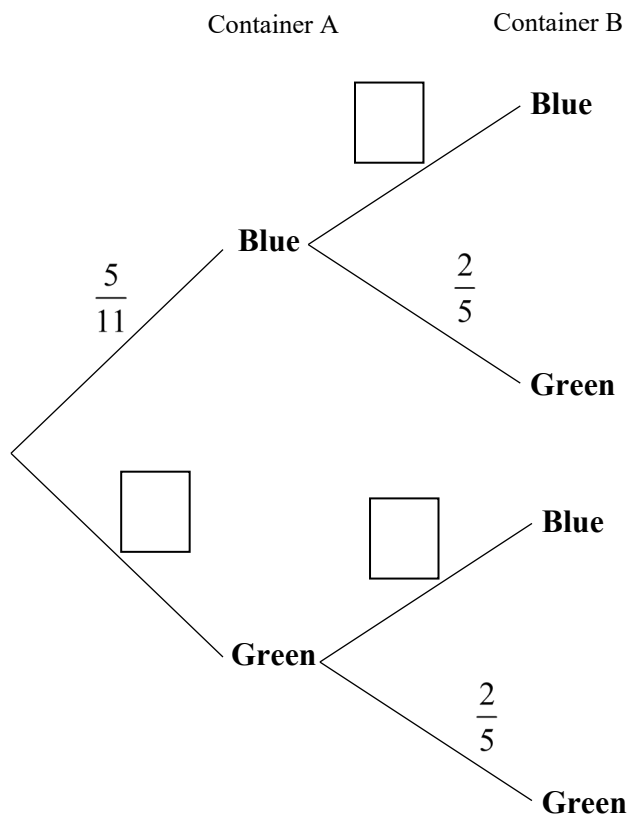
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Question 13 (4 marks)

There are two containers of identically sized balls. Container A contains 5 blue and 6 green balls whilst Container B contains 3 blue and 2 green balls.

Amy randomly chose one ball from each container.

- (a) Complete the probability tree below by inserting the missing probabilities 1



- (b) What is the probability that both balls are green? 1

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- (c) What is the probability that both balls are not the same colour? 2

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Question 14 (4 marks)

Differentiate the following functions:

(a) $y = (2x^2 - 3)^5$ 2

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(b) $y = \frac{2x+3}{4x-9}$ 2

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Question 15 (2 marks)

Find the gradient of the tangent of the function $y = x^2 e^{5x}$ at the point where $x = 1$ 2

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Question 16 (2 marks)

Solve $2 \cos \theta = -\sqrt{3}$ for $[0, 2\pi]$

2

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Question 17 (2 marks)

Let $f(x) = \sqrt{x}$ and $g(x) = x - 4$.

(a) Write an expression for $f(g(x))$.

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(b) State the domain of $f(g(x))$

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Question 18 (5 marks)

Let $f(x) = x^2 - 6x + 5$

- (a) Solve $f(x) = 0$ **1**

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- (b) By completing the square, express $f(x)$ in the form of $f(x) = (x - h)^2 + k$ **2**

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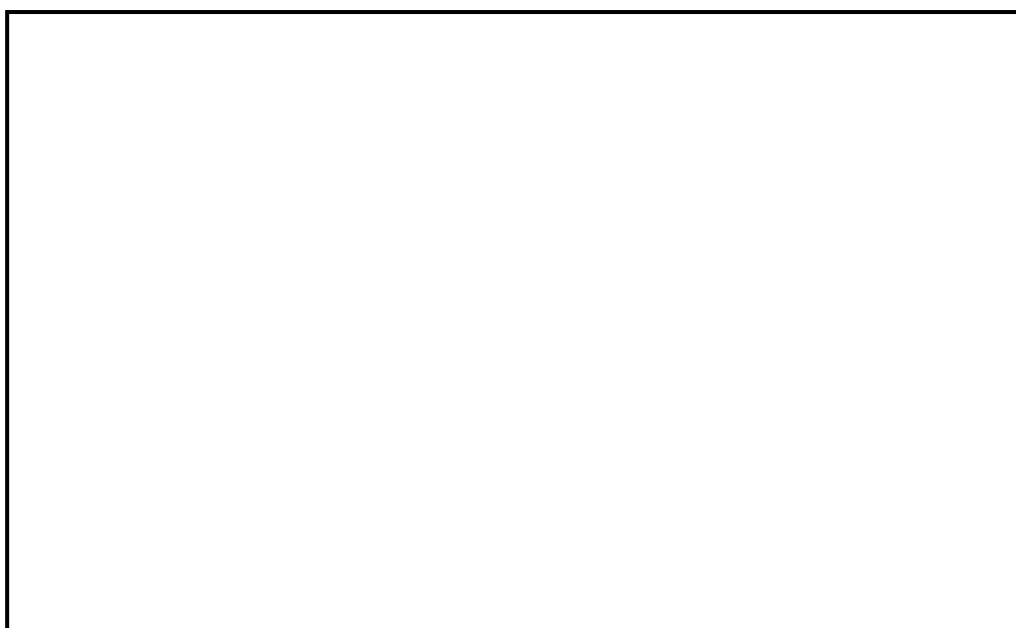
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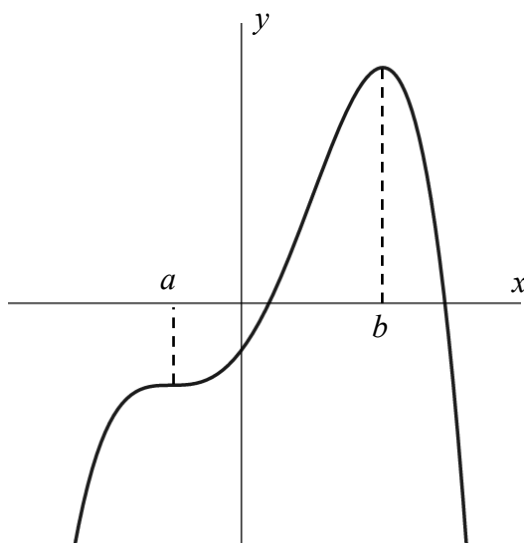
- (c) Sketch the graph of $y = f(-x)$, labelling the vertex and x - intercepts **2**



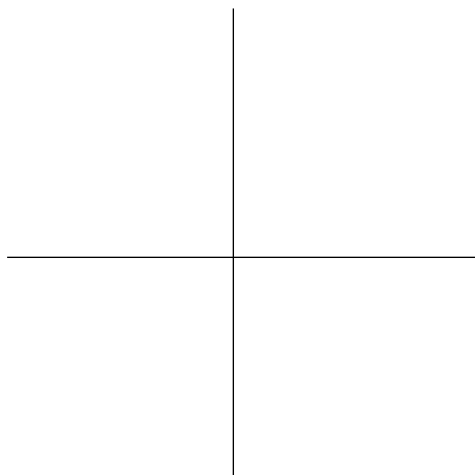
Question 19 (2 marks)

The graph of $y = f(x)$ is shown below.

2



On the number plane below sketch $y = f'(x)$



Question 20 (2 marks)

Given $\log_a 4 = x$ and $\log_a 6 = y$, find an expression for $\log_a 96$ in terms of x and y .

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Question 21 (3 marks)

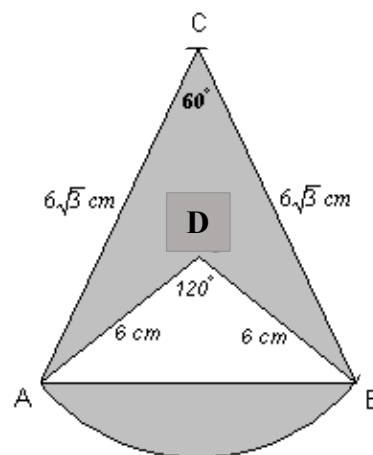
The design drawing for a company logo is shown.

$\triangle ADB$ is inscribed in sector ACB

$$AC = CB = 6\sqrt{3}\text{ cm}$$

$$AD = DB = 6\text{ cm}$$

$$\angle ADB = 120^\circ \text{ and } \angle ACB = 60^\circ$$



- (a) Find the exact area of the sector ACB

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- (b) Find the exact area of the shaded region

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Question 22 (4 marks)

50 married men were asked whether they gave their wife flowers or chocolates for their last birthday. The results were: 31 gave chocolates, 12 gave flowers and 5 gave both chocolates and flowers.

- (a) Draw a Venn diagram representing the information given. 2

- (b) If one married man was chosen at random, determine the probability that he gave his wife:

- (i) chocolates or flowers 1

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- (ii) flowers if it is known that he did **NOT** give her chocolates 1

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Question 23 (5 marks)

The number of birds, N , in a park at time t days is modelled by the formula:

$$N = 100\,000 e^{-0.15t}$$

- (a) What is the number of birds in the park when $t = 40$? **1**
Give your answer to the nearest whole number.

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- (b) What is the rate of change of the number of birds in the park when $t = 40$? **2**
Give your answer to the nearest whole number.

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- (c) How many days would it take for there to be 100 birds in the park? **2**
Give your answer to the nearest day.

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Question 24 (3 marks)

It is known that for dependent events A and B, $P(A) = 0.22$, $P(B) = 0.54$ and

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$$P(A \cup B) = 0.72$$

Find $P(A|B)$ to 3 decimal places.

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Question 25 (3 marks)

Solve the equation $\log_{10} x + \log_{10} (x - 3) = \log_{10} 18$

3

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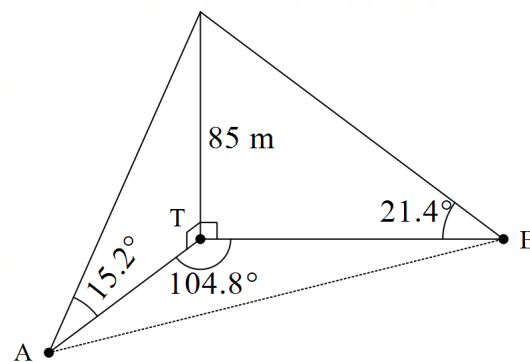
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Question 26 (3 marks)

A radio tower at point T is 85 metres tall and stands perpendicular to the ground.

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Gemma is at point A where the angle of elevation of the tower is 15.2° . Erin is at point B where the angle of elevation of the tower is 21.4° . The angle $\angle ATB$ is 104.8° .

Find AB, the distance of Gemma from Erin in a straight line to the nearest metre.

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Question 27 (4 marks)

- (a) Prove the following identity: $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta + 1} = \sec \theta$ 2

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- (b) Hence solve the equation $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta + 1} = 2$ for $[-\pi, \pi]$ 2

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Question 28 (5 marks)

Sound intensity, I , is measured in watts per square metre, W / m^2 .

The loudness of a sound, B , in decibels (dB), is measured on a logarithmic scale and can be calculated with the following formula:

$$B = 10 \log_{10} \frac{I}{10^{-12}}, \quad \text{where } I \text{ is the sound intensity.}$$

For example: If a sound had the intensity of $I = 1.5 W / m^2$ then the loudness would be calculated by:

$$\begin{aligned} B &= 10 \log_{10} \frac{1.5}{10^{-12}} \\ &= 122 \text{ dB (to the nearest dB)} \end{aligned}$$

- (a) Occupational health and safety regulations state that hearing protection must be worn if loudness levels in an area exceed 85 dB. **1**

An industrial machine has a sound intensity of $0.05 W / m^2$. Determine if workers should be required to wear hearing protection when working near this machine.

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Question 29 continues on the next page

Question 29 continued

- (b) Sound intensity also depends on the distance from the sound source. A law of physics states that intensity varies inversely with the square of the distance from the sound source, as in the equation. 3

$$I = \frac{k}{d^2}$$

where k is a constant and d is the distance from the sound source in metres.

If B_1 and B_2 are the decibel levels of two sounds at a distance of d_1 and d_2 respectively, show that:

$$B_2 = B_1 + 20 \log_{10} \frac{d_1}{d_2}$$

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- (c) Assuming the sound intensity of the machine in part (a) was measured at a distance of 1 metre from the machine, determine whether workers who work in an area 20 metres Away from the machine should be required to wear hearing protection. 1

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End of paper



SCEGGS Darlinghurst

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Student Number

SOLUTIONS

2022

YEAR 11 SEMESTER 2 EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Answer Section I on the multiple-choice answer sheet provided
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Total marks:

71

Section I - 7 marks

- Attempt Questions 1-7
- Allow about 12 minutes for this section

Section II - 64 marks

- Attempt Questions 8-28
- Allow about 1 hour 48 minutes for this section

Section I – Multiple Choice

7 marks

Attempt Questions 1–7

Allow about 12 minutes for this section

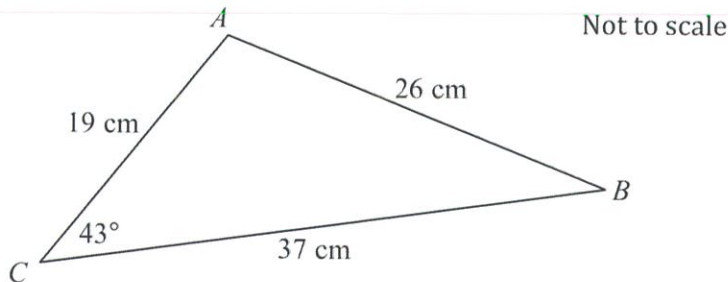
Use the multiple-choice answer sheet for Questions 1–7.

- 1 What is the exact value of $(1 + \sqrt{3})(2 - \sqrt{3})$?

- A. $-1 + \sqrt{3}$ $= 2 - \sqrt{3} + 2\sqrt{3} - 3$
B. $-1 + 3\sqrt{3}$ $= -1 + \sqrt{3}$
C. $5 + \sqrt{3}$
D. $5 + 3\sqrt{3}$

F&A-1

- 2 Which of the following is the correct expression involving $\angle ABC$ in $\triangle ABC$?

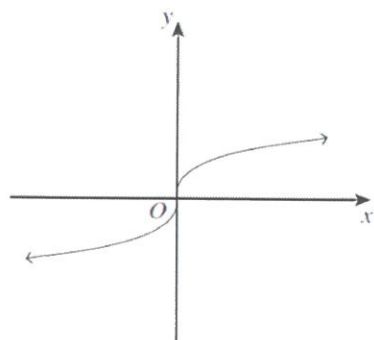


- A. $\frac{19}{\sin \angle ABC} = \frac{26}{\sin 43^\circ}$
B. $\frac{\sin \angle ABC}{26} = \frac{\sin 43^\circ}{19}$
C. $19^2 = 26^2 + 37^2 + 2 \times 26 \times 37 \cos \angle ABC$
D. $\cos \angle ABC = \frac{26^2 + 37^2 - 19^2}{2 \times 37 \times 19}$

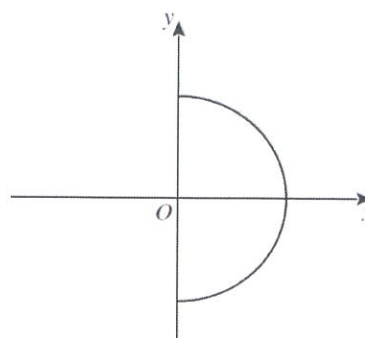
Trig-1

3 Which of the following graphs represent a one-to-many relationship?

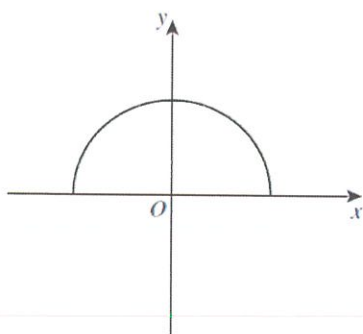
A.



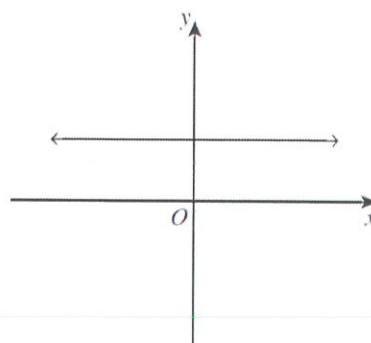
B.



C.



D.



F4K-1

4 If $y = \frac{-3}{x^2}$ then what is $\frac{dy}{dx}$?

A. $\frac{-6}{x^3}$

B. $\frac{-3}{x}$

C. $\frac{3}{x}$

D. $\frac{6}{x^3}$

$$y = -3x^{-2}$$

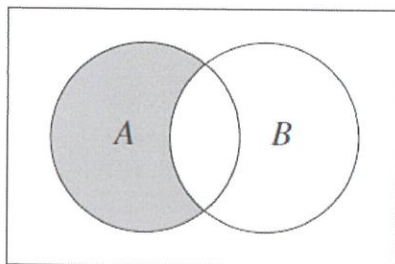
$$\frac{dy}{dx} = 6x^{-3}$$

$$= \frac{6}{x^3}$$

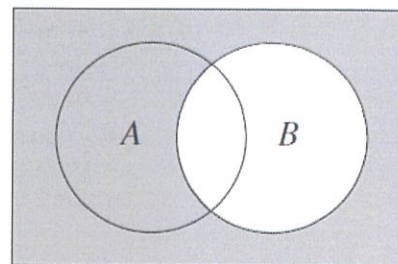
Calc-1

5 Which of the following Venn diagrams represents $A \cup \bar{B}$?

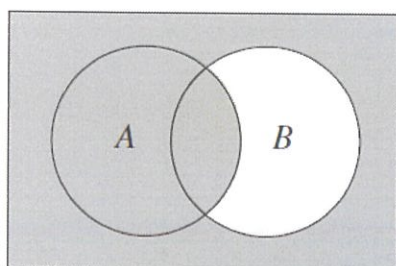
A.



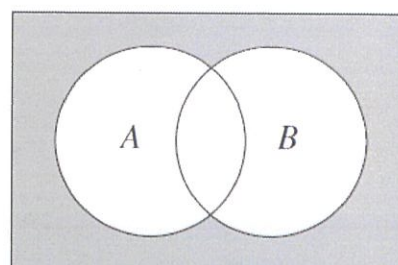
B.



C.



D.



Prob-1

6 How many solutions of the equation $5\sin^2 \theta + 2\sin \theta - 3 = 0$ lie between 0° and 360° ?

A. 2

B. 3

C. 4

D. 5

$$(5\sin \theta - 3)(\sin \theta + 1) = 0$$

$$\sin \theta = \frac{3}{5}$$



2 solutions

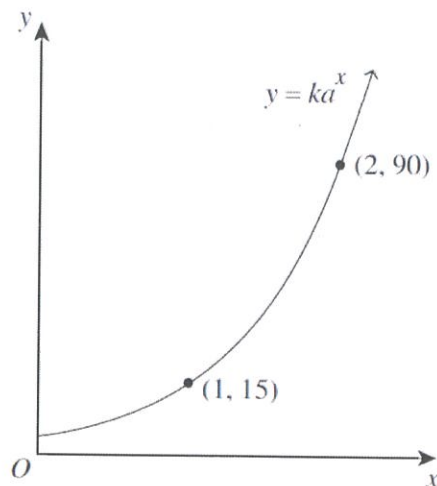
$$\sin \theta = -1$$



1 solution

Trig-1

- 7 The graph shows $y = ka^x$.



What are the values of a and k ?

- A. $a = 15, k = 6$
B. $a = 6, k = 15$
C. $a = 2.5, k = 6$
D. $a = 6, k = 2.5$

$$15 = ka^1 \dots \textcircled{1}$$

$$90 = ka^2 \dots \textcircled{2}$$

$$\textcircled{2} \div \textcircled{1}$$

$$\frac{ka^2}{ka} = \frac{90}{15}$$

$$a = 6$$

$$\text{using } \textcircled{1}$$

$$15 = k \times 6$$

$$k = 2.5$$

End Of Section I

L4E-1

Question 8 (2 marks)

Solve:

$$|3x+1|=8$$

2

$$3x+1=8 \quad \text{or} \quad 3x+1=-8$$

$$3x=7$$

$$3x=-9$$

$$x=\frac{7}{3}$$

$$x=-3$$

FdA -2

Question 9 (2 marks)

Simplify:

$$\frac{2x^2}{y^3} \times (4x^2y)^2$$

2

$$= \frac{2x^2}{y^3} \times 16x^4y^2$$

$$= \frac{32x^6}{y}$$

FdA -2

Question 10 (3 marks)

Find the equation of the tangent to the curve $y = 2x^2 - 4x + 3$ at the point $(2, 3)$.

3

$$\frac{dy}{dx} = 4x - 4$$

$$\text{when } x=2 \quad \frac{dy}{dx} = 4 \times 2 - 4 = 4$$

$$\therefore m_{\text{tangent}} = 4$$

$$y - 3 = 4(x - 2)$$

$$y - 3 = 4x - 8$$

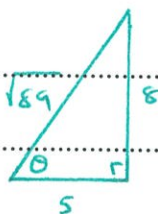
$$y = 4x - 5 \quad (4x - y - 5 = 0)$$

Calc - 3

Question 11 (2 marks)

Given that $\tan \theta = -\frac{8}{5}$ and $90^\circ \leq \theta \leq 180^\circ$, find the exact value of $\sin \theta$.

2



θ lies in the 2nd quadrant

so $\sin \theta$ is positive

$$\therefore \sin \theta = \frac{8}{\sqrt{89}}$$

Try - 2

Question 12 (2 marks)

State the domain and range for the function $y = 2^x - 3$.

2

Domain: $(-\infty, \infty)$

Range: $(-3, \infty)$

LE - 2

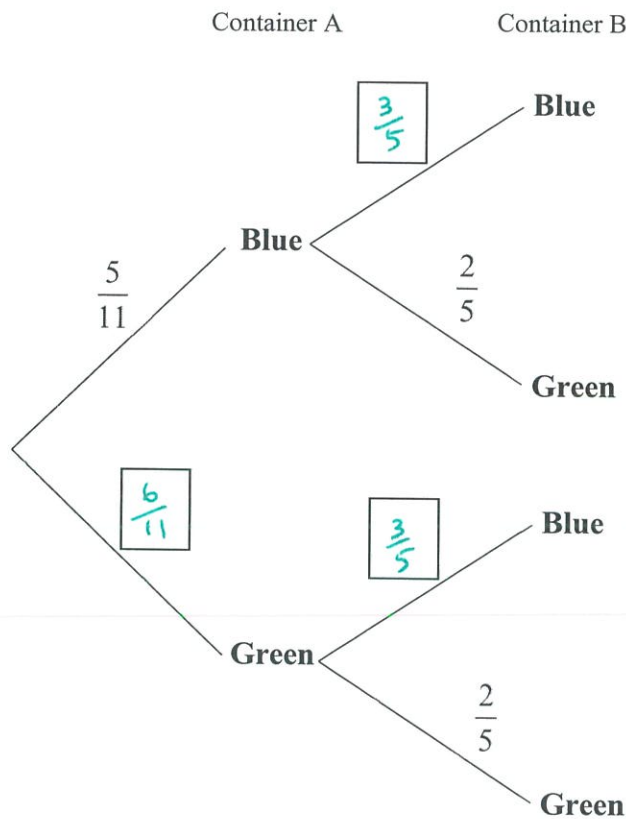
Question 13 (4 marks)

There are two containers of identically sized balls. Container A contains 5 blue and 6 green balls whilst Container B contains 3 blue and 2 green balls.

Amy randomly chose one ball from each container.

- (a) Complete the probability tree below by inserting the missing probabilities.

1



- (b) What is the probability that both balls are green?

1

$$P(GG) = \frac{6}{11} \times \frac{2}{5}$$

$$= \frac{12}{55}$$

- (c) What is the probability that both balls are not the same colour?

2

$$P(BG \text{ or } GB) = \frac{5}{11} \times \frac{2}{5} + \frac{6}{11} \times \frac{3}{5}$$

$$= \frac{28}{55}$$

Prob - 4

Question 14 (4 marks)

Differentiate the following functions:

(a) $y = (2x^2 - 3)^5$

2

$$\frac{dy}{dx} = 5(2x^2 - 3)^4 \times 4x$$

$$= 20x(2x^2 - 3)^4$$

(b) $y = \frac{2x+3}{4x-9}$ $\frac{u}{v}$

2

$$u = 2x+3 \quad v = 4x-9$$

$$u' = 2 \quad v' = 4$$

$$\frac{dy}{dx} = \frac{(4x-9) \times 2 - (2x+3) \times 4}{(4x-9)^2}$$

$$= \frac{8x-18-8x-12}{(4x-9)^2}$$

$$= \frac{-30}{(4x-9)^2}$$

Calc - 4

Question 15 (2 marks)Find the gradient of the tangent of the function $y = x^2 e^{5x}$ at the point where $x = 1$.

2

$$u = x^2 \quad v = e^{5x}$$

$$u' = 2x \quad v' = 5e^{5x}$$

$$\frac{dy}{dx} = x^2 \times 5e^{5x} + 2x \times e^{5x}$$

$$= xe^{5x}(5x+2)$$

when $x=1$

$$\frac{dy}{dx} = 1 \times e^{5 \times 1} (5 \times 1 + 2)$$

$$= 7e^5$$

 $\therefore$ gradient of the tangent is $7e^5$

Calc - 2

Question 16 (2 marks)

Solve $2 \cos \theta = -\sqrt{3}$ for $0^\circ \leq \theta \leq 360^\circ$.

2

$$\cos \theta = -\frac{\sqrt{3}}{2}$$

θ lies in the 2nd and 3rd quadrants

$$\text{ref. angle} = 30^\circ$$

$$\theta = 180^\circ - 30^\circ$$

$$= 150^\circ$$

$$\theta = 180^\circ + 30^\circ$$

$$= 210^\circ$$

Trig-2

Question 17 (2 marks)

Let $f(x) = \sqrt{x}$ and $g(x) = x - 4$.

- (a) Write an expression for $f(g(x))$.

1

$$f(g(x)) = \sqrt{x-4}$$

- (b) State the domain of $f(g(x))$.

1

$$x - 4 > 0$$

$$\therefore x > 4$$

F&A-2

Question 18 (5 marks)

Let $f(x) = x^2 - 6x + 5$.

- (a) Solve $f(x) = 0$.

1

$$x^2 - 6x + 5 = 0$$

$$(x-5)(x-1) = 0$$

$$x = 5 \quad x = 1$$

- (b) By completing the square, express $f(x)$ in the form of $f(x) = (x-h)^2 + k$.

2

$$x^2 - 6x = -5$$

$$x^2 - 6x + 9 = -5 + 9$$

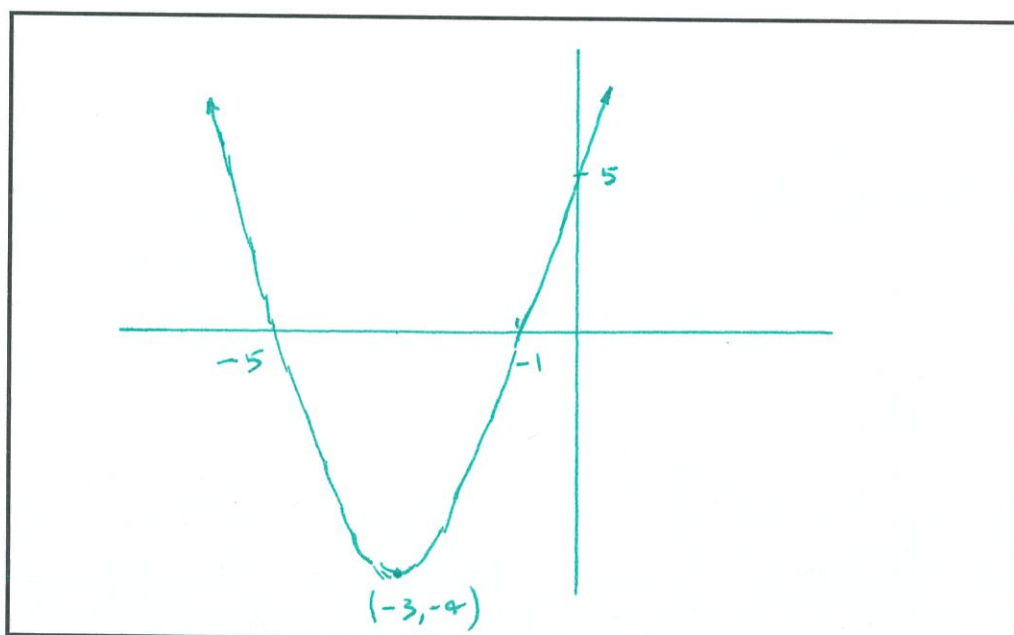
$$(x-3)^2 = 4$$

$$(x-3)^2 - 4$$

$$\therefore f(x) = (x-3)^2 - 4$$

- (c) Sketch the graph of $y = f(x)$, labelling the vertex and x -intercepts.

2

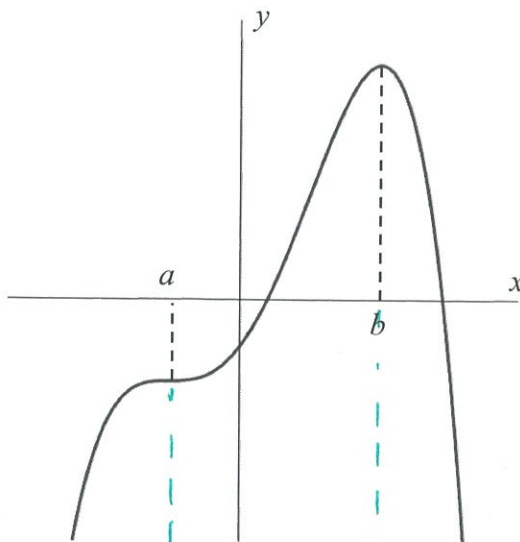


$f(x) = x^2 - 6x + 5$

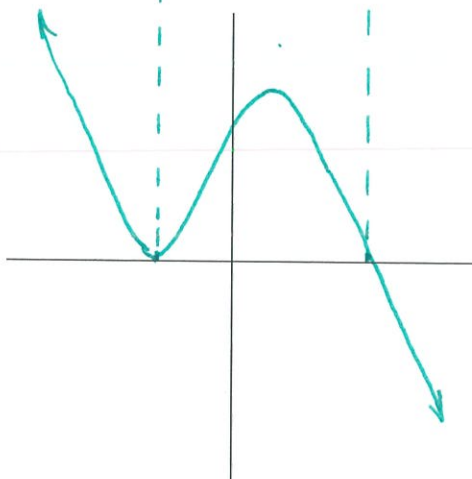
Question 19 (2 marks)

The graph of $y = f(x)$ is shown below.

2



On the number plane below sketch $y = f'(x)$.



Calc - 2

Question 20 (2 marks)

Given $\log_a 4 = x$ and $\log_a 6 = y$, find an expression for $\log_a 96$ in terms of x and y .

2

$$\begin{aligned}\log_a 96 &= \log_a (16 \times 6) \\ &= \log_a (4^2 \times 6) \\ &= \log_a 4^2 + \log_a 6 \\ &= 2\log_a 4 + \log_a 6 \\ &= 2x + y\end{aligned}$$

LE - 2

Question 21 (3 marks)

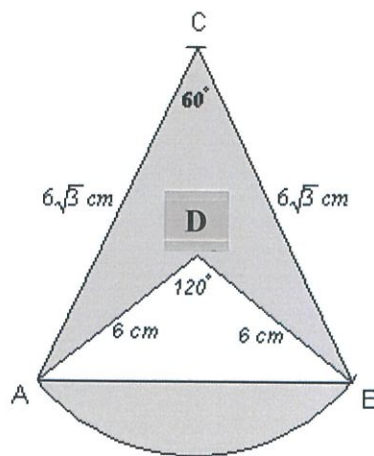
The design drawing for a company logo is shown.

$\triangle ADB$ is inscribed in sector ABC

$$AC = CB = 6\sqrt{3} \text{ cm}$$

$$AD = DB = 6 \text{ cm}$$

$$\angle ADB = 120^\circ \text{ and } \angle ACB = 60^\circ$$



- (a) Find the exact area of the sector ABC .

1

$$60^\circ = \frac{\pi}{3} \text{ radians}$$

$$\text{Area of sector} = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \times (6\sqrt{3})^2 \times \frac{\pi}{3}$$

$$= \frac{1}{2} \times 108 \times \frac{\pi}{3}$$

$$= 18\pi \text{ cm}^2$$

- (b) Find the exact area of the shaded region.

2

$$\text{Area of } \triangle ADB = \frac{1}{2} \times 6 \times 6 \times \sin 120^\circ$$

$$= \frac{1}{2} \times 36 \times \frac{\sqrt{3}}{2}$$

$$= 9\sqrt{3} \text{ cm}^2$$

$$\therefore \text{Area of shaded region} = 18\pi - 9\sqrt{3} \text{ cm}^2$$

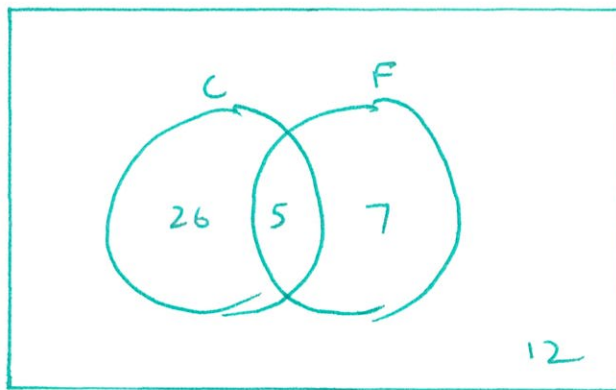
Trig-3

Question 22 (4 marks)

50 married men were asked whether they gave their wife flowers or chocolates for their last birthday. The results were: 31 gave chocolates, 12 gave flowers and 5 gave both chocolates and flowers.

- (a) Draw a Venn diagram representing the information given.

2



- (b) If one married man was chosen at random, determine the probability that he gave his wife:

- (i) chocolates or flowers.

1

$$\frac{38}{50} = \frac{19}{25}$$

- (ii) flowers if it is known that he did **NOT** give her chocolates.

1

$$\frac{7}{19}$$

Prob-4

Question 23 (5 marks)

The number of birds, N , in a park at time t days is modelled by the formula:

$$N = 100\,000 e^{-0.15t}$$

- (a) What is the number of birds in the park when $t = 40$?
Give your answer to the nearest whole number.

1

$$N = 100\,000 e^{-0.15 \times 40}$$

$$= 247.87...$$

$$\therefore 248 \text{ birds}$$

- (b) What is the rate of change of the number of birds in the park when $t = 40$?
Give your answer to the nearest whole number.

2

$$\frac{dN}{dt} = -15000 e^{-0.15t}$$

$$\text{when } t = 40$$

$$\frac{dN}{dt} = -15000 e^{-0.15 \times 40}$$

$$= -37.18...$$

$$= -37 \text{ birds/day}$$

- (c) How many days would it take for there to be 100 birds in the park?
Give your answer to the nearest day.

2

$$100 = 100\,000 e^{-0.15t}$$

$$0.001 = e^{-0.15t}$$

$$\ln(0.001) = -0.15t$$

$$t = \frac{\ln(0.001)}{-0.15}$$

$$= 46.05...$$

$$\therefore 46 \text{ days}$$

L4E-5

Question 24 (3 marks)

It is known that for dependent events A and B, $P(A) = 0.22$, $P(B) = 0.54$ and $P(A \cup B) = 0.72$. Find $P(A|B)$ to 3 decimal places.

3

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.72 = 0.22 + 0.54 - P(A \cap B)$$

$$0.72 = 0.76 - P(A \cap B)$$

$$P(A \cap B) = 0.04$$

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{0.04}{0.54} \\ &= 0.074 \end{aligned}$$

Prob-3

Question 25 (3 marks)

Solve the equation $\log_{10} x + \log_{10} (x-3) = \log_{10} 18$.

3

$$\log_{10} [x(x-3)] = \log_{10} 18$$

$$x^2 - 3x = 18$$

$$x^2 - 3x - 18 = 0$$

$$(x-6)(x+3) = 0$$

$$x = 6 \quad x = -3$$

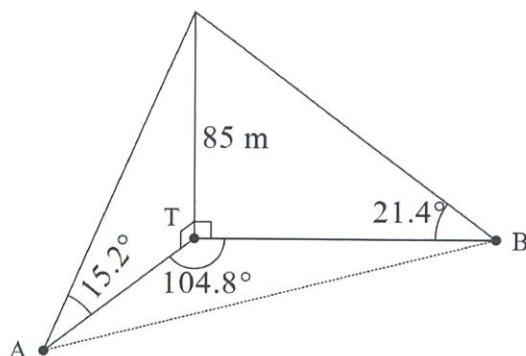
$$x = 6 \text{ is only solution}$$

L4E-3

Question 26 (3 marks)

A radio tower at point T is 85 metres tall and stands perpendicular to the ground.

3



Gemma is at point A where the angle of elevation of the tower is 15.2° . Erin is at point B where the angle of elevation of the tower is 21.4° . The angle $\angle ATB$ is 104.8° .

Find AB, the distance of Gemma from Erin in a straight line to the nearest metre.

$$\tan 68.6^\circ = \frac{TB}{85}$$

$$TB = 85 \times \tan 68.6^\circ$$

$$\text{similarly, } TA = 85 \tan 74.8^\circ$$

$$\therefore AB^2 = (85 \tan 68.6^\circ)^2 + (85 \tan 74.8^\circ)^2 - 2(85 \tan 68.6^\circ)(85 \tan 74.8^\circ) \cos 104.8^\circ$$

$$= 179586.53...$$

$$AB = \sqrt{179586.53...}$$

$$= 423.77...$$

$$\therefore AB \approx 424 \text{ m}$$

Trig-3

Question 27 (4 marks)

- (a) Prove the following identity: $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta + 1} = \sec \theta$.

2

$$\text{LHS} = \frac{\sin \theta (\sin \theta + 1) + \cos \theta \times \cos \theta}{\cos \theta (\sin \theta + 1)}$$

$$= \frac{\sin^2 \theta + \sin \theta + \cos^2 \theta}{\cos \theta (\sin \theta + 1)}$$

$$= \frac{1 + \cancel{\sin \theta}}{\cos \theta (1 + \cancel{\sin \theta})}$$

$$= \frac{1}{\cos \theta}$$

$$= \sec \theta$$

- (b) Hence solve the equation $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta + 1} = 2$ for $[-\pi, \pi]$.

2

$$\therefore \sec \theta = 2$$

$$\cos \theta = \frac{1}{2}$$

θ lies in the 1st and 4th quadrants

$$\therefore \theta = \frac{\pi}{3} \text{ or } \theta = -\frac{\pi}{3}$$

Trig - 4

Question 28 (5 marks)

Sound intensity, I , is measured in watts per square metre, W / m^2 .

The loudness of a sound, B , in decibels (dB), is measured on a logarithmic scale and can be calculated with the following formula:

$$B = 10 \log_{10} \frac{I}{10^{-12}}, \quad \text{where } I \text{ is the sound intensity.}$$

For example: If a sound had the intensity of $I = 1.5 W / m^2$ then the loudness would be calculated by:

$$\begin{aligned} B &= 10 \log_{10} \frac{1.5}{10^{-12}} \\ &= 122 \text{ dB (to the nearest dB)} \end{aligned}$$

- (a) Occupational health and safety regulations state that hearing protection must be worn if loudness levels in an area exceed 85 dB. 1

An industrial machine has a sound intensity of $0.05 W / m^2$. Determine if workers should be required to wear hearing protection when working near this machine.

$$\begin{aligned} B &= 10 \times \log_{10} \frac{0.05}{10^{-12}} \\ &= 107 \text{ dB} \\ \therefore \text{the workers need to wear protection} \end{aligned}$$

- (b) Sound intensity also depends on the distance from the sound source. A law of physics states that intensity varies inversely with the square of the distance from the sound source, as in the equation. 3

$$I = \frac{k}{d^2}$$

where k is a constant and d is the distance from the sound source in metres.

Question 28(b) continues on the next page

Question 28(b) (continued)

If B_1 and B_2 are the decibel levels of two sounds at a distance of d_1 and d_2 respectively, show that:

$$B_2 = B_1 + 20 \log_{10} \frac{d_1}{d_2}$$

$$B_1 = 10 \log_{10} \left(\frac{k}{d_1^2} \right) \quad B_2 = 10 \log_{10} \left(\frac{k}{d_2^2} \right)$$

$$B_2 - B_1 = 10 \log_{10} \left(\frac{k}{d_2^2} \right) - 10 \log_{10} \left(\frac{k}{d_1^2} \right)$$

$$= 10 \log_{10} \left[\frac{\cancel{k} d_1^2}{d_2^2 \cancel{k}} \times \frac{10^{20}}{10^{20}} \right]$$

$$= 10 \log_{10} \left[\left(\frac{d_1}{d_2} \right)^2 \right]$$

$$= 20 \log_{10} \left(\frac{d_1}{d_2} \right)$$

- (c) Assuming the sound intensity of the machine in part (a) was measured at a distance of 1 metre from the machine, determine whether workers who work in an area 20 metres away from the machine should be required to wear hearing protection. 1

$$B_2 = 107 + 20 \log_{10} \frac{1}{20}$$

$$= 80.979..$$

$$= 81 \text{ dB}$$

$\therefore$ hearing protection not needed

End of paper

L4E-5