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Examination Number:

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Set:

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2023

YEAR 11 YEARLY EXAMINATION

Mathematics Advanced

September 1

**General
Instructions**

- Reading time – 10 minutes
- Working time – 2 hours
- Write using a non erasable black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

**Total marks:
70**

Section I – 10 marks (pages 2–8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 9–40)

- Attempt Questions 11–29
- Allow about 1 hour and 45 minutes for this section

Note: Any time you have remaining should be spent reviewing your answers.

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Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1–10.

1 What is the solution of the equation $x^2 + 3x - 4 = 0$?

A. $x = -4$ and $x = 1$

B. $x = 4$ and $x = -1$

C. $x = 1$ and $x = -\frac{4}{3}$

D. $x = -1$ and $x = \frac{4}{3}$

2 Which of the following expressions is equal to $\frac{a^2 - b^2}{a^2b - ab^2}$?

A. $\frac{a+b}{a-b}$

B. $\frac{1}{a} - \frac{1}{b}$

C. $\frac{1}{ab}$

D. $\frac{a+b}{ab}$

3 A fair die is rolled twice. What is the probability of tossing two even numbers?

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{2}{3}$

4 What is the value of $\frac{\log_a 81}{\log_a 3}$?

A. $\log_a 78$

B. $\log_a 27$

C. 27

D. 4

5 If $y = \frac{1}{\sqrt{x}}$ what is an expression for $\frac{dy}{dx}$?

A. $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

B. $\frac{dy}{dx} = \frac{-1}{2\sqrt{x^3}}$

C. $\frac{dy}{dx} = \frac{-1}{2\sqrt{x}}$

D. $\frac{dy}{dx} = \frac{1}{2\sqrt{x^3}}$

6 Given $f(x) = \log_e(x-3)$ and $g(x) = x+4$, what is the domain of $f(g(x))$?

A. $x > -4$

B. $x > -1$

C. $x > -3$

D. $x > 3$

7 Which of the following is a simplified form of $-\sec(180^\circ - \theta)\sin \theta$?

A. $-\tan \theta$

B. $\tan \theta$

C. 1

D. -1

8 If $y = \frac{e^x}{2-e^x}$ which of the following is correct?

A. $x = \ln\left(\frac{2y}{1+y}\right)$

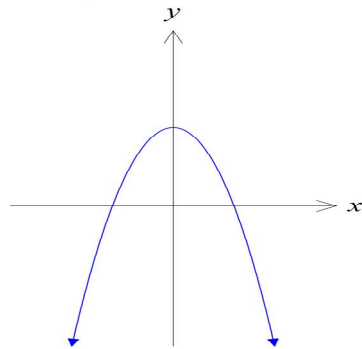
B. $x = \ln\left(\frac{1+y}{2y}\right)$

C. $x = \ln\left(\frac{2y}{1-y}\right)$

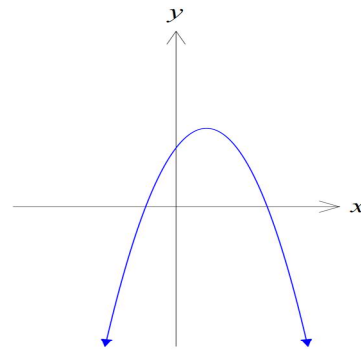
D. $x = \ln\left(\frac{1-y}{2y}\right)$

- 9 Which of the following could be the graph of $y = ax^2 + bx + c$, given that c is a positive constant and a and b are negative constants?

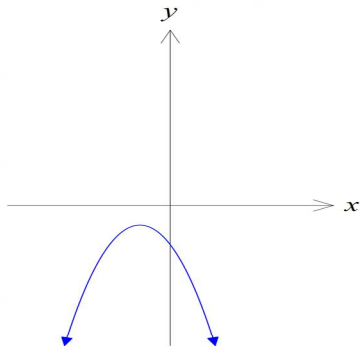
A.



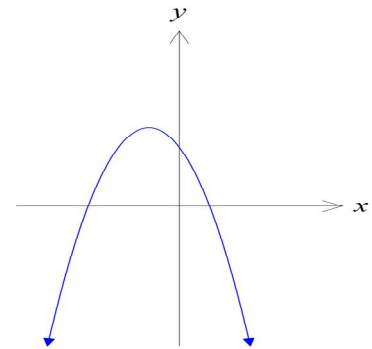
B.



C.



D.



- 10 A point lies on the circle $x^2 + y^2 - 12x + 16y + 91 = 0$.
What is the furthest distance, in units, the point could be from the origin?

- A. 9
- B. 11
- C. 13
- D. 14

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2023

YEAR 11 YEARLY EXAMINATION

Examination Number:

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Mathematics Advanced

Section II Answer Booklet

A

Section II

60 marks**Attempt Questions 11–28****Allow about 1 hour and 45 minutes for this section****Booklet A — Attempt Questions 11–15 (16 marks)****Booklet B — Attempt Questions 16–19 (15 marks)****Booklet C — Attempt Questions 20–24 (15 marks)****Booklet D — Attempt Questions 25–28 (14 marks)**

Instructions

- Write your Examination Number and Set at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on the back of the booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

Show that $\frac{4}{\sqrt{13}+2} = \frac{4\sqrt{13}-8}{9}$.

2

Question 12 (2 marks)

Simplify the expression $\left(\frac{2y^3}{x^3}\right) \div (2x^3y)^2$.

2

Question 13 (4 marks)

- (a) Determine the equation of the tangent to the curve $y = x^2 + 2x + 3$ at $x = 1$. **2**

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- (b) The tangent above meets the x -axis at A and the y -axis at B . If O is the origin, determine the area of $\triangle AOB$. **2**

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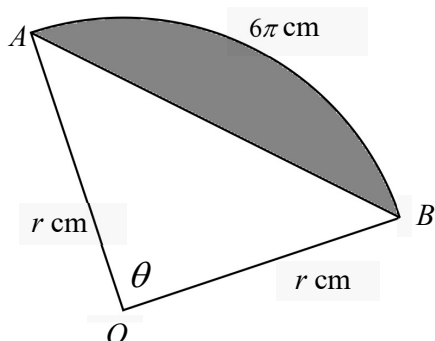
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Question 14 (5 marks)

The diagram shows a sector with an angle θ at the centre and radius r cm, where $6 < r < 12$. The arc length is 6π cm.



NOT TO
SCALE

- (a) Show why θ is an obtuse angle.

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Question 14 continues on page 13

Question 15 (3 marks)

- (a) Consider the function $f(x) = 3^{-x} - 1$.

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Sketch the curve $y = f(x)$, clearly showing any asymptote(s), intercept(s), and one other point.

- (c) Using your graph above, explain why $f(x) = 3^{-x} - 1$ is a function.

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Mathematics Advanced

Section II Answer Booklet

B**Booklet B — Attempt Questions 16–19 (15 marks)**

Instructions

- Write your Examination Number and Set at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
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Please turn over

Question 16 (3 marks)

Differentiate the following with respect to x .

(a) $y = 15x^3 - x^{-2} + 2$

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(b) $y = \frac{1}{5x^2 + 1}$

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Question 17 (3 marks)

Solve the equations below leaving answers in exact form.

(a) $3^x = 17$

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(b) $\log_3 x - \log_3 (x+2) + 1 = 0$

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Question 18 (5 marks)

Four cards, numbered 3, 4, 5 and 6 are used in a game.

The four cards are placed face down and each player pays \$1 to take a turn to draw two cards, one at a time without replacement. The two cards make a 2-digit number, with the first card drawn being the first digit of their number.

If the cards form a number over 60, the player wins \$3 back (i.e. an additional \$2). Otherwise they receive nothing back.

The four cards are then shuffled and replaced for the next turn.

- (a) Use a diagram to show all the 2-digit numbers that could be drawn. **2**

- (b) What is the probability that a player will win on their first turn? **1**

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Question 18 continues on page 21

- (c) Calculate the probability that a player who brings \$2 to play will have exactly \$3 left after two turns.

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Question 19 (4 marks)

(a) Given that $y = \frac{2x+1}{x^2+2}$, show that $\frac{dy}{dx} = \frac{2(2-x-x^2)}{(x^2+2)^2}$.

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Question 19 continues on page 23

- (b) Hence, or otherwise, determine the x coordinates of the point(s) on the curve $y = \frac{2x+1}{x^2+2}$ where the tangent(s) are horizontal.

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Section II Booklet B extra writing space

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Mathematics Advanced

Section II Answer Booklet

C

Booklet C — Attempt Questions 20–24 (15 marks)

Instructions

- Write your Examination Number and Set at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
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Please turn over

Question 20 (2 marks)

Solve $\cos^2 x = \cos x$ for $0^\circ \leq x \leq 360^\circ$.

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Question 21 (3 marks)

Data from a student survey is tabulated below. The data gathered indicates which hand they write with and their favourite colour.

	Green	Red	Blue	Total
Left	17	8	21	46
Right	44	71	39	154
Total	61	79	60	

A person is selected at random.

- (a) Find the probability that the pupil is right handed. **1**

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- (b) Find the probability that the pupil prefers green, given that they are left handed. **1**

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- (c) Are the events being left-handed and preferring green independent? **1**
Justify your answer with a calculation.

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Question 22 (5 marks)

A 20 kg block of radioactive isotope is decaying, and the amount, M kg, remaining after t years is given by the equation

$$M = 20e^{-0.05t}.$$

- (a) Find the mass of the isotope remaining after 2 years. Give your answer correct to 3 significant figures. **1**

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- (b) How long until half the isotope has decayed? Give your answer correct to 3 significant figures. **2**

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- (c) Find the rate of decay when 10 kg of the isotope remains. **2**

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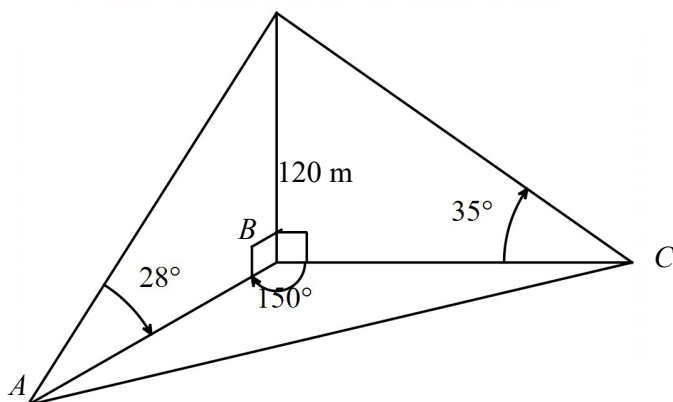
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Question 23 (3 marks)

A building at Point B is 120 metres tall and stands perpendicular to the ground.

3



At point A the angle of elevation to the top of the building is 28° . At point C the angle of elevation to the top of the building is 35° .

The angle $\angle ABC$ is 150° .

The owners want to landscape the area at the front of the building that is in the shape of the triangle ABC . Determine the area of the triangle ABC to the nearest square metre.

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Question 24 (2 marks)

If $a = \log_{\frac{1}{x}} \left(\frac{1}{N} \right)$ such that $x > 0$ and $N > 0$, show that $a = \log_x N$.

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Section II Booklet C extra writing space

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Mathematics Advanced

Section II Answer Booklet

D**Booklet D — Attempt Questions 25–28 (14 marks)**

Instructions

- Write your Examination Number and Set at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on the back of the booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 25 (1 marks)

On a set of axes, draw a graph that is continuous, but not differentiable at $x=1$.

1

Question 26 (5 marks)

(a) Show that $\frac{\cos \theta}{1 + \sin \theta} + \tan \theta = \sec \theta$.

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(b) Hence, or otherwise, solve

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$\frac{\cos \theta}{1 + \sin \theta} + \tan \theta = 2$ for $0 \leq \theta \leq 2\pi$.

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Question 27 (5 marks)

The displacement of a particle is described by the equation $x = 4te^{-t} + 3$ where x is the displacement from the origin in cm and t is the time in seconds.

- (a) Find the initial displacement of the particle.

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- (b) Find an equation for the particle's velocity.

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Question 27 continues on page 37

- (c) Find the time at which the particle has zero velocity and its exact displacement at that time.

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The diagram below shows a right angled triangle ABC where $AC = \sqrt{2}$ units long. **3**
Point E lies on the hypotenuse BC such that $AE = EC$.

Calculate the possible value(s) of $\angle BDE$, labelled θ in the diagram.

Diagram illustrating a geometric configuration involving a triangle ABC and a point E on the base BC .

- Point D lies on the side AB .
- Line segments AE and DE are drawn.
- The angle $\angle ADE$ is labeled θ .
- The length of the segment DE is labeled 1 .
- The length of the side AC is labeled $\sqrt{2}$.
- The diagram is labeled "NOT TO SCALE".

NOT TO
SCALE

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Section II Booklet D extra writing space

If you use this space, clearly indicate which question you are answering.

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Mathematics Advanced

September 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using a non erasable black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks:
70

Section I – 10 marks (pages 2–8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 9–40)

- Attempt Questions 11–29
- Allow about 1 hour and 45 minutes for this section

Note: Any time you have remaining should be spent reviewing your answers.

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Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1–10.

1 What is the solution of the equation $x^2 + 3x - 4 = 0$?

A. $x = -4$ and $x = 1$

B. $x = 4$ and $x = -1$

C. $x = 1$ and $x = -\frac{4}{3}$

D. $x = -1$ and $x = \frac{4}{3}$

$$\begin{aligned}x^2 + 3x - 4 &= 0 \\(x + 4)(x - 1) &= 0 \\x &= -4, 1\end{aligned}$$

2 Which of the following expressions is equal to $\frac{a^2 - b^2}{a^2b - ab^2}$?

A. $\frac{a+b}{a-b}$

B. $\frac{1}{a} - \frac{1}{b}$

C. $\frac{1}{ab}$

D. $\frac{a+b}{ab}$

$$\frac{(a+b)(\cancel{a-b})}{ab(\cancel{a-b})}$$

- 3 A fair die is rolled twice. What is the probability of tossing two even numbers?

A. $\frac{1}{2}$

B. $\frac{1}{3}$

☒ C. $\frac{1}{4}$

D. $\frac{2}{3}$

$$P(EE) = \frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$$

- 4 What is the value of $\frac{\log_a 81}{\log_a 3}$?

A. $\log_a 78$

B. $\log_a 27$

C. 27

☒ D. 4

$$\frac{\log_a 3^4}{\log_a 3} = \frac{4 \log_a 3}{\log_a 3}$$

- 5 If $y = \frac{1}{\sqrt{x}}$ what is an expression for $\frac{dy}{dx}$?

A. $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

☒ B. $\frac{dy}{dx} = \frac{-1}{2\sqrt{x^3}}$

C. $\frac{dy}{dx} = \frac{-1}{2\sqrt{x}}$

D. $\frac{dy}{dx} = \frac{1}{2\sqrt{x^3}}$

$$\begin{aligned} y &= x^{-1/2} \\ y' &= -\frac{1}{2} x^{-3/2} \\ &= \frac{-1}{2\sqrt{x^3}} \end{aligned}$$

6 Given $f(x) = \log_e(x-3)$ and $g(x) = x+4$, what is the domain of $f(g(x))$?

A. $x > -4$

☒ B. $x > -1$

C. $x > -3$

D. $x > 3$

$$\begin{aligned} f(g(x)) &= f(x+4) \\ &= \ln(x+4-3) \\ &= \ln(x+1) \\ \therefore \text{Domain } x &> -1 \end{aligned}$$

7 Which of the following is a simplified form of $-\sec(180^\circ - \theta) \sin \theta$?

A. $-\tan \theta$

☒ B. $\tan \theta$

C. 1

D. -1

$$\begin{aligned} &-\sec(180^\circ - \theta) \sin \theta \\ &= \frac{-1}{\cos(180^\circ - \theta)} \sin \theta \\ &= \frac{\sin \theta}{\cos \theta} = \tan \theta \end{aligned}$$

8 If $y = \frac{e^x}{2 - e^x}$ which of the following is correct?

☒ A. $x = \ln\left(\frac{2y}{1+y}\right)$

B. $x = \ln\left(\frac{1+y}{2y}\right)$

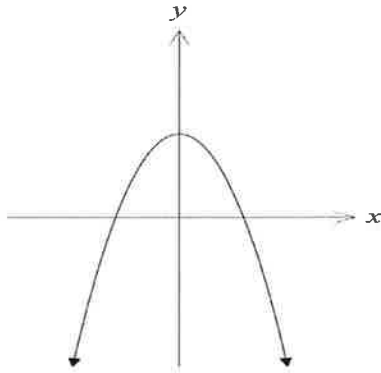
C. $x = \ln\left(\frac{2y}{1-y}\right)$

D. $x = \ln\left(\frac{1-y}{2y}\right)$

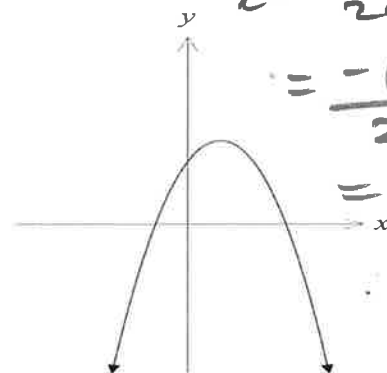
$$\begin{aligned} y &= \frac{e^x}{2 - e^x} \\ 2y - ye^x &= e^x \\ 2y &= e^x(y+1) \\ \frac{2y}{y+1} &= e^x \\ x &= \ln\left(\frac{2y}{y+1}\right) \end{aligned}$$

- 9 Which of the following could be the graph of $y = ax^2 + bx + c$, given that c is a positive constant and a and b are negative constants?

A.



B.



$c > 0 \therefore \text{not } \underline{\underline{C}}$

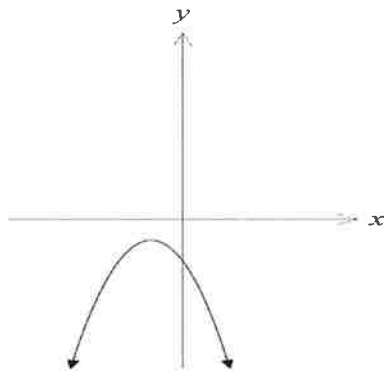
$x = \frac{-b}{2a}$ $a \text{ and } b < 0$

$= \frac{-(-ve)}{2 \times (-ve)}$

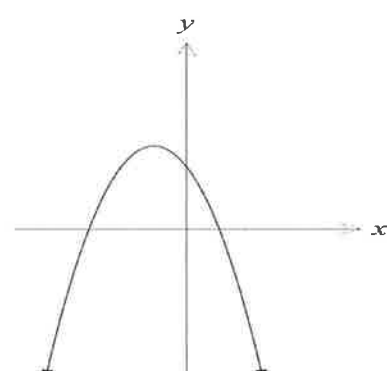
$= -ve$

$\therefore \underline{\underline{D}}$

C.



D.



- 10 A point lies on the circle $x^2 + y^2 - 12x + 16y + 91 = 0$.
What is the furthest distance, in units, the point could be from the origin?

A. 9

B. 11

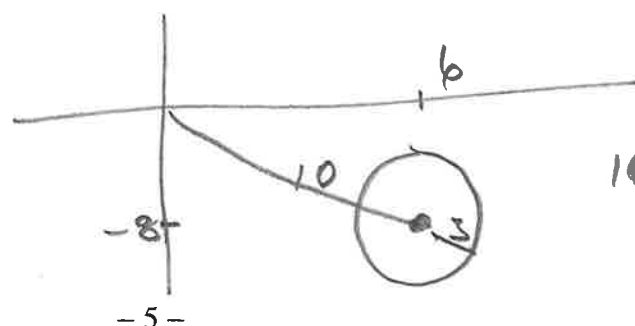
C. 13

D. 14

$x^2 - 12x + y^2 + 16y = -91$

$x^2 - 12x + 36 + y^2 + 16y + 64 = 9$

$(x-6)^2 + (y+8)^2 = 9$



$10 + 3 = 13$

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2023**YEAR 11 YEARLY EXAMINATION**

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Mathematics Advanced

Section II Answer Booklet

A

Section II

60 marks**Attempt Questions 11–28****Allow about 1 hour and 45 minutes for this section****Booklet A — Attempt Questions 11–15 (16 marks)****Booklet B — Attempt Questions 16–19 (15 marks)****Booklet C — Attempt Questions 20–24 (15 marks)****Booklet D — Attempt Questions 25–28 (14 marks)**

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Please turn over

Question 11 (2 marks)

Show that $\frac{4}{\sqrt{13}+2} = \frac{4\sqrt{13}-8}{9}$.

2

$$\frac{4}{\sqrt{13}+2} \times \frac{\sqrt{13}-2}{\sqrt{13}-2} = \frac{4\sqrt{13}-8}{13-4}$$
$$= \frac{4\sqrt{13}-8}{9}$$

Question 12 (2 marks)

Simplify the expression $\left(\frac{2y^3}{x^3}\right) \div (2x^3y)^2$.

2

$$\frac{2y^3}{x^3} \times \frac{1}{(2x^3y)^2}$$
$$= \frac{2y^3}{x^3} \times \frac{1}{4x^6y^2}$$
$$= \frac{y}{2x^3}$$

Question 13 (4 marks)

- (a) Determine the equation of the tangent to the curve $y = x^2 + 2x + 3$ at $x = 1$.

2

$$y = x^2 + 2x + 3$$

$$y' = 2x + 2$$

$$\text{@ } x=1 \quad m_T = 2+2 = 4$$

$$\text{@ } x=1 \quad y = 1+2+3 = 6$$

$$y = mx + b$$

$$b = 4 + b$$

$$\underline{\underline{2 = b}}$$

$$y = 4x + 2$$

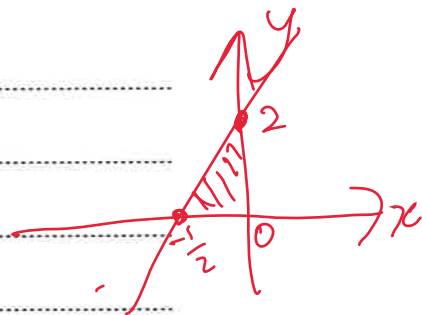
- (b) The tangent above meets the x -axis at A and the y -axis at B . If O is the origin, determine the area of $\triangle AOB$.

2

$$y = 4x + 2$$

$$\text{@ } x=0 \quad y=2$$

$$\text{@ } y=0 \quad x = -\frac{1}{2}$$



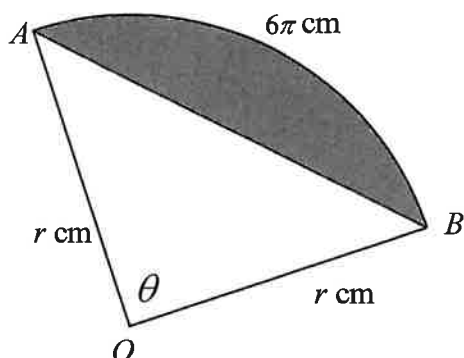
$$A = \frac{1}{2}bh$$

$$= \frac{1}{2} \times \frac{1}{2} \times 2$$

$$= \frac{1}{2}$$

Question 14 (5 marks)

The diagram shows a sector with an angle θ at the centre and radius r cm, where $6 < r < 12$. The arc length is 6π cm.



NOT TO
SCALE

- (a) Show why θ is an obtuse angle.

2

$$l = r\theta$$

$$\text{If } r=6 \quad 6\pi = 6\theta$$

$$\theta = \pi$$

$$\text{If } r=12 \quad 6\pi = 12\theta$$

$$\theta = \frac{\pi}{2}$$

$$\therefore \frac{\pi}{2} < \theta < \pi$$

θ is obtuse.

Question 14 continues on page 13

- (b) Calculate the exact area of the shaded region when $\theta = \frac{3\pi}{4}$.

3

$$l = r\theta$$

$$6\pi = \frac{3\pi}{4} r$$

$$r = 8$$

$$A = \frac{1}{2} r^2 (\theta - \sin \theta)$$

$$= \frac{1}{2} \times 8^2 \left(\frac{3\pi}{4} - \sin \frac{3\pi}{4} \right)$$

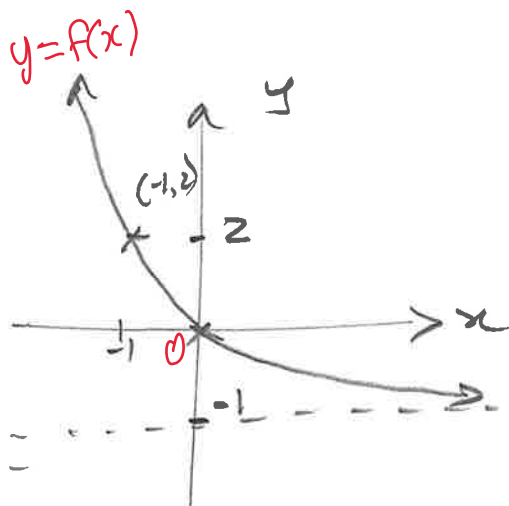
$$= 32 \left(\frac{3\pi}{4} - \frac{1}{\sqrt{2}} \right) \text{ u}^2.$$

Question 15 (3 marks)

- (a) Consider the function $f(x) = 3^{-x} - 1$.

2

Sketch the curve $y = f(x)$, clearly showing any asymptote(s), intercept(s), and one other point.



- 6 Using your graph above, explain why $f(x) = 3^{-x} - 1$ is a function.

1

It is a function as for each value of x there is only one value of y assigned.

Section II Booklet A extra writing space

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Mathematics Advanced

Section II Answer Booklet

B

Booklet B — Attempt Questions 16–19 (15 marks)

Instructions

- Write your Examination Number and Set at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
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Please turn over

Question 16 (3 marks)

Differentiate the following with respect to x .

(a) $y = 15x^3 - x^{-2} + 2$

1

$$y = 15x^3 - x^{-2} + 2$$

$$\frac{dy}{dx} = 45x^2 + 2x^{-3}$$

(b) $y = \frac{1}{5x^2 + 1}$

2

$$y = \frac{1}{5x^2 + 1}$$

$$= (5x^2 + 1)^{-1}$$

$$\frac{dy}{dx} = -1(5x^2 + 1)^{-2} \times 10x$$

$$= \underline{\underline{-10x(5x^2 + 1)^{-2}}}$$

Question 17 (3 marks)

Solve the equations below leaving answers in exact form.

(a) $3^x = 17$

1

$$3^x = 17$$
$$x = \log_3 17$$

(b) $\log_3 x - \log_3 (x+2) + 1 = 0$

2

$$\log_3 x - \log_3 (x+2) = -1$$

$$\log_3 \frac{x}{x+2} = -1$$

$$\frac{x}{x+2} = 3^{-1}$$

$$3x = x+2$$

$$2x = 2$$

$$x = 1$$

$$\underline{\underline{x = 1}}$$

Question 18 (5 marks)

Four cards, numbered 3, 4, 5 and 6 are used in a game.

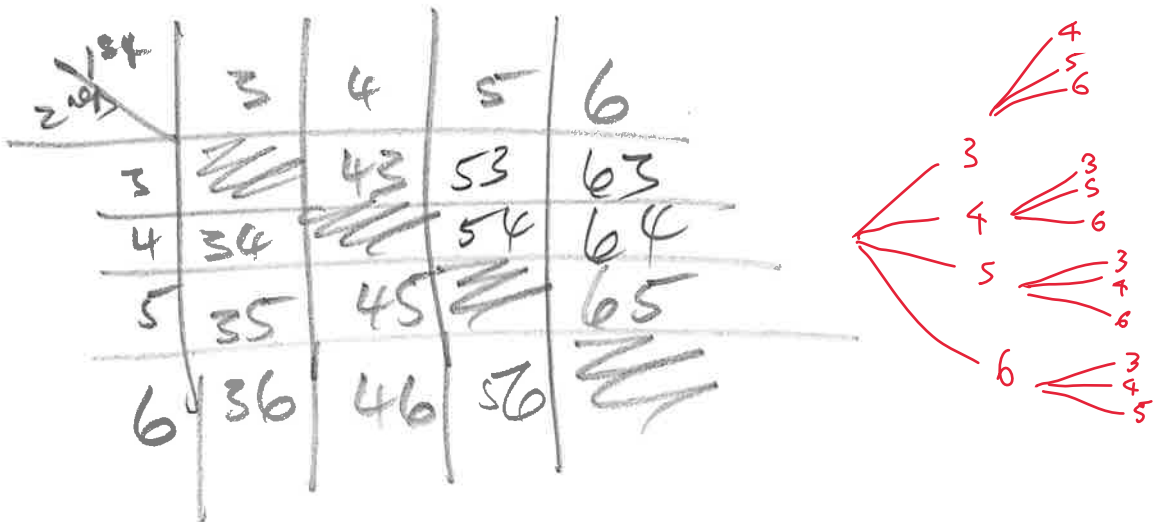
The four cards are placed face down and each player pays \$1 to take a turn to draw two cards, one at a time without replacement. The two cards make a 2-digit number, with the first card drawn being the first digit of their number.

If the cards form a number over 60, the player wins \$3 back (i.e. an additional \$2). Otherwise they receive nothing back.

The four cards are then shuffled and replaced for the next turn.

- (a) Use a diagram to show all the 2-digit numbers that could be drawn.

2



- (b) What is the probability that a player will win on their first turn?

1

$$P(W) = \frac{3}{12}$$

$$= \frac{1}{4}$$

Question 18 continues on page 21

- (c) Calculate the probability that a player who brings \$2 to play will have exactly \$3 left after two turns.

2

This means the player
will win then lose or
lose then win.

$$P(WL) + P(LW) = \frac{1}{2} \times \frac{3}{4} + \frac{3}{4} \times \frac{1}{4}$$

$$= \frac{3}{10} \times 2$$

$$= \frac{3}{5}$$

Question 19 (4 marks)

- (a) Given that $y = \frac{2x+1}{x^2+2}$, show that $\frac{dy}{dx} = \frac{2(2-x-x^2)}{(2+x^2)^2}$.

2

$$y = \frac{2x+1}{x^2+2} \quad \frac{u}{v}$$
$$y' = \frac{u'v - uv'}{v^2}$$

$$\begin{aligned} u &= 2x+1 \\ u' &= 2 \\ v &= x^2+2 \\ v' &= 2x \end{aligned}$$

$$= \frac{2(x^2+2) - (2x+1)2x}{(x^2+2)^2}$$

$$= \frac{2x^2+4 - 4x^2 - 2x}{(x^2+2)^2}$$

$$= \frac{-2x^2 - 2x + 4}{(x^2+2)^2}$$

$$= \frac{2(2-x-x^2)}{(x^2+2)^2}$$

Question 19 continues on page 23

- (b) Hence, or otherwise, determine the x coordinates of the point(s) on the curve

2

$y = \frac{2x+1}{x^2+2}$ where the tangent(s) are horizontal.

this means $\frac{dy}{dx} = 0$

$$0 = \frac{2(2-x-x^2)}{(x^2+2)^2}$$

$$0 = x^2 + x - 2$$

$$0 = (x+2)(x-1)$$

$$x = -2, 1$$

Section II Booklet B extra writing space

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Mathematics Advanced

Section II Answer Booklet

C

Booklet C — Attempt Questions 20–24 (15 marks)

Instructions

- Write your Examination Number and Set at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
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Please turn over

Question 20 (2 marks)

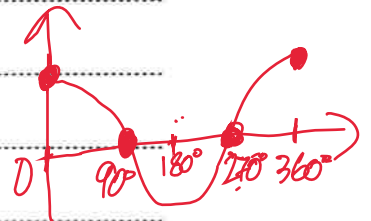
Solve $\cos^2 x = \cos x$ for $0^\circ \leq x \leq 360^\circ$.

2

$$\cos^2 x = \cos x$$

$$\cos^2 x - \cos x = 0$$

$$\cos x (\cos x - 1) = 0$$



$$\cos x = 0$$

$$\cos x = 1$$

$$x = 90^\circ, 270^\circ, 0^\circ, 360^\circ$$

Question 21 (3 marks)

Data from a student survey is tabulated below. The data gathered indicates which hand they write with and their favourite colour.

	Green	Red	Blue	Total
Left	17	8	21	46
Right	44	71	39	154
Total	61	79	60	

A person is selected at random.

- (a) Find the probability that the pupil is right handed.

1

$$P(RH) = \frac{154}{200} = 0.77$$

- (b) Find the probability that the pupil prefers green, given that they are left handed.

1

$$P(G|L) = \frac{17}{46}$$

- (c) Are the events being left-handed and preferring green independent? Justify your answer with a calculation.

1

$$P(G \text{ and } L) = \frac{17}{200} \quad P(G) \times P(L) = \frac{61}{200} \times \frac{46}{200}$$

as $P(G|L) \neq P(G) \times P(L)$ they are not independent.

~~$P(G)$~~ OR $P(G|L) \neq P(G)$

Question 22 (5 marks)

A 20 kg block of radioactive isotope is decaying, and the amount, M kg, remaining after t years is given by the equation

$$M = 20e^{-0.05t}$$

- (a) Find the mass of the isotope remaining after 2 years. Give your answer correct to 3 significant figures. 1

$$M = 20e^{-0.05 \times 2}$$
$$= 18.1 \text{ kg}$$

- (b) How long until half the isotope has decayed? Give your answer correct to 3 significant figures. 2

$$10 = 20e^{-0.05t}$$
$$\frac{1}{2} = e^{-0.05t}$$
$$\ln \frac{1}{2} = -0.05t$$
$$t = \frac{-\ln \frac{1}{2}}{0.05}$$
$$= 13.9 \text{ yrs}$$

- (c) Find the rate of decay when 10 kg of the isotope remains. Give your answer correct to 1 significant figure. 2

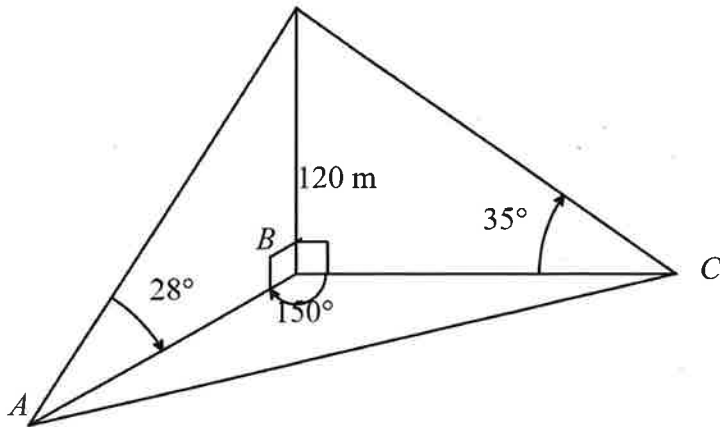
$$\frac{dM}{dt} = -0.05 \times 20e^{-0.05t}$$

$$\frac{dM}{dt} = -k \times M$$
$$= -0.05 \times 20e^{-0.05 \times 13.9}$$
$$= -0.5 \text{ kg/yr}$$

Question 23 (3 marks)

A building at Point B is 120 metres tall and stands perpendicular to the ground.

3



At point A the angle of elevation to the top of the building is 28° . At point C the angle of elevation to the top of the building is 35° .

The angle $\angle ABC$ is 150° .

The owners want to landscape the area at the front of the building that is in the shape of the triangle ABC . Determine the area of the triangle ABC to the nearest square metre.

$$\tan 62^\circ = \frac{AB}{120}$$

$$AB = 120 \tan 62^\circ$$

$$= 225.68$$

$$= 226 \text{ m}$$

$$\tan 55^\circ = \frac{BC}{120}$$

$$BC = 120 \tan 55^\circ$$

$$= 171 \text{ m}$$

$$A = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 226 \times 171 \sin 150^\circ$$

$$= 9662 \text{ m}^2$$

Question 24 (2 marks)

If $a = \log_{\frac{1}{x}} \left(\frac{1}{N} \right)$ such that $x > 0$ and $N > 0$, show that $a = \log_x N$.

2

$$a = \log_{\frac{1}{x}} \left(\frac{1}{N} \right)$$

$$\left(\frac{1}{x} \right)^a = \frac{1}{N}$$

$$x^a = N$$

By definition

$$x^a = N \Rightarrow \log_x N = a$$

Section II Booklet C extra writing space

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Mathematics Advanced

Section II Answer Booklet

D**Booklet D — Attempt Questions 25–28 (15 marks)**

Instructions

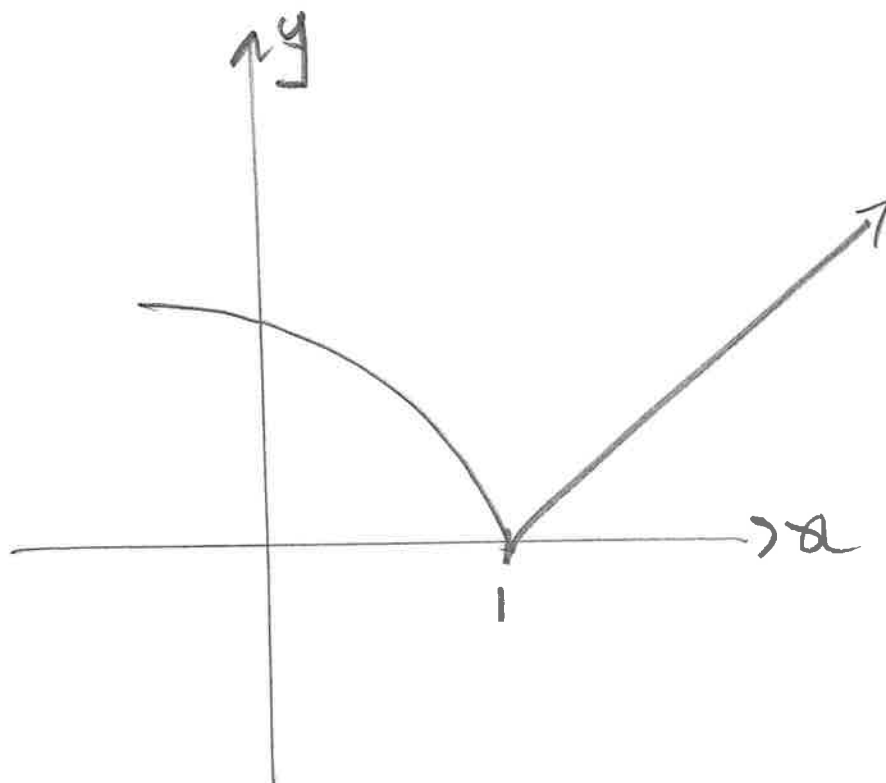
- Write your Examination Number and Set at the top of this page.
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Please turn over

Question 25 (1 marks)

On a set of axes, draw a graph that is continuous, but not differentiable at $x=1$.

1



Question 26 (5 marks)

- (a) Show that $\frac{\cos \theta}{1 + \sin \theta} + \tan \theta = \sec \theta$.

3

$$\text{LHS} = \frac{\cos \theta}{1 + \sin \theta} + \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\cos^2 \theta + \sin \theta (1 + \sin \theta)}{(1 + \sin \theta) \cos \theta}$$

$$= \frac{\cos^2 \theta + \sin^2 \theta + \sin \theta}{(1 + \sin \theta) \cos \theta}$$

$$= \frac{1 + \sin \theta}{(1 + \sin \theta) \cos \theta} \quad \text{but } \sin^2 \theta + \cos^2 \theta = 1$$

$$= \frac{1}{\cos \theta}$$

$$= \sec \theta$$

$\therefore \text{RHS} \quad \therefore \text{true}$

- (b) Hence, or otherwise, solve

2

$$\frac{\cos \theta}{1 + \sin \theta} + \tan \theta = 2 \text{ for } 0 \leq \theta \leq 2\pi.$$

$$\therefore \frac{\cos \theta}{1 + \sin \theta} + \tan \theta = 2$$

$$\therefore \sec \theta = 2$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\begin{array}{c|c} 5 & A \\ \hline 1 & C \end{array}$$

$$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Question 27 (5 marks)

The displacement of a particle is described by the equation $x = 4te^{-t} + 3$ where x is the displacement from the origin in cm and t is the time in seconds.

- (a) Find the particle's displacement at the start of its motion.

1

$$t=0 \quad x = 4 \times 0 e^0 + 3$$

$$= \underline{\underline{3 \text{ cm}}}$$

- (b) Find an equation for the particle's velocity.

2

$$x = 4te^{-t} + 3$$

u v

$$u'v + uv'$$

$$u = 4t \quad v = e^{-t}$$

$$u' = 4 \quad v' = -e^{-t}$$

$$\dot{x} = 4e^{-t} + 4te^{-t}$$

$$= 4e^{-t}(1+t)$$

Question 27 continues on page 37

- (c) Find the time at which the particle has zero velocity and its exact displacement at that time.

2

$$v = 4e^{-t}(1-t)$$

$$e^{-t} \neq 0 \quad \therefore t = 1 \text{ sec.}$$

at that time

$$x = 4te^{-t} + 3$$

$$= \frac{4}{e} + 3 \text{ cm.}$$

Question 28 (4 marks)

The diagram below shows a right angled triangle ABC where $AC = \sqrt{2}$ units long. Point E lies on the hypotenuse BC such that $AE = EC$.

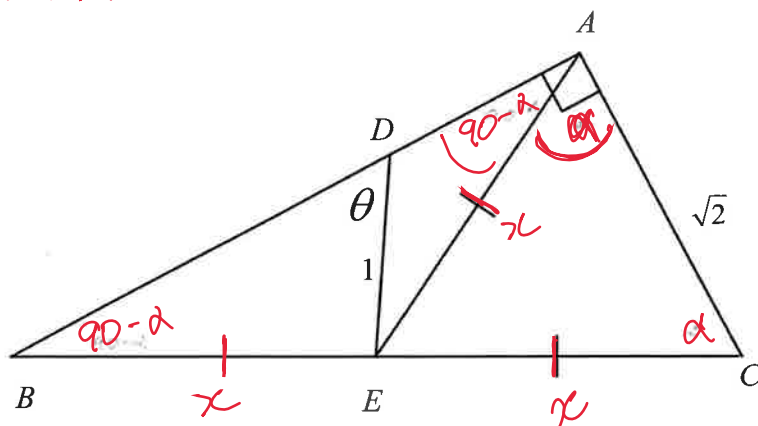
~~X~~ 3

Point D lies on AB such that $DE = 1$.

Calculate the size of $\angle BDE$, labelled θ in the diagram.

1 or 2

Hint: Let $\angle EAC = x$



NOT TO SCALE

$$\text{Let } \angle EAC = x \quad \angle AEC = x$$

$$\angle BAE = 90 - x \quad \angle CBA = 90 - x$$

$\triangle ABE$ is isosceles $AE = BE = CE = x$

$$\sin(90 - x) = \frac{\sqrt{2}}{2x} \quad (1) \quad (\triangle ABC)$$

$$= \frac{1}{\sqrt{2}x} \quad (\text{as } BC = 2x)$$

$$\therefore \triangle BDE: \frac{\sin \theta}{x} = \frac{\sin(90 - x)}{1}$$

$$\frac{\sin \theta}{x} = \frac{1}{\sqrt{2}x}$$

$$\sin \theta = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

using (1)

OR $45^\circ, 135^\circ$

Section II Booklet D extra writing space

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