Name: _____

Teacher: _____

St George Girls High School

Year 11 Assessment Task 3 2019

Mathematics Advanced

General Instructions

- Working time: **120 minutes**.
- Reading Time: **10 minutes**
- Write using black pen
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- For questions in **Section I,** use the multiple-choice answer sheet provided.
- For questions in **Section II:**
  - Answer the questions in the space provided.
  - Show relevant mathematical reasoning and/or calculations.
  - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
  - Marks may not be awarded for incomplete or poorly presented solutions

Total marks: 81

Section I – 8 marks (pages 3 – 5)

- Attempt Questions 1 - 8
- Allow about 12 minutes for this section

Section II – 73 marks (pages 6 – 23)

- Attempt all questions
- Allow about 1 hour and 48 minutes for this section

Q1 – 8	/8
Q9 – Q12	/9
Q13 – Q15	/10
Q16 – Q17	/9
Q18 – Q19	/9
Q20 – Q22	/9
Q23 – Q25	/9
Q26 – Q27	/9
Q28 – Q29	/9
Total	**/81**
	%

BLANK PAGE

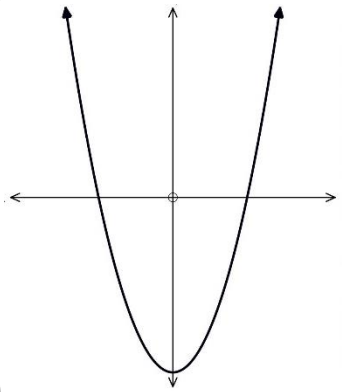
Section I

8 marks
Attempt Questions 1 – 8
Allow about 12 minutes for this section

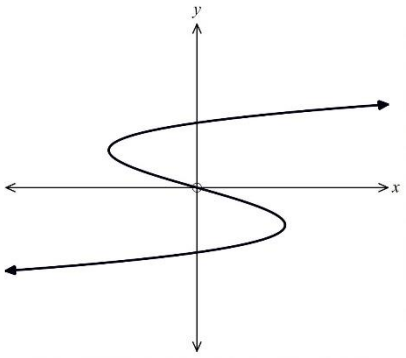
Use the multiple-choice answer sheet provided for Questions 1 to 8.

1. Which graph represents a one to many relation?

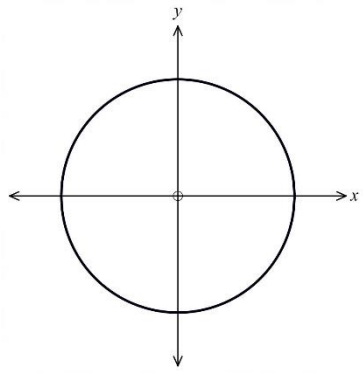
(A)



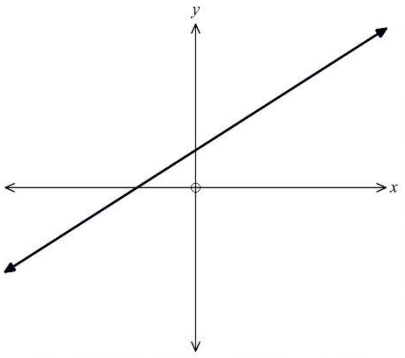
(B)



(C)



(D)



2. Given the equation $2x^2 - x + 5 = 0$, which of the following statements is correct?

(A) The equation has equal roots.

(B) The equation has no real roots.

(C) The equation has distinct real roots.

(D) The equation has rational roots.

3. Simplify $\dfrac{\cos\left(\frac{\pi}{2}-x\right)}{\cos x}$.

(A) -1

(B) $-\tan x$

(C) $\tan x$

(D) 1

4. Consider the function $f(x) = |x+2| + 1$.

Which of the following is true at the point (– 2, 1) of this function?

(A) The function is continuous and differentiable.

(B) The function is discontinuous but differentiable.

(C) The function is continuous but not differentiable.

(D) The function is discontinuous and not differentiable.

5. Given that $\left(\frac{1}{2}\right)^x \geq \frac{1}{1\,024}$.

Which of the following is correct?

(A) $\log_{10} x \leq 1$

(B) $\log_{10} x \geq 1$

(C) $x \geq 10$

(D) $x \leq 10$

6. A polynomial which has a leading coefficient of 3 has a single root at $x = 4$, triple root at $x = 2$ and double root at $x = -3$. Which is the polynomial function that best describes the information ?

(A) $y = 3(x-4)(x-3)^2(x-2)^3$.

(B) $y = 4(x-3)(x-2)^2(x-3)^3$.

(C) $y = 3(x-4)(x+3)^2(x+2)^3$.

(D) $y = 3(x-4)(x+3)^2(x-2)^3$.

7. If $y = \frac{x^2}{x-1}$ then $\frac{dy}{dx} =$

(A) $\frac{x^2-2x}{(x-1)^2}$

(B) $\frac{x^2-x}{(x-1)^2}$

(C) $\frac{x^2+2x}{(x-1)^2}$

(D) $\frac{x^2+x}{(x-1)^2}$

8. If $f(x) = \sqrt{x}$ and $g(x) = 16 - x^2$ then which of the following is true for $f(g(x))$?

(A) Domain is $[0,4]$ and range is $[0,4]$.

(B) Domain is $[-4,4]$ and range is $(0,4)$.

(C) Domain is $(-4,4)$ and range is $[0,4]$.

(D) Domain is $[-4,4]$ and range is $[0,4]$.

End of Section I

Section II

73 marks
Attempt Questions 9 – 29
Allow about 108 minutes for this section

Answer the questions in the spaces provided.
Your answers should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of this booklet.

Marks

Question 9 (2 marks)

Simplify the following expression with a rational denominator.

$$\frac{1}{\sqrt{3}} - \frac{1}{9}$$ 2

……

……

……

……

……

……

……

Question 10 (2 marks)

Show that $\log_3\left(\frac{1}{9}\right) - \log_3\left(\sqrt{3}\right) + \log_3(27) = \frac{1}{2}$. 2

……

……

……

……

……

……

Question 11 (2 marks)

Express $\sin(\pi - \alpha) - \sin(2\pi - \alpha)$ in terms of $\sin \alpha$. 2

……

……

……

Question 12 (3 marks)

Given that $\sqrt{2}\cos x = -1$, where $\frac{\pi}{2} < x < \pi$:

(a) solve for x. 1

……

……

……

……

……

……

……

(b) find the exact value of $\tan x$. 2

……

……

……

……

……

……

……

Question 13 (2 marks)

Simplify $\dfrac{6x^3 \times 5x^{-1}}{2x^6 \times 10x^{-2}}$. 2

……

……

……

……

Question 14 (3 marks)

Differentiate $y = 2x - 3x^2$ from first principles. 3

……

……

……

……

……

……

……

……

……

……

……

……

Question 15 (5 marks)

Differentiate the following expressions with respect to x.

(a) $y = 2x^6 - 3x + \frac{7}{x}$ 1

……

……

……

……

(b) $y = (2x^3 - 5x)^4$ 2

……

……

……

……

(c) $f(x) = 3x\sqrt{x^3}$ 2

……

……

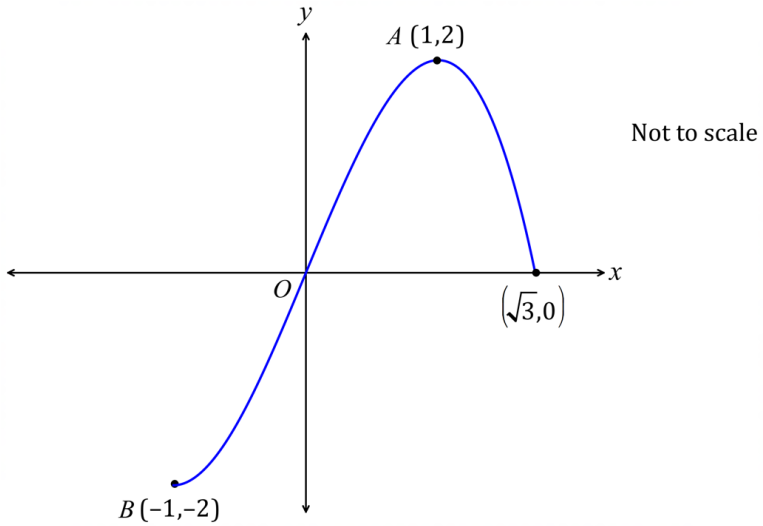
……

……

……

Question 16 (4 marks)

The sketch of $y = f(x)$ is shown below.



(a) Use the graph of $y = f(x)$ to draw a neat sketch of $y = f(-x)$. 2

(b) Use the graph of $y = f(x)$ to draw a neat sketch of $y = -f(-x)$. 2

Question 17 (5 marks)

Solve the following equations for x.

(a) $\log_a x - \log_a (x-2) = \frac{2}{3}\log_a 27$ 3

……

……

……

……

……

……

……

……

……

……

(b) $3^x = 0.024$ (leave your answer correct to 2 decimal places). 2

……

……

……

……

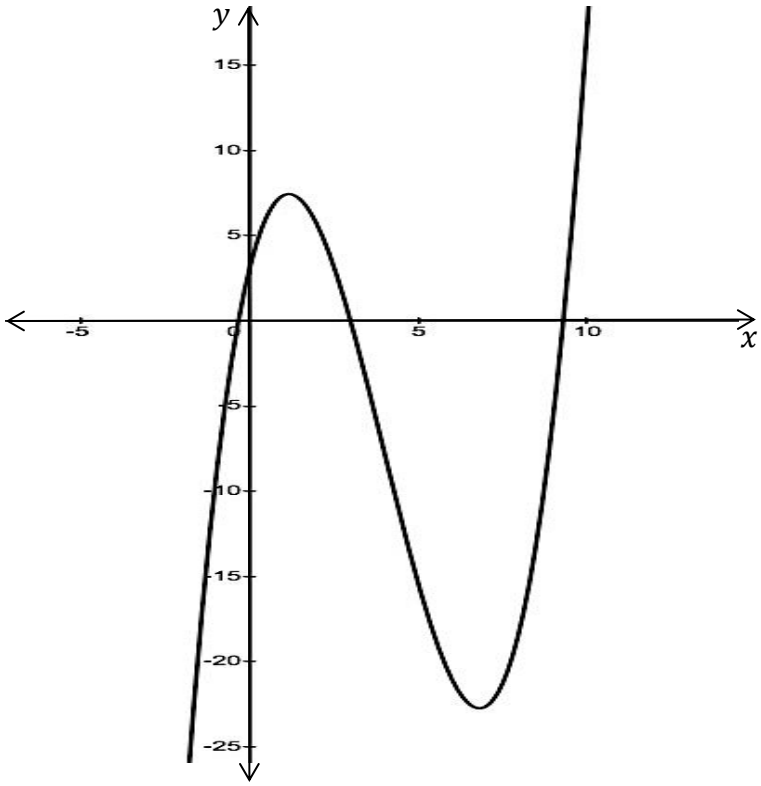
……

……

……

Question 18 (5 marks)

The curve shown below has the equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$.



The point P has the coordinates (3, 0).

(a) Show that P lies on the curve. 1

……

……

……

……

……

……

……

……

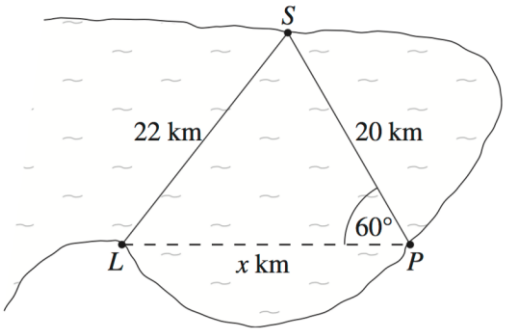
(b) Find the equation of the tangent at P. 2

(c) Another point Q also lies on the curve. The tangent to the curve at Q is parallel to the tangent at P. Find the coordinates of Q. 2

Question 19 (4 marks)

In the diagram below, the surf shop at S is 20 km across the bay from post office at P. The distance from the surf shop to the lighthouse at L is 22 km and $\angle SPL$ is 60°.

Let the distance PL be x kilometres.



(a) Using the cosine rule, show that $x^2 - 20x - 84 = 0$. 2

……

……

……

……

……

……

(b) Hence, find the distance from the post office to the lighthouse. 2
Give your answer to 1 decimal place.

……

……

……

……

……

……

……

……

Question 20 (2 marks)

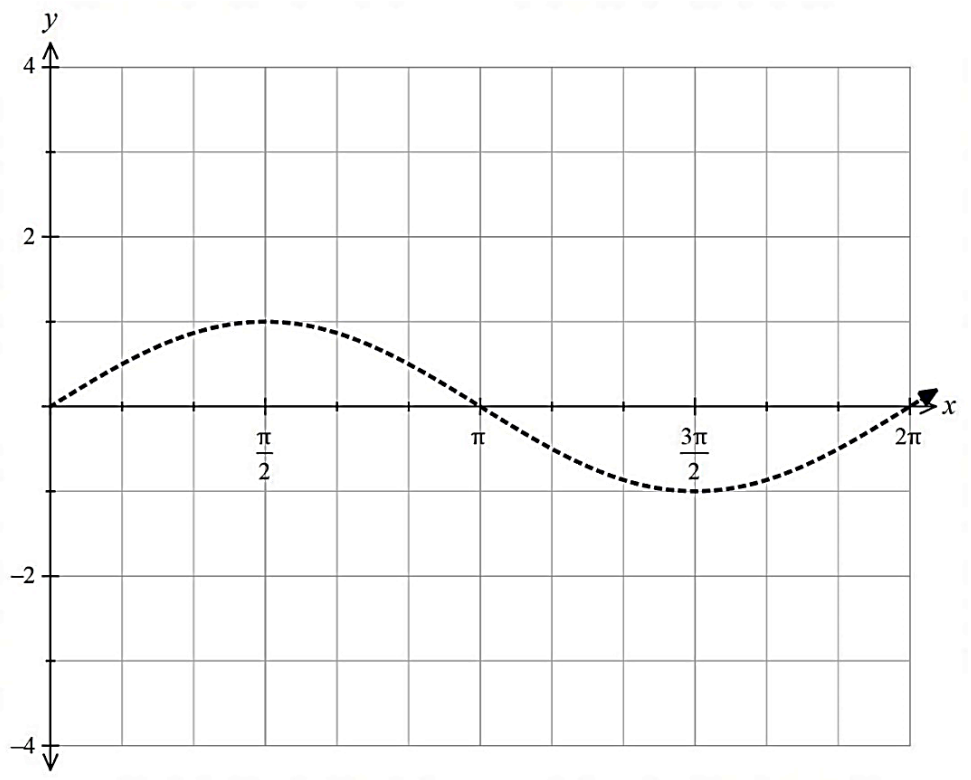
Sketch the function of $f(x) = 2(3^x) + 1$. 2

Question 21 (2 marks)

2

The graph of $y = \sin x$ for $0 \leq x \leq 2\pi$ is drawn on the axes below.

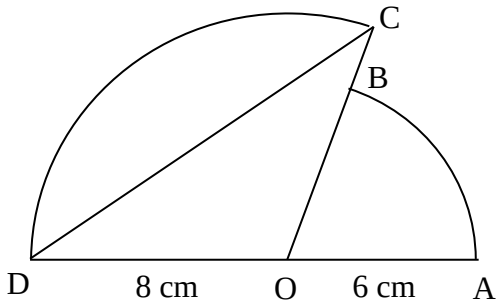
Draw and label a neat sketch of $y = \text{cosec } x$ for $0 \leq x \leq 2\pi$.



Question 22 (5 marks)

The diagram shows two sectors OAB and OCD with the same centre O, such that OBC and AOD are two straight lines. The radius OA of the smaller sector OAB is 6 cm, while the radius OD of the larger sector is 8 cm.

The area of triangle OCD is $16\sqrt{3}$ cm^2.



NOT TO SCALE

(a) Find the size of the obtuse angle COD in radians. 2

……

……

……

……

……

……

……

(b) Find the exact length of the arc CD. 1

……

……

……

(c) Find the area of sector OAB. 2

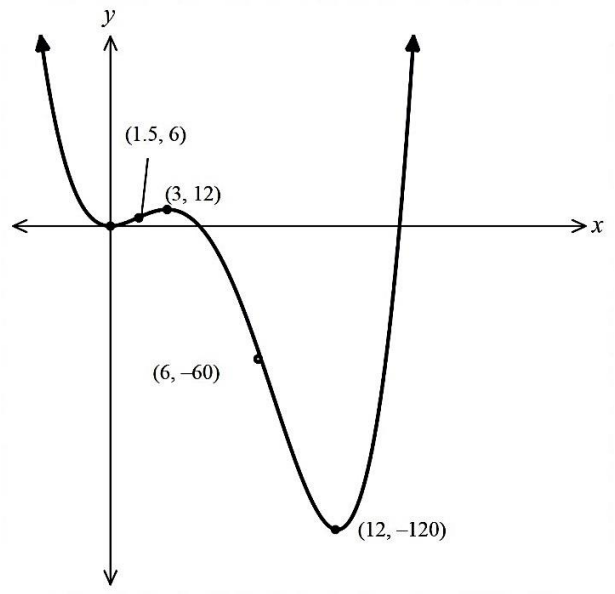
……

……

……

Question 23 (3 marks)

The function $y = f(x)$ below has turning points at the origin and at (3, 12) and (12, –120).



(a) For what $x-$ values is the function increasing? 1

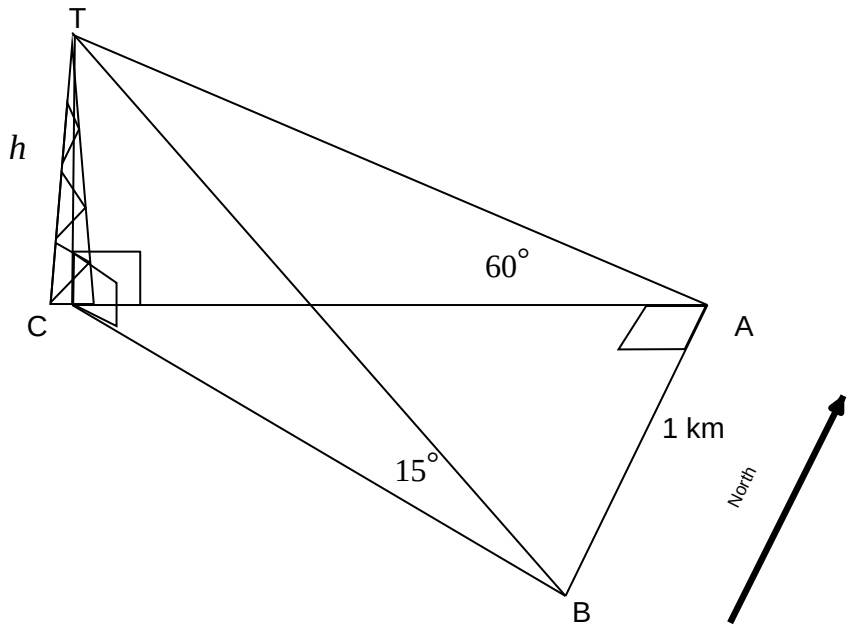
……

……

(b) Sketch the gradient function of $y = f(x)$. 2

Question 24 (3 marks)

The angle of elevation of the top of a tower (T) from a point A due East of the tower is 60°. From a point *B* due South of *A,* the angle of elevation of T is 15°. *A* and *B* are at the same elevation as the base of the tower. If the distance *AB* = 1 km, find the height (*h*) of the tower. 3



……

……

……

……

……

……

……

……

……

……

……

……

Question 25 (3 marks)

Show that if $y = x\sqrt{2x-1}$ then $\frac{dy}{dx} = \frac{3x-1}{\sqrt{2x-1}}$. 3

Question 26 (4 marks)

A particle moves along the x-axis such that its displacement from the origin (in cm) at a time t (seconds) is given by the equation: 4

$$x = 2t^3 + \sqrt{t}.$$

Find the velocity and acceleration of the particle after 9 seconds.

……

……

……

……

……

……

……

……

……

Question 27 (5 marks)

A water tank is being filled at a variable rate. The height of the water, h cm, at any time, t minutes can be described by,

$$h(t) = 2t^2 - 4t + 50.$$

Find:

(a) the initial height of water in the tank. 1

……

……

……

……

……

……

(b) the instantaneous rate at which the height is changing at 4 minutes. 2

……

……

……

……

(c) the average rate at which the height has changed over the first 4 minutes. 2

……

……

……

……

……

……

……

……

……

……

Question 28 (4 marks)

On the Richter scale, the magnitude R of an earthquake of intensity I is modelled by the formula $R = \log_{10}\left(\frac{I}{I_0}\right)$, where I_0 is a reference intensity used for comparisons.

(a) Find the magnitude of an earthquake that is 2.1 million times more intense than the reference intensity. Give your answer to 1 decimal place. 2

……

……

(b) In Japan an earthquake was recorded to have of magnitude 7.2 on the Richter scale in 2002. 2

How many times more intense is this than the reference intensity? Give your answer to 3 significant figures.

……

……

……

……

Question 29 (5 marks)

Boyles Law is a gas law that states that the pressure P in a container is inversely proportional to the volume V of the container, assuming the gas is at the same temperature at all times. For a particular canister, the volume is 28 cm^3 and the pressure is 2.5 kg/cm,

(a) Show that the equation that models the above information is: 2

$$P = \frac{70}{V}.$$

……

……

……

……

……

……

(b) Find the exact volume of the canister if the pressure is 3.75 kg/cm. 1

……

……

……

……

……

(c) If the volume in (b) is halved, what effect will this have on the pressure? Verify your answer by finding the pressure. 2

……

……

……

……

……

……

……

……

End of Assessment Task

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

BLANK PAGE

BLANK PAGE

Name: _____

Teacher:_____

Section I

Year 11 Course

Mathematics Advanced

Assessment Task 3

2019

Multiple-choice Answer Sheet - Questions 1 – 8

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct ↙

A ⊗ B ⊗ C ○ D ○

---

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○

Section I

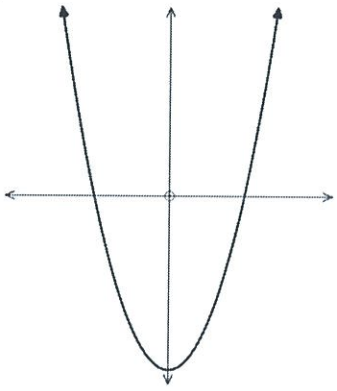
Solutions
Yr 11 Maths Advanced.
Task 3 _ 2019

8 marks
Attempt Questions 1 – 8
Allow about 12 minutes for this section

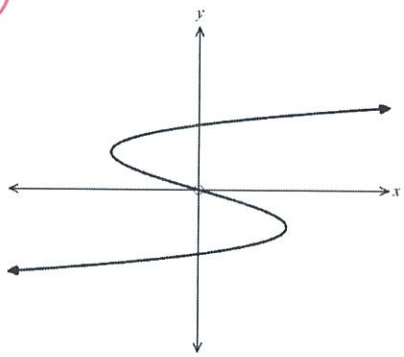
Use the multiple-choice answer sheet provided for Questions 1 to 8.

1. Which graph represents a one to many relation?

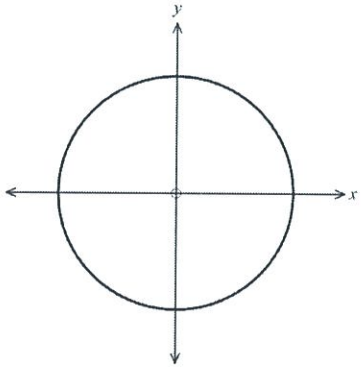
(A)



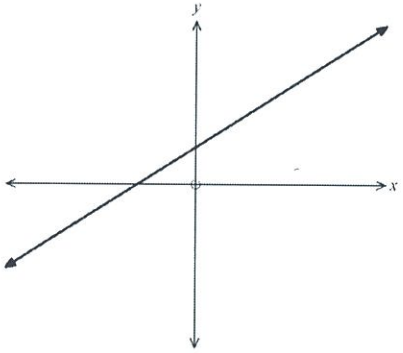
(B)



(C)



(D)



2. Given the equation $2x^2 - x + 5 = 0$, which of the following statements is correct?

(A) The equation has equal roots.

(B) The equation has no real roots.

(C) The equation has distinct real roots.

(D) The equation has rational roots.

$\Delta = b^2 - 4ac$
$= 1 - 4 \times 2 \times 5$
$= 1 - 40$
$= -39$
< 0
$\therefore$ no real roots

3. Simplify $\dfrac{\cos\left(\frac{\pi}{2}-x\right)}{\cos x}$.

(A) -1

(B) $-\tan x$

(C) $\tan x$

(D) 1

$$\frac{\cos\left(\frac{\pi}{2}-x\right)}{\cos x} = \frac{\sin x}{\cos x} = \tan x$$

4. Consider the function $f(x) = |x+2| + 1$.

Which of the following is true at the point $(-2, 1)$ of this function?

(A) The function is continuous and differentiable.

(B) The function is discontinuous but differentiable.

(C) The function is continuous but not differentiable.

(D) The function is discontinuous and not differentiable.

5. Given that $\left(\frac{1}{2}\right)^x \geq \frac{1}{1\,024}$.

Which of the following is correct?

(A) $\log_{10} x \leq 1$

(B) $\log_{10} x \geq 1$

(C) $x \geq 10$

(D) $x \leq 10$

Take $\log_{10}$ to both sides

$\log_{10}\left(\frac{1}{2}\right)^x \geq \log_{10}\frac{1}{1024}$

$x \log\left(\frac{1}{2}\right) \geq \log_{10}\frac{1}{1024}$

(change sign as $\log \frac{1}{2} < 0$)

$x \leq \dfrac{\log_{10}\frac{1}{1024}}{\log_{10}\frac{1}{2}}$

$x \leq 10$

6. A polynomial which has a leading coefficient of 3 has a single root at $x = 4$, triple root at $x = 2$ and double root at $x = -3$. Which is the polynomial function that best describes the information ?

(A) $y = 3(x-4)(x-3)^2(x-2)^3$.

(B) $y = 4(x-3)(x-2)^2(x-3)^3$.

(C) $y = 3(x-4)(x+3)^2(x+2)^3$.

(D) $y = 3(x-4)(x+3)^2(x-2)^3$.

7. If $y = \frac{x^2}{x-1}$ then $\frac{dy}{dx} =$

(A) $\frac{x^2-2x}{(x-1)^2}$

(B) $\frac{x^2-x}{(x-1)^2}$

(C) $\frac{x^2+2x}{(x-1)^2}$

(D) $\frac{x^2+x}{(x-1)^2}$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$
$$= \frac{(x-1).2x - x^2 \times 1}{(x-1)^2}$$
$$= \frac{2x^2 - 2x - x^2}{(x-1)^2}$$
$$= \frac{x^2 - 2x}{(x-1)^2}$$

8. If $f(x) = \sqrt{x}$ and $g(x) = 16 - x^2$ then which of the following is true for $f(g(x))$?

(A) Domain is $[0,4]$ and range is $[0,4]$.

(B) Domain is $[-4,4]$ and range is $(0,4)$.

(C) Domain is $(-4,4)$ and range is $[0,4]$.

(D) Domain is $[-4,4]$ and range is $[0,4]$.

End of Section I

Section II

73 marks
Attempt Questions 9 – 29
Allow about 108 minutes for this section

Answer the questions in the spaces provided.
Your answers should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of this booklet.

Marks

Question 9 (2 marks)

Simplify the following expression with a rational denominator.

$$\frac{1}{\sqrt{3}} - \frac{1}{9}$$

2

Method 1

$\frac{1}{\sqrt{3}} - \frac{1}{9}$

$= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \frac{1}{9}$

$= \frac{\sqrt{3}}{3} \times 3 - \frac{1}{9}$ ①

$= \frac{3\sqrt{3}-1}{9}$ ①

OR

$\frac{9-\sqrt{3}}{9\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ — ①

$= \frac{9\sqrt{3}-3}{27}$

$= \frac{3(3\sqrt{3}-1)}{27}$

$= \frac{3\sqrt{3}-1}{9}$ ①

Question 10 (2 marks)

Show that $\log_3\left(\frac{1}{9}\right) - \log_3(\sqrt{3}) + \log_3(27) = \frac{1}{2}$.

2

LHS $= \log_3\left(\frac{1}{9}\right) - \log_3(\sqrt{3}) + \log_3 27$

$= \log_3 3^{-2} + \log_3 3^{\frac{1}{2}} + \log_3 3^3$

$= -2\log_3 3 + \frac{1}{2}\log_3 3 + 3\log_3 3$

$= -2 + \frac{1}{2} + 3$

$= \frac{1}{2}$

$=$ RHS

$\therefore \log_3\left(\frac{1}{9}\right) - \log_3(\sqrt{3}) + \log_3(27) = \frac{1}{2}$

EXAMINERS COMMENT

Question 9

- generally well done
- some students incorrectly tried to cancel $\frac{3\sqrt{3}-1}{9}$ (lost $\frac{1}{2}$)
- many students multiplied by $\frac{9\sqrt{3}}{9\sqrt{3}}$ to rationalise rather than $\frac{\sqrt{3}}{\sqrt{3}}$

Question 10

- students lost $(\frac{1}{2})$ a mark if didn't present using LHS & RHS notation correctly.
- many students lost marks as they skipped steps - probably because they didn't know what to do

  eg $\log_3\left(\frac{27}{9\sqrt{3}}\right) = \frac{1}{2}$
- some methods were more difficult than others.
- most successful candidates used the method demonstrated in the solutions
- a significant number of students tried to use their calculator and the change of base formula - they did not receive full marks.

Question 11 (2 marks)

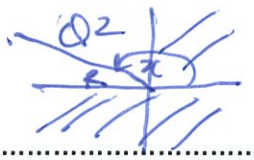
Express $\sin(\pi - \alpha) - \sin(2\pi - \alpha)$ in terms of $\sin\alpha$. 2

$= \sin\alpha - -\sin\alpha$ } ①
$= \sin\alpha + \sin\alpha$
$= 2\sin\alpha.$ } 1

Question 12 (3 marks)

Given that $\sqrt{2}\cos x = -1$, where $\frac{\pi}{2} < x < \pi$:

(a) solve for x.



$\cos x = -\frac{1}{\sqrt{2}}$ where $\frac{\pi}{2} < x < \pi$ $\therefore$ soln in Q2!

2 ↑ 2 marks

Cos x will be negative in S | A / T | C Quadrants 2 and 3, however the domain means solution is in quadrat 2

Cos (Related angle to x) $= \frac{+1}{\sqrt{2}}$ $\therefore$ Related angle $= \frac{\pi}{4}$

$\therefore x = \pi - \frac{\pi}{4}$
$= \frac{4\pi}{4} - \frac{1\pi}{4}$
$= \frac{3\pi}{4}$

(b) find the exact value of $\tan x$. 1

$x = \frac{3\pi}{4}$ $\tan\frac{3\pi}{4} = -\tan\frac{\pi}{4}$
$= -1$

NB the related angle is $\frac{\pi}{4}$

EXAMINERS COMMENT

Question 12

a) Many students did not work in Q2 only and worked in degrees. This was acceptable if students then wrote answer in radians. They did not receive full marks.

(b) generally 1 mark or zero.
Students needed to be aware x lies in Quadrant 2. and in Q2, tan is negative to receive any marks.

Question 28 (4 marks)

On the Richter scale, the magnitude R of an earthquake of intensity I is modelled by the formula $R = \log_{10}\left(\frac{I}{I_0}\right)$, where I_0 is a reference intensity used for comparisons.

(a) Find the magnitude of an earthquake that is 2.1 million times more intense than the reference intensity. Give your answer to 1 decimal place. 2

$R = \log_{10} 2100000$ (1)

$= 6.32221\ldots$

$= 6.3$ (to 1 dp) (1)

(b) In Japan an earthquake was recorded to have of magnitude 7.2 on the Richter scale in 2002. 2

How many times more intense is this than the reference intensity? Give your answer to 3 significant figures.

$7.2 = \log_{10}\frac{I}{I_0}$ (½)

$10^{7.2} = \frac{I}{I_0}$ (½)

$I = 10^{7.2} \times I_0$ $10^{7.2} = 1.584893 \times 10^7$ (½)

The earthquake was 1.58×10^7 (¼) more intense than the reference intensity

Question 29 (5 marks)

Boyles Law is a gas law that states that the pressure P in a container is inversely proportional to the volume V of the container, assuming the gas is at the same temperature at all times. For a particular canister, the volume is 28 cm³ and the pressure is 2.5 kg/cm,

(a) Show that the equation that models the above information is: 2

$$P = \frac{70}{V}.$$

$P \propto \frac{1}{V}$

$P = \frac{k}{V}$ (1)

when $P = 2.5$, $V = 28$

$2.5 = \frac{k}{28}$ (½)

$k = 2.5 \times 28$

$= 70$ $\therefore P = \frac{70}{V}$ (½)

$\therefore P = \frac{70}{V}$

EXAMINERS COMMENT

28 a) This question was done quite well however some students did not take into the account that 2.1 million actually is written as 2100 000, and only used 2.1 in their calculation.
(−1 mark if students wrote $R = \log_{10} 2.1$

b) Some students left their answer as $10^{7.2}$ and did not give their answer to 3 significant figures
For an answer of only $10^{7.2} \Rightarrow$ ½ mark

Q29 a) Many students just substituded into the formula and showed that the two values satisfied it.
That was not what the question asked.
∴ Zero marks were given.

(b) Find the exact volume of the canister if the pressure is 3.75 kg/cm. 1

$3.75 = \frac{70}{V}$

$V = \frac{70}{3.75}$ ½

$= 18\frac{2}{3}\ cm^3$ ½

(c) If the volume in (b) is halved, what effect will this have on the pressure? Verify your answer by finding the pressure. 2

$V = \frac{1}{2} \times 18\frac{2}{3}$

$= 9\frac{1}{3}$

$\therefore P = \frac{70}{9\frac{1}{3}}$

$= 7.5$ 1

∴ When Volume is halved, Pressure is doubled.

(since $7.5 = 2 \times 3.75$) 1

End of Assessment Task

EXAMINERS COMMENT

28 b) This question was well done, however care needs to be taken when asked to find the exact volume. No rounding off is necessary.

$-\frac{1}{2}$ mark if the answer given was $18.67\,cm^3$

c) A bold answer of 'Pressure is doubled' was only given 1 mark. Students needed to substitute half the volume from (b) to find P and show it was doubled.

Question 13 (2 marks)

Simplify $\dfrac{6x^3 \times 5x^{-1}}{2x^6 \times 10x^{-2}}$. 2

$= \dfrac{30x^{3-1}}{20x^{6-2}}$

$= \dfrac{3x^2}{2x^4}$

$= \dfrac{3}{2x^2}$

Question 14 (3 marks)

Differentiate $y = 2x - 3x^2$ from first principles. 3

Let $f(x) = 2x - 3x^2$ $\quad f(x+h) = 2(x+h) - 3(x+h)^2$

$= 2x + 2h - 3(x^2 + 2xh + h^2)$

$= 2x + 2h - 3x^2 - 6xh - 3h^2$

$\dfrac{dy}{dx} = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h \to 0} \dfrac{2x + 2h - 3x^2 - 6xh - 3h^2 - (2x - 3x^2)}{h}$

$= \lim\limits_{h \to 0} \dfrac{2x + 2h - 3x^2 - 6xh - 3h^2 - 2x + 3x^2}{h}$

$= \lim\limits_{h \to 0} \dfrac{2h - 6xh - 3h^2}{h}$

$= \lim\limits_{h \to 0} \dfrac{h(2 - 6x - 3h)}{h}$

$= \lim\limits_{h \to 0} 2 - 6x - 3h$

$= 2 - 6x$

EXAMINERS COMMENT

Q13 One mark was given for getting the constants simplified to $\frac{3}{2}$ and one mark for simplifying the algebra to $\frac{1}{x^2}$

Q14 One mark for setting up the limit

$$\lim_{h\to 0} \frac{2x+2h-3x^2-6xh-3h^2-2x-3x^2}{h}$$

One mark for getting to

$$\lim_{h\to 0} \frac{2h-6xh-3h^2}{h}$$

One mark for using the limit correctly

Question 15 (5 marks)

Differentiate the following expressions with respect to x.

(a) $y = 2x^6 - 3x + \frac{7}{x}$ 1

$y = 2x^6 - 3x + 7x^{-1}$

$\frac{dy}{dx} = 12x^5 - 3 - 7x^{-2}$

$= 12x^5 - 3 - \frac{7}{x^2}$

(b) $y = (2x^3 - 5x)^4$ 2

Let $u = 2x^3 - 5x$ $\quad y = u^4$

$\frac{du}{dx} = 6x^2 - 5$ $\quad \frac{dy}{du} = 4u^3$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= 4u^3(6x^2 - 5)$

$= 4(2x^3 - 5x)^3(6x^2 - 5)$

(c) $f(x) = 3x\sqrt{x^3}$ 2

Let $u = 3x$ $\quad v = x^{\frac{3}{2}}$

$\frac{du}{dx} = 3$ $\quad \frac{dv}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

$f'(x) = x^{\frac{3}{2}} \cdot 3 + 3x \cdot \frac{3}{2}x^{\frac{1}{2}}$

$= 3x^{\frac{3}{2}} + \frac{9}{2}x^{\frac{3}{2}}$

$= \frac{15}{2}x^{\frac{3}{2}}$

OR $f(x) = 3x^{\frac{5}{2}}$

$f'(x) = \frac{15}{2}x^{\frac{3}{2}}$

EXAMINERS COMMENT

Q15 a) One mark for correct answer

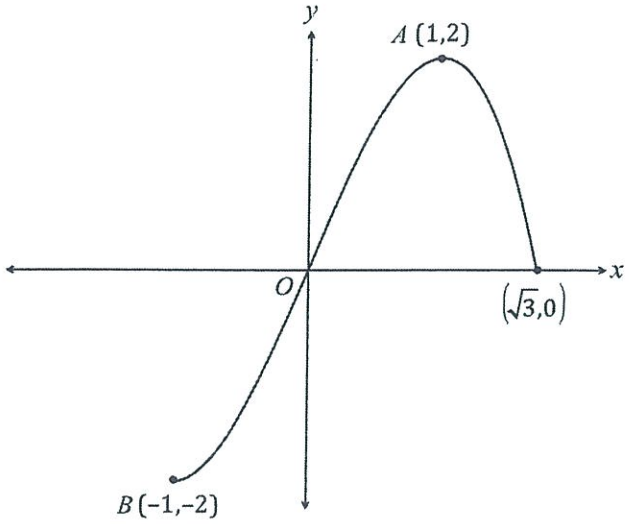
b) Marks deducted for not including the factors correctly

c) One mark for positive progress towards derivative

One mark for correct answer.

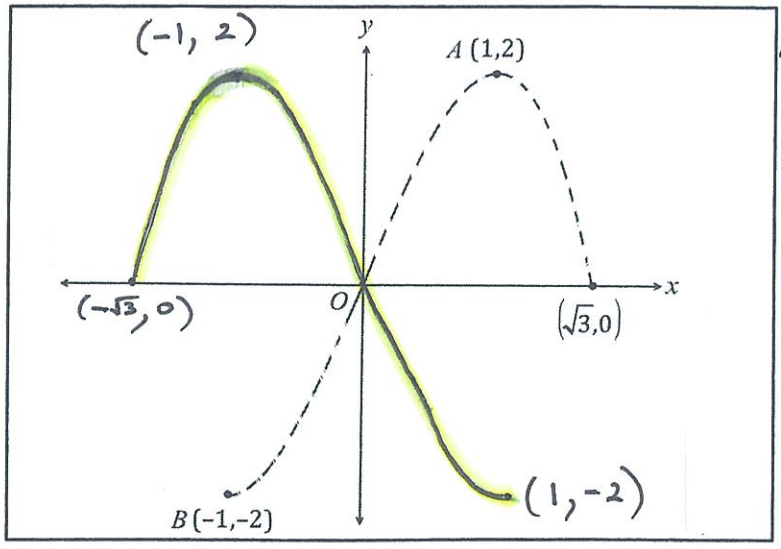
Question 16 (4 marks)

The sketch of $y = f(x)$ is shown below.



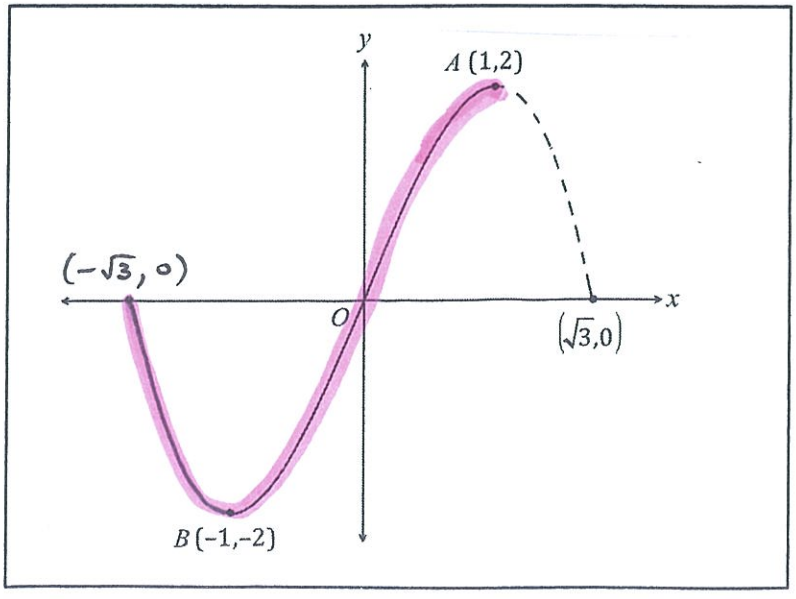
Not to scale

(a) Use the graph of $y = f(x)$ to draw a neat sketch of $y = f(-x)$. 2



• original graph is reflected in the y-axis

(b) Use the graph of $y = f(x)$ to draw a neat sketch of $y = -f(-x)$. 2



• (a) graph reflected in the x-axis.

MATHEMATICS ADVANCED - QUESTION 16

Q16 was mostly well done.

- Be careful with "shape" of graph. The size doesn't change, it's just been reflected in the appropriate axes.
- No "straight" line parts (students lost marks here if graphs didn't look like parabolas and half-parabolas!
- label points and axes.

(a) (2) marks - provides correct graph

(1) mark - correct shape and passing through the origin

(1) mark for all 3 points labelled.

$(-\frac{1}{2})$ for each point not labelled.

$(-\frac{1}{2})$ for distorted shape, straight lines, horizontal point of inflexion at (0,0), etc.

(b) as per part (a).

- Nice smooth graphs are important, not "shaky" and "distorted graphs".

Question 17 (5 marks)

Solve the following equations for x.

(a) $\log_a x - \log_a (x-2) = \frac{2}{3}\log_a 27$ 3

$\log_a \left(\frac{x}{x-2}\right) = \log_a 27^{\frac{2}{3}}$

$\log_a \left(\frac{x}{x-2}\right) = \log_a 9$ — (1)

$\frac{x}{x-2} = 9$

$x = 9(x-2)$

$x = 9x - 18$ — (1)

$8x = 18$

$x = \frac{18}{8}$ — (2½) to this step

$x = \frac{9}{4}$

$x = 2\frac{1}{4}$ — (1)

$x = 2.25$

(b) $3^x = 0.024$ (leave your answer correct to 2 decimal places). 2

$\log_{10} 3^x = \log_{10} 0.024$

$x \log_{10} 3 = \log_{10} 0.024$ — (1)

$x = \frac{\log_{10} 0.024}{\log_{10} 3}$

$= -3.394920562$ — (1½)

$\therefore x = -3.39$ — (1)

OR

$x = \log_3 0.024$

$= \frac{\log_{10} 0.024}{\log_{10} 3}$ — (1)

$= -3.394920562$ — (1½)

$\therefore x = -3.39$ — (1)

MATHEMATICS ADVANCED - QUESTION 17

- Mostly well done.
- learn log laws for part (a)

(a) (3) provides correct answer with working

(2½) - if solution correct down to $x = \frac{18}{8}$ (simplify all answers)

(-1) for some error made, then CFPA.

- show all steps so the marker can reward you accordingly.

(b) - learn how to go from exponential form to log form if you are going to use this method!

(2) provides correct solution

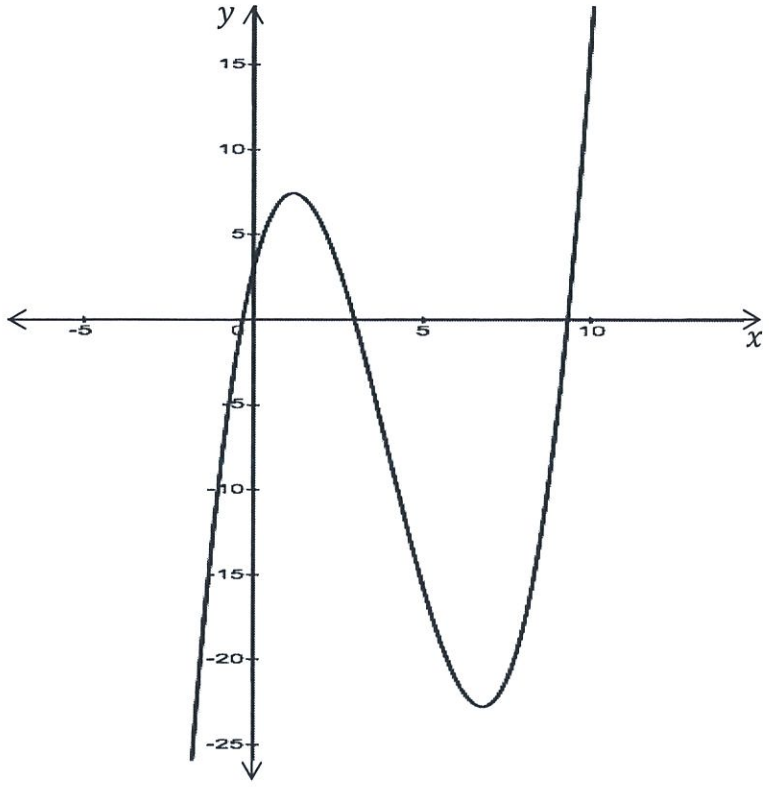
(-½) if rounded incorrectly, but showed whole calculator displayed answer (it pays to do this !!!)

(-1) if rounded incorrectly (ie bald answer from

$$x = \frac{\log_{10} 0.024}{\log_{10} 3}$$

Question 18 (5 marks)

The curve shown below has the equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$.



The point P has the coordinates (3, 0).

(a) Show that P lies on the curve. 1

Substitute $x = 3$ and $y = 0$ for point P into the curve

$y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$

LHS $= y = 0$

RHS $= \frac{1}{3}x^3 - 4x^2 + 8x + 3$

$= \frac{1}{3}(3)^3 - 4(3)^2 + 8(3) + 3$

$= 0 =$ LHS

$\therefore$ P(3, 0) satisfies the equation of the curve

$\therefore$ P lies on the curve

EXAMINERS COMMENT

Q18a)

- Most students gained this mark, but many did not set out completely properly.
- Students needs to learn how to set out a 'show that' question and prove that the LHS equals the RHS.
- Students were not penalised if they didn't do this properly, as it had already been penalised for in Question 10, but they may lose marks next time.
- Students did lose a $\frac{1}{2}$ mark for not explaining the substitution.
- Students who did not conclude that P lies on the curve also lost $\frac{1}{2}$ a mark.

(b) Find the equation of the tangent at P. 2

To find the gradient of the tangent, find $\frac{dy}{dx}$ at $x=3$

$\frac{dy}{dx} = x^2 - 8x + 8$

at $x = 3$

$m_T = (3)^2 - 8(3) + 8 = -7$ — ① mark

Equation of the tangent at $P(3,0)$ is:

$y - 0 = -7(x-3)$

$y = -7x + 21$ — ① mark

or

$7x + y - 21 = 0$

(c) Another point Q also lies on the curve. The tangent to the curve at Q is parallel to the tangent at P. Find the coordinates of Q. 2

Gradient of the tangent at Q = gradient of tangent at P

$\therefore$ m of tangent at Q $= -7$

To find x-value of Q, let $\frac{dy}{dx} = -7$

$x^2 - 8x + 8 = -7$ — $(\frac{1}{2})$ mark

$x^2 - 8x + 15 = 0$

$(x-5)(x-3) = 0$

$\therefore x = 5$ or $x = 3$

But $x = 3$ relates to the point $P(3,0)$

$\therefore$ Q is at $x = 5$ — $(\frac{1}{2})$ mark

When $x = 5$, $y = \frac{1}{3}(5)^3 - 4(5)^2 + 8(5) + 3$ ← $(\frac{1}{2})$ mark

$= -\frac{46}{3}$

$\therefore$ Q is $(5, -\frac{46}{3})$ OR $(5, -15\frac{1}{3})$ ← $(\frac{1}{2})$ mark

EXAMINERS COMMENT

Q18 part b)

- Students lost $\frac{1}{2}$ a mark if they made an error multiplying -7 and -3, resulting in $y = -7x - 21$.
- If students got the wrong number for the gradient, they were allowed to use this error in the equation without further penalty.
- If students use $m = x^2 - 8x + 8$ without substituting $x = 3$, they usually got a cubic for the tangent. They were awarded zero, as they did not understand the concept of the tangent needing to be a line.

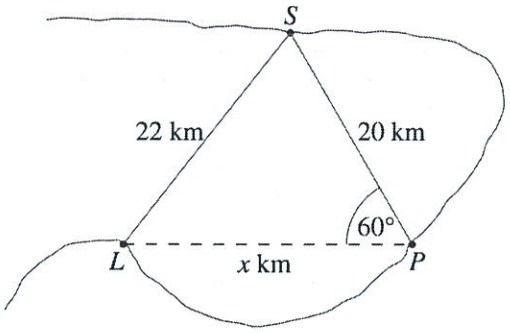
Q18 part c)

- If students could get $x^2 - 8x + 8 = -7$, they usually could get to the answer.
- Students should make sure they explain when $x = 5$ and not $x = 3$.
- Students lost $\frac{1}{2}$ a mark if they did not write Q as a pair of coordinates.

Question 19 (4 marks)

In the diagram below, the surf shop at S is 20 km across the bay from post office at P. The distance from the surf shop to the lighthouse at L is 22 km and $\angle SPL$ is 60°.

Let the distance PL be x kilometres.



(a) Using the cosine rule, show that $x^2 - 20x - 84 = 0$. 2

$22^2 = x^2 + 20^2 - 2(x)(20)\cos 60^\circ$

$484 = x^2 + 400 - 40x \times \frac{1}{2}$

$84 = x^2 - 20x$

$\therefore \quad x^2 - 20x - 84 = 0$

(b) Hence, find the distance from the post office to the lighthouse. 2
Give your answer to 1 decimal place.

$x = \dfrac{20 \pm \sqrt{20^2 - 4 \times 1 \times -84}}{2}$

$x = \dfrac{20 \pm \sqrt{400 + 336}}{2}$

$x = \dfrac{20 \pm 4\sqrt{46}}{2} = 10 \pm 2\sqrt{46}$

But x is a distance and so $x > 0$

$\therefore \quad x = 10 + 2\sqrt{46} \doteq 23.6$ km

EXAMINERS COMMENT

19 a)

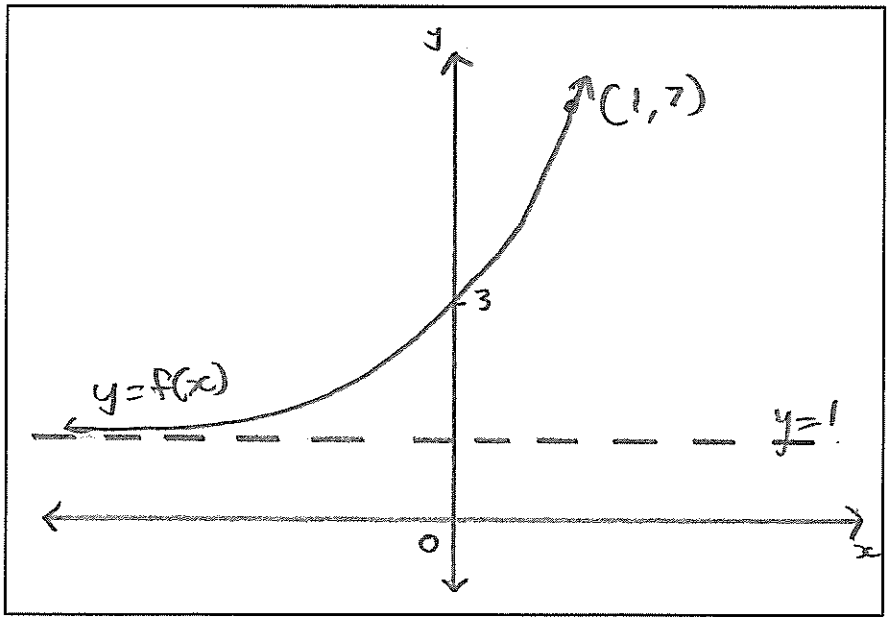
- Students needed to provide a proof that showed exactly how they got from the cosine rule to the required equation.
- Students lost $\frac{1}{2}$ a mark if they did not show $\cos 60°$ in the first line and went straight to $\frac{1}{2}$.
- Students lost $\frac{1}{2}$ a mark if they wrote $\cos P$ instead of $\cos 60°$.
- Students lost 1 mark if they skipped the first line of working entirely.

19 b)

- students who gave both a positive and negative answer got 1 out of 2.
- Students lost $\frac{1}{2}$ a mark if there answer was left as $10+2\sqrt{46}$ or the decimal was rounded incorrectly.
- Students should have mentioned eliminating the negative answer - but no marks were lost.
- If students got the right answer but did not use $x^2-20x-84=0$ to find it, they lost $\frac{1}{2}$ a mark.
- If students showed the right answer but no working, they achieved only 1 mark out of 2.

Question 20 (2 marks)

Sketch the function of $f(x) = 2(3^x) + 1$. 2

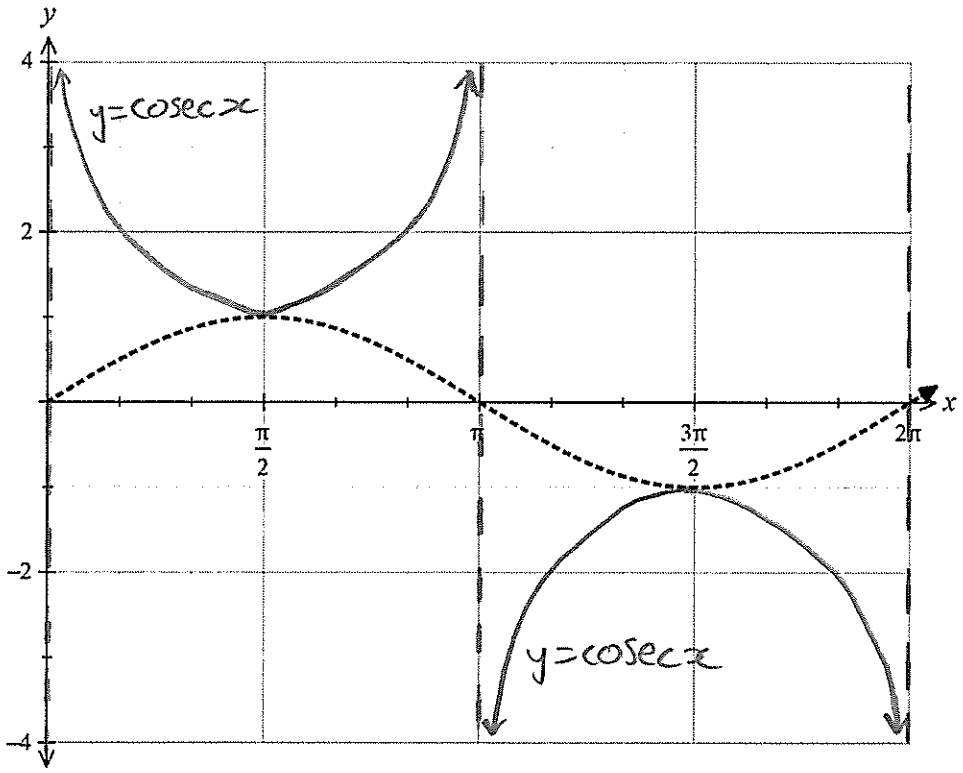


Question 21 (2 marks)

2

The graph of $y = \sin x$ for $0 \leq x \leq 2\pi$ is drawn on the axes below.

Draw and label a neat sketch of $y = \operatorname{cosec} x$ for $0 \leq x \leq 2\pi$.



EXAMINERS COMMENT

2 marks: correct answer with all details
1½ marks: correct answer with most details
1 mark: correct general idea (eg positive exponential function)

Required features: intercept, asymptote, one more point.

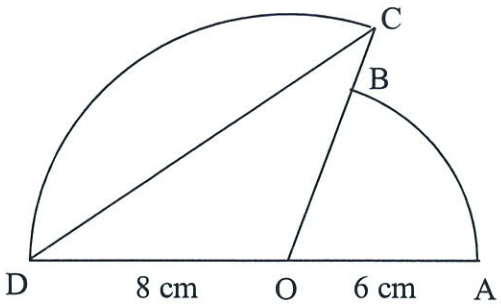
2 marks: correct answer with all details
1½ marks: correct answer with most details
1 mark: correct general idea (eg reciprocal graph)

Common mistakes were
- not meeting the original graph at $y = \pm 1$
- not approaching the vertical asymptotes.

Question 22 (5 marks)

The diagram shows two sectors OAB and OCD with the same centre O, such that OBC and AOD are two straight lines. The radius OA of the smaller sector OAB is 6 cm, while the radius OD of the larger sector is 8 cm.

The area of triangle OCD is $16\sqrt{3}$ cm^2.

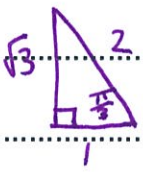


NOT TO SCALE

(a) Find the size of the obtuse angle COD in radians. 2

$A = \frac{1}{2}ab\sin C$

$\therefore 16\sqrt{3} = \frac{1}{2} \times 8 \times 8 \times \sin\theta$



$\sin\theta = \frac{\sqrt{3}}{2}$

$\therefore$ related angle is $\frac{\pi}{3}$

But $\angle COD$ is obtuse

$\therefore \angle COD = \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

(b) Find the exact length of the arc CD. 1

$l = r\theta$

$= 8 \times \frac{2\pi}{3}$

$= \frac{16\pi}{3}$ cm

(c) Find the area of sector OAB. 2

$A = \frac{1}{2}r^2\theta$

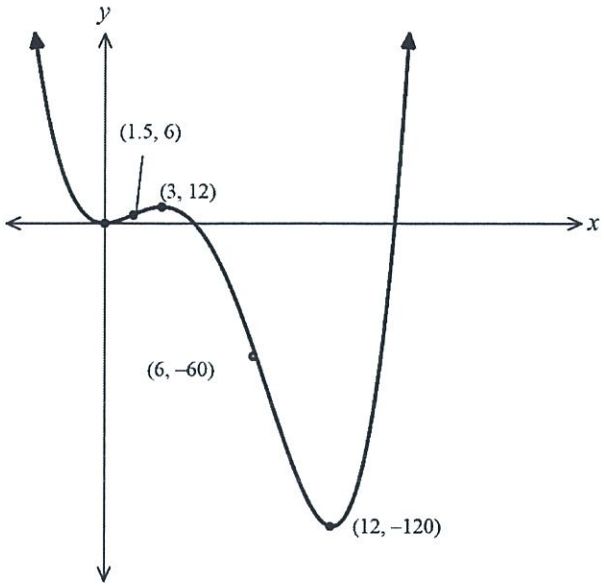
$= \frac{1}{2} \times 6^2 \times \left(\pi - \frac{2\pi}{3}\right)$

$= 6\pi$ cm^2

EXAMINERS COMMENT
a) 2 mark: correct solution
$1\frac{1}{2}$ marks: correct solution, but in degrees or acute angle correctly found
1 mark: correct substitution into correct formula
b) 1 mark: correct answer
$\frac{1}{2}$ mark: correct substitution into correct formula
c) 2 marks: correct solution
1 mark: correct substitution into correct formula or correctly finding the area of the other sector

Question 23 (3 marks)

The function $y = f(x)$ below has turning points at the origin and at (3, 12) and (12, –120).

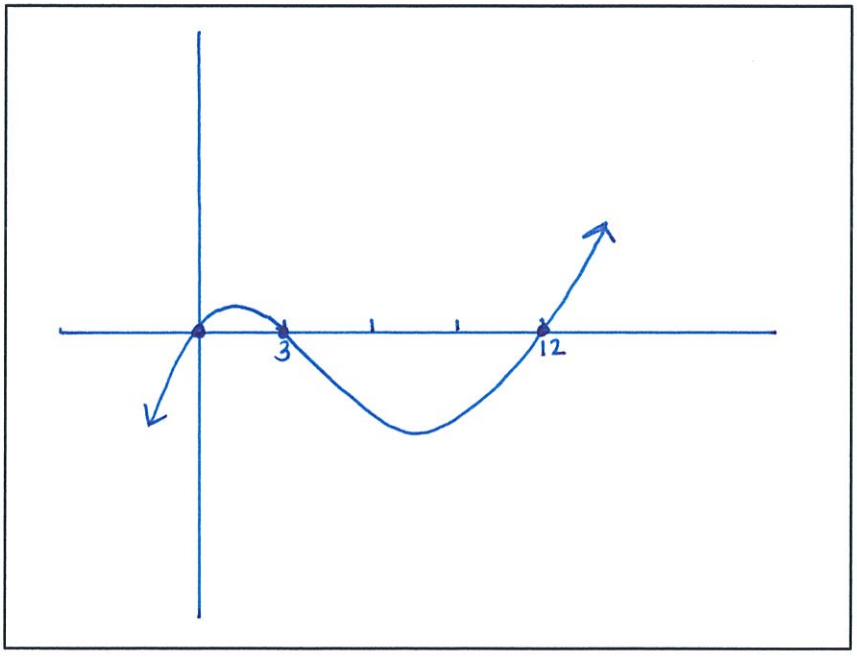


(a) For what x – values is the function increasing? 1

(-½) $0 < x < 3$ and $x > 12$ (-½)

Also if inequality sign had "equal" to then (½)/1

(b) Sketch the gradient function of $y = f(x)$. 2

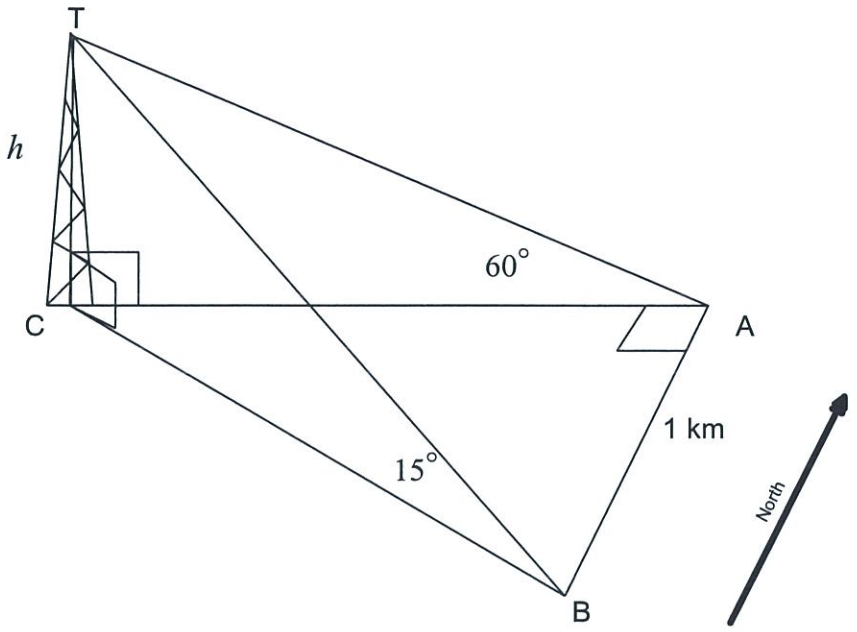


(1) shape

(1) for intercepts at 0, 3, 12

Question 24 (3 marks)

The angle of elevation of the top of a tower (T) from a point A due East of the tower is 60°. From a point *B* due South of *A*, the angle of elevation of T is 15°. *A* and *B* are at the same elevation as the base of the tower. If the distance *AB* = 1 km, find the height (*h*) of the tower. 3



$\tan 60° = \frac{h}{AC}$ $\tan 15° = \frac{h}{BC}$ — (1)

$AC = \frac{h}{\tan 60°} = \frac{h}{\sqrt{3}}$ $BC = \frac{h}{\tan 15°} = \frac{h}{2-\sqrt{3}}$

or $= h\cot 60°$ or $= h\cot 15°$

or $AC = h\tan 30°$ or $BC = h\tan 75$

$BC^2 = AC^2 + AB^2$

$\left(\frac{h}{\tan 15°}\right)^2 = \left(\frac{h}{\tan 60°}\right)^2 + (1)^2$ — (1)

Students should not use cosine rule - its a right angled Δ !

$\frac{h^2}{\tan^2 15°} - \frac{h^2}{\tan^2 60°} = 1$

$h^2\left(\frac{1}{\tan^2 15°} - \frac{1}{\tan^2 60°}\right) = 1$

$h^2 = 1 \div \left(\frac{1}{\tan^2 15°} - \frac{1}{\tan^2 60°}\right)$

$h > 0$ $h = \sqrt{1 \div \left(\frac{1}{\tan^2 15°} - \frac{1}{\tan^2 60°}\right)}$ (1)

$h = 0.2712$ km (to 4 d.p)

or 271.2142299 m.

Question 25 (3 marks)

Show that if $y = x\sqrt{2x-1}$ then $\frac{dy}{dx} = \frac{3x-1}{\sqrt{2x-1}}$. 3

$y = x\sqrt{2x-1}$

$y = uv$ where $u = x$ $v = (2x-1)^{1/2} = \sqrt{2x-1}$

$\frac{du}{dx} = 1$ $\frac{dv}{dx} = \frac{1}{2}(2x-1)^{-1/2} \times 2$

$= \frac{1}{\sqrt{2x-1}}$

Must be shown (-1/2) if not shown

$\frac{dy}{dx} = vu' + uv'$

$= \sqrt{2x-1} + \frac{x}{\sqrt{2x-1}}$ – (1) apply product rule correctly → BUT

$= \frac{(\sqrt{2x-1})(\sqrt{2x-1})}{\sqrt{2x-1}} + \frac{x}{\sqrt{2x-1}}$ (1) Common denominator

$= \frac{2x-1+x}{\sqrt{2x-1}}$ (1)

$= \frac{3x-1}{\sqrt{2x-1}}$

MATHEMATICS – QUESTION 26

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$x = 2t^3 + \sqrt{t}$		Some students need to re-visit differentiation
Velocity $\dot{x} = 6t^2 + \frac{1}{2}t^{-1/2}$	1	1mk for correct differentiation
When $t = 9$		
$\dot{x} = 6(9)^2 + \frac{1}{2\sqrt{9}}$		
$= 486 + \frac{1}{6}$		1mk for appropriate substitution and evaluation.
$= 486\frac{1}{6}$ cm/s	1	
or $\frac{2917}{6}$ cm/s or		
or 486.17 cm/s or 486.16̇ cm/s		
It was really disappointing to see that about 20% of the students did not know that x represents displacement, $\dot{x}$ represents velocity and $\ddot{x}$ represents acceleration.		
Acceleration		
$\ddot{x} = 12t - \frac{1}{4}t^{-3/2}$	1	1mk for correct differentiation
When $t = 9$		
$\ddot{x} = 12(9) - \frac{1}{4}(9)^{-3/2}$		
$= \frac{11663}{108}$ cm/s^2	1	1mk for appropriate substitution and evaluation.
or 107.990740...%		
$\simeq 107.99$ cm/s^2		
Students could have simply substituted the value of t into $\ddot{x} = 12t - \frac{1}{4}t^{-3/2}$ rather than changing $t^{-3/2}$ to a surdic form and then making mistakes.		

MATHEMATICS – QUESTION 27

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $h(0) = 2(0)^2 - 4(0) + 50$	1	1 mark for
$= 50$ cm		appropriate
Generally well attempted.		substitution.
b) $\frac{dh}{dt} = 4t - 4$	1	1 mk for
		correct
when $t = 4$		differentiation
$\frac{dh}{dt} = 4(4) - 4$		
$= 16 - 4$		
$= 12$ cm/min	1	1 mk for
		correct
		substitution
		and evaluation
c) When $t = 4$, $h = 32 - 16 + 50$		
$= 66$		
Average rate $= \frac{h(4) - h(0)}{t_4 - t_1}$	1	
$= \frac{66 - 50}{4 - 0}$		
$= \frac{16}{4}$		
$= 4$ cm/min	1	
A number of students swapped the answers for (b) and (c) which simply shows their lack of understanding of the difference between instantaneous rate and average rate. No marks were awarded in this case.		