Name: ________________________________

Teacher: ______________________________

St George Girls High School

Mathematics Advanced

2020 Year 11 End of Course Examination

General Instructions

- Reading Time: **10 minutes**
- Working time: **2 hours**.
- Write using black pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- For questions in **Section I,** use the multiple-choice answer sheet provided.
- For questions in **Section II:**
  - Answer the questions in the space provided.
  - Show relevant mathematical reasoning and/or calculations.
  - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
  - Marks may not be awarded for incomplete or poorly presented solutions

Total marks: 81

Section I – 10 marks (pages 3 – 7)

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II – 71 marks (pages 8 – 24)

- Attempt all questions
- Allow about 1 hour and 45 minutes for this section

Q1 – 10	/10
Q11 – Q14	/8
Q15 – Q17	/9
Q18 – Q20	/10
Q21 – Q23	/10
Q24 – Q26	/9
Q27 – Q28	/8
Q29 – Q30	/8
Q31 – Q32	/9
Total	**/81**
	%

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Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet provided for Questions 1 to 10.

1. What is the solution to the equation $3x^2 - x - 5 = 0$?

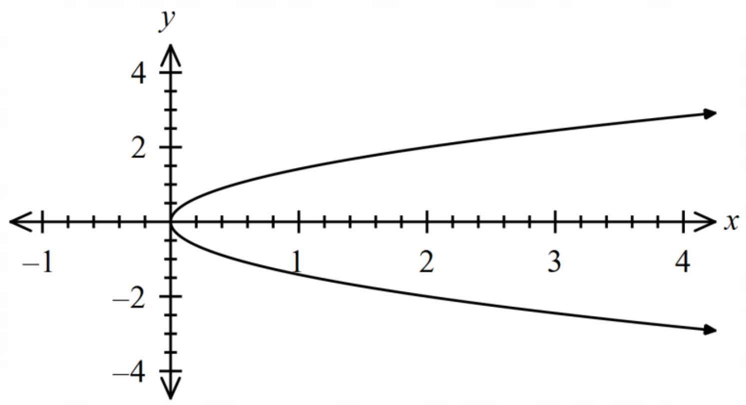
(A) $x = \frac{1 \pm \sqrt{61}}{6}$

(B) $x = \frac{1 \pm \sqrt{37}}{6}$

(C) $x = \frac{-1 \pm \sqrt{37}}{6}$

(D) $x = \frac{-1 \pm \sqrt{61}}{6}$

2. Which type of relation is shown?



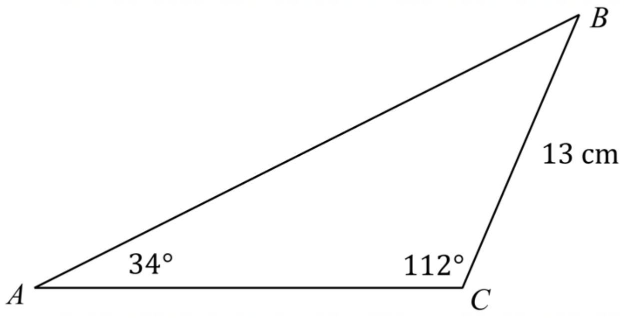
(A) one to one

(B) one to many

(C) many to one

(D) many to many

3.



Not to scale

What is the length of AB, correct to one decimal place?

(A) 13.0 cm

(B) 20.9 cm

(C) 21.6 cm

(D) 40.6 cm

4. The solution to $3^{x+1} = 15$, correct to two decimal places is:

(A) 1.21

(B) 1.46

(C) 1.36

(D) 1.84

5. The solutions to the equation $\left|\frac{x}{3} + 2\right| - 4 = 0$ are:

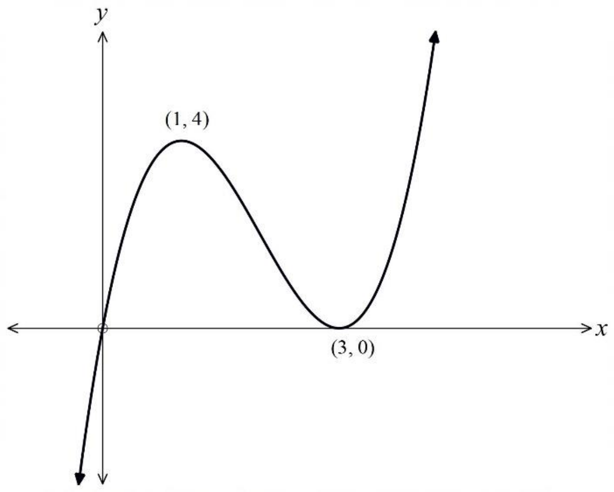
(A) $x = 6$ or $x = 18$

(B) $x = -6$ or $x = 18$

(C) $x = -6$ or $x = -18$

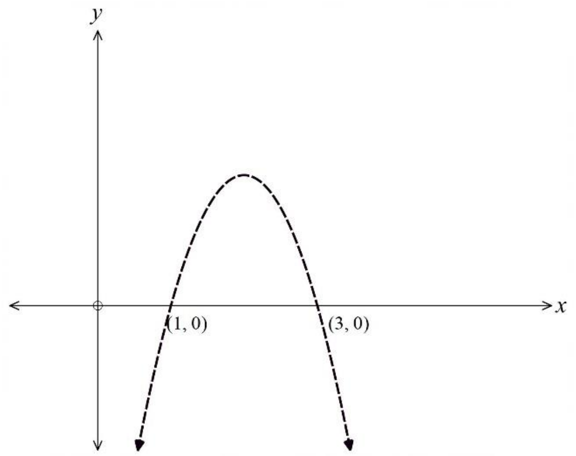
(D) $x = 6$ or $x = -18$

6. The graph of $y = f(x)$ is shown below.

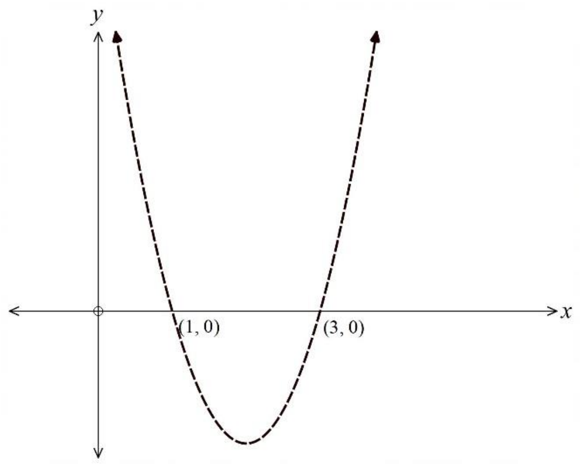


Which graph could represent $y = f'(x)$?

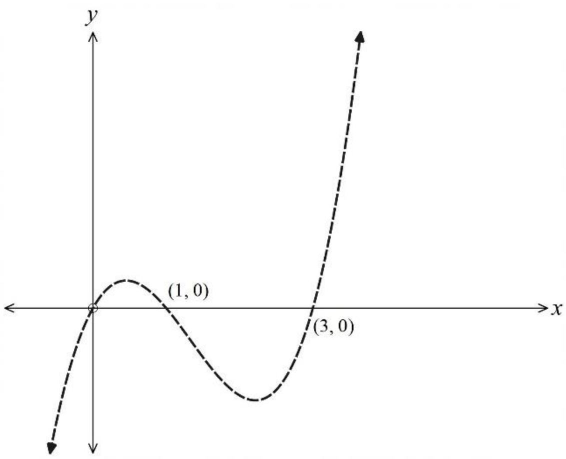
(A)



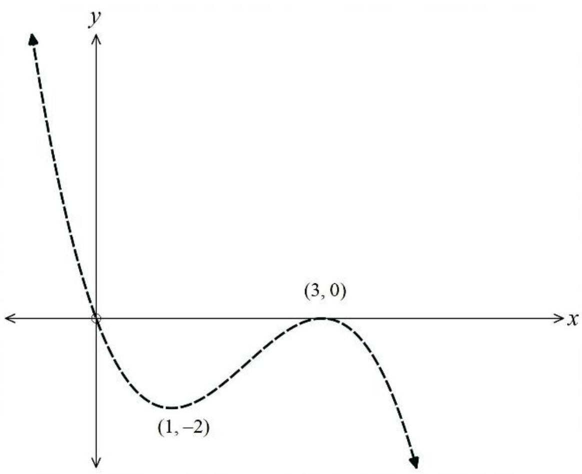
(B)



(C)



(D)



7. A sector of radius 12 cm subtends an angle of $60°$ at the centre. What is the value of the perimeter P of the sector?

(A) $(4\pi + 24)$ cm

(B) 744 cm

(C) 4π cm

(D) 720 cm

8. Find $f'(-2)$ if $f(x) = \sqrt{(x^3 + 12)^3}$.

(A) 25

(B) -72

(C) -18

(D) 36

9. Given $\log_b 5 = a$ and $\log_b 2 = c$, the value of $\log_b 100$ is:

(A) $2a + c$

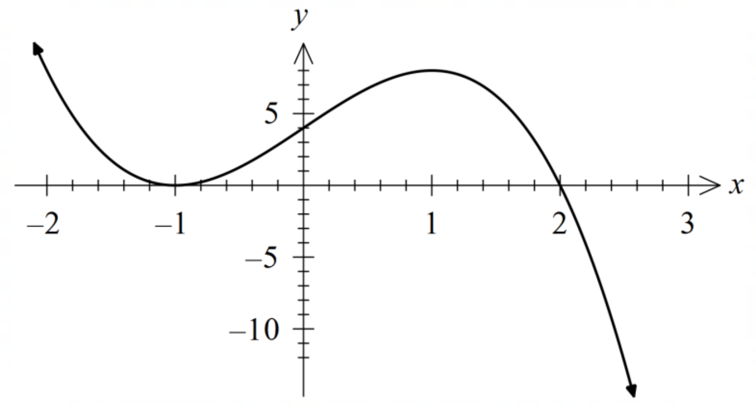
(B) $2a + 2c$

(C) $2a - 2c$

(D) $2ac$

10. The diagram below shows a curve passing through (-1,0), (2,0) and (0, 4).

What is the equation of the curve?



(A) $y = (x + 1)^2(2 - x)$

(B) $y = 2(x + 1)^2(x - 2)$

(C) $y = -2(x + 1)^2(x - 2)$

(D) $y = -4(x + 1)^2(x - 2)$

End of Section I

Section II

71 marks
Attempt Questions 11 – 32
Allow about 105 minutes for this section

Answer the questions in the spaces provided.
Your answers should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of this booklet.

Marks

Question 11 (2 marks)

Simplify $\frac{3x^2-x-2}{x^2-1}$. 2

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Question 12 (2 marks)

What angle does the line $-x + 4y + 12 = 0$ make with the positive $x-$axis? 2
Round to the nearest minute.

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Question 13 (2 marks)

Express $\left(2\sqrt{3}+1\right)\left(2-\sqrt{3}\right)$ in the form $a+b\sqrt{3}$, where a and b are integers. 2

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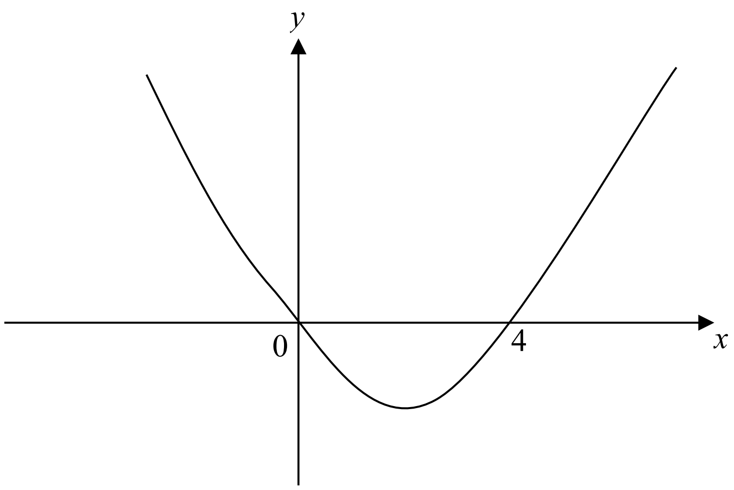
Question 14 (2 marks)

The sketch below shows the function $y = f(x)$. On the same number plane draw $y = f(-x)$. 2



Question 15 (5 marks)

Consider the functions $f(x) = x^2$ and $g(x) = 2x + 1$.

(a) Find the composite function $f\left(g(x)\right)$. 1

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(b) What is the domain of the composite function? 1

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(c) What is the range of the composite function? 1

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(d) For what values of x is $f\left(g(x)\right) = g(x)$? 2

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Question 16 (2 marks)

Given that $\cot\beta = -\frac{5}{9}$, and that $\sin\beta < 0$ find the value of $\cos\beta$. 2

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Question 17 (2 marks)

Find the radius of the circle $x^2 - 4x + y^2 - 2y - 2 = 0$ in exact form. 2

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Question 18 (5 marks)

Find $\frac{dy}{dx}$ if:

(a) $y = 10x^3 - x + 7$ 1

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(b) $y = \frac{1}{(2-3x)^2}$ 2

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(c) $y = 3x^4\sqrt{x-1}$ 2

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Question 19 (3 marks)

Prove $\dfrac{\tan\alpha}{\sec\alpha - 1} - \dfrac{\tan\alpha}{\sec\alpha + 1} = 2\cot\alpha$. 3

Question 20 (2 marks)

Graph $y = \tan x$ in the domain $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$, showing all intercepts and asymptotes. 2

Question 21 (5 marks)

Solve

(a) $\log_3 x^3 = 6$ 2

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(b) $\log_2(x+2) - \log_2(x-5) = 3.$ 3

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Question 22 (3 marks)

Solve $(\sin x + 2)(2\sin x + 1) = 0$ in the domain $[0, 2\pi]$. 3

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Question 23 (2 marks)

Find the values of k for which the equation $(k+2)x^2 + 2x + 4 = 0$ has no real roots. 2

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Question 24 (4 marks)

A function is defined by $f(x) = \frac{2x-1}{3x+2}$.

(a) Show that $f'(x) = \frac{7}{(3x+2)^2}$. 2

(b) Hence, find the equation of the normal at the point where $x = -1$. 2
(Leave your answer in general form)

Question 25 (2 marks)

Find the exact value of cot 330°. 2

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Question 26 (3 marks)

Find the coordinates of the point on the curve $y = x^2 - 2x - 8$ at which the tangent is perpendicular to the line $x - 2y = 6$. 3

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Question 27 (5 marks)

A particle moves from rest at a point A in a straight line, so that t seconds after leaving A, its displacement, x metres, is given by $x = 8t^2 - t^4$.

Find:

(a) the time(s) for the particle to come to rest and the displacement at these time(s). 3

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(b) the distance travelled in the first 2 seconds. 2

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Question 28 (3 marks)

Using the method of first principles, find $f'(x)$ if $f(x) = 2x^2 + 3x + 6$. 3

Question 29 (4 marks)

A small tank of liquid which contains L litres of a toxic chemical is being drained.
The amount of chemical in the tank over time t minutes, is given by:

$$L = 192 - 96t + 12t^2$$

(a) At what rate is the chemical draining out of the tank after 6 minutes? 2

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(b) How long will it take for the tank to be completely empty? 2

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Question 30 (4 marks)

The intensity I Watts/m^2 of a sound of loudness L decibels is given by the formula

$$I = I_o(10)^{\frac{1}{10}L}$$

where $I_o = 10^{-12}$ Watts/m^2 is the intensity of the Threshold of Hearing.

(a) Show that $L = 10(\log_{10} I - \log_{10} I_o)$. 2

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(b) Hence, find the loudness, in decibels, of noise at a Justin Bieber concert whose intensity is 10^{12} times greater that the Threshold of Hearing. 2

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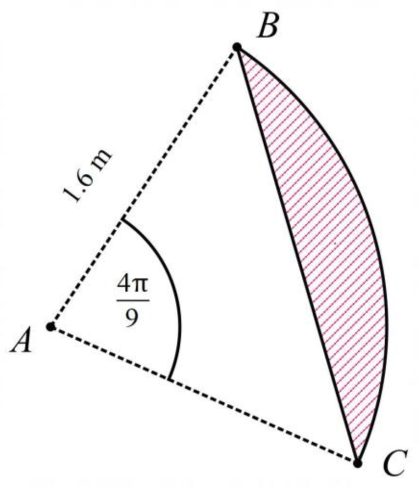
Question 31 (3 marks)

The shaded segment shown is formed by an arc BC of a circle radius 1.6 m, which subtends an angle of $\frac{4\pi}{9}$ radians at its centre A. 3



Find the area of the shaded segment, correct to 3 significant figures.

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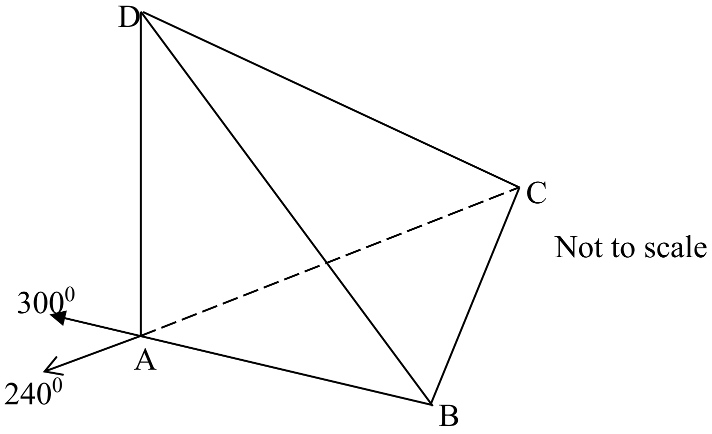
Question 32 (6 marks)

The diagram below shows Jackie standing at B on level ground, whilst Fiona is standing 2000 m away from Jackie at C on the same level ground. They both take the bearing and elevation of an aeroplane D at the same instant. Jackie finds the bearing is 300^0T and the angle of elevation 25^0, whilst Fiona finds the bearing to be 240^0T and the angle of elevation to be 22^0.

(a) Complete the following diagram, showing all the information given. 1



(b) Show that if the height DA of the plane is h metres, then

$AB = h \cot 25^0$ and $AC = h \cot 22^0$ 2

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(c) Show that if the height DA of the plane is h metres, then 3

$$h = \frac{2000}{(\cot^2 25^0 + \cot^2 22^0 - \cot 25^0 \cot 22^0)^{\frac{1}{2}}}$$

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End of Assessment Task

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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Name: ____________________________

Teacher:____________________________

Section I

Year 11

Mathematics Advanced

End of Course Examination

2020

Multiple-choice Answer Sheet - Questions 1 – 10

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ● C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ● B ● C ○ D ○

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

EXAMINER'S COMMENTS

Year 11 Advanced - Task 2 - Multiple Choice (2020)

1. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 3 \times -5}}{2 \times 3}$

$= \frac{1 \pm \sqrt{1 + 60}}{6}$

$= \frac{1 \pm \sqrt{61}}{6}$ (A)

2. (B)

3 $\frac{c}{\sin 112} = \frac{13}{\sin 34}$

$c = \frac{13 \times \sin 112}{\sin 34}$

$= 21.55497689...$

$\doteqdot 21.6\text{m}$ (1 d.p.) (C)

EXAMINER'S COMMENTS

4. $3^{x+1} = 15$

$\ln 3^{x+1} = \ln 15$

$(x+1)\ln 3 = \ln 15$

$x+1 = \dfrac{\ln 15}{\ln 3}$

$x = \dfrac{\ln 15}{\ln 3} - 1$

$= 1.464973521\ldots$

$\doteqdot 1.46$ (B)

5. $\left|\frac{x}{3}+2\right| - 4 = 0$

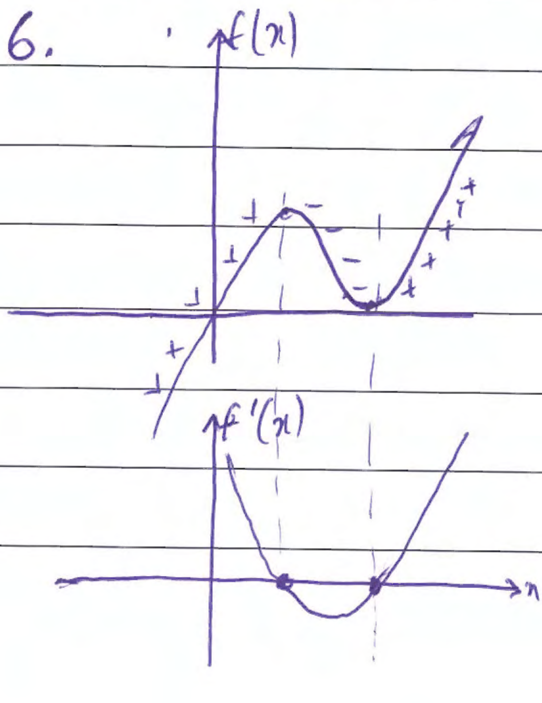
$\left|\frac{x}{3}+2\right| = 4$

$\frac{x}{3}+2 = 4$ or $\frac{x}{3}+2 = -4$

$\frac{x}{3} = 2$ $\frac{x}{3} = -6$

$x = 6$ or $x = -18$ (D)

6.



$\therefore$ (B)

EXAMINER'S COMMENTS

7. Perimeter $= 12\text{ cm} + 12\text{ cm} + r\theta$

$= 24 + \left(12 \times \frac{\pi}{3}\right)$ $\theta = \frac{\pi}{3}$

$= (24 + 4\pi)\text{ cm}$

(A)

8. $f(x) = \sqrt{(x^3+12)^3}$

$= (x^3+12)^{3/2}$

$f'(x) = \frac{3}{2}(x^3+12)^{1/2} \times 3x^2$

$= \frac{9x^2\sqrt{x^3+12}}{2}$

$f'(-2) = \frac{9(-2)^2\sqrt{(-2)^3+12}}{2}$

$= \frac{36\sqrt{-8+12}}{2}$

$= 36$ (D)

9. $\log_b 100 = \log_b(5^2 \times 4)$

$= \log_b 5^2 + \log_b 2^2$

$= 2\log_b 5 + 2\log_b 2$

$= 2a + 2c$ (B)

10. (C)

Section II

71 marks
Attempt Questions 11 – 32
Allow about 105 minutes for this section

Answer the questions in the spaces provided.
Your answers should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of this booklet.

Marks

Question 11 (2 marks)

Simplify $\frac{3x^2-x-2}{x^2-1}$. 2

$3x$ 2
x -1

$= \frac{(3x+2)(x-1)}{(x-1)(x+1)}$ — $\frac{1}{2}$ factorise numerator
— $\frac{1}{2}$ factorise denominator

$= \frac{3x+2}{x+1}$ — 1 Correct answer
OR, carried over error that incorporated cancelling.

Question 12 (2 marks)

What angle does the line $-x+4y+12=0$ make with the positive $x-$axis? 2
Round to the nearest minute.

$-x+4y+12=0$
$4y = x-12$
$y = \frac{1}{4}x - 3$
$m_T = \frac{1}{4}$ — 1
(positive gradient ∴ angle of inclination is acute)

$m = \tan\theta$
$\frac{1}{4} = \tan\theta$
$\theta = 14°2'$ — 1
(If the obtuse angle was found ½ mark deducted)

EXAMINER'S COMMENTS

Q11. Some students need to improve their factorisation of trinomials.

Otherwise, completed well.

Q12. Some students did too much work to find the gradient of the line. They sketched it, found intercepts and then found $\frac{\text{rise}}{\text{run}}$. Much easier to put it in the form $y=mx+b$ and then simply state $m=\frac{1}{4}$.

Question 13 (2 marks)

Express $(2\sqrt{3}+1)(2-\sqrt{3})$ in the form $a+b\sqrt{3}$, where a and b are integers. 2

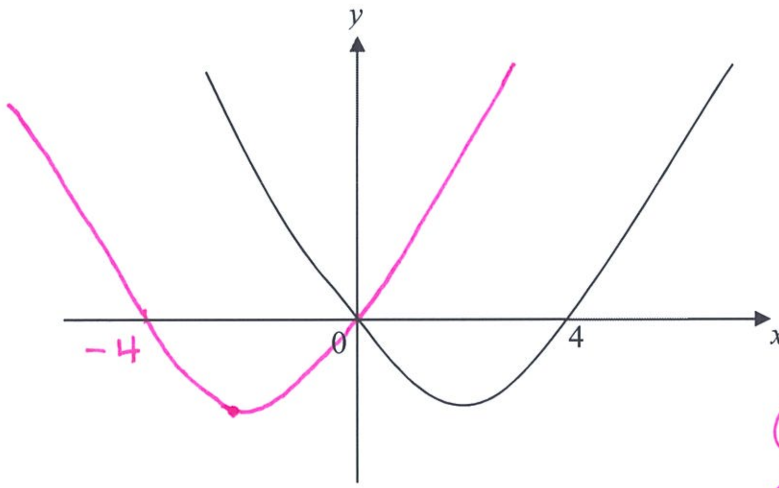
$= 4\sqrt{3} - 2(3) + 2 - \sqrt{3}$ 1

$= 3\sqrt{3} - 4$

$= -4 + 3\sqrt{3}$ 1 Either order

Question 14 (2 marks)

The sketch below shows the function $y = f(x)$. On the same number plane draw $y = f(-x)$. 2



1 - -4

some ½ marks if the minus was missing.

1 - reflection
- shape

some half marks here for incorrect shape and not fully reflecting.

EXAMINER'S COMMENTS

Q13. Please read the question carefully. The question asked for the answer to be expressed in the form $a + b\sqrt{3}$ NOT to state a and b as the final answer.

Also, take care to put the constant first and the surd second.

Q14. In future, be more carefull to reflect the curve and to not "assume" it is a parabola. No arrows required, as the original did not have arrows.

Question 15 (5 marks)

Consider the functions $f(x) = x^2$ and $g(x) = 2x + 1$.

(a) Find the composite function $f(g(x))$. 1

$f(g(x)) = (2x+1)^2$

$= 4x^2 + 4x + 1$

(b) What is the domain of the composite function? 1

domain $(-\infty, \infty)$ or $\{x : x \in \mathbb{R}\}$ or all real x

(c) What is the range of the composite function? 1

range $[0, \infty)$ or $\{y : y \in \mathbb{R}, y \geq 0\}$

or all real $y \geq 0$

(d) For what values of x is $f(g(x)) = g(x)$? 2

$(2x+1)^2 = 2x+1$

$4x^2 + 4x + 1 = 2x + 1$

$4x^2 + 2x = 0$

$2x(2x+1) = 0$

Either $2x = 0$ or $2x + 1 = 0$

$x = 0$ $\quad$ $x = -\frac{1}{2}$

Question 16 (2 marks)

Given that $\cot\beta = -\frac{5}{9}$, and that $\sin\beta < 0$ find the value of $\cos\beta$. 2

$\tan\beta = -\frac{9}{5}$

tan is negative in

2nd and 4th quadrants

$\sin\beta < 0$

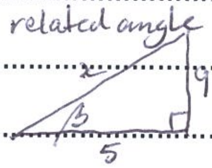
sin is negative in

3rd and 4th quadrants

$\therefore$ 4th quadrant

In 4th quadrant cos is positive

related angle



$x^2 = 9^2 + 5^2$

$= 81 + 25$

$= 106$

$x = \sqrt{106}$

$\cos\beta = \frac{5}{\sqrt{106}}$

MARKER'S COMMENTS - QUESTIONS 15, 16

Q15 a) 1 mark was given for $(2x+1)^2$.

$\frac{1}{2}$ mark was deducted if the expression was expanded incorrectly.

b) No half marks

c) No half marks except if the inequality used was $>$ instead of $\geq$

d) If the expansion was done incorrectly in a) no marks were deducted otherwise $\frac{1}{2}$ mark was deducted for incorrect expansion

$\frac{1}{2}$ mark for getting to the third line

$\frac{1}{2}$ mark for factorisation

$\frac{1}{2}$ mark each for the two solutions

Q16 $\frac{1}{2}$ mark for β in the 4th quadrant or for explaining why $\cos\beta > 0$

$\frac{1}{2}$ mark for $\sqrt{106}$

1 mark for $\cos\beta = \frac{5}{\sqrt{106}}$ or $\frac{5\sqrt{106}}{106}$

Question 17 (2 marks)

Find the radius of the circle $x^2 - 4x + y^2 - 2y - 2 = 0$ in exact form. 2

$x^2 - 4x + 4 + y^2 - 2y + 1 - 4 - 1 - 2 = 0$

$(x-2)^2 + (y-1)^2 - 7 = 0$

$(x-2)^2 + (y-1)^2 = 7$

This is the equation of a circle with centre $(2,1)$ and radius $\sqrt{7}$

EXAMINER'S COMMENTS

Q.17 1 mark for getting the equation into the form $(x-h)^2 + (y-k)^2 = r^2$

1 mark for taking the radius from the equation

Question 18 (5 marks)

Find $\frac{dy}{dx}$ if:

(a) $y = 10x^3 - x + 7$ 1

$\frac{dy}{dx} = 30x^2 - 1$ — 1 mk

(b) $y = \frac{1}{(2-3x)^2}$ 2

$y = (2-3x)^{-2}$

$\frac{dy}{dx} = -2(2-3x)^{-3} \times -3$ — 1 mk for correct application of the chain rule

$= 6(2-3x)^{-3}$

$= \frac{6}{(2-3x)^3}$ $\Big\}$ or — 1 mk

or Quotient Rule $\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$u = 1$ $v = (2-3x)^2$

$u' = 0$ $v' = 2(2-3x) \times -3 = -6(2-3x)$

$= \frac{(2-3x)^2(0) - 1(-6(2-3x))}{(2-3x)^4}$

$= \frac{0 + 6(2-3x)}{(2-3x)^4}$ — 1 mk or Correct application of quotient rule

$= \frac{6(2-3x)}{(2-3x)^4}$

$= \frac{6}{(2-3x)^3}$ — 1 mk for appropriate expansion and simplification

(c) $y = 3x^4\sqrt{x-1}$

Use product rule

Let $u = 3x^4$ $v = (x-1)^{1/2}$

1 mk $u' = 12x^3$ $v' = \frac{1}{2}(x-1)^{-1/2}$

$\frac{dy}{dx} = uv' + vu'$

$= 3x^4 \cdot \frac{1}{2}(x-1)^{-1/2} + (x-1)^{1/2} \cdot 12x^3$ $\Big\}$ or 1 mk

$= \frac{3x^4}{2\sqrt{x-1}} + 12x^3\sqrt{x-1}$

MATHEMATICS – QUESTION 18

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) Generally, well done.		
b) As usual, a few students failed to differentiate the inner function $(x-3)$ Some students used the quotient rule which was a much harder way of doing the question. A few students need to re-visit the quotient rule. It is on the reference sheet. A few students forgot to write either $\frac{dy}{dx}$ or y'.		
c) Some students attempted to get the common denominator but in the process made a mistake. Please do not oversimplify your answers for product rule and quotient rule questions.		

Question 19 (3 marks)

Prove $\dfrac{\tan\alpha}{\sec\alpha - 1} - \dfrac{\tan\alpha}{\sec\alpha + 1} = 2\cot\alpha.$ 3

$$\text{LHS} = \frac{\tan\alpha}{\sec\alpha - 1} - \frac{\tan\alpha}{\sec\alpha + 1}$$

$$= \frac{\tan\alpha(\sec\alpha + 1) - \tan\alpha(\sec\alpha - 1)}{\sec^2\alpha - 1} \quad \text{— 1mk}$$

$$= \frac{\tan\alpha\sec\alpha + \tan\alpha - \tan\alpha\sec\alpha + \tan\alpha}{\tan^2\alpha} \quad \text{— 1mk}$$

$1 + \tan^2\alpha = \sec^2\alpha$

$\tan^2\alpha = \sec^2\alpha - 1$

Very important to be present.

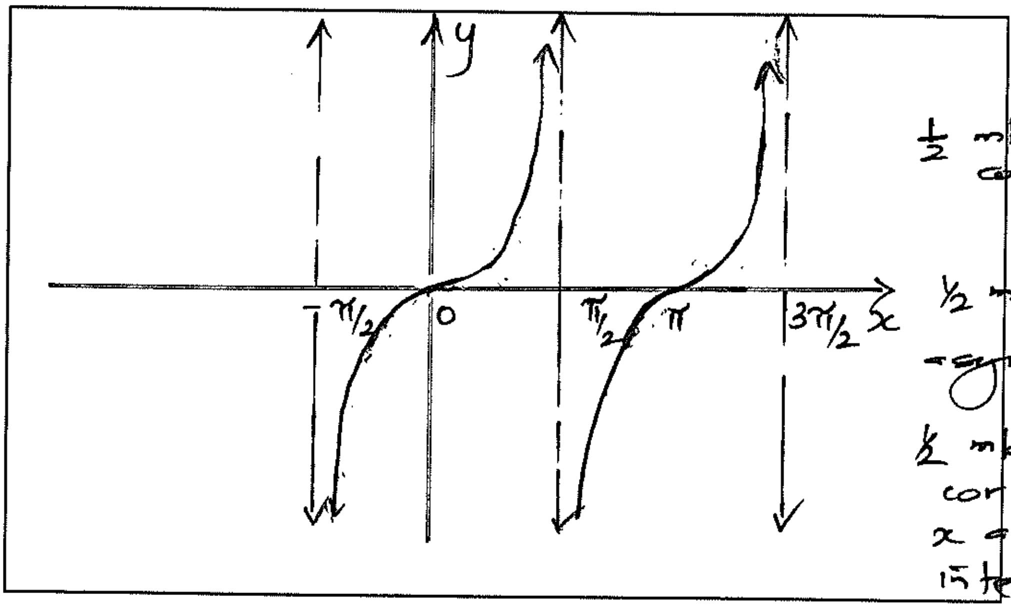
$$= \frac{2\tan\alpha}{\tan^2\alpha}$$

$$= \frac{2}{\tan\alpha} \quad \text{— 1mk — correct from previous working out.}$$

$$= 2\cot\alpha$$

Question 20 (2 marks)

Graph $y = \tan x$ in the domain $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$, showing all intercepts and asymptotes. 2



$\frac{1}{2}$ mk for correct shape

$\frac{1}{2}$ mk – correct asymptotes

$\frac{1}{2}$ mk – correct x and y-intercepts

$\frac{1}{2}$ – correct domain in radians

MATHEMATICS – QUESTION 19 + 20		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Q.19 The easiest way is displayed in the model answer. Some students used $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ and found the common denominator which was a longer method. A few students made a mistake with the identity: $1+\tan^2\alpha = \sec^2\alpha$		
Q20) A few students left their answer in degrees. If the domain is given in radians then the graph should be done in radians rather than degrees. Please label the origin. The graphs should approach the asymptotes and so should be shown accordingly. Students should label one point on the graph, place arrows on the graph, label the axes and the graph itself.		

Question 21 (5 marks)

Solve

(a) $\log_3 x^3 = 6$ 2

$\log_3 x^3 = 6$

$3^6 = x^3$ 1 mark

$x = (3^6)^{\frac{1}{3}}$

$x = 3^2$

$= 9$ 1 mark

(b) $\log_2(x+2) - \log_2(x-5) = 3.$ 3

$\log_2(x+2) - \log_2(x-5) = 3$

$\log_2\left(\frac{x+2}{x-5}\right) = 3$ 1 mark

$\frac{x+2}{x-5} = 2^3$ 1 mark

$x+2 = 8(x-5)$

$x+2 = 8x-40$

$8x - x = 42$

$7x = 42$

$x = 6$ 1 mark

EXAMINER'S COMMENTS

Q21 a) Alternative solution: $3\log_3 x = 6$ --- ½ mark

$\log_3 x = 2$ --- ½ mark

$x = 3^2 = 9$ ---- 1 mark

A bold answer of ---→ 1 mark

b) A few notes of what a few students did when solving this question

$\log_2\left(\frac{x+2}{x-5}\right) = 3$ → Many students did not put the brackets here, but were still awarded full marks

$\frac{x+2}{x-5} = 3$ → Many students made this error. They needed to apply the log definition correctly as

$\log_a b = c$

$b = a^c$

Those students that proceeded to solve the equation correctly and get $x = 17/2$ were still give 2 marks.

NOTE: When solving equations it is always useful to check the validity of your solutions.

Sub $x=6$ into equation

$LHS = \log_2 8 - \log_2 1$

$= \log_2 2^3$

$= 3\log_2 2$

$= 3$

$= RHS$

$\therefore$ a solution

Sub $x = \frac{17}{2}$

$LHS = \log_2 \frac{21}{2} - \log_2 \frac{7}{2}$

$= \log_2 7$

$= \frac{\log_{10} 7}{\log_{10} 2}$

$\doteqdot 2.807$

$\neq 3$ $\therefore$ not a solution

Question 22 (3 marks)

Solve $(\sin x + 2)(2\sin x + 1) = 0$ in the domain $[0, 2\pi]$. 3

$\sin x + 2 = 0$ or $2\sin x + 1 = 0$

$\sin x = -2$ $\quad$ $2\sin x = -1$

No solution since $\quad$ $\sin x = -\frac{1}{2}$ $\quad$ $\frac{1}{2}$ mark

$-1 \leq \sin x \leq 1$ --- $\frac{1 \text{ mark}}{2}$ $\quad$ related angle $x = \frac{\pi}{6}$

$\therefore x = \pi + \frac{\pi}{6}$ or $x = 2\pi - \frac{\pi}{6}$

$x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$

1 mark $\quad$ 1 mark

Question 23 (2 marks)

Find the values of k for which the equation $(k + 2)x^2 + 2x + 4 = 0$ has no real roots. 2

$\Delta = b^2 - 4ac$

$= 2^2 - 4(k+2) \times 4$

$= 4 - 16k - 32$

$= -16k - 28$

For no real roots, $\Delta < 0$

$-16k - 28 < 0$ $\quad$ 1 mark

$16k + 28 > 0$

$16k > -28$

$k > \frac{-28}{16}$

$> -7/4$ $\quad$ 1 mark

EXAMINER'S COMMENTS

Q22.

This question was not done very well

- Many students expanded the LHS and made errors along the way. It was not necessary to expand the LHS
- Also many students ___ left their final answer in degrees when it should have been in radians

If students gave an answer of

$210°$ or $330°$ ----→ $2\frac{1}{2}$ marks

Q23

Overall, this question was done very well.

Question 24 (4 marks)

A function is defined by $f(x) = \frac{2x-1}{3x+2}$.

(a) Show that $f'(x) = \frac{7}{(3x+2)^2}$. 2

$f(x) = \frac{2x-1}{3x+2}$ Let $u = 2x-1$, $v = 3x+2$; $u' = 2$, $v' = 3$ $\frac{1}{2}$

$f'(x) = \frac{vu' - uv'}{v^2}$

$= \frac{(3x+2)\cdot 2 - (2x-1)\cdot 3}{(3x+2)^2}$ 1

$= \frac{6x+4-6x+3}{(3x+2)^2}$ $\frac{1}{2}$

$= \frac{7}{(3x+2)^2}$

(b) Hence, find the equation of the normal at the point where $x = -1$. 2
(Leave your answer in general form)

m OF TANGENT AT $x=-1$ $= \frac{7}{(3(-1)+2)^2} = 7$

$\therefore$ m OF NORMAL at $x=-1$ $= -\frac{1}{7}$ (negative reciprocal) — $\frac{1}{2}$

Also when $x=-1$, $y = f(-1) = \frac{2(-1)-1}{3(-1)+2} = \frac{-3}{-1} = 3$

$\therefore$ Equation of normal is: $y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{7}(x - {-1})$ — 1

$7y - 21 = -1(x+1)$

$7y - 21 = -x - 1$

$x + 7y - 20 = 0$ — $\frac{1}{2}$

(general form)

EXAMINER'S COMMENTS

Q24

a) Question completed quite well.
Show that question required students to show understanding of which formula to use and how to apply it.
* $\frac{1}{2}$ mark for preparing for substitution
* 1 mark for correct substituition.
* $\frac{1}{2}$ mark for correct simplification.

Q24

b) * $\frac{1}{2}$ mark for finding the gradient of normal.
* 1 mark for substitution correctly and finding the equation of the line in any form.
* $\frac{1}{2}$ mark for showing line in correct general form : $ax+by+c=0$ where a is a positive whole number.
- The form $by+ax+c=0$ was not accepted.
- CFPE allowed for last step.

Question 25 (2 marks)

Find the exact value of cot 330°. 2

$\cot 330^\circ = \cot(360^\circ - 30^\circ)$ (related angle $= 30^\circ$)

$= -\cot 30^\circ$

$= -\dfrac{1}{\tan 30^\circ}$ (1)

$= -\dfrac{1}{1/\sqrt{3}}$

$= -1 \div \dfrac{1}{\sqrt{3}}$

$= -\sqrt{3}$ (1)

Question 26 (3 marks)

Find the coordinates of the point on the curve $y = x^2 - 2x - 8$ at which the tangent is perpendicular to the line $x - 2y = 6$. 3

Gradient of line: $x - 2y = 6 \Rightarrow -2y = -x + 6$

$y = \frac{1}{2}x - 3$

$\therefore$ M of line $= \frac{1}{2}$ — $\left(\frac{1}{2}\right)$

$\therefore$ Gradient perpendicular to the line $= -2$ (negative reciprocal)

$\therefore$ Gradient of tangent $= -2$ — $\left(\frac{1}{2}\right)$

Gradient function of parabola $y = x^2 - 2x - 8$

is $y' = 2x - 2$ — $\left(\frac{1}{2}\right)$

To find the coordinates of the point on

$y = x^2 - 2x - 8$ where the tangent has $m = 2$,

let $y' = -2$

$\therefore 2x - 2 = -2 \Rightarrow x = 0$ — (1)

when $x = 0$, $y = (0)^2 - 2(0) - 8$

$= -8$

$\therefore$ Coordinates of the point are $(0, -8)$ — $\left(\frac{1}{2}\right)$

EXAMINER'S COMMENTS

Q25

* 1 mark for changing the ratio into a correct equivalent using the acute related angle.
* 1 mark for giving the correct answer as an exact value.
  - CFPE was allowed only in cases where in the first step students attempted to write an equivalent using an acute related angle.

Q26

- $\frac{1}{2}$ mark for finding gradient of line
- $\frac{1}{2}$ mark for finding gradient of tangent
- $\frac{1}{2}$ mark for differentiating parabola function
- 1 mark for finding the x-value of the point.
- $\frac{1}{2}$ mark for finding the y-value of the point.

- A good portion of students need to revise the concepts in the topic "applications of differentiation".

Question 27 (5 marks)

A particle moves from rest at a point A in a straight line, so that t seconds after leaving A, its displacement, x metres, is given by $x = 8t^2 - t^4$.

Find:

(a) the time(s) for the particle to come to rest and the displacement at these time(s). 3

$x = 8t^2 - t^4$

$\dot{x} = 16t - 4t^3$

At rest when $\dot{x} = 0$

$16t - 4t^3 = 0$

$4t(2-t)(2+t) = 0$

$\therefore t = 0, t = 2, t = -2$

but $t \geq 0$ (time) $\therefore$ at rest at $t = 0, t = 2$

when $t = 0$, $x = 0$

when $t = 2$, $x = 16(2) - 4(2)^3$

$= 16$

$\therefore$ at rest at $x = 0$ when $t = 0$, and at $x = 16$ when $t = 2$

(b) the distance travelled in the first 2 seconds. 2

when $t = 2$, $x = 16$

when $t = 0$, $x = 0$

$\therefore$ travelled 16 m

EXAMINER'S COMMENTS

1 mark for finding $\dot{x} = 16t - 4t^3$

1 mark for solving $\dot{x} = 0$ to find $t = 0$, $t = 2$

1 mark for the full solution

If you didn't attempt to find velocity, you couldn't earn any marks.

Full marks were only awarded where candidates rejected $t = -2$ and expressed their answers as matching pairs of time and displacement (i.e. it was not enough to say that the solutions were $x = 16$, $x = 0$, $t = 0$, $t = 2$ – they needed to be paired).

1 mark for find $x = 16$ at $t = 2$

1 mark for having "travelled 16m" as your final answer.

Many students found displacement at $t = 1$ and $t = 2$, and added them to find the "total", an illogical process.

Question 28 (3 marks)

Using the method of first principles, find $f'(x)$ if $f(x) = 2x^2 + 3x + 6$. 3

$f(x) = 2x^2 + 3x + 6$

$f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \dfrac{2(x+h)^2 + 3(x+h) + 6 - (2x^2 + 3x + 6)}{h}$

$= \lim_{h \to 0} \dfrac{2x^2 + 4xh + 2h^2 + 3x + 3h + 6 - 2x^2 - 3x - 6}{h}$

$= \lim_{h \to 0} \dfrac{4xh + 3h + 2h^2}{h}$

$= \lim_{h \to 0} \dfrac{h(4x + 3 + 2h)}{h}$

$= \lim_{h \to 0} 4x + 3 + 2h$

$= 4x + 3$

EXAMINER'S COMMENTS

1 mark for correct substitution into correct formula ie. $\dfrac{2(x+h)^2+3(x+h)+6-(2x^2+3x+6)}{h}$

(with or without the correct limit statement)

1 mark for correct use of limits throughout the whole solution

1 mark for the full solution

Candidates who took the limit too early, or too late, earned half the second mark.

You cannot take the lim until you have an expression without h in the denominator. $h \to 0$

Question 29 (4 marks)

A small tank of liquid which contains L litres of a toxic chemical is being drained.
The amount of chemical in the tank over time t minutes, is given by:

$$L = 192 - 96t + 12t^2$$

(a) At what rate is the chemical draining out of the tank after 6 minutes? 2

$\frac{dL}{dt} = -96 + 24t$

at $t = 6$

$\frac{dL}{dt} = -96 + 24 \times 6$

$= 48L/\text{min}$

as the rate of change is positive, it means the liquid in the tank is filling up & not draining out

from part (b) below, we can see that the tank is completely empty at $t = 4$

$\therefore$ the tank has nothing left to drain out at $t = 6$

$\therefore$ the rate of chemical draining out of tank is $0L/\text{min}$.

(b) How long will it take for the tank to be completely empty? 2

The tank will be completely empty when $L = 0$

$192 - 96t + 12t^2 = 0$

$t^2 - 8t + 16 = 0$

$(t-4)^2 = 0$

$t - 4 = 0$

$\therefore t = 4$ min

$\therefore$ tank will be completely empty after 4 minutes.

EXAMINER'S COMMENTS

Question 29.

(a) This question was declared invalid and was not marked.

(b) As part (a) was declared invalid, part (b) was not marked.

Question 30 (4 marks)

The intensity I Watts/m^2 of a sound of loudness L decibels is given by the formula

$$I = I_o(10)^{\frac{1}{10}L}$$

where $I_o = 10^{-12}$ Watts/m^2 is the intensity of the Threshold of Hearing.

(a) Show that $L = 10(\log_{10} I - \log_{10} I_o)$. 2

Method 1

$I = I_0 (10)^{1/10 L}$

$\frac{I}{I_0} = 10^{1/10 L}$

$\log_{10} 10^{1/10 L} = \log_{10}(I/I_0)$

$\frac{1}{10}L = \log_{10} I - \log_{10} I_0$

$L = 10(\log_{10} I - \log_{10} I_0)$

as required.

Method 2

taking $\log_{10}$ on both sides.

$\log_{10} I = \log_{10}(I_0\, 10^{1/10 L})$

$\log_{10} I = \log_{10} I_0 + \log_{10} 10^{\frac{1}{10}L}$

$\log_{10} I - \log_{10} I_0 = \frac{1}{10} L$

$L = 10(\log_{10} I - \log_{10} I_0)$

as required.

(b) Hence, find the loudness, in decibels, of noise at a Justin Bieber concert whose intensity is 10^{12} times greater that the Threshold of Hearing. 2

Let $I_0 = 1$

$\therefore I = 1 \times 10^{12}$

$L = 10(\log_{10} I - \log_{10} I_0)$

$= 10(\log_{10} 10^{12} - \log_{10} 1)$

$= 10 \times \log_{10}\left(\frac{10^{12}}{1}\right)$

$= 10 \times \log_{10} 10^{12}$

$= 10 \times 12$

$= 120$ times louder.

EXAMINER'S COMMENTS

Question 30.

(a) There are 4 distinct steps to solving & each part was allocated ½ mark.
If the students started off in a direction which was not making any sense, then no mark was allocated.
It was hard for students to get part mark in this question. Most students got zero or 2 mark.

(b) Many students considered $I_0 = 10^{-12}$

& $I = 10^{12} \div 10^{-12}$

$\therefore$ I became 10^{24} times louder.

For students who made the above error & got the answer 240 times louder, 1 mark was awarded.

Question 31 (3 marks)

The shaded segment shown is formed by an arc BC of a circle radius 1.6 m, which subtends an angle of $\frac{4\pi}{9}$ radians at its centre A. 3

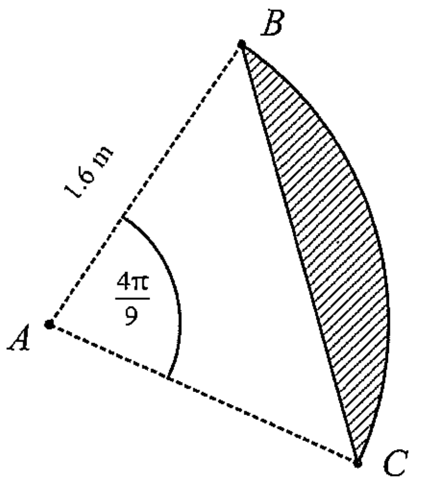
use Reference sheet:

$A = \frac{1}{2}r^2\theta$

$A = \frac{\theta}{360} \times \pi r^2$

$A = \frac{1}{2}ab\sin C$

$\frac{4\pi}{9} = 80°$



Find the area of the shaded segment, correct to 3 significant figures.

$$\text{Area of shaded segment} = \frac{1}{2}r^2(\theta - \sin\theta)$$
$$= \frac{1}{2} \times 1.6^2\left(\frac{4\pi}{9} - \sin\frac{4\pi}{9}\right)$$
$$= 0.52666323$$
$$= 0.527 \text{ m}^2$$

OR Area of shaded segment = Area of sector ABC − Area of $\triangle ABC$

$$= \frac{80}{360} \times \pi \times 1.6^2 - \frac{1}{2} \times 1.6 \times 1.6 \times \sin 80°$$
$$= 1.787217154 - 1.260553924$$
$$= 0.52666323$$
$$= 0.527 \text{ m}^2$$

Q 31

EXAMINER'S COMMENTS

- this question showed that many students have poor calulator skills and rounding to 3 signficant figures
- show all calulator displays befre rounding so I can check!
- learn when to put your calulator into DEG OR RAD mode!
- $\sin \frac{4\pi}{9} \Rightarrow$ RAD mode
- $\sin 80^\circ \Rightarrow$ DEG mode

3 marks – provides correct solution with correct working

2 marks – correct substitution into correct formula for area of a sector & area of a triangle (use Reference sheet!)

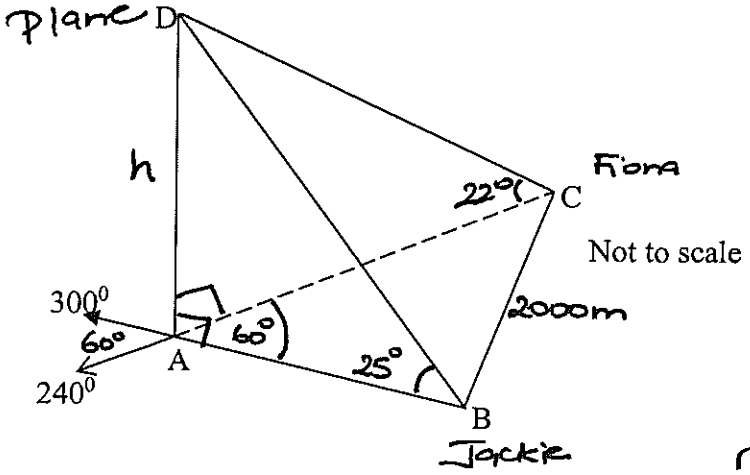
1 mark – for some productive progress.

ie 1 for correct area of sector
1 for correct area of triangle
1 for correct answer to 3 sig. figs.

Question 32 (6 marks)

The diagram below shows Jackie standing at B on level ground, whilst Fiona is standing 2000 m away from Jackie at C on the same level ground. They both take the bearing and elevation of an aeroplane D at the same instant. Jackie finds the bearing is 300^0T and the angle of elevation 25^0, whilst Fiona finds the bearing to be 240^0T and the angle of elevation to be 22^0.

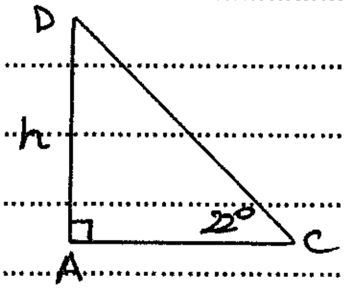
(a) Complete the following diagram, showing all the information given. **1**



Note: ΔABC is <u>not</u> rightangled.

(b) Show that if the height DA of the plane is h metres, then
$AB = h\cot 25^0$ and $AC = h\cot 22^0$ **2**

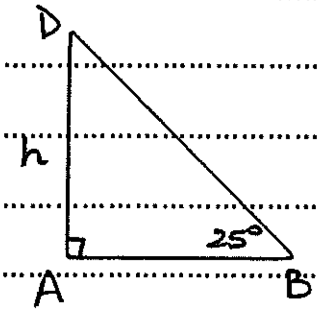
In Δ DAC: $\tan 22° = \dfrac{h}{AC}$ OR $\tan 22° = \dfrac{h}{AC}$



$\cot 22° = \dfrac{AC}{h}$ $\qquad AC = \dfrac{h}{\tan 22°}$

$\therefore AC = h\cot 22°$ $\qquad \therefore AC = h\cot 22°$

In Δ DAB: $\tan 25° = \dfrac{h}{AB}$ OR $\tan 25° = \dfrac{h}{AB}$



$\cot 25° = \dfrac{AB}{h}$ $\qquad AB = \dfrac{h}{\tan 25°}$

$\therefore AB = h\cot 25°$ $\qquad \therefore AB = h\cot 25°$

Q32 (a) **EXAMINER'S COMMENTS**

- needed to see 22°, 25° and 2000 m all in the correct positions on your diagram for the 1 mark

Show questions for (b) & (c):

In a show question, you must show all sequential steps. Do not expect the examiner to join the dots for you. Many students left out substitutions and crucial steps (especially in (c)).

Remember: In a "show" question it must be clear how one line is obtained from another.

(b) the tan ratio must be stated.

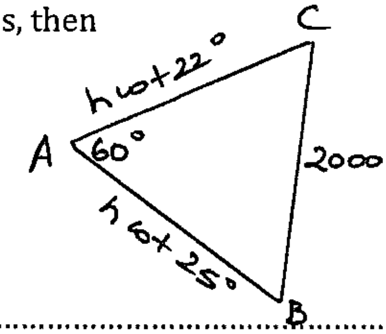
1 mark for showing $AB = h\cot 25^\circ$ with correct working (no marks for working backwards).

1 mark for showing $AC = h\cot 22^\circ$

(no half marks)

(c) Show that if the height DA of the plane is h metres, then **3**

$$h = \frac{2000}{(\cot^2 25^0 + \cot^2 22^0 - \cot 25^0 \cot 22^0)^{\frac{1}{2}}}$$



Using Cosine Rule in ΔCAB:

$$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos\hat{A}$$

$$2000^2 = (h\cot 25°)^2 + (h\cot 22°)^2 - 2(h\cot 25°)(h\cot 22°)\cos 60°$$

$$2000^2 = h^2\cot^2 25° + h^2\cot^2 22° - \cancel{2} \times h^2 \times \cot 25° \times \cot 22° \times \cancel{\tfrac{1}{2}}$$

1 for correct substitution

$$2000^2 = h^2\left[\cot^2 25° + \cot^2 22° - \cot 25° \cot 22°\right]$$

$$h^2 = \frac{2000^2}{\cot^2 25° + \cot^2 22° - \cot 25° \cot 22°}$$

1 to this step

$$h = \sqrt{\frac{2000^2}{\cot^2 25° + \cot^2 22° - \cot 25° \cot 22°}} \quad (h > 0)$$

1 for showing the square root.

$$\therefore h = \frac{2000^2}{(\cot^2 25° + \cot^2 22° - \cot 25° \cot 22°)^{\frac{1}{2}}}$$

as required.

NOTE: ΔABC is not right-angled, & all of you that used Pythagora's Theorem "fudged" your way through the proof, hence no marks awarded !!!

End of Assessment Task

Q32 **EXAMINER'S COMMENTS**

(c) A <u>show</u> question : it must be clear how one line is obtained from another.

(1) mark for substituting into the cosine rule correctly (wrong formula – no marks!)

(2) marks for all steps leading to

$$h^2 = \frac{2000^2}{\cot^2 25^\circ + \cot^2 22^\circ - \cot 25^\circ \cot 22^\circ}$$

and final mark was showing the square root <u>OR</u> raising both sides to the power of $\frac{1}{2}$

eg

$$h = \sqrt{\frac{2000^2}{\cot^2 25^\circ + \cot^2 22^\circ - \cot 25^\circ \cot 22^\circ}} \quad (h>0)$$

$$= \frac{2000^2}{(\cot^2 25^\circ + \cot^2 22^\circ - \cot 25^\circ \cot 22^\circ)^{\frac{1}{2}}}$$

as required

<u>OR</u> $$(h^2)^{\frac{1}{2}} = \left(\frac{2000^2}{\cot^2 25^\circ + \cot^2 22^\circ - \cot 25^\circ \cot 22^\circ}\right)^{\frac{1}{2}}$$

$$\therefore h = \frac{2000}{(\cot^2 25^\circ + \cot^2 22^\circ - \cot 25^\circ \cot 22^\circ)^{\frac{1}{2}}}$$

as required.