THIS PAGE IS FOR TEACHER USE ONLY:

Student Name:

St George Girls High School

Mathematics Advanced

2021 Year 11 Assessment Task 3

General Instructions

- Working Time – 50 minutes.
- No reading time.
- Write using black pen.
- Calculators approved by NESA may be used.
- You can use your reference sheet.
- To answer the questions
  - Answer each question on a separate A4 sheet of paper, which includes your full name, teacher's name and question number.
  - Show relevant mathematical reasoning and/or calculations.
  - Marks may not be awarded for incomplete or poorly presented solutions.

Total marks: 38

- Attempt Questions 1 – 8

Q1	/5
Q2	/5
Q3	/5
Q4	/5
Q5	/5
Q6	/5
Q7	/4
Q8	/4
Total	**/38**
	%

Question 1 (5 marks)

a) Differentiate with respect to x:

i) $y = 8x + \frac{1}{x^2}$. 1

ii) $y = \frac{4}{\sqrt{3x-1}}$. 2

b) Simplify the following expression: $\frac{1 - \cos^2\theta}{\tan\theta}$. 2

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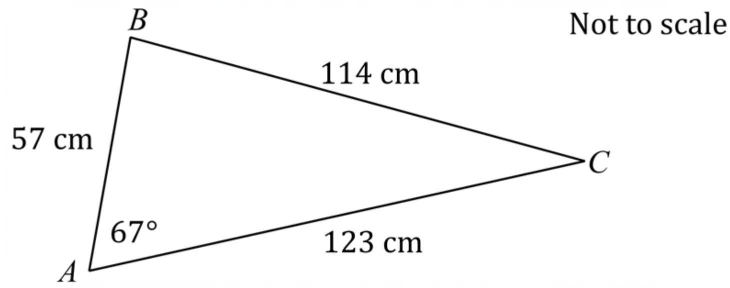
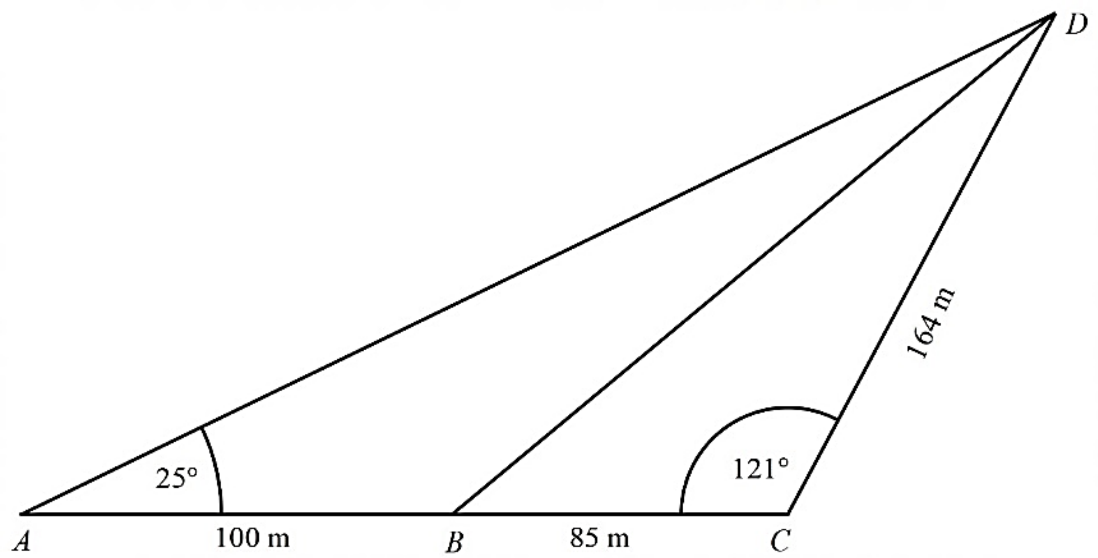
Question 2 (5 marks) – Start your answer on a new page.

a)



What is the area of triangle ABC to the nearest square centimetre? 1

b) The diagram below shows four cafés in a park, labelled A, B, C and D.
There are straight paths joining the cafés.



i) Show by calculations that the distance BD is 220m (to the nearest metre). 1

ii) Find the size of $\angle ADB$, correct to the nearest degree. 1

iii) Lisa walks from A to B to D. How many extra metres does she walk compared to if she had walked directly from A to D? Answer to the nearest metre. 2

Continued on next page

Question 3 (5 marks) – Start your answer on a new page.

a) Differentiate $f(x) = 4x^2 - 5x$, using the method of first principles. 3

b) If $y = (2x+1)(3x+4)^3$, find $\dfrac{dy}{dx}$. 2

Question 4 (5 marks) – Start your answer on a new page.

A function is defined by $f(x) = \dfrac{2x-1}{3x+2}$.

i) Show that $f'(x) = \dfrac{7}{(3x+2)^2}$. 2

ii) Find $f'(-1)$. 1

iii) Find the equation of the tangent at the point where $x = -1$. 2

Continued on next page

Question 5 (5 marks) – Start your answer on a new page.

a) A particle is moving along the x-axis.
Its displacement x centimetres at time t seconds is given by:

$$x = 4 + 12t^2 - t^3 \text{ for } t \geq 0.$$

i) What is the initial displacement of the particle? 1

ii) At what time(s) is the particle at rest? 2

b) Solve the equation $2\cos\alpha - \sqrt{3} = 0$ for $0 \leq \alpha \leq 2\pi$. 2

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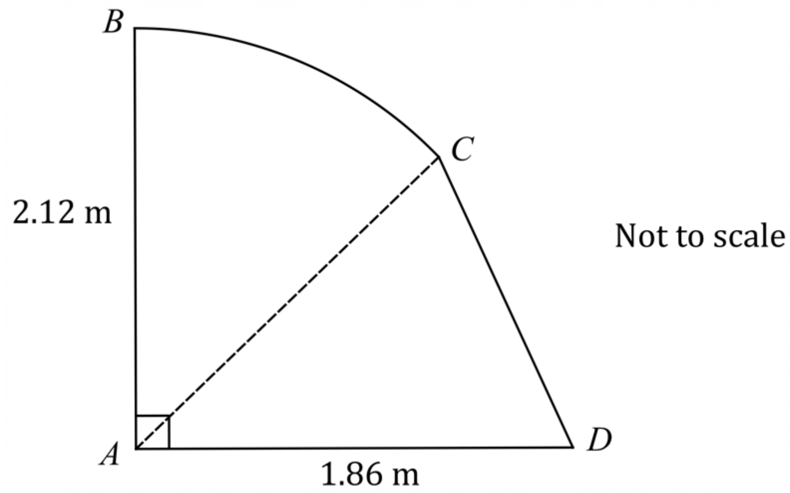
Question 6 (5 marks) – Start your answer on a new page.

a)



A plane shape $ABCD$ is shown above.
The straight line AB is vertical and has length 2.12 metres.
The straight line AD is horizontal and has length 1.86 metres.
The curve BC is an arc of a circle with centre A, and CD is a straight line.
The size of $\angle BAC$ is 0.65 radians.

i) Find the length of the arc BC. Answer correct to two decimal places. 1

ii) What is the area of the sector BAC? 1
Answer correct to two decimal places.

iii) Determine the size of $\angle CAD$, in radians, correct to two decimal places. 1

b) If $\sin\theta = \dfrac{7}{24}$ and $\cos\theta < 0$, find the exact value of $\sec\theta$. 2

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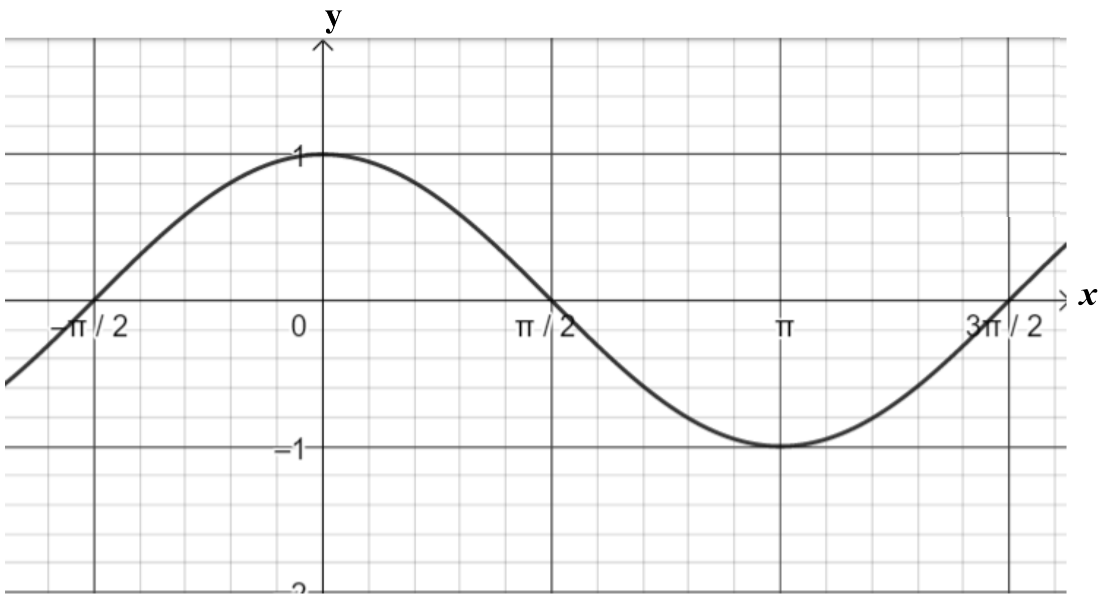
Question 7 (4 marks) – Start your answer on a new page.

a) Write an equation for the graph below. 1



b) A block of ice of mass 8 kg melts according to the formula $M = 8 - \dfrac{t^2}{32}$, 3

where M is the mass remaining, in kilograms, after t minutes.

Find the rate at which the ice is melting when 2kg of ice has melted.

Continued on next page

Question 8 (4 marks) – Start your answer on a new page.

a) Let $f(x) = |x - 2|$.

i) Explain why this function is continuous at $x = 2$. 1

ii) Is this function differentiable at $x = 2$? Explain your answer. 1

b) The volume of water in a container after t minutes is given by the equation 2
$V(t) = t^2 + 4t + 30$ where V is measured in litres.
It is known that the average rate of change in volume over the first k minutes is 10 litres per minute. Find the value of k.

End of Assessment Task

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

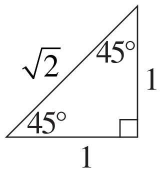
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

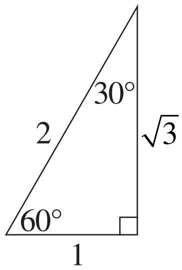
$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \dfrac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

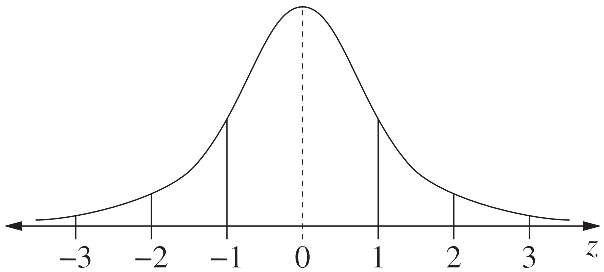
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

2021 Yr 11 Advanced CT3

MATHEMATICS ADVANCED – QUESTION 1

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
1 a) Differentiate w.r.t.x		
(i) $y = 8x + x^{-2}$		
$y = 8x + x^{-2}$ $y' = 8 - 2x^{-3}$	1	No half marks given. generally well done
(ii) $y = \dfrac{4}{\sqrt{3x-1}}$	2	
$y = 4(3x-1)^{-\frac{1}{2}}$ $y' = 4 \times -\frac{1}{2}(3x-1)^{-\frac{1}{2}-1} \times 3$		1 mark for this line
$y' = -2(3x-1)^{-\frac{3}{2}} \times 3$ $y' = -6(3x-1)^{-3/2}$ or $= -\dfrac{6}{\left(\sqrt{3x-1}\right)^3}$		1 mark answer
		many students forgot to multiply by 3 $\frac{d}{dx}(3x-1)$
(b) Simplify $\dfrac{1-\cos^2\theta}{\tan\theta}$	2	
$= \dfrac{\sin^2\theta}{\tan\theta}$ OR $\dfrac{\sin^2\theta}{\tan\theta}$		① mark to here well done generally
$= \dfrac{\sin^2\theta}{\frac{\sin\theta}{\cos\theta}} \times \dfrac{\cos\theta}{\cos\theta}$ $= \sin^2\theta \div \tan\theta$ $= \sin^2\theta \div \dfrac{\sin\theta}{\cos\theta}$ $= \dfrac{\sin^2\theta\cos\theta}{\sin\theta}$ $= \sin^2\theta \times \dfrac{\cos\theta}{\sin\theta}$ $= \sin\theta\cos\theta$ $= \sin\theta\cos\theta$		A number of students didn't use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

MATHEMATICS ADVANCED – QUESTION 2

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
2a) $A = \frac{1}{2} bc \sin A$		Some students
$= \frac{1}{2} \times 123 \times 57 \times \sin 67$	1	forgot to use the
$\doteqdot 3227 \text{ cm}^2$		formula.
		Some did not
		realise the formula
		means 2 sides
		and an included
		angle, and so
		proceeded to find
		$\angle C$ before using
		formula.
2b) i) Using $\triangle BDC$:		
cosine rule:		If students
$c^2 = a^2 + b^2 - 2ab \cos C$		miscalculated
$\therefore BD^2 = 85^2 + 164^2 - 2 \times 85 \times 164 \times \cos 121^\circ$	1	and left another
$\therefore BD \doteqdot 220 \text{ m}$		answer for BD,
		then ½ was
		deducted.
2b) ii) most used $\triangle ADB$ as follows		
$\frac{\sin \angle ADB}{100} = \frac{\sin 25^\circ}{220}$		
$\therefore \angle ADB \doteqdot 11^\circ$ (nearest degree)	1	
There was an error in the question,		
and $\angle DAB$ is not equal to 25°.		
Therefore the following methods		
were also suitable, but resulted in		

different values.

MATHEMATICS ADVANCED – QUESTION 2

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
2 b) ii) continued.		
Option 2		
$\angle ADC = 180 - \angle DAC - \angle DCA$		
$= 180 - 25 - 121$		
$= 34^\circ$		
$\dfrac{\sin \angle BDC}{85} = \dfrac{\sin 121}{220}$		
$\therefore \angle BDC = 19^\circ 20'$		
$\therefore \angle ADB = \angle ADC - \angle BDC$		
$= 34^\circ - 19^\circ 20'$		
$= 14^\circ 40'$		
$\doteqdot 15^\circ$ (nearest degree)	1	
Option 3		
$\dfrac{\sin \angle DBC}{164} = \dfrac{\sin 121^\circ}{220}$		
$\angle DBC = 39^\circ 43'$		
$\therefore \angle ABD = 180 - 39^\circ 43'$		
$= 140^\circ 17'$		
$\therefore \angle ADB = 180 - \angle DAB - \angle ABD$		
$= 180 - 25 - 140^\circ 17'$		
$= 14^\circ 43'$		
$\doteqdot 15^\circ$	1	
2 b) iii) – Due to the error of 25° in the diagram, this part of the question became infeasible to mark. 2 marks were awarded to all students.	2	

Yr 11 Adv Task 3 – 2021 /5

MATHEMATICS ADVANCED – QUESTION 3 Solutions and marking criteria		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $f(x) = 4x^2 - 5x$ $f(x+h) = 4(x+h)^2 - 5(x+h)$		
$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$		
$= \lim_{h\to 0} \frac{4(x+h)^2 - 5(x+h) - [4x^2 - 5x]}{h}$		
$= \lim_{h\to 0} \frac{4(x^2 + 2xh + h^2) - 5x - 5h - 4x^2 + 5x}{h}$ $= \lim_{h\to 0} \frac{\cancel{4x^2} + 8xh + 4h^2 - \cancel{5x} - 5h - \cancel{4x^2} + \cancel{5x}}{h}$		1 mark
$= \lim_{h\to 0} \frac{8xh + 4h^2 - 5h}{h}$ $= \lim_{h\to 0} \frac{\cancel{h}(8x + 4h - 5)}{\cancel{h}}$		1 mark
$= \lim_{h\to 0} 8x + 4h - 5$		
$= 8x - 5$	1	
		3
Note: • Many students did not have the limit sign using this principle $\therefore$ -1 mark or they had it appearing once or placed before the equal sign. • A hand ful of students did not include the vinculum to cover the whole expression $\therefore$ $-\frac{1}{2}$ mark •		Finding the derivative from first principles needs to be reviewed by many students.

MATHEMATICS ADVANCED – QUESTION 3		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

b)

$$y=(2x+1)(3x+4)^3$$

$u = 2x+1 \qquad v=(3x+4)^3$

$u' = 2 \qquad v' = 3(3x+4)^2 \times 3$

$\qquad\qquad = 9(3x+4)^2$

$$\frac{dy}{dx} = vu' + uv'$$

$= (3x+4)^3 \times 2 + (2x+1) \times 3(3x+4)^2 \times 3$

$= 2(3x+4)^3 + (2x+1) \times 9(3x+4)^2$ — 2 marks

1 1

$= (3x+4)^2 \left[2(3x+4) + 9(2x+1)\right]$

$= (3x+4)^2 (6x+8+18x+9)$

$= (3x+4)^2 (24x+17)$ #

NOTE:

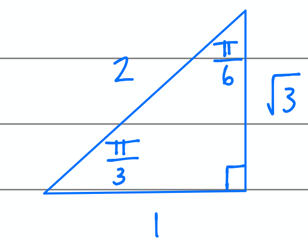
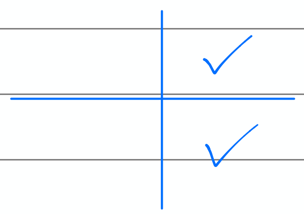
Many students successfully got to the last line by factorising and simplifying correctly however there were a handful of students who need a lot of practising this skill. Only the first two lines of working gave you the full marks though.

2

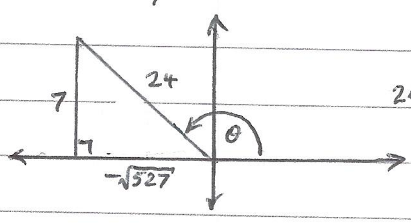
MATHEMATICS ADVANCED – QUESTION 4 Year 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
i) $f(x) = \dfrac{2x-1}{3x+2}$		Some students did not have the correct quotient rule formula. It is on the reference sheet. Others made mistakes in the
$f'(x)$ – Use quotient rule.		
Let $u = 2x-1$ $\quad v = 3x+2$		
$\dfrac{du}{dx} = 2 \quad \dfrac{dv}{dx} = 3$		
$f'(x) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$		
$= \dfrac{2(3x+2) - 3(2x-1)}{(3x+2)^2}$	1mk Correct application of quotient rule	
$= \dfrac{6x+4-6x+3}{(3x+2)^2}$	1mk for expansion and simplification.	
$= \dfrac{7}{(3x+2)^2}$ As required		
ii) $f'(-1) = \dfrac{7}{(3(-1)+2)^2}$		Generally well attempted.
$= \dfrac{7}{(-3+2)^2}$		
$= \dfrac{7}{(-1)^2}$		
$= 7$	1mk	Many computational errors were made by students, otherwise well done.
iii) When $x=-1$, $f(-1) = \dfrac{2(-1)-1}{3(-1)+2}$		
$= \dfrac{-3}{-1} = 3$	1mk	
$y-3 = 7(x--1)$		
$y-3 = 7x+7$		
$y = 7x+10$		
or $y-7x-10=0$	1mk	

MATHEMATICS ADVANCED – QUESTION 5

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(a) $x = 4 + 12t^2 - t^3$		
(i) when $t = 0$,		(i) "Initial" means
$x = 4 + 12(0)^2 - (0)^3$		when $t = 0$.
$\therefore x = 4$ cm	1	Students should also
$\therefore$ Initial displacement is 4cm.		show the substitution
(ii) $v = \frac{dx}{dt} = 24t - 3t^2$	1	(ii) "At rest" means
particle is at rest when $v = 0$.		when the particle is
$24t - 3t^2 = 0$		stationary $\Rightarrow v = 0$.
$3t(8 - t) = 0$		
$\therefore t = 0, \ t = 8$	1	Be careful to use the
$\therefore$ Particle is at rest at 0 seconds and		correct notation:
8 seconds.		$\cancel{\frac{dy}{dx}}$ $v = \frac{dx}{dt} = \dot{x}$
(b) $2\cos\alpha - \sqrt{3} = 0$ for $0 \le \alpha \le 2\pi$		- If the domain is
$2\cos\alpha = \sqrt{3}$		$0 \le \alpha \le 2\pi$, the
$\cos\alpha = \frac{\sqrt{3}}{2}$	1	angles are measured
 related angle $= \frac{\pi}{6}$		in RADIANS.
		- Some students
Solutions exist in 1st and 4th quadrants.		forgot about the
		solution in the
		4th quadrant.
$\alpha = \frac{\pi}{6}, \ 2\pi - \frac{\pi}{6}$		
$\therefore \alpha = \frac{\pi}{6}, \ \frac{11\pi}{6}$	1	- Be careful with
		the notation:
		Not $\cancel{\cos\alpha = \frac{\pi}{6}}$
		but $\alpha = \frac{\pi}{6}$

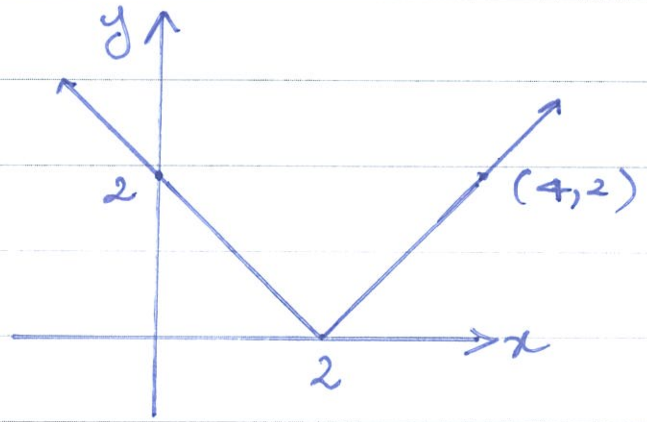
MATHEMATICS ADVANCED – QUESTION 6

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) (i) $l = r\theta \qquad r = 2.12,\ \theta = 0.65$		
$= 2.12 \times 0.65$		Some students were
$= 1.378$		confused with the
$= 1.38$ correct to 2 dec. pl.	1	use of radians
(ii) $\text{Area} = \frac{1}{2} r^2 \theta$		
$= \frac{1}{2} \times 2.12^2 \times 0.65$		
$= 1.46068$		
$= 1.46$ correct to 2 dec. pl.	1	Same as for (i)
(iii) $\angle BAD = \frac{\pi}{2}$ radians		
$\angle CAD = \angle BAD - \angle BAC$		
$= \frac{\pi}{2} - 0.65$		
$= 0.9207963\ldots$		
$= 0.92$ correct to 2 dec. pl.	1	
b) $\sin\theta = \frac{7}{24} \qquad \cos\theta < 0$		
second quadrant		
 $24^2 - 7^2 = 527$		1 mark for finding the length of the adjacent side and setting up the secant ratio
$\cos\theta = \frac{-\sqrt{527}}{24}$		
$\sec\theta = \frac{-24}{\sqrt{527}}$	2	1 mark for making the answer negative

MATHEMATICS – QUESTION 7

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $y = \cos x$	1	Correct Answer. ½ mark if there was no $y = \cos x$ (left off $y=$) and ½ mark if it was not $\cos \underline{x}$ (ie $\cos \underline{\theta}$)
b) $M = 8 - \frac{t^2}{32}$ $6 = 8 - \frac{t^2}{32}$ $t^2 = 64$ $t = 8$ ($t \geq 0$ as t is time) ∴ It takes 8 min for the 8kg to melt to 6kg. NOTE: M is the mass "REMAINING". Mass remaining after 2kg has melted is $8 - 2 = 6$ kg	1 1	Realising $M = 6$ kg Finding the time (get used to justifying that $t \geq 0$ as t is time) no marks deducted this time.
$M = 8 - \frac{1}{32}t^2$ $\frac{dM}{dt} = -\frac{2}{32}t$ $\frac{dM}{dt} = -\frac{1}{16}t$	½	Correct differentiation
$= -\frac{1}{16} \times 8$ $= -\frac{1}{2}$ kg/min ∴ The rate at which the ice is melting when 2kg of ice has melted is 0·5 kg/min.	½	Correct Answer (rate must be evident with units kg/min) Also if you say "melting" at 0·5kg/min, the word "melting" means the negative sign. (no marks deducted for this as long at $\frac{dM}{dt} = -0{\cdot}5$ kg/min

MATHEMATICS ADVANCED – QUESTION 8 (4 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(a)(i) $f(x) = \|x-2\|$ is continuous because it can be drawn without lifting your pen from the paper.	1	for correct explaination.
OR $f(x) = \|x-2\|$ is continuous because it is made up of two lines $y = x-2$ and $y = -x+2$ joining at $x = 2$.		1 for correct explaination
OR 		
The graph as shown does not have any breaks or gaps.	1	for correct explaination with correct graph drawn

MATHEMATICS ADVANCED – QUESTION 8		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
OR using Formal definition:		
$f(x) = \|x-2\| = \begin{cases} +(x-2) = x-2 & \text{for } x-2 \geq 0,\ x \geq 2 \\ -(x-2) = -x+2 & \text{for } x-2 < 0,\ x < 2 \end{cases}$		
$\lim_{x \to 2^+} f(x) = 2-2 = 0$	①	for correct explaination.
$\lim_{x \to 2^-} f(x) = -2+2 = 0$		
Since, $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = 0$, $f(x) = \|x-2\|$ is continuous at $x=2$.		
(ii) No, $f(x) = \|x-2\|$ is not differentiable at $x=2$, because there is a sharp corner at that point.	①	for correct explaination.
OR Not differentiable at $x=2$, since the function is made up of 2 lines with different gradients	①	for correct explaination.

MATHEMATICS ADVANCED – QUESTION 8		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
OR using formal definition:		
$f(x) = \begin{cases} x-2 & \text{for } x \geq 2 \\ -x+2 & \text{for } x < 2 \end{cases}$		
$\therefore f'(x) = \begin{cases} 1 & \text{for } x \geq 2 \\ -1 & \text{for } x < 2 \end{cases}$		
$\lim_{x \to 2^-} f'(x) = -1$	1	provides correct explaination.
$\lim_{x \to 2^+} f'(x) = 1$		
Since $\lim_{x \to 2^-} f'(x) \neq \lim_{x \to 2^+} f'(x)$, then $f(x) = \lvert x-2 \rvert$ is not differentiable at $x=2$.		

MATHEMATICS ADVANCED – QUESTION 8		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(b) when $t=0$ (initially):		
$v(0) = (0)^2 + 4(0) + 30 = 30$ litres		
when $t=k$:		
$v(k) = k^2 + 4k + 30$		
Average rate of change: $\dfrac{v(k)-v(0)}{k-0} = 10$ L/min		
$\dfrac{k^2+4k+30-30}{k} = 10$		(2) marks provides correct solution
$k^2+4k = 10k$		(1) mark for some worthwhile progress
$k^2-6k=0$		
$k(k-6)=0$		
$k=0$ or $k=6$		
$\therefore k=6$ minutes (since $k>0$)		
• students need to be mindful of their algebra skills.		
Remember: Average rate of change $= \dfrac{\text{change in } y\text{-value}}{\text{change in time}}$		