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Name:
Teacher:

St George Girls High School

Mathematics Advanced

2022 Year 11 End of Course Examination

General Instructions

- Reading time – 10 minutes.
- Working time – 2 hours.
- Write using black pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- For questions in **Section I**, use the Multiple-Choice answer sheet provided.
  For questions in **Section II:**
  - Answer the questions in the 2 booklets provided.
  - Show relevant mathematical reasoning and/or calculations.
  - Marks may not be awarded for incomplete or poorly presented solutions, or where multiple solutions are provided.

Total marks: 79

Section I – 9 marks (pages 3 – 7)

- Attempt Questions 1 – 9
- Allow about 15 minutes for this section

Section II – Allow about 1 hour and 45 minutes

Booklet 1: 40 marks (pages 9 – 18)

- Attempt Questions 10 – 25 for this section

Booklet 2: 30 marks (pages 21 – 29)

- Attempt Questions 26 – 34 for this section

Q1 – Q9	/9
Q10 – Q14	/11
Q15 – Q18	/10
Q19 – Q22	/9
Q23 – Q25	/10
Q26 – Q28	/10
Q29 – Q31	/10
Q32 – Q34	/10
Total	**/79**
	%

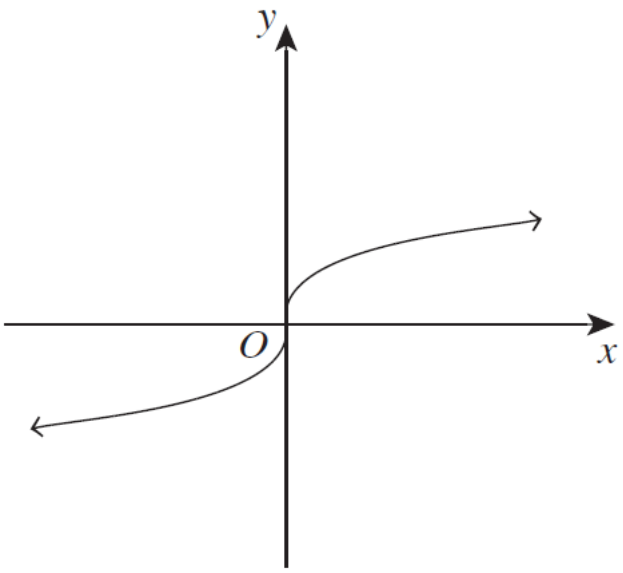
Section I

9 marks
Attempt Questions 1 – 9
Allow about 15 minutes for this section

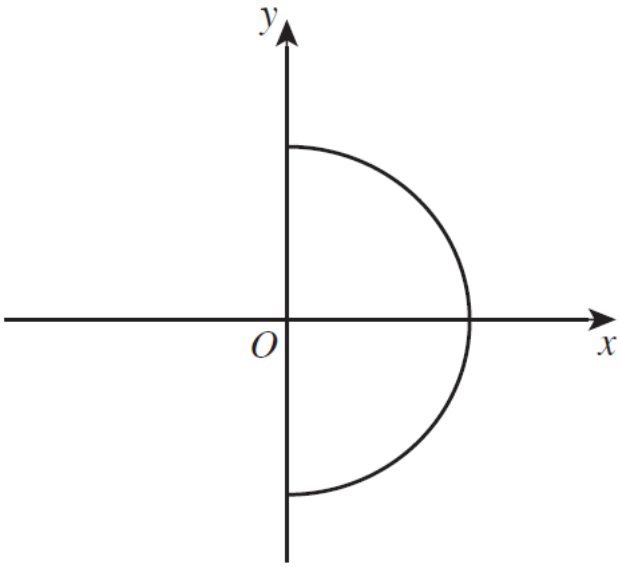
Use the multiple-choice answer sheet provided for Questions 1 to 9.

1. Which of the following graphs represents a one-to-many relationship?

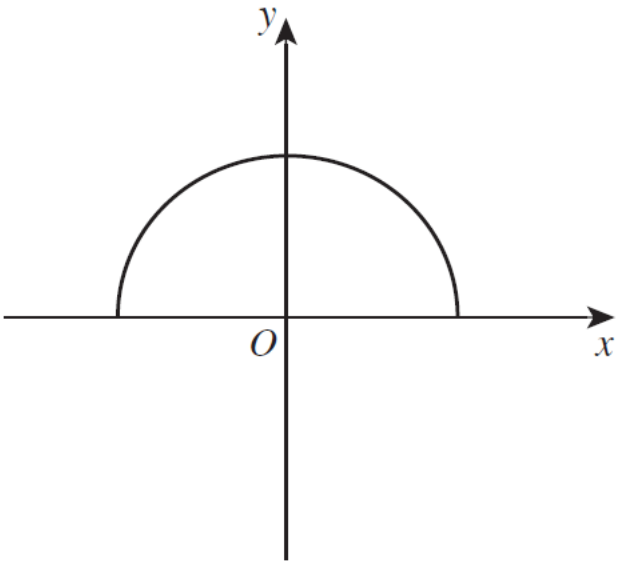
(A) .



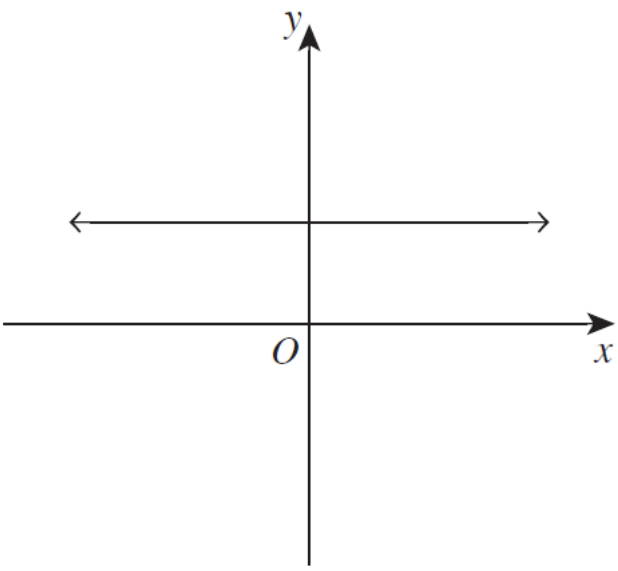
(B) .



(C) .



(D) .



2. What is the value of $\log_2 \frac{1}{4} + \log_2 \sqrt{2}$?

(A) $-\frac{5}{2}$

(B) $-\frac{3}{2}$

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

3. If $y = 2x\sqrt{x}$ what is an expression for $\frac{dy}{dx}$?

(A) $\frac{dy}{dx} = \sqrt{x}$

(B) $\frac{dy}{dx} = 2\sqrt{x}$

(C) $\frac{dy}{dx} = 3\sqrt{x}$

(D) $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$

4. The equation $2x + 3y + 4 = 0$ makes an angle with the positive x-axis. Which of the following is closest to the size of the angle?

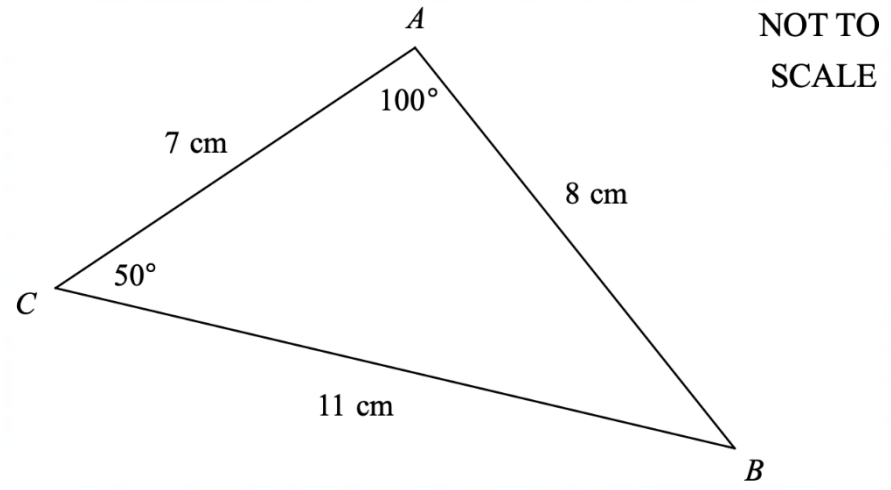
(A) $34°$

(B) $56°$

(C) $124°$

(D) $146°$

5. A triangle ABC is pictured below.



Which of the following is the correct expression for the area of ΔABC?

(A) $A = \frac{1}{2} \times 7 \times 8 \times sin30°$

(B) $A = \frac{1}{2} \times 7 \times 8 \times sin50°$

(C) $A = \frac{1}{2} \times 8 \times 11 \times sin30°$

(D) $A = \frac{1}{2} \times 8 \times 11 \times sin50°$

6. What is the gradient of the tangent to the curve $y = 5 + 2x - x^2$ at the point (–2, –3)?

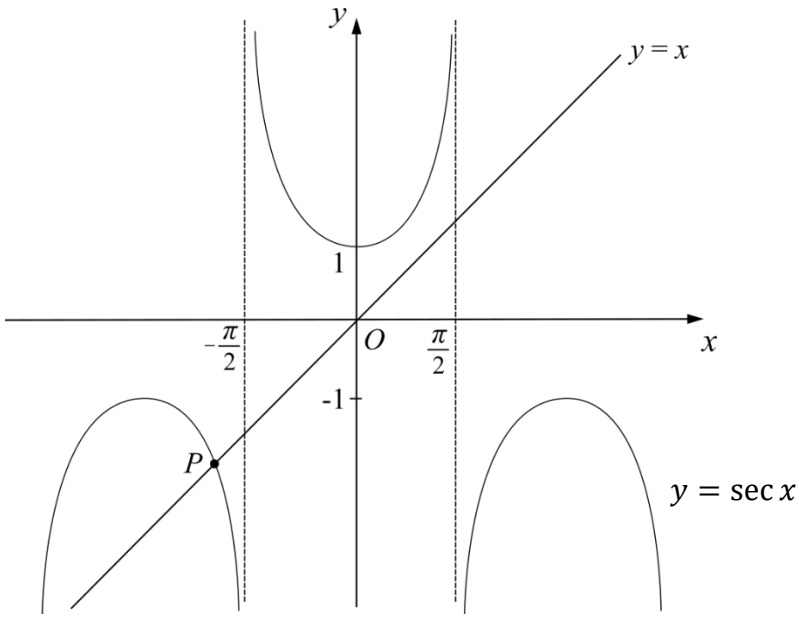
(A) -4

(B) -3

(C) 6

(D) 8

7. The graph of a trigonometric function and linear function are shown below.



Which equation can be used to find the x-coordinate of the point of intersection of the two functions?

(A) $x = \sin x$

(B) $x = \cos x$

(C) $1 = x\cos x$

(D) $1 = x\sin x$

8. If $y = \frac{e^x}{e^x+1}$ which of the following is correct?

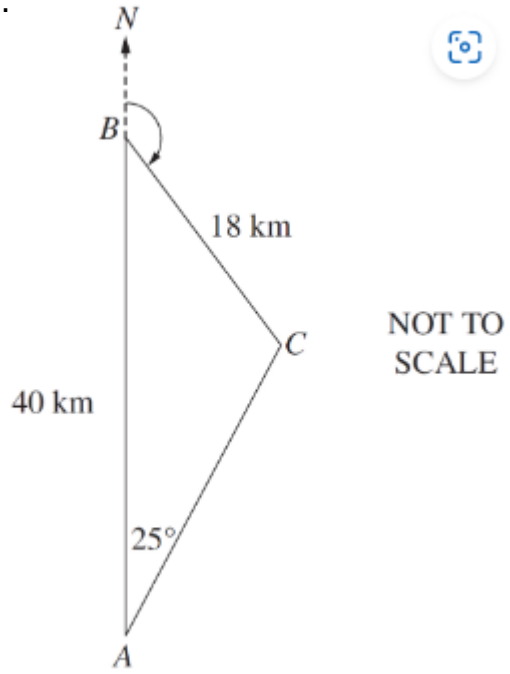
(A) $x = ln\left(\frac{1-y}{y}\right)$

(B) $x = ln\left(\frac{y}{1-y}\right)$

(C) $x = ln\left(\frac{y}{1+y}\right)$

(D) $x = ln\left(\frac{1+y}{y}\right)$

9. The diagram shows towns A, B and C. Town B is 40 km due North of town A. The distance from town B to town C is 18 km and the bearing of town C from town A is 025° .



What is the bearing of town C from town B, given that $\angle BCA$ is obtuse?

(A) 070°

(B) 095°

(C) 110°

(D) 135°

END OF SECTION I

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Mathematics Advanced

Section II Answer Booklet 1

Name:
Teacher:

Section II
70 marks
Attempt Questions 10 – 34
Allow about 1 hour and 45 minutes for this section

Booklet 1 – Attempt Question 10 – 25 (40 marks)
Booklet 2 – Attempt Question 26 – 34 (30 marks)

Instructions

- Write your Name and your Teacher's Name at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 19 and 20 of Booklet 1. If you use this space, clearly indicate which question you are answering.

Booklet 1	
Q10 – Q14	/11
Q15 – Q18	/10
Q19 – Q22	/9
Q23 – Q25	/10

Please turn over

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Question 10 (2 marks)

Solve $2^x = 5$ correct to 3 significant figures. 2

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Question 11 (2 marks)

State the domain and range of $y = 1 - 2x^2$.

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Question 12 (2 marks)

If $f(x) = x^2 + e^{2x}$, find $f'(x)$. 2

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Question 13 (2 marks)

If $\cos\theta = \frac{2}{5}$ and $\sin\theta < 0$, find the value of $\tan\theta$. 2

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Question 14 (3 marks)

A function is given by the formula $f(x) = 2x^2 - 32ax$ where a is a constant.

a) Evaluate $f(-2)$. 1

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b) Find the value of a given that $f'(4) = 0$. 2

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Question 15 (2 marks)

Simplify $\frac{6^n - 4^n}{2^n}$. 2

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Question 16 (3 marks)

Solve $\log_2 x - \log_4 x = 3$. 3

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Question 17 (2 marks)

A function is defined by the rule

$$f(x)=\begin{cases}1-x, & x\le -2\\ -1, & -1<x<1\\ x+1, & x\ge 1\end{cases}$$

Evaluate: $f(2)+f(-2)$. 2

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Question 18 (3 marks)

Solve $2\sin\theta+\dfrac{1}{\sin\theta}=3$ for $[0,2\pi]$ 3

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Question 19 (2 marks)

Differentiate $f(x) = x^2 + 5x$ from first principles. 2

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Question 20 (2 marks)

Differentiate $3e^{5x}(x + 4)$ with respect to x. 2

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Question 21 (2 marks)

Show whether the function $f(x) = \frac{2x}{x^2-1}$ is odd, even or neither. 2

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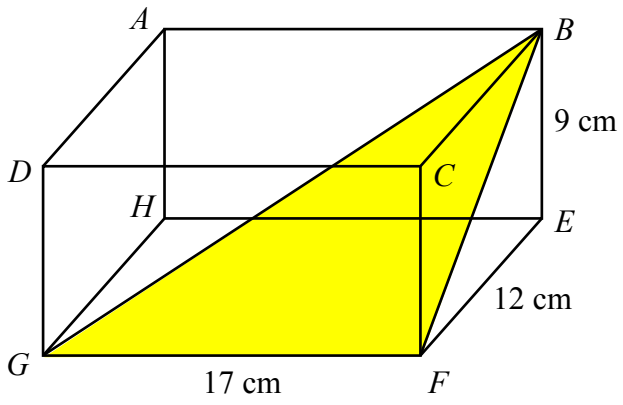
Question 22 (3 marks)

The diagram shows a rectangular prism with dimensions as shown. 3



Find the size of $\angle GBF$ to the nearest degree.

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Question 23 (2 marks)

Differentiate $\sqrt{4-x}$ with respect to x. 2

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Question 24 (5 marks)

A function is defined by $f(x) = \frac{2x-1}{3x+2}$.

a) Find $f(-1)$ 1

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b) Show that $f'(x) = \frac{7}{(3x+2)^2}$ 2

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Question 24 continued.

c) Find the equation of the tangent at the point where $x = -1$. 2

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Question 25 (3 marks)

Show that $\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = 2\sec\theta$. 3

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End of Question 25

Proceed to Booklet 2 for Questions 26-34

Extra writing space.
If you use this space, clearly indicate which question you are answering.

Extra writing space.
If you use this space, clearly indicate which question you are answering.

Mathematics Advanced

Name:

Section II Answer Booklet 2

Teacher:

Section II

Booklet 2 – Attempt Question 26 – 34 (30 marks)

Instructions

- Write your Name and your Teacher's Name at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 30 of Booklet 2. If you use this space, clearly indicate which question you are answering.

Booklet 2	
Q26 – Q28	/10
Q29 – Q31	/10
Q32 – Q34	/10

Please turn over

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Question 26 (3 marks)

Consider the functions $f(x) = x^2 - 4$ and $g(x) = 4 - 2x$. 3

Solve $f\big(g(x)\big) = 0$.

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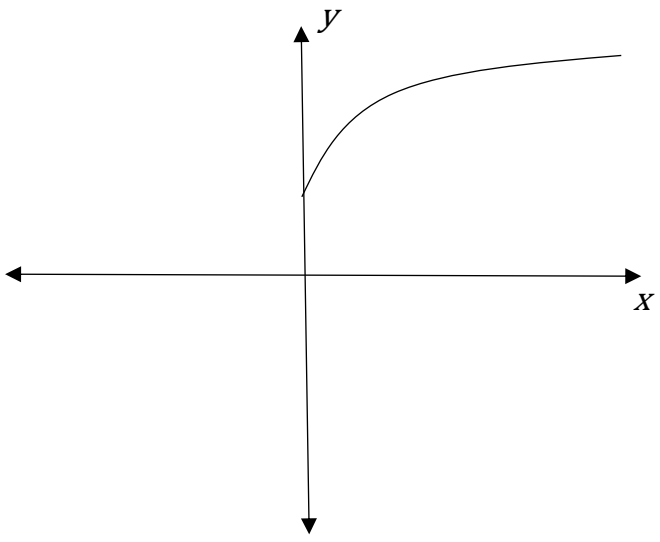
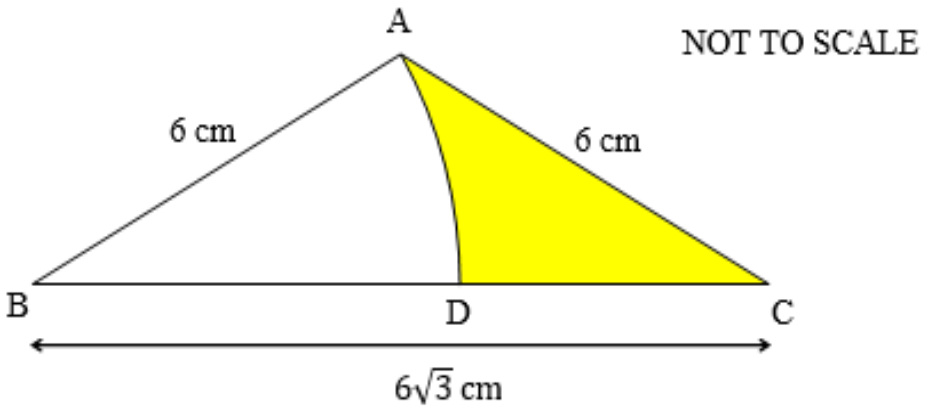
Question 27 (2 marks)

The diagram shows the graph $y = f(x)$. On the same diagram sketch the graph of $y = -f(-x)$. 2



Question 28 (5 marks)

The diagram shows an isosceles triangle ABC with equal sides $AB = AC = 6$ cm and base $BC = 6\sqrt{3}$ cm.
A sector ABD with centre B and a radius AB is inscribed in triangle ABC.



a) Find the size of angle ABC in radians. 2

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b) Find the exact area of the shaded part of the triangle ABC. 3

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Question 29 (4 marks)

A particle moves from rest at a point A in a straight line, so that t seconds after leaving A its displacement, x metres, is given by $x = 16t^2 - 2t^4$.

a) Find what time the particle next comes to rest. 2

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b) Hence, find the acceleration at this time. 2

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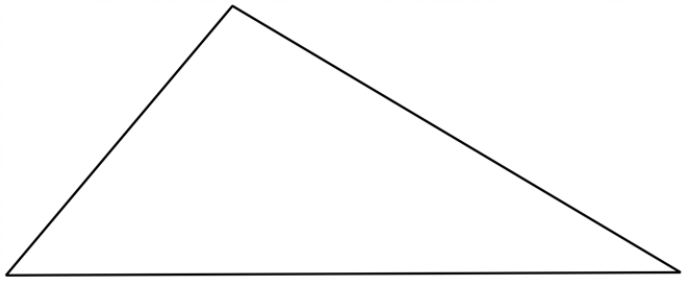
Question 30 (3 marks)

Aria and Ben are flying kites in the local park when their kites collide mid-air. The angles of elevation of Aria and Ben's kites are 50° and 31° respectively. Aria's kite string was 13 metres and Ben's kite string was 19 metres long at the time of impact.

a) Label the above information on the diagram below. 1



Not to scale

b) Use the cosine rule to find distance between Aria and Ben when their kites collide. Give your answer correct to the nearest whole number. 2

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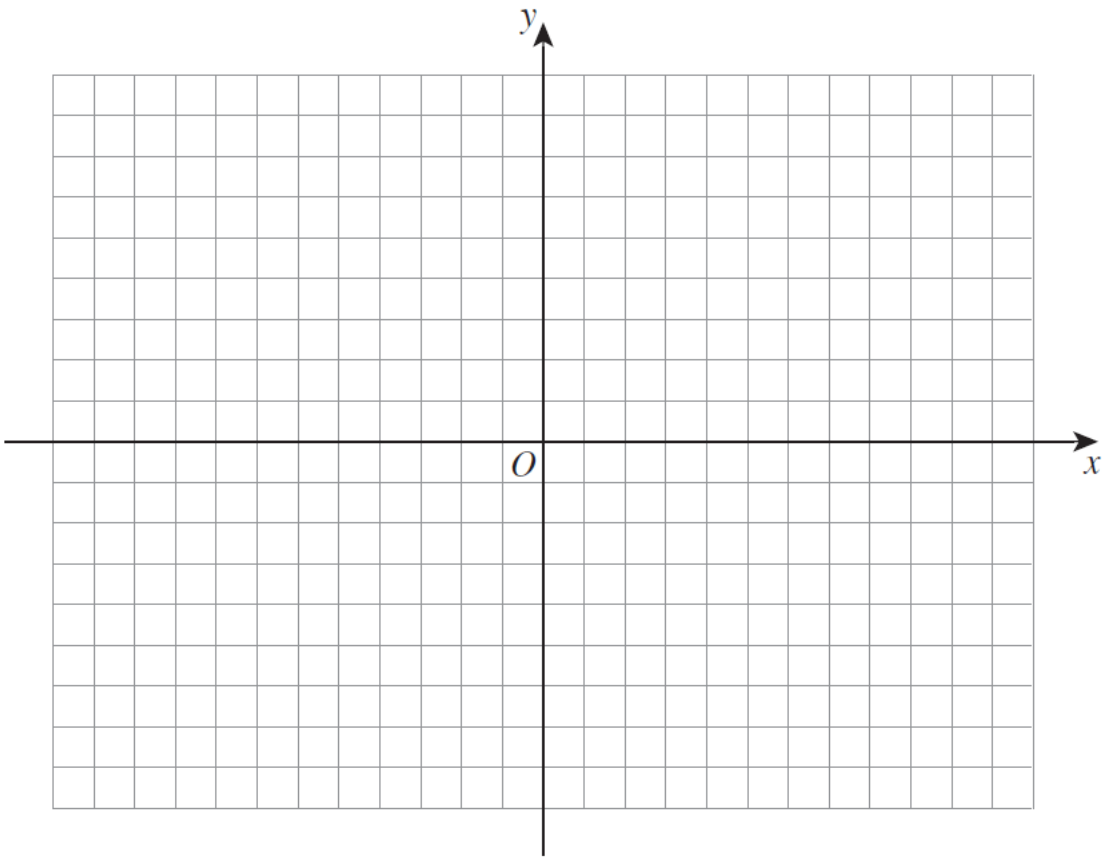
Question 31 (3 marks)

Sketch the graph of $y = \ln x - 2$, labelling the x-intercept and vertical asymptote. 3



Question 32 (3 marks)

Solve the equation $(4e^x - 2)^2 = 4$. 3

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Question 33 (2 marks)

Given $\log_a 3 = x$ and $\log_a 5 = y$, find an expression for $\log_a \frac{25}{3}$ in terms of x and y. 2

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Question 34 (5 marks)

The number of people entering a stadium is approximated by the formula

$$N = 416t^2 - 13t^4 + 88400$$

Where N is the number of people in the stadium and t is the time in hours after 11 a.m. $(0 \leq t \leq 10)$.

a) How many people are in the stadium at 2 p.m.? 1

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Question 34 continued.

b) What is the average rate of people per hour entering the stadium between 11 a.m. and 2 p.m.? 2

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c) Find the rate at which the number of people in the stadium is changing at 2 p.m.? 2

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END OF PAPER

Extra writing space.
If you use this space, clearly indicate which question you are answering.

Name:

Teacher:

Section I

Year 11 Mathematics Advanced

End of Course Examination

2022

Multiple-Choice Answer Sheet - Questions 1 – 9

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct ↙

A ⊗ B ⊗ C ○ D ○

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1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○

Name:	SOLUTIONS
Teacher:	

St George Girls High School

Mathematics Advanced

2022 Year 11 End of Course Examination

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- Attempt Questions 10 – 25 for this section

Booklet 2: 30 marks (pages 21 – 29)

- Attempt Questions 26 – 34 for this section

Q1 – Q9	/9
Q10 – Q14	/11
Q15 – Q18	/10
Q19 – Q22	/9
Q23 – Q25	/10
Q26 – Q28	/10
Q29 – Q31	/10
Q32 – Q34	/10
Total	**/79**
	%

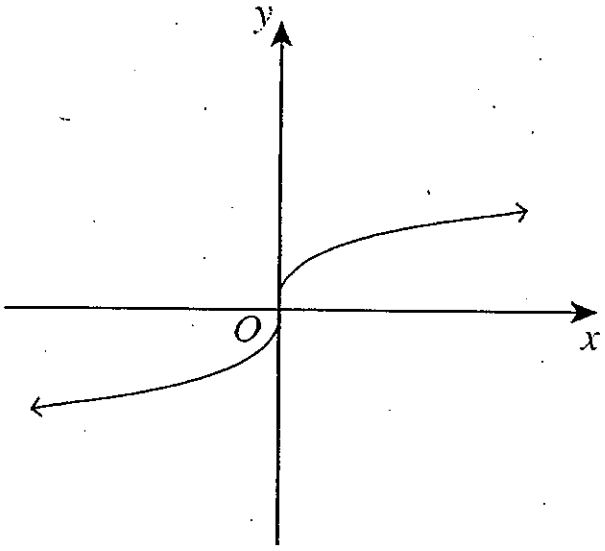
Section I

9 marks
Attempt Questions 1 – 9
Allow about 15 minutes for this section

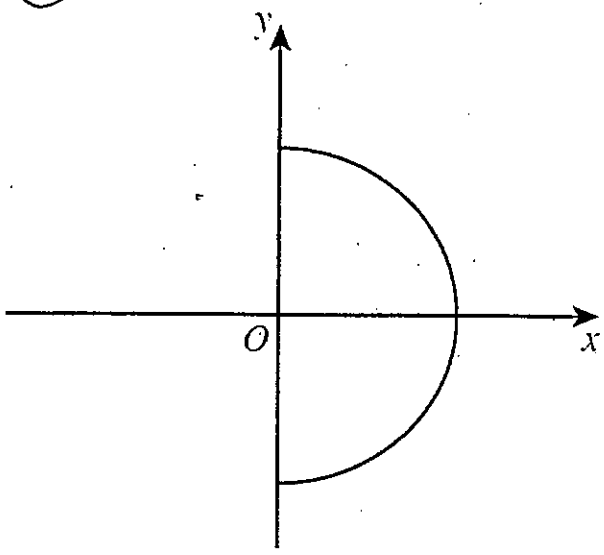
Use the multiple-choice answer sheet provided for Questions 1 to 9.

1. Which of the following graphs represents a one-to-many relationship?

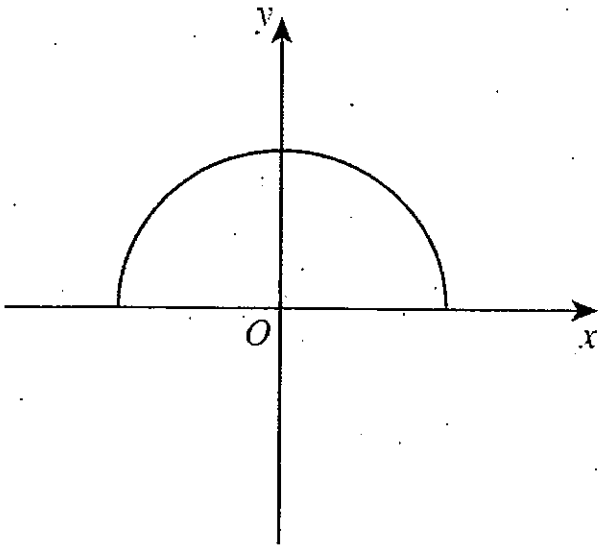
(A)



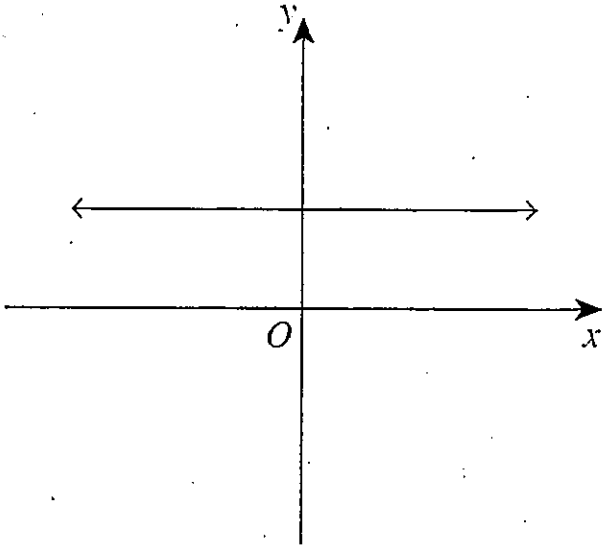
(B)



(C)



(D)



2. What is the value of $\log_2 \frac{1}{4} + \log_2 \sqrt{2}$?

(A) $-\frac{5}{2}$

(B) $-\frac{3}{2}$

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

$\log_2 2^{-2} + \log_2 2^{\frac{1}{2}}$

$= -2\log_2 2 + \frac{1}{2}\log_2 2$

$= -2 + \frac{1}{2}$

$= -1\frac{1}{2}$

3. If $y = 2x\sqrt{x}$ what is an expression for $\frac{dy}{dx}$?

(A) $\frac{dy}{dx} = \sqrt{x}$

(B) $\frac{dy}{dx} = 2\sqrt{x}$

(C) $\frac{dy}{dx} = 3\sqrt{x}$

(D) $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$

$y = 2x^{\frac{3}{2}}$

$= \frac{3}{2} \times 2x^{\frac{1}{2}}$

$= 3x^{\frac{1}{2}}$

4. The equation $2x + 3y + 4 = 0$ makes an angle with the positive x-axis. Which of the following is closest to the size of the angle?

(A) 34°

(B) 56°

(C) 124°

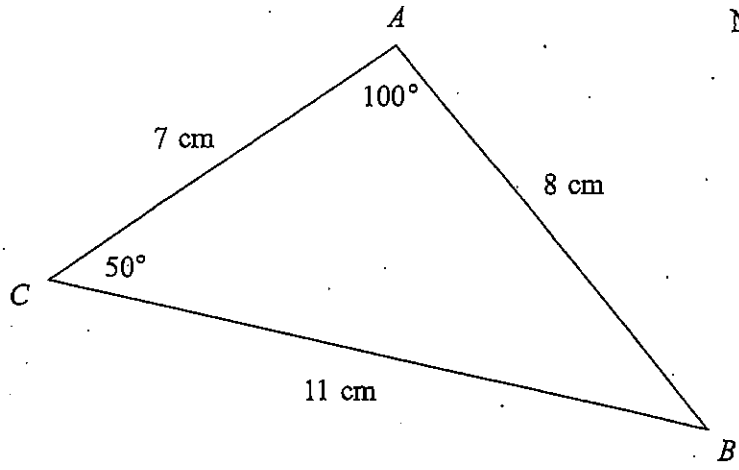
(D) 146°

$y = -\frac{2}{3}x - \frac{4}{3}$

$\tan\theta = -\frac{2}{3}$

$\theta = 146.31°$

5. A triangle ABC is pictured below.



NOT TO SCALE

Which of the following is the correct expression for the area of ΔABC?

(A) $A = \frac{1}{2} \times 7 \times 8 \times sin30°$

(B) $A = \frac{1}{2} \times 7 \times 8 \times sin50°$

(C) $A = \frac{1}{2} \times 8 \times 11 \times sin30°$

(D) $A = \frac{1}{2} \times 8 \times 11 \times sin50°$

$180 - (50 + 100)$
$= 30$

6. What is the gradient of the tangent to the curve $y = 5 + 2x - x^2$ at the point $(-2, -3)$?

(A) -4

(B) -3

(C) 6

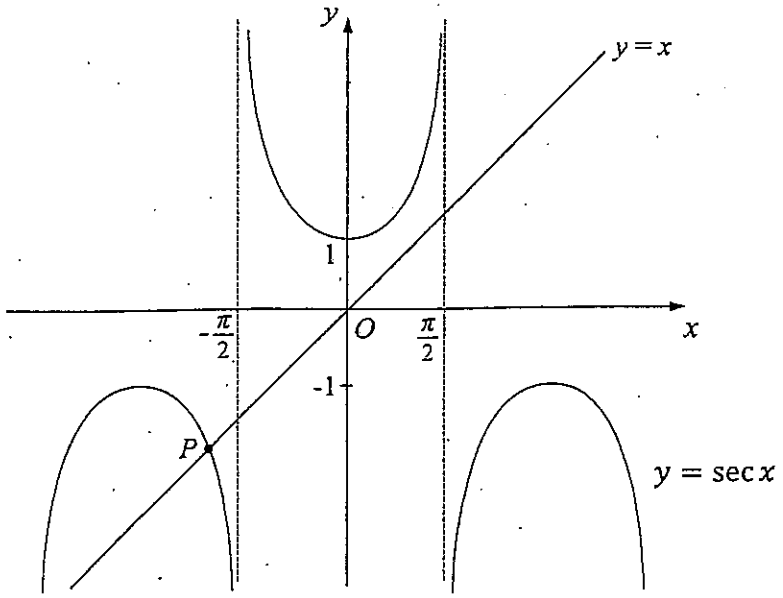
(D) 8

$\frac{dy}{dx} = 2 - 2x$

when $x = -2$

$\frac{dy}{dx} = 2 + 4$
$= 6$

7. The graph of a trigonometric function and linear function are shown below.



Which equation can be used to find the x-coordinate of the point of intersection of the two functions?

(A) $x = \sin x$

(B) $x = \cos x$

(C) $1 = x\cos x$

(D) $1 = x\sin x$

$y = x$

$y = \sec x$

$x = \sec x$

$= \frac{1}{\cos x}$

$x\cos x = 1$

8. If $y = \frac{e^x}{e^x+1}$ which of the following is correct?

(A) $x = ln\left(\frac{1-y}{y}\right)$

(B) $x = ln\left(\frac{y}{1-y}\right)$

(C) $x = ln\left(\frac{y}{1+y}\right)$

(D) $x = ln\left(\frac{1+y}{y}\right)$

$y(e^x+1) = e^x$

$e^x y + y = e^x$

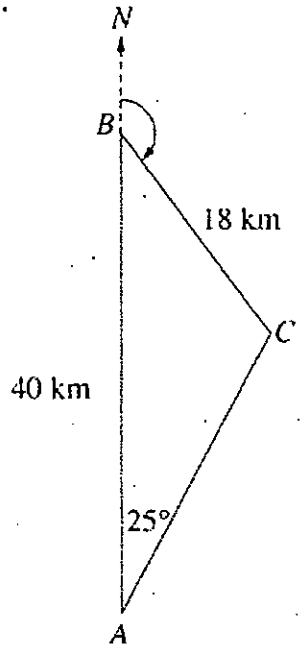
$y = e^x - e^x y$

$= e^x(1-y)$

$\frac{y}{1-y} = e^x$

$x = \ln\left(\frac{y}{1-y}\right)$

9. The diagram shows towns A, B and C. Town B is 40 km due North of town A. The distance from town B to town C is 18 km and the bearing of town C from town A is 025°.



NOT TO SCALE

What is the bearing of town C from town B, given that $\angle BCA$ is obtuse?

(A) 070°

(B) 095°

(C) 110°

(D) 135°

$\frac{\sin 25^\circ}{18} = \frac{\sin C}{40}$

$\sin C = \frac{40 \sin 25^\circ}{18}$

$C = 70^\circ$ or 110°

$\angle ABC = 180 - (110 + 25)$

$= 45^\circ$

$\angle NBC = 135^\circ$

END OF SECTION I

Question 10 (2 marks)

Solve $2^x = 5$ correct to 3 significant figures. 2

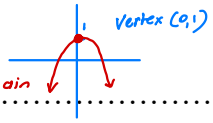
$x = \log_2 5$ (1) Re-write using log law definition

$= \frac{\log_{10} 5}{\log_{10} 2}$

$\doteq 2.32$ (to 3 s.f.) (1) Correct answer

Question 11 (2 marks)

State the domain and range of $y = 1 - 2x^2$. 2



D: $(-\infty, \infty)$ (1) correct domain

R: $(-\infty, 1]$ (1) correct range

(OR) D: all real x (OR) $x \in \mathbb{R}$

R: $y \leq 1$

Question 12 (2 marks)

If $f(x) = x^2 + e^{2x}$, find $f'(x)$. 2

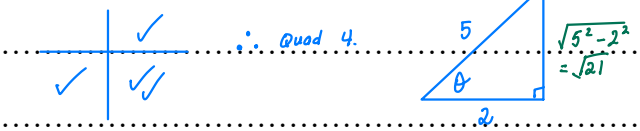
$f'(x) = 2x + 2e^{2x}$ (1) correct differentiation of x^2; correct differentiation of $2e^{2x}$

MATHEMATICS ADVANCED – QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Q10 Some students need to revise significant figures.		
Overall - well done.		
Q11. Students had difficulty with the range.		
Please write from smallest to largest → $(-\infty, 1]$ NOT $[1, -\infty)$		
Q12 Alot of students do not know how to differentiate e^{2x}		
many incorrectly thought it was $2xe^{2x}$ (OR) $2xe^{2x-1}$.		

Question 13 (2 marks)

If $\cos\theta = \frac{2}{5}$ and $\sin\theta < 0$, find the value of $\tan\theta$. 2



$\tan\theta = -\frac{\sqrt{21}}{2}$ ① for correct ratio

① for the negative sign

Question 14 (3 marks)

A function is given by the formula $f(x) = 2x^2 - 32ax$ where a is a constant.

a) Evaluate $f(-2)$. 1

$f(-2) = 2(-2)^2 - 32a(-2)$

$= 8 + 64a$ ① correct answer

b) Find the value of a given that $f'(4) = 0$. 2

$f'(x) = 4x - 32a$ ① correct differentiation

$0 = 4(4) - 32a$

$0 = 16 - 32a$

$32a = 16$

$a = \frac{1}{2}$ ① correct answer

MATHEMATICS ADVANCED – QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Q13 Many students did not take into account the quadrant and they forgot the negative		
Q14 a) Well done - however some students thought it was an equation to solve instead of an expression. Half a mark was taken off.		
b) Many students did not differentiate. A carried over error was awarded if the substitution was correctly done.		

Question 15 (2 marks)

Simplify $\frac{6^n-4^n}{2^n}$. 2

$= \frac{2^n \times 3^n - 2^n \times 2^n}{2^n}$

$= \frac{2^n(3^n-2^n)}{2^n}$ — ① correct factorisation or equivalent

$= 3^n - 2^n$ — ① correct final solution

OR

$\frac{6^n-4^n}{2^n} = \frac{6^n}{2^n} - \frac{4^n}{2^n}$

$= \left(\frac{6}{2}\right)^n - \left(\frac{4}{2}\right)^n$

$= 3^n - 2^n$

Question 16 (3 marks)

Solve $\log_2 x - \log_4 x = 3$. 3

$\log_2 x - \frac{\log_2 x}{\log_2 4} = 3$ — ① change of base or equivalent

$\log_2 x - \frac{1}{2}\log_2 x = 3$

$\frac{1}{2}\log_2 x = 3$ — ①

$\log_2 x = 6$

$x = 2^6$ — ① CFPE applied

$= 64$

OR

$\frac{\log_{10} x}{\log_{10} 2} - \frac{\log_{10} x}{\log_{10} 4} = 3$

$\frac{2\log_{10} x - \log_{10} x}{\log_{10} 4} = 3$

$2\log_{10} x - \log_{10} x = 3\log_{10} 4$

$\log_{10} x = \log_{10} 4^3$

$\therefore x = 4^3$

$= 64$

EXAMINER'S COMMENTS

Question 15

Many students had trouble with this question. Please make sure to review your work on indices and index laws. For example:

$6^n = (2 \times 3)^n$ and $2^{2n} \div 2^n = 2^{2n-n}$

$= 2^n \times 3^n$ $= 2^n$

Use your index laws - don't just cancel out!

Question 16

This question was also poorly attempted by many students.
The simplest way to solve was to change the base of just one of the logarithms and solve accordingly.

Students need to ensure that they are fluent with using logarithms as it is an important concept for the HSC course. Make sure to review your log laws!

1 mark was awarded for a bald answer $x = 64$ with no relevant working shown.

Question 17 (2 marks)

A function is defined by the rule

$$f(x)=\begin{cases}1-x, & x\le -2\\ -1, & -1<x<1\\ x+1, & x\ge 1\end{cases}$$

Evaluate: $f(2)+f(-2)$. 2

$f(2) = 2+1$ $\quad f(-2) = 1-(-2)$

$= 3$ $\quad = 3$ — ① correct use of function notation to find $f(2)$ and $f(-2)$

$\therefore f(2)+f(-2) = 3+3$

$= 6$ — ① correct final solution

Question 18 (3 marks)

Solve $2\sin\theta + \dfrac{1}{\sin\theta} = 3$ for $[0, 2\pi]$ 3

$2\sin^2\theta + 1 = 3\sin\theta$

$2\sin^2\theta - 3\sin\theta + 1 = 0$

Let $u = \sin\theta$,

$2u^2 - 3u + 1 = 0$

$(2u-1)(u-1) = 0$ — ① correct factorisation of quadratic equation

$\therefore u = \frac{1}{2}$ or $u = 1$

$\therefore \sin\theta = \frac{1}{2}$ or $\sin\theta = 1$

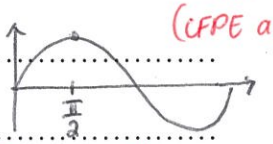
when $\sin\theta = \frac{1}{2}$,

$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$= \frac{\pi}{6}, \frac{5\pi}{6}$ — ① correct solutions in radians (CFPE awarded)

when $\sin\theta = 1$,

$\theta = \frac{\pi}{2}$



① correct solution in radians (CFPE awarded)

$\therefore \theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

EXAMINER'S COMMENTS

Question 17

This question was answered well by majority of students.

Question 18

many students did not know how to solve an equation written in this form or were unable to correctly arrive at the quadratic equation.

Pay attention to the domain given in the question - $[0, 2\pi]$ means that your answer(s) need to be in radians.

$\frac{1}{2}$ mark was deducted if answers given in degrees.

A sketch of $y = \sin\theta$ should be drawn to solve $\sin\theta = 1$.

$\frac{1}{2}$ mark was deducted for every extra solution given.

Question 19 (2 marks)

Differentiate $f(x) = x^2 + 5x$ from first principles. 2

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$ (1/2)

$= \lim_{h\to 0} \dfrac{(x+h)^2 + 5(x+h) - (x^2+5x)}{h}$

$= \lim_{h\to 0} \dfrac{x^2+2xh+h^2+5x+5h-x^2-5x}{h}$ (1)

$= \lim_{h\to 0} \dfrac{2xh+h^2+5h}{h}$

$= \lim_{h\to 0} \dfrac{h(2x+h+5)}{h}$

$= \lim_{h\to 0} 2x+h+5$

$= 2x+5$ (1/2)

Question 20 (2 marks)

Differentiate $3e^{5x}(x+4)$ with respect to x. 2

u v

Let $u = 3e^{5x}$ $v = x+4$

$u' = \dfrac{du}{dx} = 15e^{5x}$ $v' = \dfrac{dv}{dx} = 1$

$\dfrac{dy}{dx} = vu' + uv'$ (1)

$= (x+4) \times 15e^{5x} + 3e^{5x} \times 1$ (1)

$= 15e^{5x}(x+4) + 3e^{5x}$

EXAMINER'S COMMENTS

Q19

- Students need to include the limit notation when differentiating from first principles and many either did not include it or had it in the wrong place.

  2 marks – Complete correct solution

  1½ mark – Correct solution without limit sign

  1 mark – With some correct expanding and simplifying.

- Some student gave the derivative of $x^2 + 5x$ without differentiating from first principles – Zero marks

Q20

2 marks – Applying each part of the product rule correctly

1 mark – Getting either vu' or uv' correct.
i.e., $(x+4) \times 15e^{5x}$ or $3e^{5x} \times 1$

Many students were able to use the formula to the product rule well, but there were a handful who made subsequent errors as they did not factorise correctly.

Correct way: $\frac{dy}{dx} = 15e^{5x}(x+4) + 3e^{5x}$

$= 3e^{5x}[5(x+4) + 1]$

$= 3e^{5x}[5x + 20 + 1]$

$= 3e^{5x}(5x + 21)$

No penalty for subsequent error.

Question 21 (2 marks)

Show whether the function $f(x) = \frac{2x}{x^2-1}$ is odd, even or neither. 2

$f(x) = \frac{2x}{x^2-1}$

$f(-x) = \frac{2(-x)}{(-x)^2-1}$ ——— ½

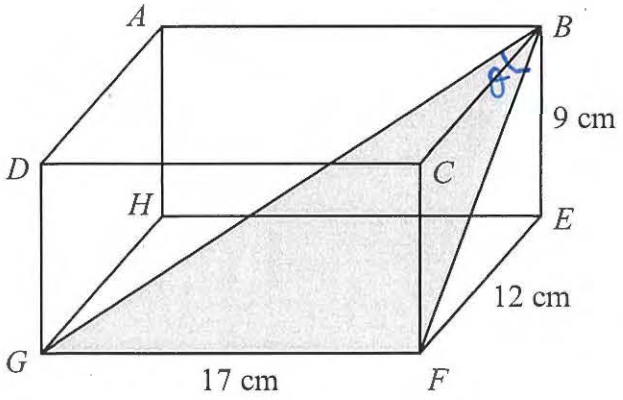
$= \frac{-2x}{x^2-1}$ ——— ½

$= -f(x)$

Since $f(-x) = -f(x)$, $f(x)$ is an odd function } 1

Question 22 (3 marks)

The diagram shows a rectangular prism with dimensions as shown. 3



Find the size of $\angle GBF$ to the nearest degree.

As $\triangle BEF$ is a right-angled triangle,

$BF^2 = 9^2 + 12^2$ (Pythagoras Thm)

$= 81 + 144$

$BF = \sqrt{225} = 15$ ——— 1

Let $\angle GBF = \theta$, $\tan\theta = \frac{17}{15}$ ——— 1

$\theta = 48.57633\ldots$

$= 49°$ (to the nearest degree) } 1

EXAMINER'S COMMENTS

Q21

- As it is a SHOW question many students skipped the step of substituting $-x$ in $f(x)$, so they were not given that ½ mark.
- Some students started the proof with what they were asked to prove, which is not correct.

They started with $f(-x) = -f(x)$ then said

$$\frac{2(-x)}{-x^2-1} = \frac{-2x}{x^2-1}$$

Please DO NOT show whether a function is odd, even or neither this way.

Q22

Many students answered it the following way:

$GB = \sqrt{15^2+17^2}$, $BF = 15$ — (1)

$= \sqrt{514}$ — (½)

Using the cosine rule,

$$\cos\angle GBF = \frac{15^2 + (\sqrt{514})^2 - 17^2}{2\times 15 \times \sqrt{514}}$$ — (1)

$=$

$= 48.57633....$ — (½)

$= 49°$ (to the nearest degree)

The marking scale was slightly different to that suggested in the previous page.

Note: No penalty mark for incorrect rounding, but please take care.

Question 23 (2 marks)

Differentiate $\sqrt{4-x}$ with respect to x. 2

Let $f(x) = (4-x)^{\frac{1}{2}}$

$f'(x) = -\frac{1}{2}(4-x)^{-\frac{1}{2}}$

$= \frac{-1}{2\sqrt{4-x}}$

Question 24 (5 marks)

A function is defined by $f(x) = \frac{2x-1}{3x+2}$.

a) Find $f(-1)$ 1

$f(-1) = \frac{2\times(-1)-1}{3\times(-1)+2}$

$= \frac{-3}{-1}$

$= 3$

b) Show that $f'(x) = \frac{7}{(3x+2)^2}$ 2

$f(x) = \frac{2x-1}{3x+2}$ let $u = 2x-1$ $v = 3x+2$

$\frac{du}{dx} = 2$ $\frac{dv}{dx} = 3$

$f'(x) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{(3x+2)\times 2 - (2x-1)\times 3}{(3x+2)^2}$

$= \frac{6x+4-6x+3}{(3x+2)^2}$

$= \frac{7}{(3x+2)^2}$

EXAMINER'S COMMENTS

Q 23 1 mark for having $-\frac{1}{2}$ and 1 mark for $(4-x)^{-\frac{1}{2}}$

Some students did not multiply by -1 from the derivative of $4-x$. Otherwise done well by most.

Q 24 a) The majority did this well

b) Not all students showed enough working.

Other students were confused as to where to start.

Question 24 continued.

c) Find the equation of the tangent at the point where $x = -1$. 2

$f'(-1) = \frac{7}{(3+(1)+2)^2}$

$= 7$

at $(-1, 3)$ from a)

$y - 3 = 7(x+1)$

$= 7x + 7$

$7x - y + 10 = 0$

Question 25 (3 marks)

Show that $\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = 2\sec\theta$. 3

$LHS = \frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta}$

$= \frac{\cos^2\theta + (1+\sin\theta)^2}{\cos\theta(1+\sin\theta)}$

$= \frac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)}$

$= \frac{2 + 2\sin\theta}{\cos\theta(1+\sin\theta)}$ $\cos^2\theta + \sin^2\theta = 1$

$= \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$

$= \frac{2}{\cos\theta}$

$= 2\sec\theta$

$= RHS$

End of Question 25

Proceed to Booklet 2 for Questions 26-34

EXAMINER'S COMMENTS

Q24 c) 1 mark for finding the gradient

1 mark for using $(-1, 3)$ and $m = 7$ to find the equation of the line.

Done well by most students

Q25 1 mark for setting up LHS = RHS

1 mark for reasonable progress

1 mark for complete solution.

Most students did this well

Question 26 (3 marks)

Consider the functions $f(x) = x^2 - 4$ and $g(x) = 4 - 2x$. 3

Solve $f(g(x)) = 0$.

$f(g(x)) = (4-2x)^2 - 4$

$= 16 - 16x + 4x^2 - 4$

$= 4x^2 - 16x + 12$ 1 MARK.

As $f(g(x)) = 0$ $\quad 4x^2 - 16x + 12 = 0$

$x^2 - 4x + 3 = 0$

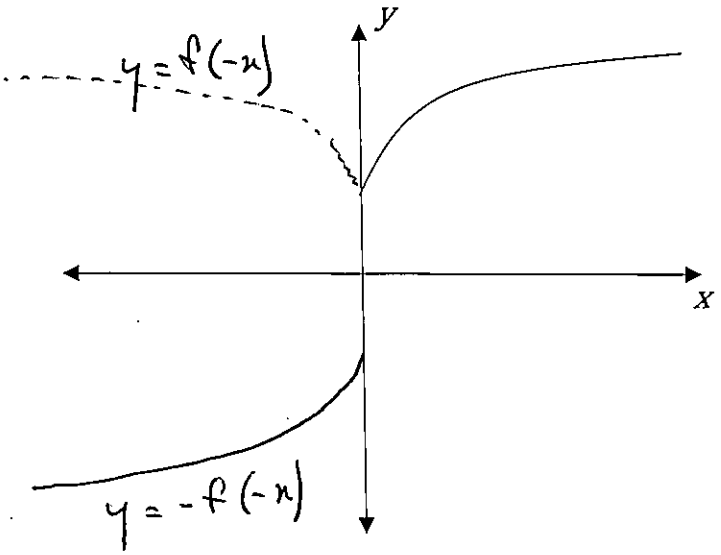
$(x-1)(x-3) = 0$ 1 MARK

$x = 1$ or 3 1 MARK.

Could also use $(4-2x)^2 - 4 = [4-2x-2][4-2x+4]$

Question 27 (2 marks)

The diagram shows the graph $y = f(x)$. On the same diagram sketch the graph of $y = -f(-x)$. 2



1 MARK for correct shape

1 MARK for correct y intercept

EXAMINER'S COMMENTS

Q26

Mostly well done, alternative methods available. Some students made basic expanding or algebraic errors.

Q27

Care needs to be given to drawing neat and accurate diagrams.

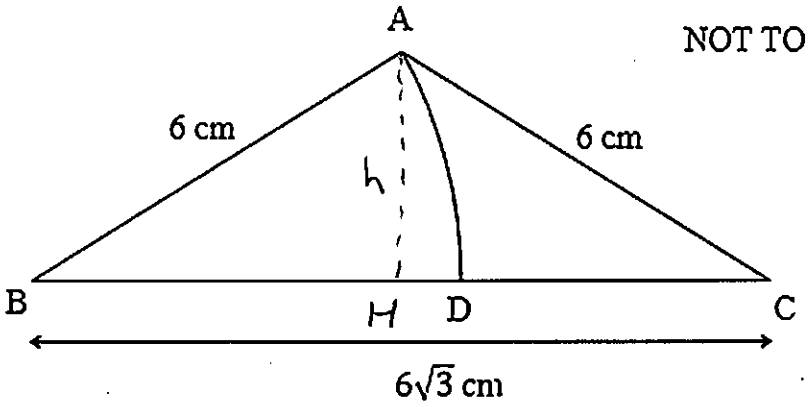
Some students did not reflect in ~~y~~ x axis using line parallel to x axis through y intercept.

Also graph should not have positive gradient for tangent at left hand end.

~~Q28~~

Question 28 (5 marks)

The diagram shows an isosceles triangle ABC with equal sides $AB = AC = 6$ cm and base $BC = 6\sqrt{3}$ cm.
A sector ABD with centre B and a radius AB is inscribed in triangle ABC.



a) Find the size of angle ABC in radians. 2

Method 1 Draw $AH \perp BC$

$\cos \angle ABC = \frac{3\sqrt{3}}{6}$

(as $\triangle ABC$ is isosceles $BH = HC$)

$\cos \angle ABC = \frac{\sqrt{3}}{2}$

$\angle ABC = \frac{\pi}{6}$ 1 MARK.

Method 2

$\cos \angle ABC = \frac{6^2 + (6\sqrt{3})^2 - 6^2}{2(6)(6\sqrt{3})}$ 1 MARK

$= \frac{108}{72\sqrt{3}}$

$= \frac{3}{2\sqrt{3}}$

$= \frac{\sqrt{3}}{2}$

$\angle ABC = \frac{\pi}{6}$ (decimal approx. $\frac{1}{2}$)

b) Find the exact area of the shaded part of the triangle ABC. 3

Area $\triangle ABC = \frac{1}{2} \times 6\sqrt{3} \times 6 \times \sin\frac{\pi}{6}$ or $2\left(\frac{1}{2} \times 3\sqrt{3} \times 3\right)$ 1 MARK.

$= 18\sqrt{3} \times \frac{1}{2}$ $= 9\sqrt{3}$

$= 9\sqrt{3}$

Area of sector $= \frac{\frac{\pi}{6}}{2\pi} \times \pi(6)^2$ or $\frac{1}{2}(6)^2\left(\frac{\pi}{6}\right)$ 1 MARK.

$= 3\pi$ $= 3\pi$

Shaded area $= (9\sqrt{3} - 3\pi)$ cm^2 1 MARK

EXAMINER'S COMMENTS

Q 28

a) Many students did not recognise the connection with ΔABC and the exact value triangle for $30°$ and $60°$.

A small number either did know the cosine rule or could not substitute into it.

0·5235... is an approximation for $\frac{\pi}{6}$ and only gained $1\frac{1}{2}$ marks.

b) Many students did not seem to know if the required area formula contained a "sin" or not.

Answer needed to be fully simplified in exact form.

EXAMINER'S COMMENTS

Q26

Mostly well done, alternative methods available. Some students made basic expanding or algebraic errors.

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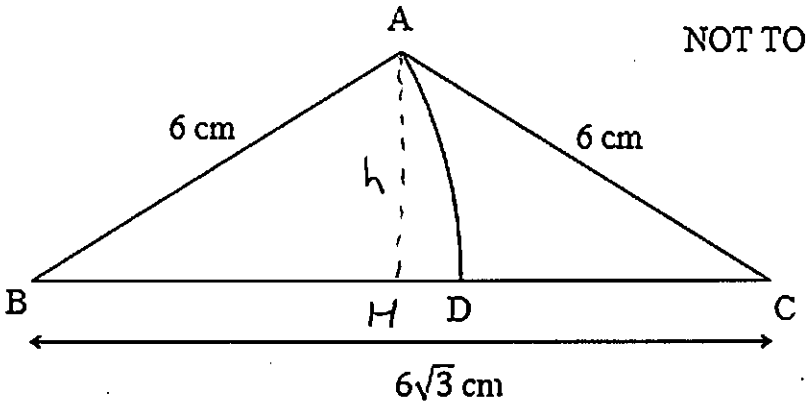

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(as $\triangle ABC$ is isosceles $BH = HC$)

$\cos \angle ABC = \frac{\sqrt{3}}{2}$

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$\cos \angle ABC = \frac{6^2 + (6\sqrt{3})^2 - 6^2}{2(6)(6\sqrt{3})}$ 1 MARK

$= \frac{108}{72\sqrt{3}}$

$= \frac{3}{2\sqrt{3}}$

$= \frac{\sqrt{3}}{2}$

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$= 18\sqrt{3} \times \frac{1}{2}$ $= 9\sqrt{3}$

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$= 3\pi$ $= 3\pi$

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EXAMINER'S COMMENTS

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A small number either did know the cosine rule or could not substitute into it.

0·5235... is an approximation for $\frac{\pi}{6}$ and only gained $1\frac{1}{2}$ marks.

b) Many students did not seem to know if the required area formula contained a "sin" or not.

Answer needed to be fully simplified in exact form.

Question 29 (4 marks)

A particle moves from rest at a point A in a straight line, so that t seconds after leaving A its displacement, x metres, is given by $x = 16t^2 - 2t^4$.

a) Find what time the particle next comes to rest. 2

$\frac{dx}{dt} = 32t - 8t^3$ 1 mk for differentiating correctly and letting the expression =0

$= 8t(4 - t^2)$

$= 8t(2-t)(2+t)$

For the particle to be at rest $\frac{dx}{dt} = 0$

$8t = 0$ $2 - t = 0$ $2 + t = 0$

$t = 0$ $t = 2$ $t = -2$ But $t \geq 0$ 1 mk for $t = 2$ seconds

$t = 2s$ is the only possible solution as the question is asking about the next time it will come to rest.

b) Hence, find the acceleration at this time. 2

$\frac{d^2x}{dt^2} = 32 - 24t^2$ 1 mk for the correct expression for $\frac{d^2x}{dt^2}$

When $t = 2$

$= 32 - 24(2)^2$

$= 32 - 96$

$= -64 m s^{-2}$ or $64 m/s^2$ 1 mk for the final answer.

EXAMINER'S COMMENTS

Q29 a) Poorly attempted as some students lost solutions when dividing by t. Please avoid dividing by a variable when solving equations. A common working out was:

$8t^3 = 32t \quad (\div 8t)$

$t^2 = 4$

$t = \pm 2$

→ What if t was 0, then division by 0 is not possible

This is INCORRECT.

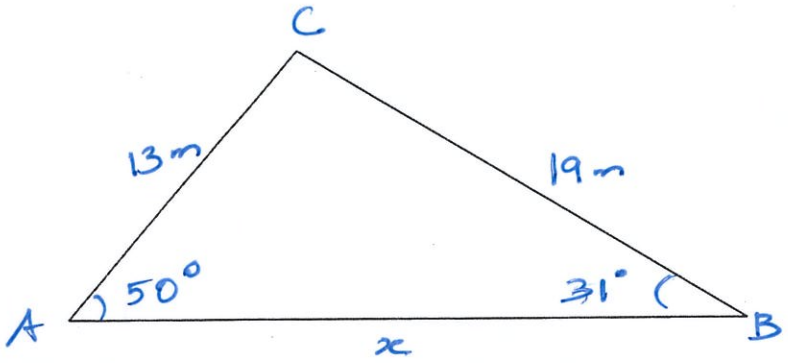
Consequently, a number of students did not have $t=0$ as one of the 'possible solutions'. Also, you need to justify why $t=-2$ is not a solution (as $t \geq 0$). Remember that t can be 0 but in this question $t > 0$ only. (Find what time the particle next comes to rest).

b) Quite well attempted. Students need to avoid making calculational errors and also remember to write down the units of measurement.

Question 30 (3 marks)

Aria and Ben are flying kites in the local park when their kites collide mid-air. The angles of elevation of Aria and Ben's kites are 50° and 31° respectively. Aria's kite string was 13 metres and Ben's kite string was 19 metres long at the time of impact.

a) Label the above information on the diagram below. 1



Not to scale

No $\frac{1}{2}$ marks were allocated for this question.

b) Use the cosine rule to find distance between Aria and Ben when their kites collide. Give your answer correct to the nearest whole number. 2

$\angle ACB = 99°$ $(180° - (50° + 31°))$

$x^2 = 13^2 + 19^2 - 2 \times 13 \times 19 \cos 99°$ — 1 mk for appropriate substitution into the formula

$= 607.2786257...$

$= 24.64302388...$

≈ 25 m (to the nearest whole number)

↳ 1 mk for the correct solution

EXAMINER'S COMMENTS

30 a) Quite well attempted except that a few students had the lengths of the strings in an incorrect position.

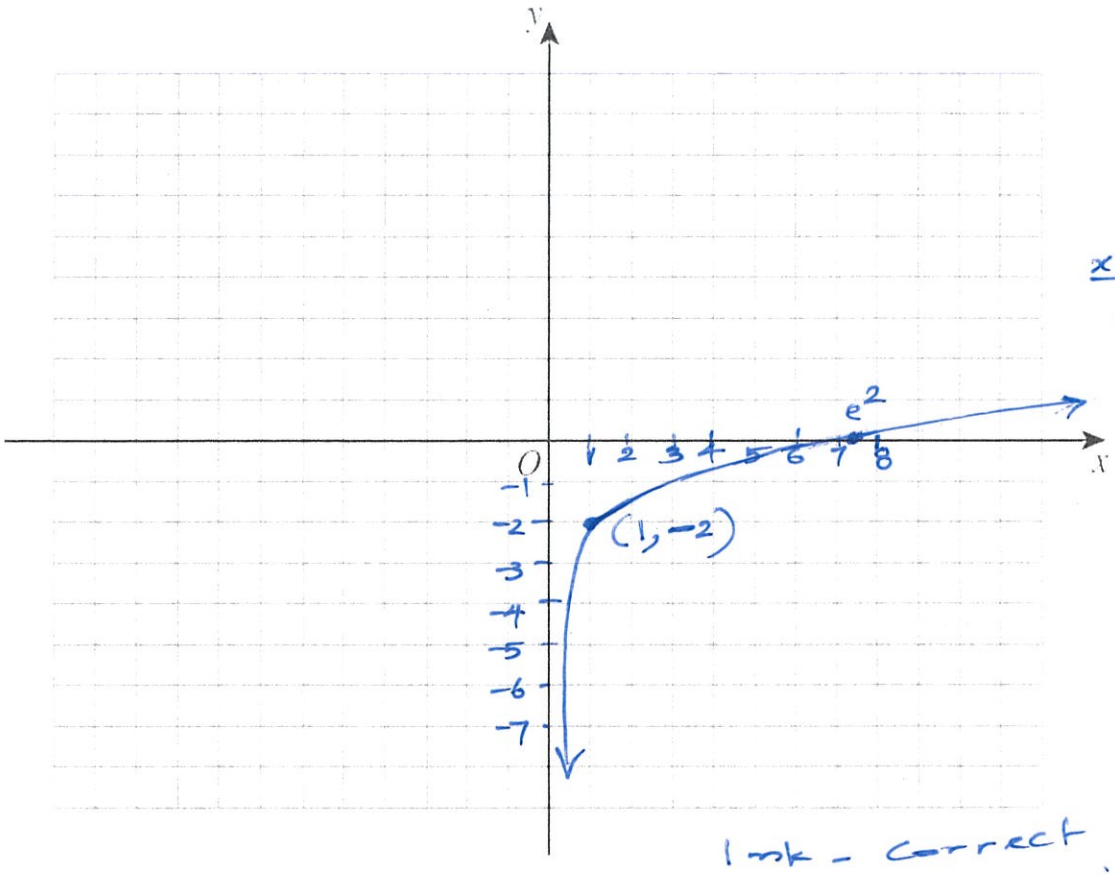
b) Generally, well done. A few students used sine rule rather than cosine rule and so were not awarded any marks as the question explicitely required you to use cosine rule rather than sine rule.

A few students forgot to take the square root of 607.2786257...

Again, a few students made calculational errors.

Question 31 (3 marks)

Sketch the graph of $y = \ln x - 2$, labelling the x-intercept and vertical asymptote. 3



x - intercept,
let $y=0$

$y = \ln x - 2$

$0 = \ln x - 2$

$\ln x = 2$

$x = e^2$

$(7.3890\ldots)$

1 mk - correct shape
1 mk - displaying (1, −2) correctly.
1 mk - x - intercept at e^2 ($\simeq 7.4$) and plotting it accurately on the number plane

Question 32 (3 marks)

Solve the equation $(4e^x - 2)^2 = 4$. 3

$(4e^x - 2)^2 = 4$

$4e^x - 2 = \pm 2$

$4e^x = 4$ or 0

$4e^x = 4$ $\quad$ $4e^x = 0$

$e^x = 1$ $\quad$ $e^x = 0$

$x = 0$ $\quad$ no solution

$\therefore x = 0$

EXAMINER'S COMMENTS

Q31. Poorly attempted. Students need to practise drawing appropriate scale diagrams. Also, the graph should have approached the y-axis as $x \to 0^+$. A smooth curve should have been drawn. A few students did not know that $e^2 \simeq 7.4$ and so did not plot it accurately on the number plane, hence lost 1 mark.

3 marks Complete solution
2 marks Finds one solution for x
1 mark Significant progress

Many students presented poor solutions to this question. Whenever you reject a branch of your working, be clear about why you are doing so. Many students showed no consideration of the negative case. Also, no matter how bad things get, don't put "math error" as your answer.

Question 33 (2 marks)

Given $\log_a 3 = x$ and $\log_a 5 = y$, find an expression for $\log_a \frac{25}{3}$ in terms of x and y. 2

$\log_a \frac{25}{3} = \log_a 25 - \log_a 3$

$= \log_a 5^2 - \log_a 3$

$= 2\log_a 5 - \log_a 3$

$= 2y - x$

Question 34 (5 marks)

The number of people entering a stadium is approximated by the formula

$$N = 416t^2 - 13t^4 + 88400$$

Where N is the number of people in the stadium and t is the time in hours after 11 a.m. $(0 \leq t \leq 10)$.

a) How many people are in the stadium at 2 p.m.? 1

When $t = 3$, $N = 416(3)^2 - 13(3)^4 + 88400$

$= 91091$

EXAMINER'S COMMENTS	
2 marks	Full solution
1 mark	Progress to simplifying $\log_a \frac{25}{3}$ by clearly applying a useful log law

Question 34 continued.

b) What is the average rate of people per hour entering the stadium between 11 a.m. and 2 p.m.? 2

$N(0) = 88400$

$N(3) = 91091$

$\therefore$ average rate $= \dfrac{91091 - 88400}{3}$

$= 897$ people/hour

c) Find the rate at which the number of people in the stadium is changing at 2 p.m.? 2

$N = 416t^2 - 13t^4 + 88400$

$\therefore \dfrac{dN}{dt} = 832t - 52t^3$

when $t = 3$, $\dfrac{dN}{dt} = 832(3) - 52(3)^3$

$= 1092$ people/hour

END OF PAPER

EXAMINER'S COMMENTS

2 marks Full solution

1 mark Finds 88400

or Correct expression for an average rate of change

Many students did not show understanding of what was meant by "average rate".

2 marks Full solution

1 mark Correctly finds $\frac{dN}{dt} = 832t - 52t^3$

The dash notation for derivatives (e.g. y') refers to a derivative with respect to x. It is not appropriate to use in this context (i.e. don't write the derivative as N')