Name:
Teacher:

St George Girls High School

Mathematics Advanced

2023 Year 11 End of Course Examination

General Instructions

- Reading time – 10 minutes.
- Working time – 2 hours.
- Write using black pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- For questions in **Section I**, use the Multiple-Choice answer sheet provided.
  For questions in **Section II:**
  - Answer the questions in the 2 booklets provided.
  - Show relevant mathematical reasoning and/or calculations.
  - Marks may not be awarded for incomplete or poorly presented solutions, or where multiple solutions are provided.

Total marks: 74

Section I – 5 marks (pages 3 – 4)

- Attempt Questions 1 – 5
- Allow about 8 minutes for this section

Section II – Allow about 1 hour and 52 minutes

Booklet 1: 35 marks (pages 5 – 14)

- Attempt Questions 6 – 18 for this section

Booklet 2: 34 marks (pages 17 – 25)

- Attempt Questions 19 – 26 for this section

Q1 – Q5	/5
Q6 – Q10	/9
Q11 – Q13	/8
Q14 – Q16	/9
Q17 – Q18	/9
Q19 – Q21	/13
Q22 – Q24	/10
Q25 – Q26	/11
Total	**/74**
	%

Section I

6 marks
Attempt Questions 1 – 6
Allow about 10 minutes for this section

Use the multiple-choice answer sheet provided for Questions 1 to 6.

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1. Simplify $\frac{\sin(90°-\theta)}{\cos(90°-\theta)}$.

   (A) 1

   (B) $\cos\theta$

   (C) $\tan\theta$

   (D) $\cot\theta$

2. If $4x + 3(x-2) = x(2-x)$ with $x > 0$ then x is equal to:

   (A) 1

   (B) $1 \pm \sqrt{5}$

   (C) $1 + \sqrt{5}$

   (D) -1

3. What is 0.5 radians in degrees, when rounded to the nearest degree?

   (A) $29°$

   (B) $45°$

   (C) $50°$

   (D) $90°$

4. What is the value of x if $\log_3(x - 30) = 4$?

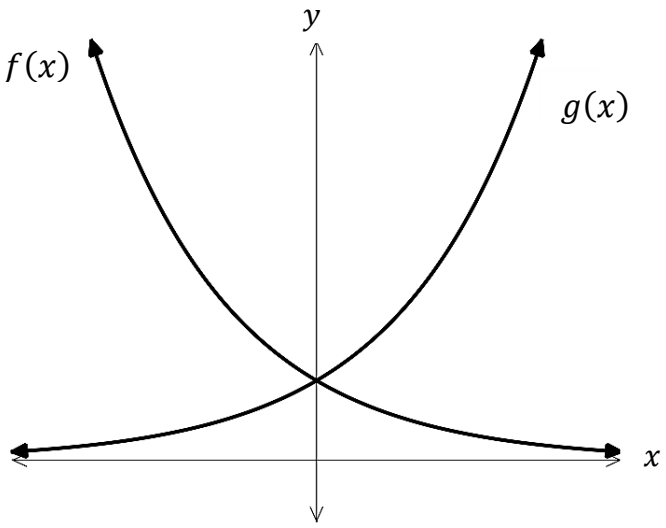
   (A) 152

   (B) 111

   (C) 81

   (D) 51

5. The diagram shows the graphs of $f(x) = a^x$ and $g(x) = b^x$, and $f(x)$ is a reflection of $g(x)$ along the y-axis.



Which of the following are true?

I. $0 < a < 1$

II. $b > 1$

III. $ab = 1$

(A) I and II

(B) I and III

(C) II and III

(D) I, II and III

END OF SECTION I

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Mathematics Advanced

Section II Answer Booklet 1

Name:
Teacher:

Section II
69 marks
Attempt Questions 6 – 26
Allow about 1 hour and 45 minutes for this section

Booklet 1 – Attempt Question 6 – 18 (35 marks)
Booklet 2 – Attempt Question 19 – 26 (34 marks)

Instructions

- Write your Name and your Teacher's Name at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 15 and 16 of Booklet 1. If you use this space, clearly indicate which question you are answering.

Booklet 1	
Q6 – Q10	/9
Q11 – Q13	/8
Q14 – Q16	/9
Q17 – Q18	/9

Question 6 (2 marks)

Find the values of x for which $|2x - 1| = 3$. 2

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Question 7 (2 marks)

If $11\sqrt{3} - 2\sqrt{27} = \sqrt{p}$, find the value of p. 2

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Question 8 (1 marks)

Find $\frac{dy}{dx}$, if $y = 6x^2 - 5x - 1$. 1

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Question 9 (2 marks)

Let $x^2 + 6x - 3 = (x - a)^2 + b$ where a and b are constants.

Find the value of a and b. 2

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Question 10 (2 marks)

Solve for x: $2^{x-3} + 2 = 7$, leave your answer correct to 2 decimal places. 2

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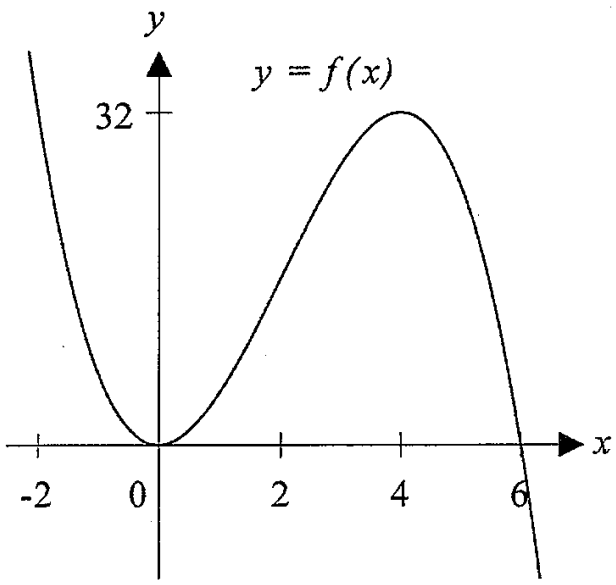
Question 11 (3 marks)

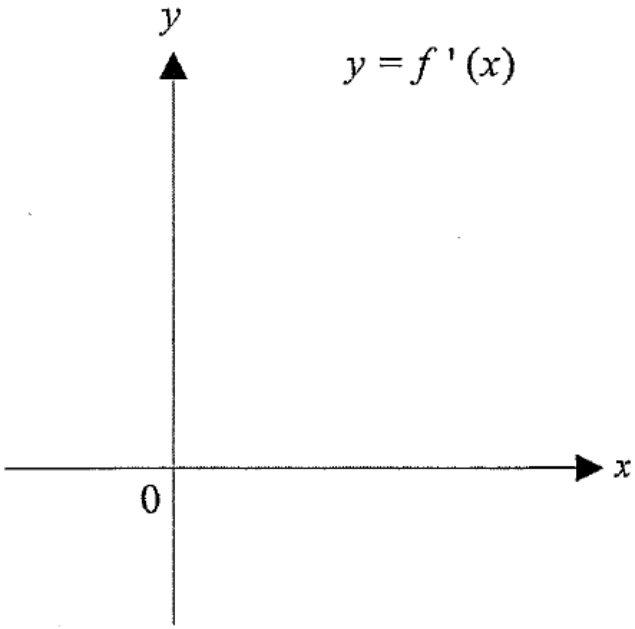
Without using calculus, sketch the following polynomial, clearly showing the x and y −intercepts. 3

$$P(x) = (x + 1)(3 - x)^2$$

Question 12 (2 marks)

The diagram shows the graph of the curve $y = f(x)$. Sketch the graph of the curve $y = f'(x)$ showing the shape of the curve and the x −intercepts. 2





Question 13 (3 marks)

If $\cos\theta = -\frac{9}{15}$ and $180° < \theta < 360°$, find the exact value of $\tan\theta$ and the possible value/s for θ to the nearest degree. 3

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Question 14 (2 marks)

Given the function $y = 10x^3 - 4$, for what value(s) of x is $\frac{dy}{dx} = 10$? 2

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Question 15 (4 marks)

(a) State the domain and range of the function $f(x) = 3^{-x} + 1$. 2

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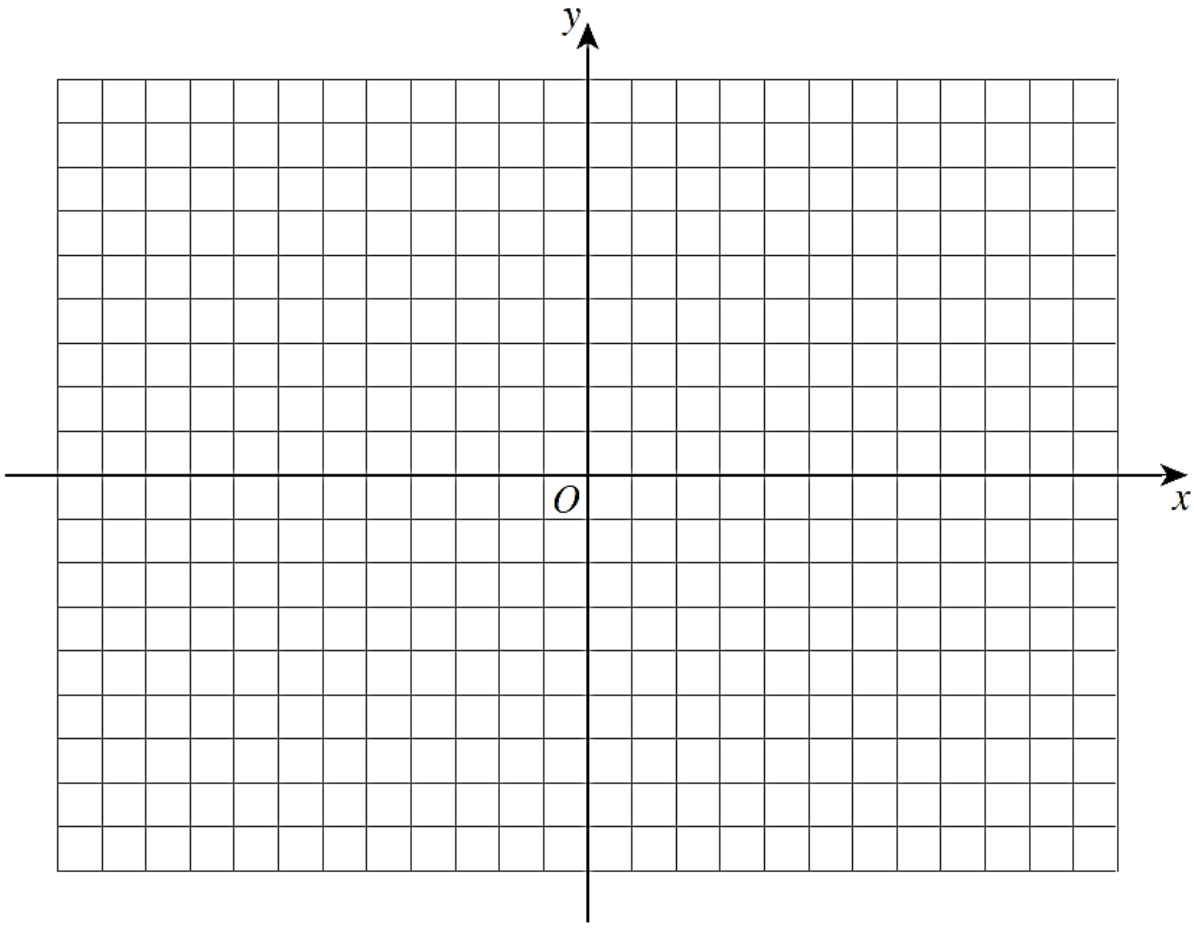
(b) Sketch the graph of $f(x) = 3^{-x} + 1$, showing all key features. 2



Question 16 (3 marks)

The intensity of a current (C) varies inversely with the resistance (R) in a circuit.

The intensity $C = 19$ when $R = 12.5$.

(a) Find the constant of variation (k). 2

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(b) What is the intensity of the current when $R = 30$? 1
(Leave your answer correct to 2 decimal places)

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Question 17 (5 marks)

(a) Differentiate the following with respect to x:

(i) $f(x) = (x^2 + 3x - 2)^4$. 1

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(ii) $f(x) = \sqrt{x} + \frac{3}{x}$. 2

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(iii) $y = \frac{2x^3+1}{x-1}$. 2

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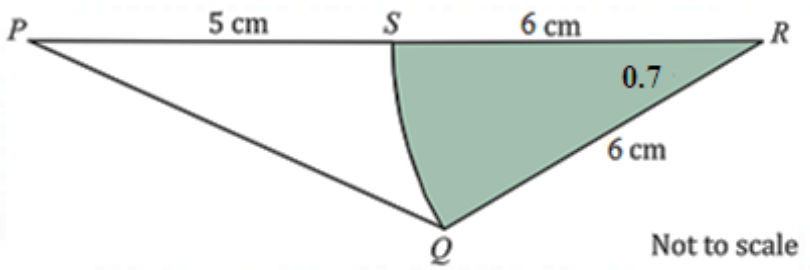
Question 18 (4 marks)



In ΔPQR, $QR = RS = 6$ cm and $\angle QRS$ is 0.7 radians.

The curve QS is an arc of a circle with centre R and radius 6 cm.

(a) What is the length of arc QS? 1

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(b) What percentage, to the nearest whole number, of the area of ΔPQR is shaded? 3

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End of Question 18

Proceed to Booklet 2 for Questions 19-26

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Mathematics Advanced

Name:

Section II Answer Booklet 2

Teacher:

Section II

Booklet 2 – Attempt Question 19 – 26 (34 marks)

Instructions

- Write your Name and your Teacher's Name at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 26 and 27 of Booklet 2. If you use this space, clearly indicate which question you are answering.

Booklet 2	
Q19 – Q21	/13
Q22 – Q24	/10
Q25 – Q26	/11

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Question 19 (4 marks)

Find the equation of the tangent to the curve $y = 3x\sqrt{2x+1}$ at the point where $x = 4$. 4

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Question 20 (3 marks)

Differentiate $f(x) = x^2 + 2x$ from first principles. 3

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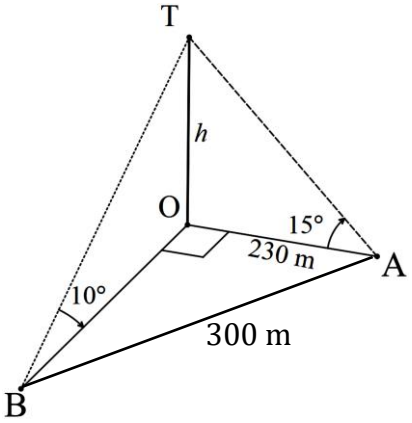
Question 21 (6 marks)

Anthony stands at point A, 230 m due east of the tower OT, of height h metres. The angle of elevation of T from A is 15°. Brian, at point B, is due south of the tower OT.
The angle of elevation of T from B is 10° and the distance between Brian and Anthony is 300 m.



(a) Show that OA $= h \tan 75°$. 2

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(b) By finding a similar expression for OB, find the height of the tower? 2

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(c) What is the bearing of Brian from Anthony to the nearest degree? 2

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Question 22 (2 marks)

If $\log_a 2 = 0.3$ and $\log_a 5 = 0.8$, what is the value of $\log_a 20$? 2

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Question 23 (4 marks)

(a) Show that $2\cos^2\theta + \sin\theta = 2 + \sin\theta - 2\sin^2\theta$. 1

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(b) Hence or otherwise solve $2\cos^2\theta + \sin\theta = 1$ given that $0 \leq \theta \leq \pi$. 3

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Question 24 (4 marks)

The diagram shows points A, B and C which represent three bus stops close to Emmy's home (H).

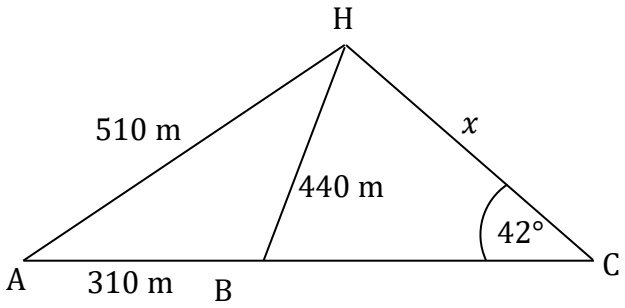

Diagram
NOT to scale

(a) Find the size of $\angle$ABH to the nearest degree. 2

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(b) Find the value of x , the distance to bus stop C from Emmy's home, correct to nearest metre. 2

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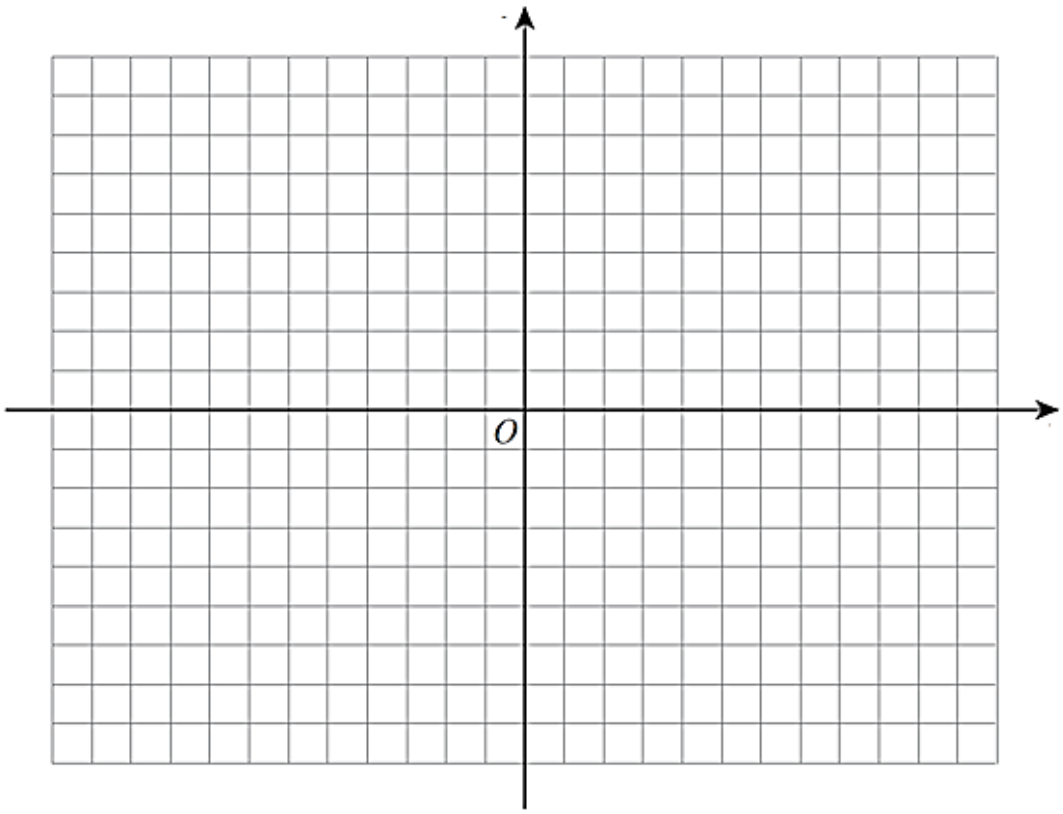
Question 25 (8 marks)

The displacement x metres of a particle moving along the $y-$ axis at a time t seconds is given by $x = 12 + 4t - t^2$.

(a) Graph the displacement x of a particle in the domain $0 \leq t \leq 6$. 2



(b) Find the average velocity of the particle in the first 3 seconds. 2

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(c) Find the instantaneous velocity of the particle when $t = 3$. 2

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(d) What is the initial velocity and initial acceleration of the particle? 2

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Question 26 (3 marks)

The functions $f(x)$ and $g(x)$ are defined for all real x and the function

$$g(x) = 3\left(f(x)\right)^4.$$

Given that $f'(-2) = -12$, $g'(-2) = -1\,152$.

What is the value of $f(-2)$? 3

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Name:

Teacher:

Section I

Year 11 Mathematics Advanced

2023 End of Course Examination

Multiple-Choice Answer Sheet - Questions 1 – 5

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct ↙

A ⊗ B ⊗ C ○ D ○

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1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○

Section I

Solutions AT3 - 2023 MATHEMATICS ADVANCED

6 marks
Attempt Questions 1 – 6
Allow about 10 minutes for this section

Use the multiple-choice answer sheet provided for Questions 1 to 6.

1. Simplify $\frac{\sin(90°-\theta)}{\cos(90°-\theta)}$.

(A) 1

(B) $\cos\theta$

(C) $\tan\theta$

(D) $\cot\theta$

2. If $4x + 3(x-2) = x(2-x)$ with $x > 0$ then x is equal to:

(A) 1

(B) $1 \pm \sqrt{5}$

(C) $1 + \sqrt{5}$

(D) -1

3. What is 0.5 radians in degrees, when rounded to the nearest degree?

(A) 29°

(B) 45°

(C) 50°

(D) 90°

SOLUTIONS & MARKING CRITERIA

MATHEMATICS ADVANCED – QUESTION Year 11 AT3 2023

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(1) $\frac{\sin(90°-\theta)}{\cos(90°-\theta)} = \frac{\cos\theta}{\sin\theta}$		
$= \cot\theta$ (D)	(1)	
(2) $4x+3(x-2) = x(2-x)$		
$4x+3x-6 = 2x-x^2$		
$x^2+5x-6 = 0$		
$x^2+6x-1x-6 = 0$		
$x(x+6)-1(x+6) = 0$		
$(x-1)(x+6) = 0$		
$x = 1$ when $(x-1) = 0$		
$x = -6$ when $(x+6) = 0$		
$x > 0$ $\therefore x \neq -6$		
$\therefore x = 1$ is the only solution.	(1)	
(A)		
(3) π radians $= 180°$		
1 radian $= \frac{180°}{\pi}$		
$\doteq 28°39'$		
$\doteq 29°$ (A)	(1)	
(4) $\log_3(x-30) = 4$		
$3^{\log_3(x-30)} = 3^4$		
$\therefore x-30 = 81$		
$x = 111$ (B)	(1)	

4. What is the value of x if $\log_3(x - 30) = 4$?

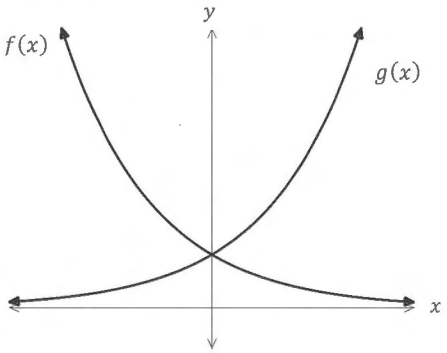
(A) 152

(B) 111

(C) 81

(D) 51

5. The diagram shows the graphs of $f(x) = a^x$ and $g(x) = b^x$, and $f(x)$ is a reflection of $g(x)$ along the y-axis.



Which of the following are true?

I. $0 < a < 1$

II. $b > 1$

III. $ab = 1$

(A) I and II

(B) I and III

(C) II and III

(D) I, II and III

END OF SECTION I

MATHEMATICS ADVANCED – QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Q5 if $g(x) = b^x$		
$f(x) = b^{-x}$ ($f(x)$ & $g(x)$ are reflection in y-axis)		
$\therefore a^x = b^{-x}$		
or $a^x = \frac{1}{b^x}$		
or $a^x = \left(\frac{1}{b}\right)^x$		
$\therefore a = \frac{1}{b}$		
$\therefore ab = 1$		
$b > 1$ otherwise $g(x)$ would not be an increasing function.		
If $a > 1$, it would be increasing function		
If $a < 0$, it would be an increasing function in Quadrant 2.		
$\therefore$ by elimination: $0 < a < 1$		
$\therefore$ at I, II & III are true		
$\therefore$ (D)		

Question 6 (2 marks)

Find the values of x for which $|2x - 1| = 3$. 2

$2x - 1 = 3$ or $2x - 1 = \pm 3$

$2x = 4$ $2x - 1 = 3$

$x = 2$ —① $2x = 4$

$-(2x - 1) = 3$ $x = 2$

$-2x + 1 = 3$

$-2x = 2$

$x = -1$ —①

$\therefore x = -1, x = 2$

Question 7 (2 marks)

If $11\sqrt{3} - 2\sqrt{27} = \sqrt{p}$, find the value of p. 2

$11\sqrt{3} - 2\sqrt{27} = 11\sqrt{3} - 2 \times 3\sqrt{3}$

$= 11\sqrt{3} - 6\sqrt{3}$

$= 5\sqrt{3}$

$= \sqrt{75}$ —① for showing correct simplification process

so $\sqrt{p} = \sqrt{75}$

$\therefore p = 75$ —①

EXAMINER'S COMMENTS

Q6) This was mostly well done by all students.

Q7) Students who simply used their calculators did not receive full marks for this question.

Some students need to revise their operations with surds.

Question 8 (1 marks)

Find $\frac{dy}{dx}$, if $y = 6x^2 - 5x - 1$. 1

$\frac{dy}{dx} = 12x - 5$

Question 9 (2 marks)

Let $x^2 + 6x - 3 = (x - a)^2 + b$ where a and b are constants.

Find the value of a and b. 2

$x^2 + 6x - 3 = x^2 + 6x + 9 - 3 - 9$

$= (x+3)^2 - 12$ —① completing the square

$\therefore a = -3, \ b = -12$

① ①

OR

$x^2 + 6x - 3 = x^2 - 2ax + a^2 + b$

$\therefore -2a = 6$

$a = -3$ —① equating coefficients

$\therefore a^2 + b = -3$

$(-3)^2 + b = -3$

$9 + b = -3$

$b = -12$ —①

Question 10 (2 marks)

Solve for x: $2^{x-3} + 2 = 7$, leave your answer correct to 2 decimal places. 2

$2^{x-3} = 5$

$x - 3 = \log_2 5$

$x = \log_2 5 + 3$ —① making x the subject either by rewriting in logarithmic form or equivalent

$= \frac{\log_{10} 5}{\log_{10} 2} + 3$

$= 5.321928...$

$= 5.32$ (2 d.p.) —① correct answer to 2 d.p. using change of base or calculator.

EXAMINER'S COMMENTS

Q8) Done well by all students.

Q9) This question was answered poorly. The correct technique was to make both sides have the same form - either both factorised (by completing the square) or both in expanded form. Once both sides of the equation are in the same form, then you can equate them.

Many students need to review their completing the square process, and make sure they know how to complete the square on an expression and also an equation.

Also, if $(x+3)^2 = (x-a)^2$, then $a = -3$ as two negatives make a positive, i.e. $-(-3) = +3$.

Q10) This question was mostly answered well.

Could also have done: $2^{x-3} = 5$

$\log_2 2^{x-3} = \log_2 5$

$\therefore x - 3 = \log_2 5$

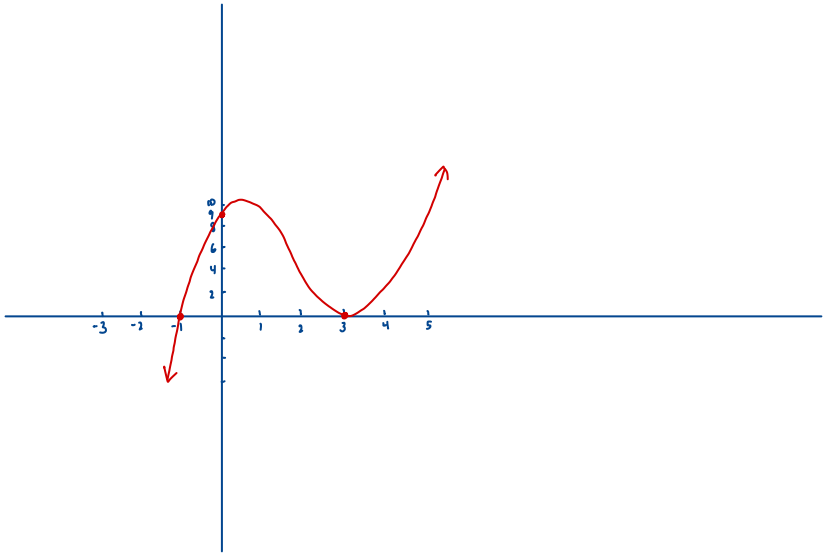
$\vdots$

For students who simply used their calculators, please keep in mind that this concept can be tested in multiple ways (e.g. a "show" question) so make sure you also practice and understand how to use the change of base formula.

Question 11 (3 marks)

Without using calculus, sketch the following polynomial, clearly showing the x and y −intercepts. 3

$$P(x) = (x+1)(3-x)^2$$



EXAMINER'S COMMENTS

1 mark - Correct x-intercepts ($\frac{1}{2}$ mark for each one)

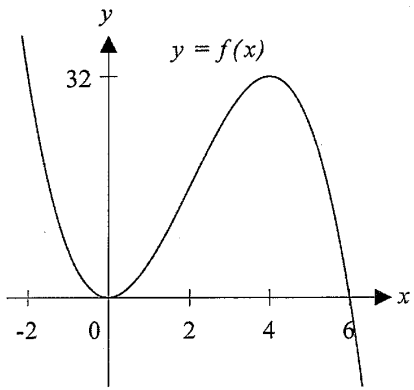
1 mark - Correct y-intercept

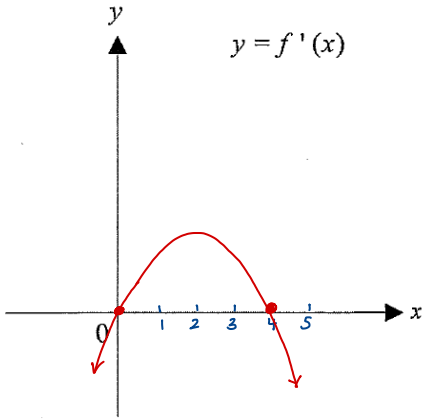
1 mark - correct concavity and shape.

Important – Take more care with the smooth shape of the curve. Do not "flick" the curve out at its ends and ensure you have a scale.

Question 12 (2 marks)

The diagram shows the graph of the curve $y = f(x)$. Sketch the graph of the curve $y = f'(x)$ showing the shape of the curve and the x –intercepts. 2



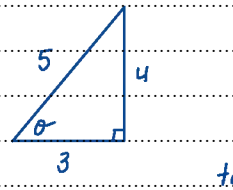


EXAMINER'S COMMENTS
1 mark – Correct shape
1 mark – Correct x-intercepts

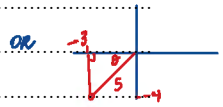
Question 13 (3 marks)

If $\cos\theta = -\frac{9}{15}$ and $180° < \theta < 360°$, find the exact value of $\tan\theta$ and the possible value/s for θ to the nearest degree. 3

$\cos\theta = -\frac{3}{5}$ $180° \leq \theta \leq 360°$



∴ quad 3



$\tan\theta = \frac{4}{3}$

$\theta = 180° + \tan^{-1}\frac{4}{3}$

$\theta = 180° + 53°8'$

$\doteq 233°$

Question 14 (2 marks)

Given the function $y = 10x^3 - 4$, for what value(s) of x is $\frac{dy}{dx} = 10$? 2

$y = 10x^3 - 4$

$y' = 30x^2$

For $30x^2 = 10$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$

$= \frac{\pm 1}{\sqrt{3}}$

$= \frac{\pm\sqrt{3}}{3}$

MATHEMATICS EXTENSION 1 – QUESTION 14

1 mark – obtain $\tan\theta = \frac{4}{3}$

1 mark – to find the acute related angle $53°8'$

1 mark – correct final answer of $233°$ in quad 3

Many mistakes for realising the answer was in quad 3.

2 marks Complete solution
1½ marks Correctly finds $x = \frac{1}{\sqrt{3}}$ or $x = \frac{-1}{\sqrt{3}}$
1 mark Correctly finds $y' = 30x^2$

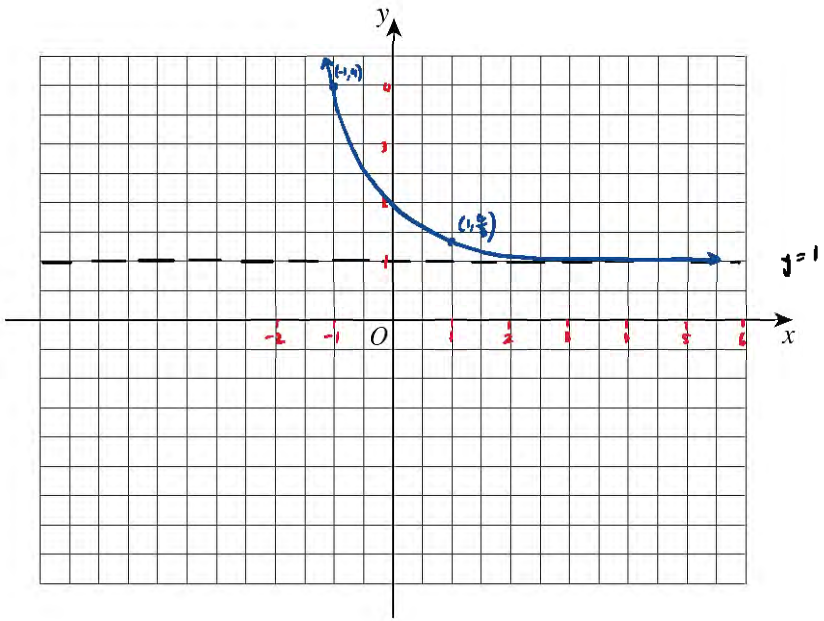
Question 15 (4 marks)

(a) State the domain and range of the function $f(x) = 3^{-x} + 1$. 2

Domain: all real x

Range: $y > 1$

(b) Sketch the graph of $f(x) = 3^{-x} + 1$, showing all key features. 2



MATHEMATICS EXTENSION 1 – QUESTION 15

2 marks Complete solution

1 mark Correctly states the domain or the range

This question was mostly well-attempted, with two notable caveats:

- A concerningly large number of candidates provided two answers for both the domain and the range. Don't do this.
- Many candidates made the curious statement: "$x \in \mathbb{R}$". This statement is read as "x is an element of the real numbers". You may have been attempting to say that x is all real numbers, but these are very different statements. Please use the statement "all real x" in these cases.

Note also:

- Domain: $(-\infty, \infty)$ and Range: $(1, \infty)$ were also acceptable.
- Half marks were awarded where the endpoints were erroneously included.

The graph must be an exponential curve to be awarded any marks.

Half a mark was awarded for each correct element:

- asymptote at $y = 1$.
- y-intercept at 2.
- a horizontally flipped exponential curve.
- good shape.

Common errors included:

- linear sections in the graph – the whole thing should be curved.
- bouncing back from the asymptotes
- only drawing half a graph. We've given you the axes, there is no reason to stop drawing part-way.

Question 16 (3 marks)

The intensity of a current (C) varies inversely with the resistance (R) in a circuit.

The intensity $C = 19$ when $R = 12.5$.

(a) Find the constant of variation (k). 2

$C = \frac{k}{R}$

$19 = \frac{k}{12.5}$

$\therefore k = 19 \times 12.5$

$= 237.5$

(b) What is the intensity of the current when $R = 30$? 1
(Leave your answer correct to 2 decimal places)

$C = \frac{237.5}{R}$

$\therefore C = \frac{237.5}{30}$

$= 7.91666\ldots$

$\doteq 7.92$

MATHEMATICS EXTENSION 1 – QUESTION 16

2 marks Complete solution

1 mark Correct substitution into correct formula

Very poorly done. Many students couldn't even write the first line of the solution. Direct and inverse variation are both in the syllabus - make sure you know how they work.

Note that half-marks were awarded for parts a) and b) for candidates who created and solved an incorrect equation of comparable difficulty. This may not be the case in future - wrong formula generally results in zero marks.

Question 17 (5 marks)

(a) Differentiate the following with respect to x:

(i) $f(x) = (x^2 + 3x - 2)^4$. 1

$f'(x) = 4(2x+3)(x^2+3x-2)^3$

$u = x^2+3x-2$
$u' = 2x+3$
$v = u^4$
$v' = 4u^3$

(ii) $f(x) = \sqrt{x} + \frac{3}{x}$. 2

$f(x) = x^{1/2} + 3x^{-1}$

$f'(x) = \frac{1}{2}x^{-1/2} - 3x^{-2}$

$= \frac{1}{2\sqrt{x}} - \frac{3}{x^2}$

(iii) $y = \frac{2x^3+1}{x-1}$. 2

$y' = \frac{u'v - uv'}{v^2}$

$= \frac{6x^2(x-1) - 1(2x^3+1)}{(x-1)^2}$

$= \frac{6x^3 - 6x^2 - 2x^3 - 1}{(x-1)^2}$

$u = 2x^3+1$
$u' = 6x^2$ (½) mark
$v = x-1$
$v' = 1$ (½) mark
(1)

EXAMINER'S COMMENTS

Question 17

(a) (i) Most common mistake was

$4(2x+3) = 8x+3$

Mark was not deducted for this mistake.

(ii) (1) mark each for the two separate parts.

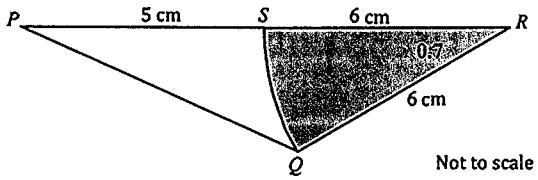
No mark deducted if there was error in the simplification after correct first step of derivation.

Full mark for finding correct derivation of the numerator & denominator function and correct substitution into the coefficient rule. No mark deducted for mistakes in simplification.

Common mistake $-(2x^3+1) = -2x^3+1$

(½) mark for correct derivation of numerator and denominator function each.

Question 18 (4 marks)



In ΔPQR, $QR = RS = 6$ cm and $\angle QRS$ is 0.7 radians.

The curve QS is an arc of a circle with centre R and radius 6 cm.

(a) What is the length of arc QS? 1

Arc length $QS = 0.7 \times 6$
$= 4.2$ cm

(b) What percentage, to the nearest whole number, of the area of ΔPQR is shaded? 3

Area of Sector $SQR = \frac{1}{2} r^2 \theta$
$= \frac{1}{2} 6^2 \times 0.7$
$\doteq 12.6 \text{ cm}^2$
Area of $\triangle PQR = \frac{1}{2} \times PR \times QR \times \sin\theta$
$= \frac{1}{2} \times 11 \times 6 \times \sin 0.7$
$\doteq 21.26 \text{ cm}^2$
Shaded area as % $= \frac{12.6}{21.26} \times 100$
$\doteq 59\%$

End of Question 18

Proceed to Booklet 2 for Questions 19-26

EXAMINER'S COMMENTS

Question 18

(a) Many students wasted time converting 0.7 radians to degrees and meny students who lost mark, it was due to incorrect conversion
i.e. 0.7 converted to 127°
not $\frac{127^\circ}{\pi}$

(b) Most common mistake was due to wrong conversion of 0.7 radians to degrees. some students had no idea of area rule of a sector using radians of even degrees. Some did not knows even the sine are rule.

Question 19 (4 marks)

Find the equation of the tangent to the curve $y = 3x\sqrt{2x+1}$ at the point where $x = 4$. 4

$y = 3x\sqrt{2x+1}$

let $u = 3x$ $v = (2x+1)^{\frac{1}{2}}$

$\frac{du}{dx} = 3$ $\frac{dv}{dx} = \frac{1}{2}(2x+1)^{-\frac{1}{2}} \times 2$

$= \frac{1}{(2x+1)^{\frac{1}{2}}} = \frac{1}{\sqrt{2x+1}}$

$\frac{dy}{dx} = 3\sqrt{2x+1} + 3x \times \frac{1}{\sqrt{2x+1}}$

$= 3\sqrt{2x+1} + \frac{3x}{\sqrt{2x+1}}$ —1mk

When $x = 4$, the gradient of tangent is $3\sqrt{2(4)+1} + \frac{3(4)}{\sqrt{2(4)+1}}$

$= 3\times 3 + \frac{12}{3}$

$= 13$ 1mk

At $x = 4$, $y = ?$

$y = 3x\sqrt{2x+1}$

$= 3(4)\sqrt{2(4)+1}$ 1mk

$= 12\sqrt{9}$

$= 36$ ∴ Point is $(4, 36)$

Now use the one-point gradient formula or gradient-intercept form of a straight line

$y - 36 = 13(x-4)$

$y - 36 = 13x - 52$

$y = 13x - 52 + 36$

$= 13x - 16$

or $13x - y - 16 = 0$ 1mk

Question 20 (3 marks)

Differentiate $f(x) = x^2 + 2x$ from first principles. 3

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{(x+h)^2 + 2(x+h) - (x^2+2x)}{h}$

$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$ —1mk

$= \lim_{h\to 0} \frac{2xh + h^2 + 2h}{h}$

$= \lim_{h\to 0} \frac{h(2x + h + 2)}{h}$ ½ mk for using the correct notation for the limit all throughout the answer.

$= \lim_{h\to 0} 2x + h + 2$ — 1mk

$= 2x + 0 + 2$

$= 2x + 2$ —½ mk

EXAMINER'S COMMENTS

Question 19

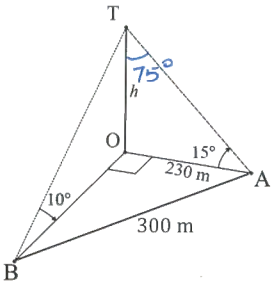
A number of students made a mistake when differentiating $\sqrt{2x+1}$ as they forgot to multiply by the derivative of the inner function $(2x+1)$. A few students need to re-visit the one-point gradient formula for a straight line.

Question 20

Generally, well done. However, a few students failed to display all the steps, particularly cancelling out the 'h's after first factorising out the terms. Also, when substituting $h = 0$ in the last step, $\lim_{h\to 0}$ should not be written down. Students are reminded to use 'short methods of differentiation' to verify their final answer.

Question 21 (6 marks)

Anthony stands at point A, 230 m due east of the tower OT, of height h metres. The angle of elevation of T from A is 15°. Brian, at point B, is due south of the tower OT.
The angle of elevation of T from B is 10° and the distance between Brian and Anthony is 300 m.



(a) Show that OA = $h \tan 75°$. 2

$\angle OTA = 90° - 15°$ (angle sum of a triangle)

$= 75°$ — 1mk

Now, $\tan 75° = \frac{OA}{h}$ — 1mk

$\therefore OA = h \tan 75°$

(b) By finding a similar expression for OB, find the height of the tower? 2

$\angle OTB = 80°$ (angle sum of a triangle)

$\tan 80° = \frac{OB}{h}$

$\therefore OB = h \tan 80°$ — ½ mk

Now, using Pythagoras' Theorem on the base triangle.

$OB^2 + OA^2 = AB^2$

$(h \tan 80°)^2 + (h \tan 75°)^2 = 300^2$ — ½ mk

$h^2 \tan^2 80 + h^2 \tan^2 75° = 300^2$

$h^2 (\tan^2 80 + \tan^2 75) = 300^2$

$\therefore h^2 = \frac{300^2}{\tan^2 80° + \tan^2 75°}$ — 1mk

$\therefore h = \frac{300}{\sqrt{\tan^2 80 + \tan^2 75°}} = 44.108592\ldots\ (h>0)$

$= 44.19$ m

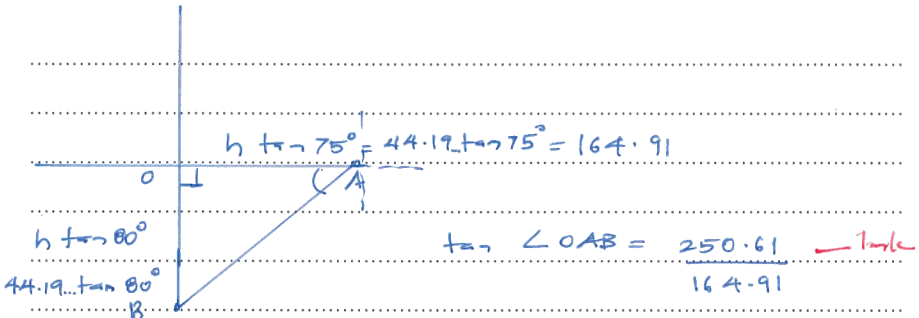
EXAMINER'S COMMENTS

Question 21

a) Generally, well done. However, a few students failed to display all the appropriate working. Remember to show all the relevant steps in show questions.

b) A few students lost ½ a mark for failing to find a similar expression for OB. In this question students were required to find the height of the tower using $OA = h \tan 75°$ and $OB = h \tan 80°$. However, no marks were deducted for using other methods (apart from the ½ mark explained earlier on). Students should evaluate their final answer if the question is asking you to calculate the height of the tower in metres rather than leaving their answer in terms of tan 80 and tan 75°. It is not fair on other students who have made an effort to work out the answer on their calculator. If the question does not specify the level of accuracy then try to leave your answer correct to two decimal places.

(c) What is the bearing of Brian from Anthony to the nearest degree? 2



$\tan \angle OAB = \frac{250.61}{164.91}$ — 1mk

$= 56°39'$

$270° - 56°39' = 213°20'50.52''$

$= 213°$ — 1mk

Question 22 (2 marks)

If $\log_a 2 = 0.3$ and $\log_a 5 = 0.8$, what is the value of $\log_a 20$? 2

$\log_a 20 = \log_a (5 \times 4)$

$= \log_a 5 + \log_a 4$

$= \log_a 5 + \log_a 2^2$

$= \log_a 5 + 2\log_a 2$ — 1mk

$= 0.8 + 2 \times 0.3$

$= 0.8 + 0.6$

$= 1.4$ — 1mk.

EXAMINER'S COMMENTS

21 c) Generally, well done. However, some students were using 15° which is on a vertical plane. Remember that bearings are measured on a horizontal plane and not on a vertical plane.

Q22 Overall this question was well done. A few students also answered the question the following way:

$\log_a 20 = \log_a (5 \times 2 \times 2)$

$= \log_a 5 + \log_a 2 + \log_a 2$

$= 0.8 + 0.3 + 0.3$

$= 1.4$

There were however a handful of student who did not use the logarithmic laws correctly. They wrote $\log_a 20$ as $\log_a 5 \times 2^2$ instead of $\log_a 20 = \log_a (5 \times 2^2)$.

Please review the log laws and their application.

Question 23 (4 marks)

(a) Show that $2\cos^2\theta + \sin\theta = 2 + \sin\theta - 2\sin^2\theta$. 1

$LHS = 2\cos^2\theta + \sin\theta$

$= 2(1-\sin^2\theta) + \sin\theta$ — ½ mark

$= 2 - 2\sin^2\theta + \sin\theta$ — ½ mark

$= 2 + \sin\theta - 2\sin^2\theta$

$= RHS$

Alternate solution

$RHS = 2 + \sin\theta - 2\sin^2\theta$

$= 2 + \sin\theta - 2(1-\cos^2\theta)$

$= 2 + \sin\theta - 2 + 2\cos^2\theta$

$= 2\cos^2\theta + \sin\theta$

$= LHS$

(b) Hence or otherwise solve $2\cos^2\theta + \sin\theta = 1$ given that $0 \le \theta \le \pi$. 3

$2\cos^2\theta + \sin\theta = 1$

$2 + \sin\theta - 2\sin^2\theta = 1$ from (a) — ½ mark

$-2\sin^2\theta + \sin\theta + 1 = 0$

$2\sin^2\theta - \sin\theta - 1 = 0$ — ½ mark

Let $u = \sin\theta$

$2u^2 - u - 1 = 0$

$2u^2 - 2u + u - 1 = 0$

$2u(u-1) + 1(u-1) = 0$

$(u-1)(2u+1) = 0$ — ½ mark

$u - 1 = 0$ or $2u + 1 = 0$

$\sin\theta - 1 = 0$ $\quad 2\sin\theta + 1 = 0$

$\sin\theta = 1$ $\quad \sin\theta = -\frac{1}{2}$

$\theta = \frac{\pi}{2}$ $\quad \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ (outside the domain)

$\therefore \theta = \frac{\pi}{2}$ only solution — ½ mark

or but $\sin\theta \neq -\frac{1}{2}$ since $0 \le \theta \le \pi$

½ mark (for final answer in radians)

EXAMINER'S COMMENTS

Q 23 a) Many students proved this well but there were also so many who had problems proving the identity the conventional way.

There were a handful of students who started off with both sides of the identity, worked their way on both sides until they found that LHS = RHS. This is NOT a correct method. Please refer to the solutions.

b) This part was not well done by a handful of students.

Many students did not factorise the quadratic well and/or did not consider the restricted domain for θ.

Please review solving quadratic equations and solving trigonometric equations.

Some students left their answer in degrees instead of radians and were given 2½ marks instead of 3 marks.

Question 24 (4 marks)

The diagram shows points A, B and C which represent three bus stops close to Emmy's home (H).

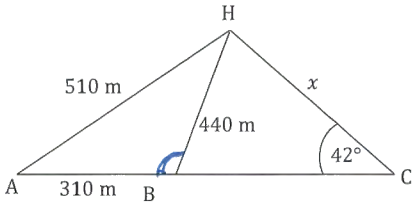


Diagram NOT to scale

(a) Find the size of ∠ABH to the nearest degree. 2

$\cos \angle ABH = \frac{440^2 + 310^2 - 510^2}{2 \times 440 \times 310}$ — 1mark

$\cos \angle ABH = \frac{37}{341}$

$\angle ABH = \cos^{-1} \frac{37}{341}$

$= 83.77089...$

$\doteq 84°$ (to the nearest degree) — 1mk.

(b) Find the value of x, the distance to bus stop C from Emmy's home, correct to nearest metre. 2

Now $\angle CBH = 180 - 84 = 96°$ — ½ mark

Using the sine rule, $\frac{x}{\sin 96} = \frac{440}{\sin 42}$ — ½ mark

$x = \frac{440 \times \sin 96°}{\sin 42}$

$= 653.9674...$

$\doteq 654$ m — 1 mark

EXAMINER'S COMMENTS

Q24

a) Overall this part was done well, however some students went further and found the supplement of 84° (ie 96°) — 1½ marks if you that.

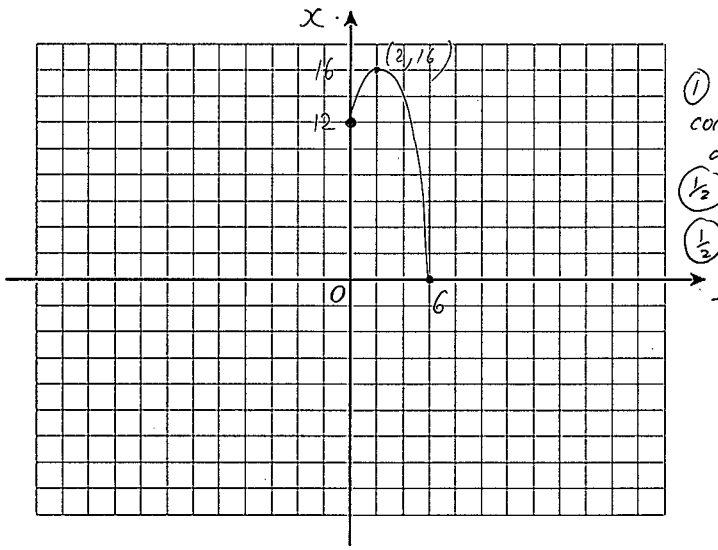
Also ½ mark off if student wrote that $\cos \theta = 84°$

b) Mostly done well.

Question 25 (8 marks)

The displacement x metres of a particle moving along the $y-$ axis at a time t seconds is given by $x = 12 + 4t - t^2$.

(a) Graph the displacement x of a particle in the domain $0 \leq t \leq 6$. 2



① Shape and correct restricted domain
(½) x-intercept
(½) y-intercept

(b) Find the average velocity of the particle in the first 3 seconds. 2

Average velocity = $\frac{15-12}{3-0}$ ①

= 1 m/s ①

(c) Find the instantaneous velocity of the particle when $t = 3$. 2

① $\dot{x} = 4 - 2t$ At $t = 3$

$= 4 - 2(3)$

$= -2$ m/s ①

EXAMINER'S COMMENTS

Q25) a. Common errors included:

- Incorrect intercepts
- Incorrect shape of parabola and location of vertex
- Failure to take into account the restricted domain

Q25) b. To find average velocity: $\frac{\text{Change in displacement}}{\text{time}}$

- Many failed to do this.

Q25) c. Well done. Don't forget units for velocity!

(d) What is the initial velocity and initial acceleration of the particle? 2

$\dot{x} = 4 - 2t$

At $t = 0$

$\dot{x} = 4 m/s$ (1)

$\ddot{x} = -2$

At $t = 0$

$\ddot{x} = -2 m/s^2$ (1)

Question 26 (3 marks)

The functions $f(x)$ and $g(x)$ are defined for all real x and the function

$g(x) = 3\left(f(x)\right)^4$.

Given that $f'(-2) = -12$, $g'(-2) = -1\,152$.

What is the value of $f(-2)$? 3

$g'(x) = 12\left(f(x)\right)^3 \times f'(x)$ (1)

$g'(-2) = 12\left(f(-2)\right)^3 \times f'(-2)$

$-1152 = 12\left(f(-2)\right)^3 \times (-12)$ (1)

$\left(f(-2)\right)^3 = 8$

$\therefore f(-2) = 2$ (1)

END OF PAPER

EXAMINER'S COMMENTS

Q25) d. Well done. Once again ensure units for velocity and acceleration are included.

Q26) Common errors included:

- Failure to differentiate using the chain rule. Many incorrectly differentiated and stated that $g'(x) = 12(f(x))^3$ forgetting to multiply by $f'(x)$.
- Issues with function notation and substitution.
- Issues with cube rooting both sides of an equation. $\pm$ is not needed