SYDNEY GIRLS HIGH SCHOOL

YEAR 11 Mathematics

Assessment Task 3

September 2015

Time allowed: 60 minutes
Plus 5 minutes reading time
Chapters : 1- 12

Instructions:

- There are Four (4) questions. Questions **are** of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work.
- Start each question on a new page. Write on one side of the paper only.

Student Name:---

Teacher:---

QUESTION ONE (15 marks)

a) Given $f(x) = x^2 - 3x$, find $f(2)$. (1)

b) Simplify $1 - \dfrac{4-a}{3}$ as single fraction (1)

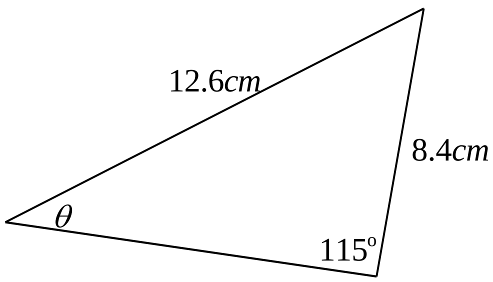
c) Factorise completely $8 - 18t^2$ (1)

d) If $(3\sqrt{12} - 2\sqrt{5})^2 = a + b\sqrt{15}$, find a and b. (2)

e) Solve $|2x-3| \le 11$. (2)

f) Find $\displaystyle\lim_{x \to 4} \frac{x-4}{x^2-16}$. (1)

g) Find the value of θ, to the nearest degree. (2)



h) Simplify $\dfrac{4}{x-3} - \dfrac{2x+1}{x^2-5x+6}$ (3)

i) Evaluate $(1 + \tan 60^\circ)^2$, in exact form. (2)

QUESTION TWO (15 marks)

a) Sketch the following, showing all the main features

i) $y = -x^2 - 5x$ (2)

ii) $y = |x+4|$ (1)

iii) $y = \sqrt{9-x^2}$ (1)

b) Solve $\sqrt{5}\sin\theta = -1$, for $0^{\circ} \le \theta \le 360^{\circ}$. (answer to the nearest degree) (2)

c) Differentiate the following

i) $y = 6x^3 + 2x^{-4} - 10$ (1)

ii) $y = (7-3x^2)^7$ (1)

iii) $y = x^2\sqrt{x}$ (1)

d) Graph the region bounded by the following $2x + y < -1$ and $y \ge x^2 - 4$ (2)

e) Show that $f(x) = \dfrac{2x}{x^2+1}$ is an odd function. (2)

f) Find the domain of $y = \sqrt{4-x} + \sqrt{-2x+7}$ (2)

QUESTION THREE (15 marks)

a) On your answer sheet plot the points P (0,10), Q(-5,0) and R (2,-2) which are the vertices of $\triangle PQR$. The origin is O (0,0).

i) Find the exact length of PQ. (1)

ii) Show that PQ has equation $2x - y + 10 = 0$. (2)

iii) Find the perpendicular distance from R to PQ. (2)

iv) Find the coordinates of S such that $PQRS$ is a parallelogram. (1)

v) Find the area of $PQRS$. (1)

vi) Find, correct to the nearest minute, the size of $\angle PQO$. (2)

b) Differentiate

i) $y = \dfrac{3x^2 - 4}{8 - 5x}$ (2)

ii) $y = (10 - 3x)\sqrt{1 - 4x^2}$ (2)

c) Show that $\dfrac{\sin\left(270^\circ - \theta\right)}{\sin\theta} = -\cot\theta$. (2)

QUESTION FOUR (15 marks)

a) Find the equation of the tangent to the curve $y = x^2 - 8x + 20$ at $x = 2$. (3)

b) Prove that $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} = 2\tan^2\theta$. (2)

c) Bella is running from Jacob at a speed of 300 km/ hr and after 10 minutes she is at a bearing of $130^\circ T$ from Jacob. Edward is waiting for Bella 37 km due north-east of Jacob.

i) Draw a diagram showing the above information. (1)

ii) Find the distance between Bella and Edward. (correct to 1d.p) (2)

iii) Calculate the bearing of Edward from Bella. (to the nearest degree) (2)

d)

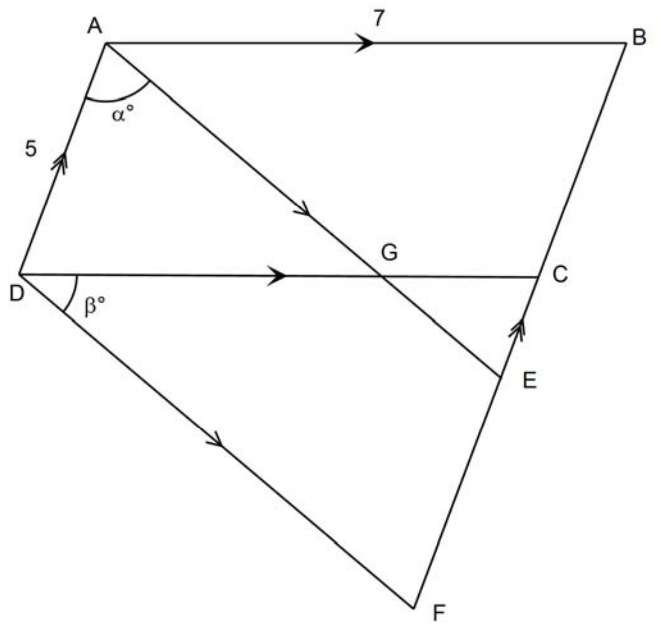


Figure not to scale

In the diagram above, ABCD and AEFD are parallelograms. AE bisects $\angle$DAB and DC bisects $\angle$FDA. Let $\angle DAG = \alpha$ and $\angle FDG = \beta$.

i) Copy the diagram on your answer sheet

ii) Show that $\triangle AGD$ is equilateral, giving reasons. (2)

iii) If $AD = 5$ units and $AB = 7$ units, find the area of the trapezium $ABFD$. (3)
(Giving your answer in simplest exact form)

THE END

2015 2unit Yr11 Yearly

Q1 a) $f(x) = x^2 - 3x$

$f(2) = 2^2 - 3 \times 2$

$f(2) = -2$

b) $1 - \frac{4-a}{3}$

Common mistake due to -ve sign

$= \frac{3}{3} - \frac{4-a}{3}$

$= \frac{3-(4-a)}{3}$

$= \frac{3-4+a}{3}$

$= \frac{a-1}{3}$

c) $8 - 18t^2$

$= 2(4 - 9t^2)$

$= 2(2+3t)(2-3t)$

d) $(3\sqrt{12} - 2\sqrt{5})^2$

$= 9 \times 12 - 12\sqrt{60} + 20$

$= 108 + 20 - 12\sqrt{60}$

$= 128 - 12\sqrt{60}$

$= 128 - 24\sqrt{15}$

$\therefore a = 128 \quad b = -24$

e) $|2x-3| \leq 11$

$-11 \leq 2x-3 \leq 11$

$-8 \leq 2x \leq 14$

$-4 \leq x \leq 7$

f) $\lim\limits_{x \to 4} \frac{(x-4)}{(x-4)(x+4)}$

Some students failed to factorise before substitution.

$= \lim\limits_{x \to 4} \frac{1}{x+4}$

$= \frac{1}{4+4} = \frac{1}{8}$

g) $\frac{\sin\theta}{8.4} = \frac{\sin 115^\circ}{12.6}$

$\sin\theta = \frac{8.4 \sin 115^\circ}{12.6}$

Students used sine rule, sine ratio which incorrect.

$\theta = 37^\circ$

h) $\frac{4}{x-3} - \frac{2x+1}{(x-3)(x-2)}$

$= \frac{4(x-2)}{(x-3)(x-2)} - \frac{2x+1}{(x-3)(x-2)}$

$= \frac{4x-8-2x-1}{(x-3)(x-2)}$

$= \frac{2x-9}{(x-3)(x-2)}$

Common mistake due to -ve sign

i) $(1+\tan 60°)^2$

$= (1+\sqrt{3})^2$

$= 1+2\sqrt{3}+3$

$= 4+2\sqrt{3}$

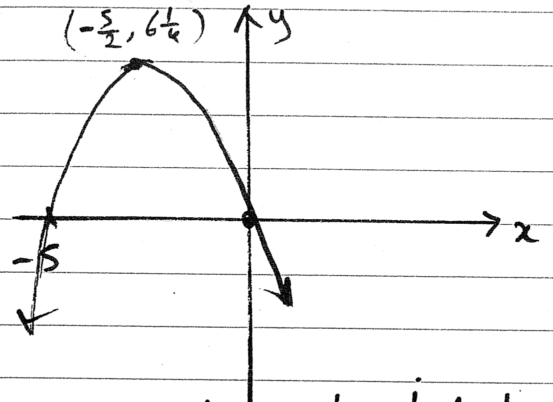
Mathematics 2015 yearly

2a)

i) $y = -x(x+5)$

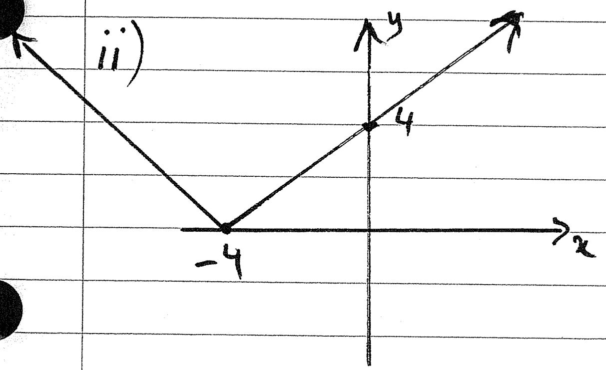
$x = 0$ or -5

at $x = \frac{-5}{2}$, $y = 6\frac{1}{4}$

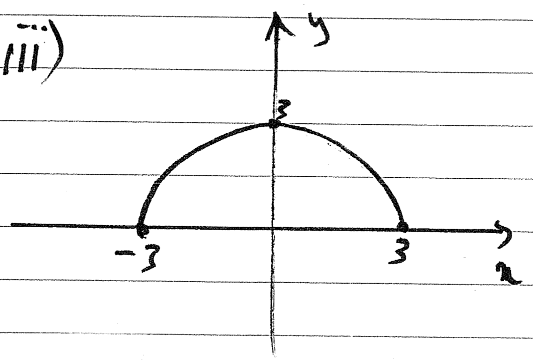


* many students didn't show the vertex

ii)



iii)



b) $\sin\theta = -\frac{1}{\sqrt{5}}$

$\theta = 206^\circ, 333^\circ$

* some students couldn't get the 2nd answer

c) i) $y' = 18x^2 - 8x^{-5}$

ii) $y' = 7(7-3x^2)^6 \cdot -6x$

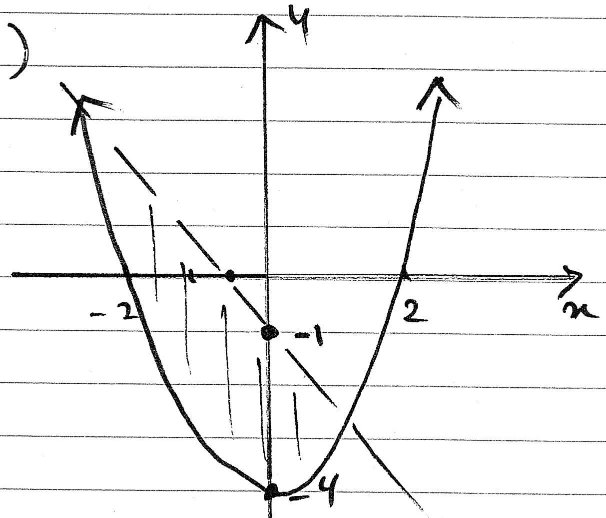
$= -42x(7-3x^2)^6$

iii) $y = x^{\frac{5}{2}}$

$y' = \frac{5}{2}x^{\frac{3}{2}}$

* some students were careless in working with indices

d)



* Some student didn't use dotted line and didn't have the correct closed region. Some didn't have any numbers on the number plane. Very Poorly done.

e) $f(-x) = \frac{2(-x)}{(-x)^2+1} = \frac{-2x}{x^2+1}$

$-f(x) = -\frac{2x}{x^2+1}$ $\quad -f(x) = f(-x)$

$\therefore$ odd

f) $4 - x \geq 0 \therefore x \leq 4$

$-2x + 7 \geq 0 \therefore x \leq 3\frac{1}{2}$

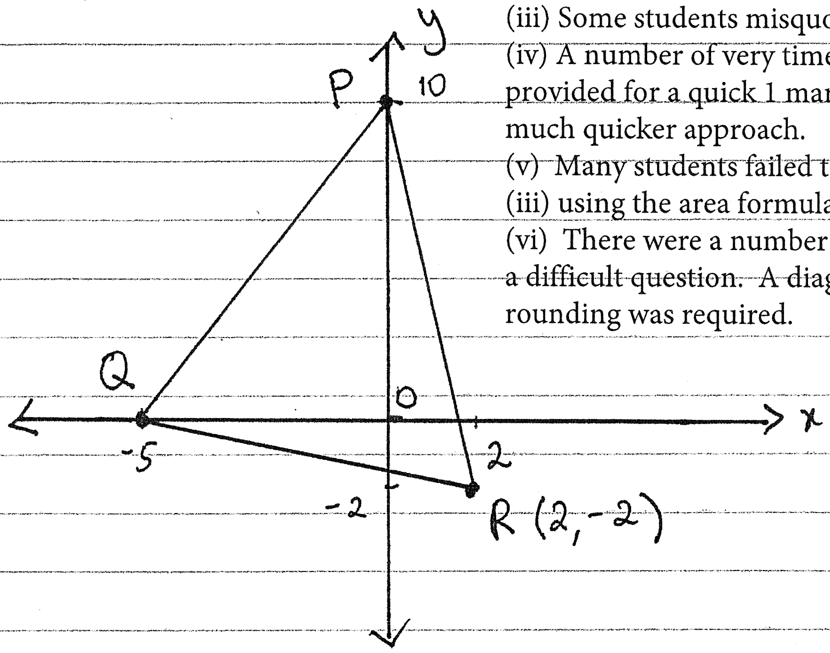
$\therefore D: x \leq 3\frac{1}{2}$ very poorly done.

Comments for part (a)

(ii) Some students needed more detail to "show" the equation of PQ.
(iii) Some students misquoted formula.
(iv) A number of very time-consuming solutions were provided for a quick 1 mark question - use of the diagram is a much quicker approach.
(v) Many students failed to connect their answers from (i) and (iii) using the area formula for a parallelogram $A=bh$.
(vi) There were a number of non-attempts - though this is not a difficult question. A diagram would be most useful. Correct rounding was required.

QUESTION THREE

(a)



(i) $PQ = \sqrt{5^2 + 10^2}$

$\therefore PQ = \sqrt{125}$ or $5\sqrt{5}$ units

(ii) From diagram $m_{PQ} = \frac{rise}{run} = \frac{10}{5} = 2$

y-int. of PQ is 10

$\therefore$ PQ has equation $y = 2x + 10$

i.e. $2x - y + 10 = 0$

(iii) $d_\perp = \frac{|2(2) - (-2) + 10|}{\sqrt{2^2 + (-1)^2}} = \frac{16}{\sqrt{5}}$ units

(iv) $S = (7, 8)$

(v) $A = bh = 5\sqrt{5} \times \frac{16}{\sqrt{5}} = 80$ units^2

(vi) $\tan\theta = m$ $\therefore$ $\tan \angle PQO = 2$

$\angle PQO \doteqdot 63^\circ 26'$

QUESTION THREE (continued)

(b) (i) $$y' = \frac{(8-5x)\times 6x - (3x^2-4)\times -5}{(8-5x)^2}$$

$$= \frac{48x - 30x^2 + 15x^2 - 20}{(8-5x)^2}$$

$$\therefore y' = \frac{48x - 15x^2 - 20}{(8-5x)^2}$$

Comments for part (b)
(i) Most students applied the quotient rule correctly - though some solutions could have been tidier.
(ii) A variety of errors - mainly due to incomplete use of the product or chain rule.

(ii) $$y = (10-3x)(1-4x^2)^{\frac{1}{2}}$$

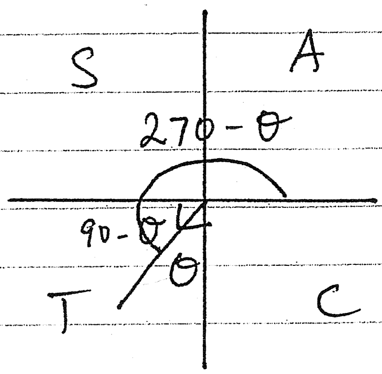
$$y' = (10-3x)\times \tfrac{1}{2}(1-4x^2)^{-\frac{1}{2}}\times -8x + (1-4x^2)^{\frac{1}{2}}\times -3$$

$$= \frac{-4x(10-3x)}{\sqrt{1-4x^2}} - 3\sqrt{1-4x^2}$$

$$= \frac{-40x + 12x^2 - 3(1-4x^2)}{\sqrt{1-4x^2}}$$

$$\therefore y' = \frac{24x^2 - 40x - 3}{\sqrt{1-4x^2}}$$

(c) $$\text{LHS} = \frac{\sin(270-\theta)}{\sin\theta}$$



$$= -\frac{\sin(90-\theta)}{\sin\theta}$$

$$= -\frac{\cos\theta}{\sin\theta}$$

$$= -\cot\theta$$

$$= \text{RHS}$$

Comments for part (c)
Some students did not provide all steps to "show" the required result. Communication of reasoning was a key element as to what was being examined in this question.

Question 4 - Year 11 - Mathematics - Task 3 2015

a) $y = x^2 - 8x + 20$ at $x = 2$, $y = 2^2 - 8(2) + 20$

$\frac{dy}{dx} = 2x - 8$ at $x = 2$

$y = 4 - 16 + 20$

$y = 8$

$\frac{dy}{dx} = 4 - 8$

$\therefore P(2, 8)$

$m = -4$

$\therefore$ Eqn of tangent: $y - y_1 = m(x - x_1)$

$y - 8 = -4(x - 2)$

$y - 8 = -4x + 8$

$4x + y - 16 = 0$

(This question was completed extremely well nearly all students got it correct)

b) $\frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta} = 2\tan^2\theta$

$L.H.S = \frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta}$

$= \frac{\sin\theta(1+\sin\theta) - \sin\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{\sin\theta + \sin^2\theta - \sin\theta + \sin^2\theta}{1 - \sin^2\theta}$

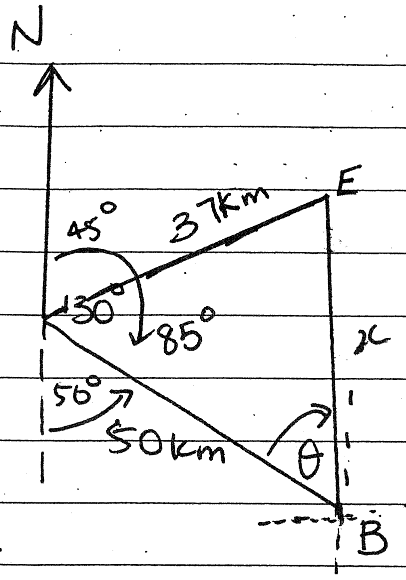
$= \frac{2\sin^2\theta}{\cos^2\theta}$

$= 2\tan^2\theta$

$= R.H.S.$

(Most students multiplied by the conjugal of each fraction, expanded & simplified very well.)

c) i)



$S = \frac{d}{t}$

$S \times t = d$

$30\cancel{0} \times \frac{10}{6\cancel{0}} = d$

$d = 50\text{km}$

i) Diagram done well
ii) Most used cosine rule some substitute incorrect angle.
iii) Angle was found very well however, the bearing was poorly done.

ii) $a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 50^2 + 37^2 - 2(50)(37)\cos 8$

$x^2 = 3546.5$

$\therefore x = \sqrt{3546.5}$

$x = 59.55\text{ km}$

iii) $\frac{a}{\sin A} = \frac{b}{\sin B}$.

$\frac{37}{\sin\theta} = \frac{59.55}{\sin 85^\circ}$

$\frac{\sin\theta}{37} = \frac{\sin 85^\circ}{59.55}$

$\sin\theta = \frac{37 \times \sin 85^\circ}{59.55}$

$\sin\theta = 0.62$.

$\theta = 38^\circ 14'$

$\therefore$ Bearing $= N(50 - 38^\circ 14')W$ (alternate $\angle$'s)

$= N\ 11^\circ 46'\ W$

OR

$348^\circ 14'\ T$

OR/ $\cos\theta = \frac{50^2 + (59.55)^2 - 37^2}{2 \times 50 \times 59.55}$

$\cos\theta = 0.7854$

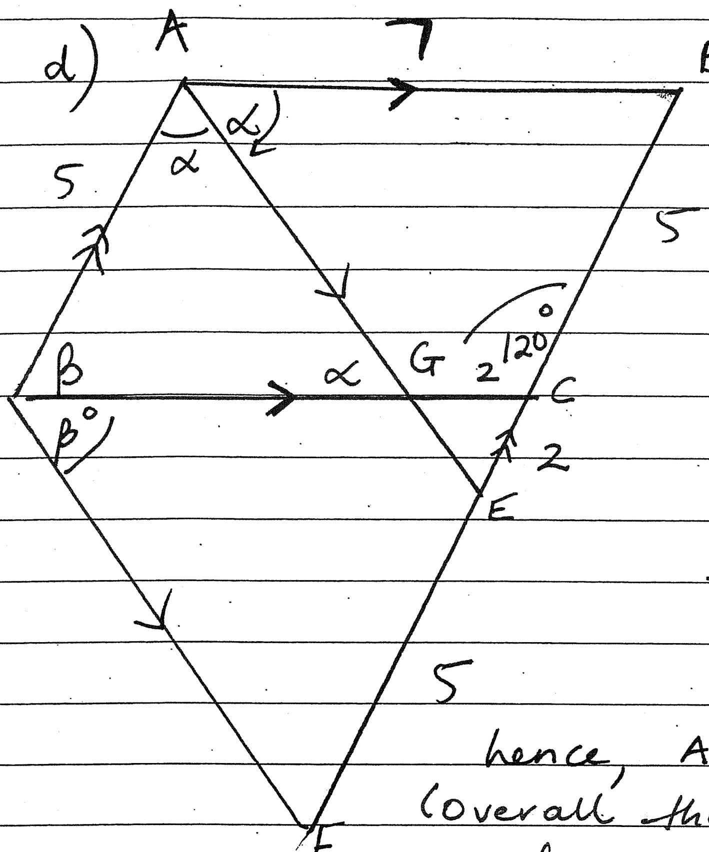
$\theta = \cos^{-1}(0.7854)$

$\theta = 38^\circ 14'$

$\therefore$ Bearing $= N(50 - 38^\circ 14')W$ (alternate $\angle$'s)

$= N\ 11^\circ 46'\ W$ or $348^\circ 14'\ T$

d)



i) $\angle BAG = \alpha$ (given)

$\angle AGD = \alpha$ (alt $\angle$'s $AB \parallel DC$)

$\angle ADC = \beta$ (given)

$\angle FDG = \angle DGA$ (alternate $\angle$'s, $DF \parallel AC$

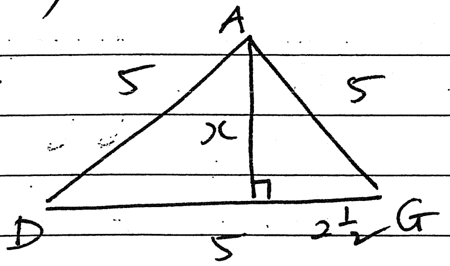
$\therefore \alpha = \beta$

$\therefore$ all angles in $\triangle ADG$ are equal $\therefore \triangle ADG$ is equilateral.

hence, $AD = AG = DG = 5$ units.

(overall this question was done poorly most found this proof challenging)

ii) In $\triangle ADG$



$x^2 = 5^2 - 2\frac{1}{2}^2$

$x^2 = \frac{75}{4}$

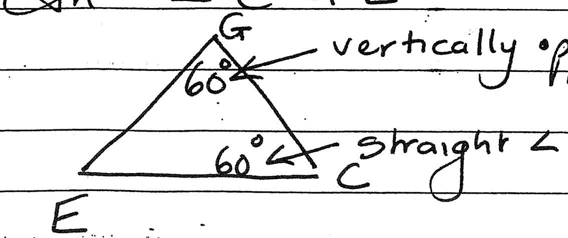
$x = \frac{5\sqrt{3}}{2}$

$DC = 7$ units opposite sides of parallelogram $ABCD$

$\therefore GC = 7 - 5$

$= 2$ units

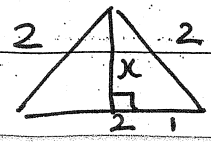
In $\triangle CGE$



$x^2 = 2^2 - 1^2$

$x = \sqrt{3}$

$\therefore \triangle CGE$ is equilateral



hence, Area of trapezium $= \frac{1}{2}h(a+b)$

$$= \frac{1}{2}\left(\frac{5\sqrt{3}}{2} + \sqrt{3}\right)\left(5 + 12\right)$$

$$= \left(\frac{5\sqrt{3}}{4} + \frac{\sqrt{3}}{2}\right) \times 17$$

$$= \left(\frac{5\sqrt{3} + 2\sqrt{3}}{4}\right) \times 17$$

$$= \frac{7\sqrt{3}}{4} \times 17$$

$$= \frac{119\sqrt{3}}{4}$$

Only a few students were successful in answering this part. Most who attempted didn't succeed. Most students ran out of time.