Sydney Girls High School

MATHEMATICS

YEAR 11

Assessment Task 3

2018

Reading time : 5 minutes

Writing time : 60 minutes

Total marks : 60

Topics: Basic Arithmetic and Algebra, Plane Geometry, Real Functions, Trigonometric Ratios, The Linear Function, Introductory Calculus: Tangent to a curve, differentiation of a function, The Quadratic Polynomial and Locus.

General Instructions:

- There are four (4) questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write using a black or blue pen.
- White-out or correction tape is NOT to be used.

Student Name:____________________________**Teacher Name:**____________________

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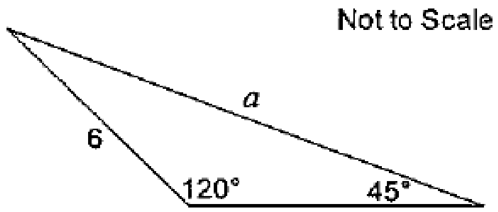
Question one (15 marks)

a) Fully factorise $4x^2 - 36$. (2)

b) Solve $|3x-2| < 1$. (2)

c) Find the values of a and b such that $\dfrac{2}{\sqrt{10}+3} = a + b\sqrt{10}$. (2)

d) Find the exact value of a in the diagram shown below: (3)



e) Simplify $\dfrac{x}{x^2-9} + \dfrac{3}{x+3}$. (2)

f) Evaluate $\lim_{x \to 2} \dfrac{x-2}{x^2+x-6}$. (2)

g) Find the gradient of the normal to the curve $y = x^3 - 7x^2 + 4x + 11$ when $x = 2$. (2)

Question two (15 marks)

a) Differentiate the following :

i) $y = 6x^3 - 2x + \dfrac{5}{x}$ (2)

ii) $y = (3x^2 - 7)^4$ (2)

iii) $y = \dfrac{3x}{x^2 + 1}$ (2)

b) Determine the value of A, B and C in the following : (3)

$$2x^2 + 6x - 9 \equiv A(x+2)(x-1) + B(x+2) + C$$

c) Solve $9^{3x-2} = \dfrac{1}{\sqrt[3]{27}}$. (2)

d) The line $x = -3$ is the axis of symmetry of the parabola $y = ax^2 + 12x + 9$.

i) Find the value of a . (1)

ii) Find the minimum value of the curve. (1)

e) Find the values of n for which the equation $x^2 + nx + 3 = 0$ has roots which differ by 2. (2)

Question three (15 marks)

a) If α and β are the roots of $5x^2-2x+6=0$ find the value of :

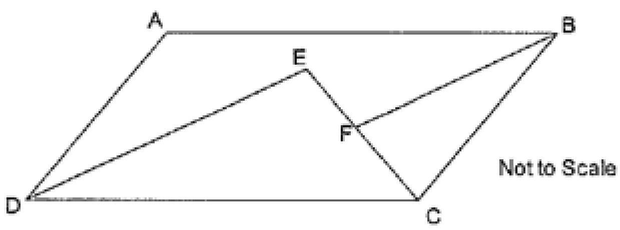
i) $\alpha+\beta$ (1)

ii) $\alpha\beta$ (1)

iii) $\dfrac{1}{\alpha^2\beta}+\dfrac{1}{\beta^2\alpha}$ (2)

iv) $\alpha^3\beta+\alpha\beta^3$ (2)

b) $ABCD$ is a parallelogram . DE bisects $\angle ADC$, EC bisects $\angle DCB$ and BF bisects $\angle ABC$. Copy the diagram on to your answer sheet.



i) Prove that ΔDEC is similar to ΔBFC. (3)

ii) Show that $\angle DEC = 90^\circ$. (2)

iii) If $DE=10cm$, $EC=6\ cm$ and $EF=FC$, find the length of BC. (2)

c) Find the coordinates of the vertex and the equation of the directrix for the parabola

$y^2+6y=8-4x$. (2)

Question Four (15 marks)

a) A is the point $(1,1)$ and B is the point $(4,1)$. A point $P(x, y)$ moves such that the ratio of PA to PB is $2:5$. Find the equation of the locus of P. (3)

b) Differentiate and express your answer in simplest factored form: (3)

$$y = (4x-5)^4(1-3x)$$

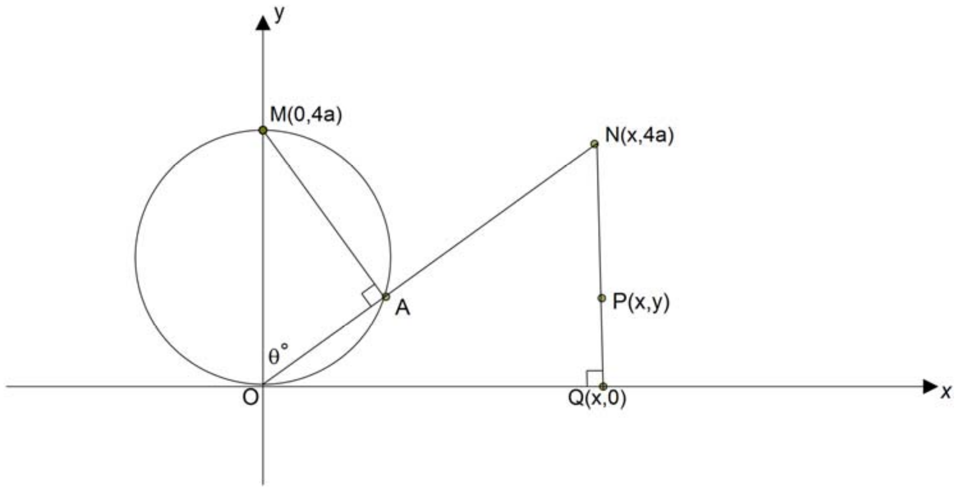
c) Find the value of k for which the quadratic equation $x^2 + (k-3)x + k = 0$ has real roots. (3)

d) Solve for x: $4^{x-1} - 3(2^x) + 8 = 0$. (2)

Question Four continued on the next page

Question Four continued:

e) A circle is drawn using the origin O and point $M(0,4a)$ as the diameter. A line segment of height $4a$ is drawn between $Q(x,0)$ and $N(x,4a)$. The line ON intersects the circle at the point A. $P(x,y)$ is a point on NQ so that A and P are the same distance from the x −axis.



The angle θ is the angle between OM and ON. $\angle OAM$ is always right angled, regardless of the location of A.

i) Show that the x coordinate of point P is given by

$$x = 4a\tan\theta \qquad (1)$$

ii) Show that the y coordinate of point P is given by

$$y = 4a\cos^2\theta \qquad (2)$$

iii) Hence, or otherwise, show that the y coordinate of P is given by the expression : (1)

$$y = \frac{64a^3}{x^2 + 16a^2}$$

The End

Year 11 – Yearly 2018

Question 1

a) $4x^2 - 36 = 4(x^2 - 9)$

$= 4(x-3)(x+3)$

Some students didn't fully factorise.

2

b) $|3x-2| < 1$

$-1 < 3x - 2 < 1$

$1 < 3x < 3$

$\frac{1}{3} < x < 1$

Poor answered, most students didn't get correct.

2

One mark was awarded if answer was written separately.

c) $\frac{2}{\sqrt{10}+3} = a + b\sqrt{10}$

$\frac{2}{\sqrt{10}+3} \times \frac{\sqrt{10}-3}{\sqrt{10}-3} = \frac{2(\sqrt{10}-3)}{(\sqrt{10})^2 - (3)^2}$

Very well done.

$= \frac{2\sqrt{10}-6}{10-9}$

2

$= 2\sqrt{10} - 6$

$= -6 + 2\sqrt{10}$

$\therefore a = -6$ and $b = 2$

d) $\frac{a}{\sin 120^\circ} = \frac{b}{\sin 45^\circ}$

$a = \frac{6 \times \sin 120^\circ}{\sin 45^\circ}$

$a = 6 \times \frac{\sqrt{3}}{2} \div \frac{1}{\sqrt{2}}$

$a = 3\sqrt{3} \times \sqrt{2}$

3

$\therefore a = 3\sqrt{6}$ units

Well done, except some student forgot exact value

e) $\frac{x}{x^2-9} + \frac{3}{x+3} = \frac{x}{(x-3)(x+3)} + \frac{3}{(x+3)}$

$= \frac{x + 3(x-3)}{(x-3)(x+3)}$

Very Well completed

$= \frac{x + 3x - 9}{(x-3)(x+3)}$

2

$= \frac{4x-9}{(x-3)(x+3)}$

f) $\lim_{x \to 2} = \frac{x-2}{x^2+x-6} = \lim_{x \to 2} \frac{\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$

$= \lim_{x \to 2} \frac{1}{x+3}$

Well completed

$= \frac{1}{2+3}$

2

$= \frac{1}{5}$

g) $y = x^3 - 7x^2 + 4x + 11$

$\frac{dy}{dx} = 3x^2 - 14x + 4$

Well completed although some students found equation of normal which was not required.

at $x = 2$

2

$\frac{dy}{dx} = 3(2)^2 - 14(2) + 4$

$\frac{dy}{dx} = -12$ (gradient of tangent at $x = 2$)

$\therefore$ Gradient of normal at $x = 2$ is: $m = \frac{1}{12}$.

$\boxed{Q2}$

a) $y = 6x^3 - 2x + \dfrac{5}{x}$

i) $y' = 18x^2 - \dfrac{5}{x^2} - 2$ ✓✓

ii) $y = (3x^2 - 7)^4$

$y' = 4(3x^2 - 7)^3 (6x)$ ✓

$= 24x(3x^2 - 7)^3$ ✓

iii) $y = \dfrac{3x}{x^2 + 1}$

$y' = \dfrac{3(x^2+1) - 2x(3x)}{(x^2+1)^2}$ ✓

$y' = \dfrac{3x^2 + 3 - 6x^2}{(x^2+1)^2}$

$y' = \dfrac{3 - 3x^2}{(x^2+1)^2}$ ✓

b) $2x^2 + 6x - 9 \equiv A(x+2)(x-1) + B(x+2) + C$

$\equiv A(x^2 - x - 2) + Bx + 2B + C$

$\equiv Ax^2 + (A+B)x - 2A + 2B + C$

$A = 2$ ✓ (1)

$A + B = 6$ (2)

$-2A + 2B + C = -9$ (3)

Subs (1) into (2) $\therefore B = 4$ ✓

$-2(2) + 2(4) + C = -9$

$C = -13$ ✓

Q2

c) $9^{3x-2} = \frac{1}{\sqrt[3]{29}}$

$3^{2(3x-2)} = 3^{-1}$

$6x - 4 = -1$

$6x = 3$

$x = \frac{1}{2}$

d) $y = ax^2 + 12x + 9$

$x = \frac{-b}{2a} = \frac{-12}{2a} = -3$

i) $-12 = -6a$

$a = 2$

ii) $y = ax^2 + 12x + 9$

$y = 2x^2 + 12x + 9$

$y = 2(-3)^2 + 12(-3) + 9$

$y = -9$ (Minimum value)

some students could not form the right equations hence, couldn't find "n".

e) $x^2 + nx + 3 = 0$

Let the roots be α and $\alpha + 2$

Sum: $2\alpha + 2 = -n$ ① $\therefore \alpha = -\frac{(n+2)}{2}$

$\alpha^2 + 2\alpha = 3$ ②

Sub ① into ② $\frac{n^2+4n+4}{4} - n - 2 = 3$

$n^2 + \cancel{4n} + 4 - \cancel{4n} - 8 - 12 = 0 \therefore n^2 = 16$ $\boxed{n = \pm 4}$

Question three (15 marks)

a) If α and β are the roots of $5x^2 - 2x + 6 = 0$ find the value of :

i) $\alpha + \beta$ (1)

ii) $\alpha\beta$ (1)

iii) $\dfrac{1}{\alpha^2\beta} + \dfrac{1}{\beta^2\alpha}$ (2)

iv) $\alpha^3\beta + \alpha\beta^3$ (2)

$$\left.\begin{aligned} a &= 5 \\ b &= -2 \\ c &= 6 \end{aligned}\right\}.$$

i) $\alpha + \beta = -\dfrac{b}{a} = \dfrac{2}{5}$ ✓

ii) $\alpha\beta = \dfrac{c}{a} = \dfrac{6}{5}$ ✓

iii) $\dfrac{1}{\alpha^2\beta} + \dfrac{1}{\beta^2\alpha}$

$= \dfrac{\beta^2\alpha + \alpha^2\beta}{\alpha^3\beta^3}$ (✱)

$= \dfrac{\cancel{\alpha\beta}(\alpha+\beta)}{(\alpha\beta)^{\cancel{3}\,2}}$

$= \dfrac{2}{5} \times \left(\dfrac{5}{6}\right)^2$ ✓

$= \dfrac{2 \times 5}{36}$

$= \dfrac{5}{18}.$ ✓

✱ Many students over-complicated $\alpha^3\beta^3$ and lost one mark

iv) $\alpha^3\beta + \alpha\beta^3$

$= \alpha\beta\,(\alpha^2 + \beta^2)$

$= \alpha\beta\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$ (✱)

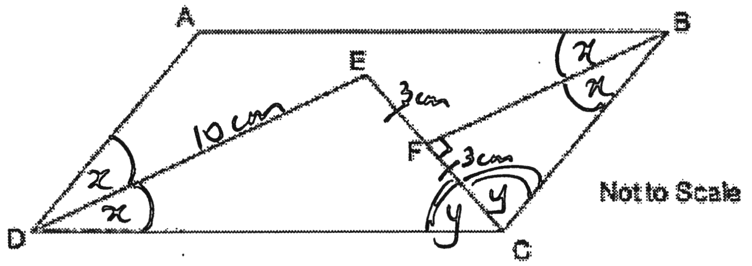
$= \dfrac{6}{5} \times \left[\left(\dfrac{2}{5}\right)^2 - 2\left(\dfrac{6}{5}\right)\right]$ ✓

$= \dfrac{6}{5} \times \left[\dfrac{4}{25} - \dfrac{12}{5}\right]$

$= \dfrac{-336}{125}.$ ✓

✱ Many students lost one mark as they did not put brackets around their work at step (✱) and so ended up with a calulation error.

b) $ABCD$ is a parallelogram. DE bisects $\angle ADC$, EC bisects $\angle DCB$ and BF bisects $\angle ABC$. Copy the diagram on to your answer sheet.



i) Prove that ΔDEC is similar to ΔBFC. (3)

ii) Show that $\angle DEC = 90^\circ$. (2)

iii) If $DE = 10cm$, $EC = 6\ cm$ and $EF = FC$, find the length of BC. (2)

i) $\angle ADC = \angle ABC$ (opp. $\angle$s of parallelogram $ABCD$) ✓

hence let $\angle ADE = \angle EDC = x$ (DE bisects $\angle ADC$)
also $\angle ABF = \angle CBF = x$ (BF bisects $\angle ABC$).

In ΔDEC and ΔBFC:
$\angle EDC = \angle CBF = x$ (as above)
$\angle ECD = \angle ECB = y$ (EC bisects $\angle DCB$) ✓
$\therefore$ $\Delta DEC \parallel\!\mid \Delta BFC$ (equiangular) ✓

3.

☆ This part was marked generously.
☆ Students need to present more structured proofs and provide more logical reasoning in future.

ii) $\angle ADC + \angle BCD = 180°$ (cointerior $\angle$s, $AD \parallel BC$ in parallelogram ABCD).

$$2x + 2y = 180°$$
$$2(x+y) = 180°$$
$$\therefore \quad x+y = 90°$$

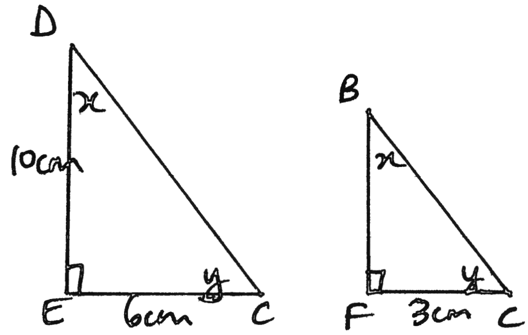
In $\triangle DEC$:

$\angle DEC + \angle EDC + \angle ECD = 180°$ ($\angle$ sum $\triangle DEC$)

$$\angle DEC + x + y = 180°$$
$$\angle DEC + 90° = 180°$$
$$\therefore \angle DEC = 90°$$

(2)

iii) Since $\triangle DEC \parallel\!\mid \triangle BFC$



$\dfrac{BC}{DC} = \dfrac{FC}{EC}$ (sides in same ratio).

$$\frac{BC}{2\sqrt{34}} = \frac{3}{6}$$

$$\therefore BC = \frac{\cancel{6}\sqrt{34}}{\cancel{6}} = \sqrt{34} \text{ cm}$$

$(= 5.83)$

$DC = \sqrt{10^2 + 6^2}$

$DC = \sqrt{136}$

$\therefore DC = 2\sqrt{34}$

(2).

If students did not provide enough reasons for parts ii) and iii) then they were deducted one mark

c) Find the coordinates of the vertex and the equation of the directrix for the parabola

$y^2 + 6y = 8 - 4x$. (2)

$$y^2 + 6y = 8 - 4x.$$

$$y^2 + 6y + 9 = 8 - 4x + 9$$

$$(y+3)^2 = 17 - 4x$$

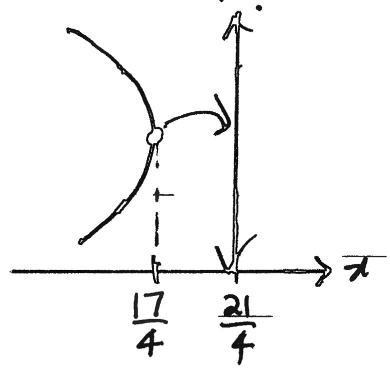
$$(y+3)^2 = -4\left(x - \frac{17}{4}\right). \Rightarrow -4a = -4$$

$$a = 1.$$

$\therefore$ vertex $= \left(\frac{17}{4}, -3\right)$ ✓

directrix $\Rightarrow$ $x = \frac{17}{4} + 1 = \frac{21}{4}$ ✓

$$\left(x = 5\tfrac{1}{4}\right)$$



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A diagram could have assisted many students to calculate the orientation of the parabola after completing the square, hence the equation of the directrix.

Q4

a) $\frac{PA}{PB} = \frac{2}{5}$

* Many students used the wrong ratio.

$5PA = 2PB$

$$5\sqrt{(x-1)^2+(y-1)^2} = 2\sqrt{(x-4)^2+(y-1)^2}$$

$$25[(x^2-2x+1)+(y^2-2y+1)] = 4[(x^2-8x+16+y^2-2y+1]$$

$$25x^2-50x+25+25y^2-50y+25 = 4x^2-32x+64+4y^2-8y+4$$

$$21x^2-18x+50+21y^2-42y-68=0$$

$$21x^2-18x+21y^2-42y-18=0$$

$$7x^2-6x+7y^2-14y-6=0$$

d) $\frac{(2^x)^2}{4} - 3(2^x) + 8 = 0$ (let $m = 2^x$)

$$\frac{m^2}{4} - 3m + 8 = 0$$

$$m^2 - 12m + 32 = 0$$

* Many students didn't use the indices rules correctly.

$$(m-4)(m-8) = 0$$

$m = 4$ or 8

$2^x = 4$ or $2^x = 8$

$x = 2$ or 3

b) $y' = 4(4x-5)^3 \cdot 4(1-3x) + (4x-5)^3 \cdot -3$

$= 16(4x-5)^3(1-3x) - 3(4x-5)^4$

$= (4x-5)^3(16(1-3x) - 3(4x-5))$

$= (4x-5)^3(16 - 48x - 12x + 15)$

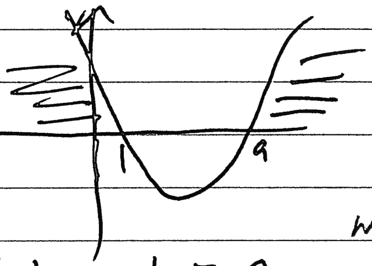
$= (4x-5)^3(31 - 60x)$

* some students didn't simplify correctly

c) $(k-3)^2 - 4k \geq 0$

$k^2 - 6k + 9 - 4k \geq 0$

$k^2 - 10k + 9 \geq 0$



$k \leq 1, \; k \geq 9$

* This question was done well

e) i) $\tan\theta = \dfrac{OQ}{NQ}$

$\angle ONQ = \theta$
alt, $\angle$'s
$OM \parallel NQ$

$= \dfrac{x}{4a}$

$x = 4a\tan\theta$

ii) $\cos\theta = \dfrac{OA}{4a}$

Draw $AH \parallel$ to NQ
$\angle OAH = \theta$

$OA = 4a\cos\theta$

$\cos\theta = \dfrac{y}{OA}$

$y = OA\cos\theta$

$y = 4a\cos\theta\cos\theta$

$= 4a\cos^2\theta$

iii) $\tan^2\theta = \dfrac{x^2}{16a^2}$

$1 + \tan^2\theta = \sec^2\theta$

$\cos^2\theta = \dfrac{y}{4a}$

$1 + \dfrac{x^2}{16a^2} = \dfrac{4a}{y}$

$y = \dfrac{4a}{1 + \dfrac{x^2}{16a^2}}$

$= \dfrac{4a(16a^2)}{16a^2 + x^2}$

$= \dfrac{64a^3}{16a^2 + x^2}$

* Many students had problems with this question