Sydney Girls High School

MATHEMATICS

YEAR 11

Assessment Task 3

2019

Reading time :	5 minutes
Writing time :	60 minutes
Total marks :	60
Topics:	Functions, Calculus, Exponential and Logarithmic Functions, Trigonometric Functions and Statistical Analysis (excluding Discrete probability distributions)

General Instructions:

- There are four (4) questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- A correct answer without working will be awarded a maximum of 1 mark.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write using a black or blue pen.
- White-out or correction tape is NOT to be used.

Student Name:________________________ **Teacher Name:**____________________

BLANK PAGE

Question One (15 marks)

a) Calculate $\log_e 7$ correct to 2 decimal places. (1)

b) Convert $\frac{8\pi}{15}$ radians into degrees. (1)

c) Solve the equation $|2x+1|=5$. (2)

d) A group of 150 people was surveyed and the results were recorded.

Survey results

	Owns a mobile phone	Does not own a mobile phone	Total
Male	42	28	70
Female	63	17	80
	105	45	150

A person is selected at random from the surveyed group. What is the probability that the person selected is male given that they don't own a mobile? (1)

e) Sketch the following graphs showing the main features:

i) $y=e^x+3$ (2)

ii) $y=(x-1)^2(x+3)$ (2)

iii) $y=2+\frac{1}{x-3}$ (2)

f) Given $f(x)=9-3x$, find a simple expression for $\frac{f(x)-f(c)}{x-c}$. (2)

g) Write down the domain and range of the following function using interval notation.

$$y=x^2-2$$ (2)

Question Two (15 marks)

a) Given $f(x)=|x|-4$, sketch $y=f(x)$ and $y=f(x+1)$ on the same number plane showing all the intercepts. (2)

b) The mass in grams of a melting Magnum ice cream is given by $M=101-t-t^2$, where t is in minutes .

i) Find the rate at which the ice-cream is melting at 5 minutes. (2)

ii) Find the average rate of change at which the ice cream is melting between 2 and 5 minutes. (2)

c) Sketch the graph of the function $\boldsymbol{y=-3+\cos x}$ for $\boldsymbol{0 \le x \le 2\pi}$.
(2)

d) Suppose $\boldsymbol{P(A \cap B)=0.12}$ and $\boldsymbol{P(A \cap \overline{B})=0.43}$. Find $\boldsymbol{P(B)}$. (1)

e) Find the values of x for which $f(x)=x^2-3x+10$, is increasing. (2)

f) Solve $\ln(x+1)=\ln 15-\ln(x+3)$. (3)

g) Shampoo A has pH 9.2 and shampoo B has pH of 10.1. The pH scale is base 10 logarithmic. How much more alkaline is shampoo B than shampoo A?
(Note that a solution is alkaline if its pH is greater than 7). (1)

Question Three (15 marks)

a) Differentiate the following:

i) $y = \sqrt[3]{x^2}$ (1)

ii) $y = \dfrac{1}{3x^2}$ (1)

iii) $y = \dfrac{e^{3x}}{2x+1}$ (2)

b) A machine produces items of which 5% are defective.

i) If three items have been chosen at random, what is the probability that they are all defective? (1)

ii) A random sample of n items is taken from the machine. Find the largest possible value of n in order that the probability that none of the items is defective is at least 0.6. (2)

c) Differentiate $y = (4x^2 + 3)^8 (9 - 3x)$, and write your answer in factored form. (3)

d) Solve the equation $\sin^2 x + 2\cos x = 1$, for $0 \le x \le 2\pi$. (2)

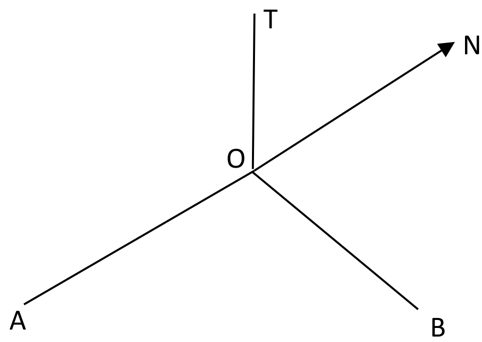
e) Given $f(x) = 3e^{2x} + e^{-2x}$, find the exact value of x, where $f'(x) = 0$. (3)

Question Four (15 marks)

a) If $y = 8 + \log_4(x+2)$, show that $x = 2(2^{2y-17} - 1)$. (2)

b) Find the equation of the normal to the curve $y = -3x^2 + 5$ where the tangent is perpendicular to the line $2x + 4y - 5 = 0$. (3)

c) From a point A due south of a tower , the angle of elevation of the top of the tower T, is 23° . From a point B , east of the tower the angle of elevation of T is 35° . The distance AB is 210 metres.

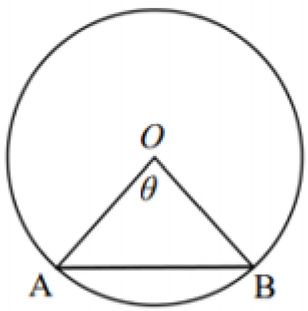


i) Copy the diagram on to your answer sheet, adding the information to your diagram. (1)

ii) Write an expression for the length of OB . (1)

iii) Hence, find the height of the tower to the nearest metre. (2)

d) The chord AB divides the area of the circle of radius r into two parts. AB subtends an angle of θ at the centre, as shown.

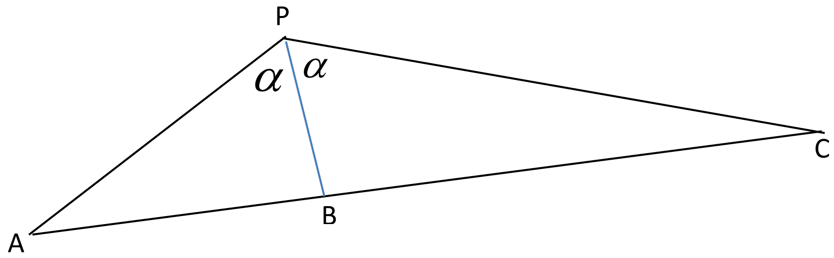


Find an algebraic expression in terms θ , for the ratio of the areas of the major segment to the minor segment. (2)

Question Four continued on the next page

Question Four continued

e) In triangle APC, B is the point on AC such that $\angle APB = \angle BPC = \alpha$.



i) Show that $AB \times PC = AP \times BC$. (2)

ii) Given that $AP = 3$ cm, $PC = 5$ cm and $\alpha = 60^{\circ}$, calculate the length of AB. (2)

The End

Q1

a) $\log_e 7 = 1.95$

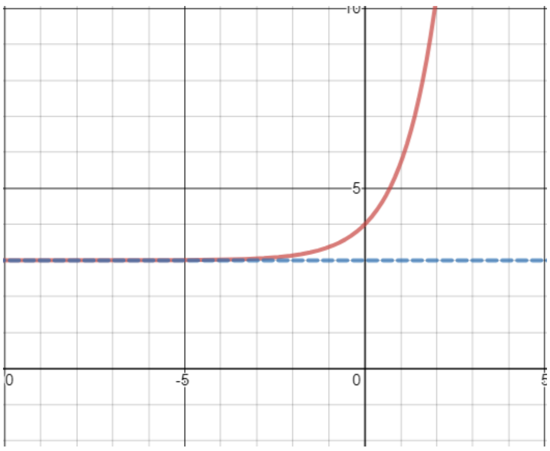
b) $\frac{8\pi}{15} = 96°$

c) $|2x + 1| = 5$

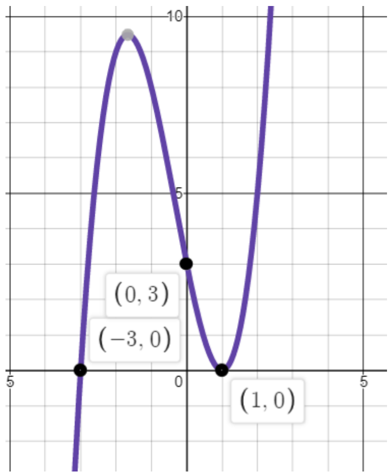
$2x + 1 = 5 \quad or \quad 2x + 1 = -5$
$2x = 4 \quad or \quad 2x = -6$
$x = 2 \quad or\ x = -3$

d) $\frac{28}{45}$

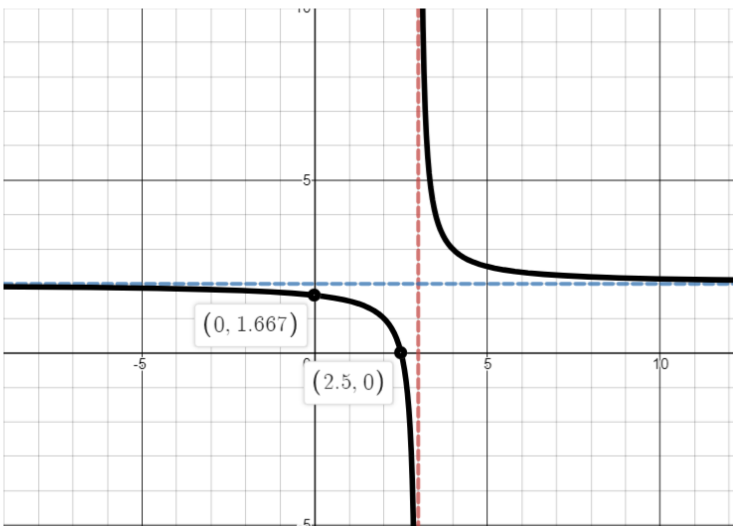
e) I)



ii)



iii)



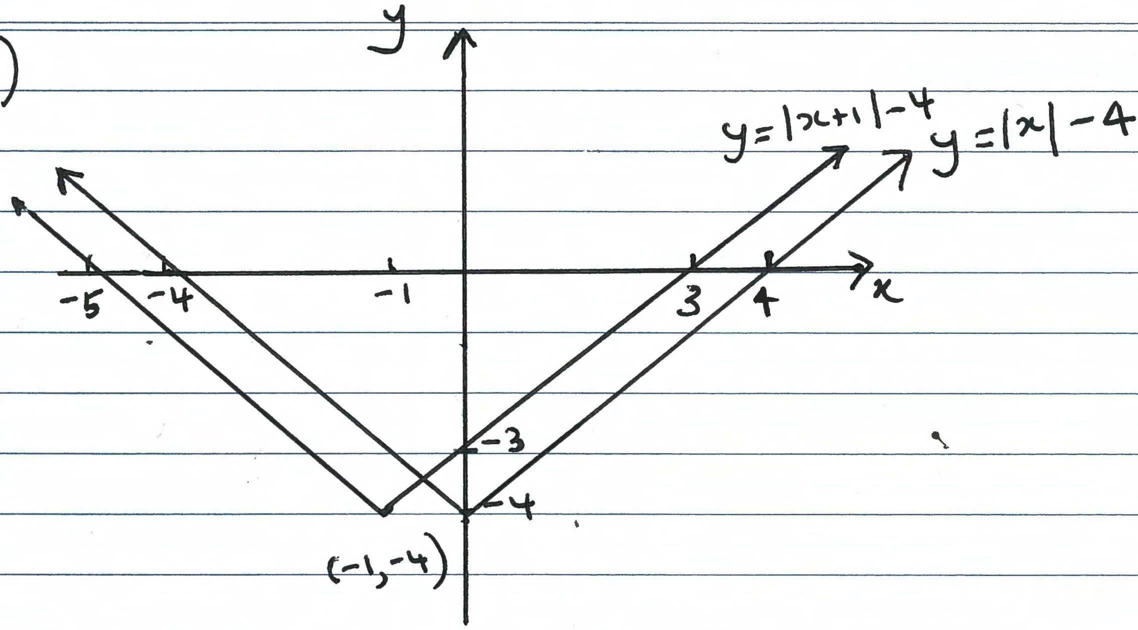
f) $f(x) = 9 - 3x\ , f(c) = 9 - 3c$

$$\frac{f(x) - f(c)}{x - c} = \frac{9 - 3x - (9 - 3c)}{x - c}$$
$$= \frac{9 - 3x - 9 + 3c}{x - c}$$
$$= \frac{-3(x - c)}{(x - 3)}$$
$$= -3$$

g) $D: (-\infty, \infty) \qquad R: [-2, \infty)$

Question 2

a)



All intercepts needed to be shown as was indicated in the question.
Graphs need to be a reasonable size, labelled correctly and a ruler should be used. Many students failed to do this.

b) i) $M = 101 - t - t^2$

$\frac{dm}{dt} = -1 - 2t$

Let $t = 5$

$\frac{dm}{dt} = -1 - 10$

$= -11$

$\therefore$ the icecream is melting at 11gm/min at 5 mins.

ii) $M(2) = 101 - 2 - 4 = 95$

$M(5) = 101 - 5 - 25 = 71$

$\therefore$ av rate of $\Delta = \frac{95-71}{3}$

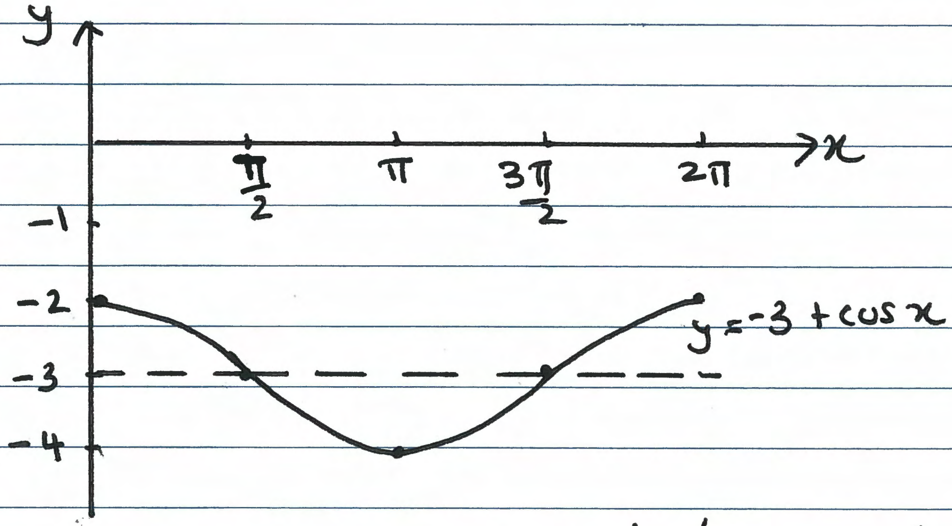
$= \frac{24}{3}$

$= 8$gm/min

(between 2min and 5 min)

* A large number of students used an incorrect process and ended up with the same answer. NO marks were awarded.

c) $y = -3 + \cos x$



Graphs needed to be clearly labelled and well shaped

d) There was a problem with this question. It couldn't be solved without additional information. All students were awarded 1 mark

e) $f(x) = x^2 - 3x + 10$

$f'(x) = 2x - 3$

$f(x)$ is increasing when $f'(x) > 0$ (not ≥ 0)

$2x - 3 > 0$

$2x > 3$

$x > 3/2$

f) $\ln(x+1) = \ln 15 - \ln(x+3)$

$\ln(x+1) = \ln\left(\frac{15}{x+3}\right)$

$\therefore\ x+1 = \frac{15}{x+3}$

$(x+1)(x+3) = 15$

$x^2 + 4x + 3 = 15$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$\therefore x = -6$ and $x = 2$ (2 marks)

however since $x+1 > 0$ and $x+3 > 0$

$x > -1$ $\quad$ $x > -3$

$\therefore x > -1$

$\therefore x = 2$ is the only possible solution

(1 mark for recognising this)

g) difference $= 10.1 - 9.2$

$= 0.9$

$\therefore$ shampoo B is $10^{0.9}$ more alkaline

ie it is $\doteqdot 7.94$ times more alkaline.

This question was poorly done. Very few students answered this correctly.

Question 3

(a)(i) $y = x^{\frac{2}{3}}$

$$\frac{dy}{dx} = \frac{2x^{-\frac{1}{3}}}{3}$$

$$= \frac{2}{3\sqrt[3]{x}}$$

(ii) $y = \frac{1}{3}x^{-2}$

$$\frac{dy}{dx} = -\frac{2}{3}x^{-3}$$

$$= -\frac{2}{3x^3}$$

(iii) $$\frac{dy}{dx} = \frac{3(2x+1)e^{3x} - 2e^{3x}}{(2x+1)^2}$$

$$= \frac{e^{3x}(6x+1)}{(2x+1)^2}$$

(b)(i) $$\left(\frac{5}{100}\right)^3 = \frac{1}{20^3}$$

$$= \frac{1}{8000}$$

(ii) $$\left(\frac{19}{20}\right)^n \geq \frac{3}{5}$$

$$n\log\left(\frac{19}{20}\right) \geq \log\left(\frac{3}{5}\right)$$

$$n \leq \frac{\log\left(\frac{3}{5}\right)}{\log\left(\frac{19}{20}\right)} \approx 9.959$$

$$n = 9$$

Many students set this problem up as an equation. So they calculated correctly but misinterpreted their solution by round up to 10.

(c) $$\frac{dy}{dx} = 8(4x^2+3)^7(8x)(9-3x) - 3(4x^2+3)^8$$

$$= (4x^2+3)^7[64x(9-3x) - 3(4x^2+3)]$$

$$= -3(4x^2+3)^7[68x^2 - 192x + 3]$$

Many students lost a mark for sloppy algebraic expansions in part (c).

(d) $$1 - \cos^2 x + 2\cos x = 1$$

$$2\cos x - \cos^2 x = 0$$

$$\cos x\,(2 - \cos x) = 0$$

Some students divided by cosx and were left with no solutions.

$$\cos x = 0 \qquad \cos x = 2$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \text{No solutions}$$

(e) $$f'(x) = 6e^{2x} - 2e^{-2x}$$

Many students differentiated correctly but had no idea of what to do next.

$$6e^{2x} - 2e^{-2x} = 0$$

$$3e^{2x} = e^{-2x}$$

$$e^{4x} = \frac{1}{3}$$

$$4x = \ln\left(\frac{1}{3}\right)$$

$$x = \frac{1}{4}\ln\left(\frac{1}{3}\right) \quad \text{or} \quad x = -\frac{\ln 3}{4}$$

Question Four (15 marks)

a) If $y = 8 + \log_4(x+2)$, show that $x = 2(2^{2y-17} - 1)$. (2)

$y - 8 = \log_4(x+2)$

$\therefore x + 2 = 4^{y-8}$ ✓

$x + 2 = 2^{2y-16}$

$x = 2^{2y-16} - 2$

$x = 2(2^{2y-16-1} - 1)$ ✓

$x - 2(2^{2y-17} - 1)$

Many students could not complete the proof for the second mark. Revision of indices is required.

b) Find the equation of the normal to the curve $y = -3x^2 + 5$ where the tangent is perpendicular to the line $2x + 4y - 5 = 0$. (3)

$2x + 4y - 5 = 0$

$4y = -2x + 5$

$y = -\frac{1}{2}x + \frac{5}{4}$

$M_L = -\frac{1}{2}$

$\therefore M_T = 2 \quad \cdots ①$

$y = -3x^2 + 5$

$\frac{dy}{dx} = -6x$ ✓

$\therefore m_T = -6x \quad \cdots ②$

① = ②: $-6x = 2$

$x = -\frac{1}{3}$

$y = -3\left(-\frac{1}{3}\right)^2 + 5 = \frac{14}{3}$.

$\therefore$ point $= \left(-\frac{1}{3}, \frac{14}{3}\right)$ ✓

$\therefore M_N = -\frac{1}{2}$

through the point $\left(-\frac{1}{3}, \frac{14}{3}\right)$

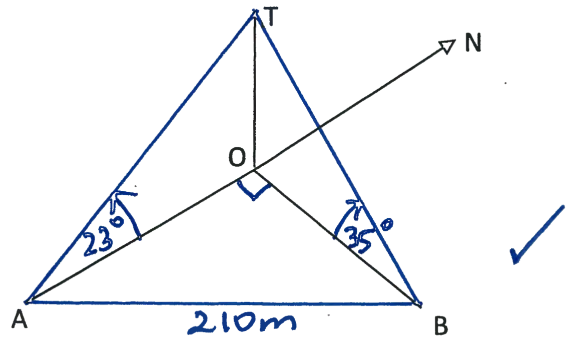
Equation of normal:

$y - \frac{14}{3} = -\frac{1}{2}\left(x + \frac{1}{3}\right)$

$y = -\frac{1}{2}x + \frac{9}{2}$ ✓

Students need to distinguish between different gradients in their solutions as confusion caused loss of marks !!

c) From a point A due south of a tower, the angle of elevation of the top of the tower T, is 23°. From a point B, east of the tower the angle of elevation of T is 35°. The distance AB is 210 metres.



i) Copy the diagram on to your answer sheet, adding the information to your diagram. (1)

ii) Write an expression for the length of OB. (1)

iii) Hence, find the height of the tower to the nearest metre. (2)

ii) $\tan 35^\circ = \dfrac{OT}{OB}$ $\quad \therefore OB = \dfrac{OT}{\tan 35^\circ}$ ✓

Many answers accepted.

iii) In ΔAOB:

$$AB^2 = OB^2 + OA^2$$

$$(210)^2 = \left(\frac{OT}{\tan 35^\circ}\right)^2 + \left(\frac{OT}{\tan 23^\circ}\right)^2 \quad ✓$$

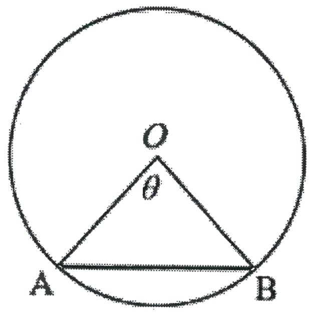
$$210^2 = OT^2\left(\frac{1}{\tan^2 35^\circ} + \frac{1}{\tan^2 23^\circ}\right)$$

$$210^2 = OT^2\left(\frac{\tan^2 23^\circ + \tan^2 35^\circ}{\tan^2 35^\circ \tan^2 23^\circ}\right)$$

$$\therefore OT = \sqrt{\frac{210^2 \tan^2 35^\circ \tan^2 23^\circ}{\tan^2 23^\circ + \tan^2 35^\circ}}$$

$OT \doteqdot 76\text{ m}$. ✓

d) The chord AB divides the area of the circle of radius r into two parts. AB subtends an angle of θ at the centre, as shown.



Find an algebraic expression in terms θ, for the ratio of the areas of the major segment to the minor segment. (2)

Minor segment $= \frac{1}{2}\,{}^{2}(\theta - \sin\theta)$

$\therefore$ Major segment $= \pi r^2 - \frac{1}{2}r^2(\theta - \sin\theta)$

$$\frac{A\ \text{major}}{A\ \text{minor}} = \frac{\pi r^2 - \frac{1}{2}r^2(\theta - \sin\theta)}{\frac{1}{2}r^2(\theta - \sin\theta)}$$

$$= \frac{\frac{1}{2}r^2\left[2\pi - (\theta - \sin\theta)\right]}{\frac{1}{2}r^2(\theta - \sin\theta)}$$

$$= \frac{2\pi - \theta + \sin\theta}{\theta - \sin\theta}$$

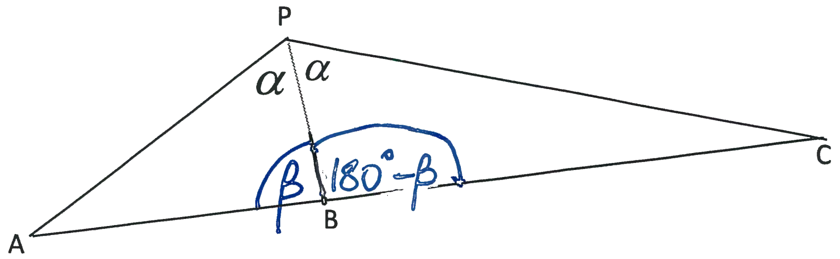
☆ Very few students achieved 2 marks, for this part

The question was marked generously.

Marks were given for the ratio of 'sectors' i terms of θ.

Ensure to read the question carefully and learn these formulae

e) In triangle APC, B is the point on AC such that $\angle APB = \angle BPC = \alpha$.



i) Show that $AB \times PC = AP \times BC$. (2)

ii) Given that $AP = 3$ cm, $PC = 5$ cm and $\alpha = 60^\circ$, calculate the length of AB. (2)

i) In ΔAPB:

$$\frac{\sin\alpha}{AB} = \frac{\sin\beta}{AP}$$

$$\sin\alpha = \frac{AB}{AP} \cdot \sin\beta \quad \ldots (1)$$

In ΔCPB

$$\frac{\sin\alpha}{BC} = \frac{\sin(180-\beta)}{PC}$$

$$\sin\alpha = \frac{BC}{PC} \cdot \sin(180-\beta)$$

✓ $[\sin\beta = \sin(180-\beta)$ ASTC$]$

$$\therefore \sin\alpha = \frac{BC}{PC} \sin\beta \quad \ldots (2)$$

(1) = (2)

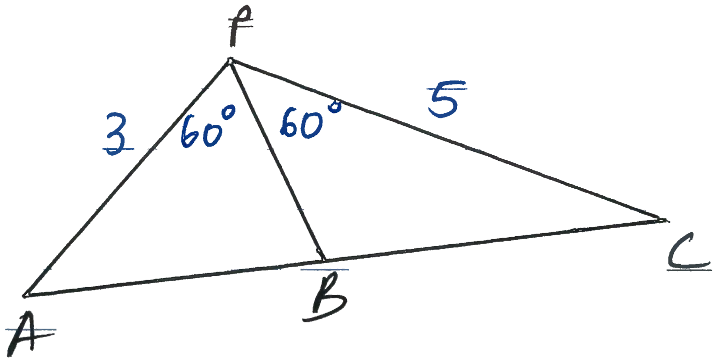
$$\frac{AB}{AP} \cdot \cancel{\sin\beta} = \frac{BC}{PC} \cancel{\sin\beta}$$

$$\therefore \frac{AB}{AP} = \frac{BC}{PC} \quad ✓$$

$$AB \times PC = AP \times BC$$

* Most students assumed incorrectly that $PB \perp AC$. No marks were awarded for this approach.

ii)



$AC^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos 120°$

$= 9 + 25 + 15$

$AC = \sqrt{49}$

$AC = 7\text{ cm}$. ✓

Now: $AB \times PC = AP \times BC$

$AB \times 5 = 3 \times BC$

$5AB = 3(AC - AB)$

$5AB = 3(7 - AB)$

$5AB = 21 - 3AB$

$8AB = 21$ ✓

$AB = \frac{21}{8} = 2\frac{5}{8} = \frac{21}{8} = 2.625$

$BC = 7 - \frac{21}{8} = 4\frac{3}{8} = 4.375$

* If students assumed that $PB \perp AC$ from part i) they were awarded zero marks, for this part of the question, due to its complexity.

An attempt to find $AC = 7\text{cm}$ and that $AB = \frac{3}{5}BC$, gained 1 mark.