Sydney Girls High School

MATHEMATICS

YEAR 11

Assessment Task 3

2020

Reading time :	5 minutes
Writing time :	60 minutes
Total marks :	60
Topics:	All Year 11 Preliminary Course Contents.

General Instructions:

- There are four (4) questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write using a black or blue pen.
- White-out or correction tape is NOT to be used.
- A correct answer without working will be awarded a maximum of 1 mark.

Student Name:__________________________ **Teacher Name:**____________________

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Question 1

a) Find the exact value of $\cos 225°$. 1

b) Expand and simplify the expression $(x-5)(2x^2-3x+4)$. 2

c) Factorise $2x^2-7x-4$. 1

d) Use the first principles method to find the derivative function of $f(x)=x^2-5x+2$. 2

e) Solve for x and sketch the solution on a number line: $2(-3x+4)\geq 2x-8$. 2

f) If $f(x)=x^4+6x^3-2x^2+x-3$, find $f'(-2)$. 2

g) Using index laws, simplify: $\left(\dfrac{a^{-2}}{b}\right)^{-3}\div\left(\dfrac{a}{b^2}\right)^4$ 2

h) Find the derivative function of: $y=3(4-5x)^6$. 2

i) Find the value of k for which the table represents a discrete probability distribution. 1

x	0	1	2	3	4
$P(X=x)$	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{8}$	k

Question 2

a) Determine the domain and range of $y=\dfrac{5}{x-3}$. Write your answer using interval notation. 2

b) Solve for x: $3^{2x}-7\times 3^{x}+12=0$. 2

c) The height h, in metres, of a ball thrown from a cliff can be modelled by the function with the rule $h(t)=20+15t-5t^{2}$, where $t\geq 0$ is the time measured in seconds.

i) Find the initial height of the ball. 1

ii) Find the time at which the ball hits the ground. 1

iii) When will the ball reach a height of 30 m? 1

iv) When will the ball again be at a height of 20 m? 1

d) Sketch $y=\log_{e}(x-3)$. 2

e) A door prize winner is to be selected by randomly choosing a letter from the word PRIZE and a number between 1 and 9 inclusive. What is the probability that the winning door prize will have a vowel and an odd number? 2

f) Find the equation of the normal to the curve $y=\dfrac{2x-3}{2-x}$ at the point $A(1,-1)$. 3

Question 3

a) Prove $\cot\theta + \tan\theta = \operatorname{cosec}\theta \sec\theta$. 2

b) Find all values of θ, where $0° \le \theta \le 360°$ for which $\sec\theta = \sqrt{2}$ 2

c) Consider the function $f(x) = \left(x^2 - 1\right)\left(x^2 - 4\right)$.

i) Sketch $y = f(x)$, showing the intercepts. 2

ii) Hence, solve $f(x) > 0$. 1

d) The random variable has the following probability distribution.

x	0	1	2	3	4
$p(X = x)$	0.1	0.3	0.3	0.1	0.2

i) Find the expected value $E(X)$. 1

ii) Find $\operatorname{Var}(X)$. 1

e) Solve the following equation for x: $\log_2(x+4) + \log_2(x-1) - \log_2(x+2) = 2$ 3

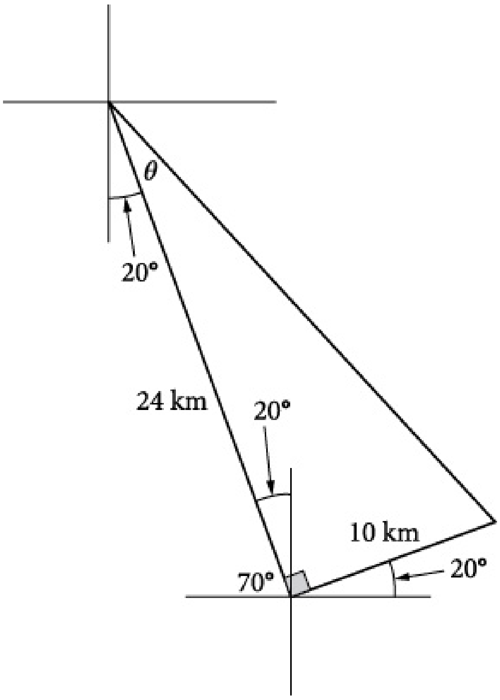
f) Solve for x: $1 = \cos x + \sin^2 x$ for $-\pi \le x \le \pi$. 3

Question 4

a) Differentiate $y = x^3 e^{-5x+2}$ 2

b) Sketch $y = -2 + e^{-x}$, showing all key features including the intercepts. 2

c) Tracey walks 10 km in the direction N70°E. Lesley starts from the same point and walks 24 km in the direction 340°T.

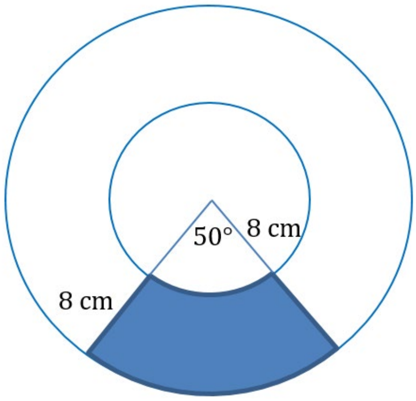


i) What is the distance between Tracey and Lesley? 1

ii) What is the true bearing of Tracey from Lesley, correct to the nearest degree? 1

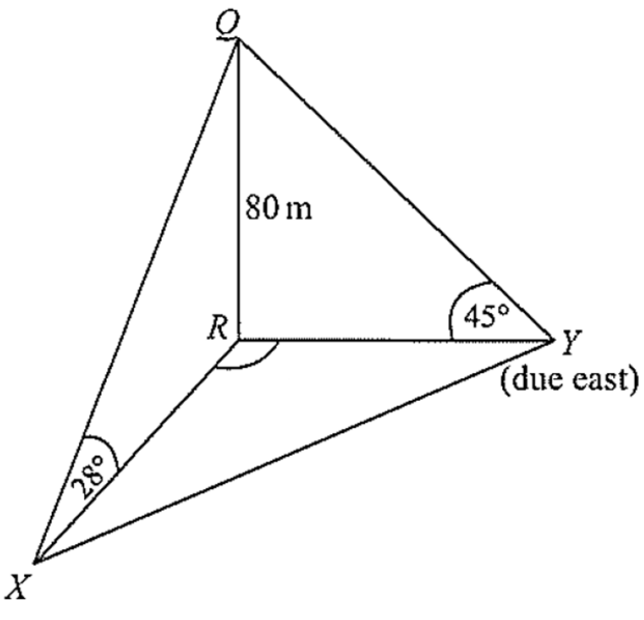
iii) How far east of Tracey is Lesley, correct to the nearest km? 1

d) Find the exact perimeter of the shaded region below. 2



e) There are 650 tickets sold in a raffle. The winning ticket is drawn and then the ticket for second prize is drawn, without replacing the winning ticket. If you buy 20 tickets, find the probability that you win at least one prize. 2

f) The angle of elevation of a tower QR of height 80 metres of a point Y due east of it is 45°
From another point X, the bearing of the tower is 030° and the angle of elevation is 28°.
The points X, Y and R are on the same level ground.



i) Show that $\angle XRY = 120^\circ$. 1

ii) Find an expression for XR 1

iii) Hence, find the distance between X and Y, giving your answer to the nearest metre. 2

The End.

2020 CT3 Year 11 Advanced

Question 1

(a) $\cos 225^\circ$

$= \cos(180+45)^\circ$

$= -\cos 45^\circ$

$= -\frac{1}{\sqrt{2}}$ /1

(b) $(x-5)(2x^2-3x+4)$

$= 2x^3 - 3x^2 + 4x - 10x^2 + 15x - 20$

$= 2x^3 - 13x^2 + 19x - 20$ /2

(c) $2x^2 - 7x - 4$

$= 2x^2 - 8x + x - 4$

$= 2x(x-4) + (x-4)$

$= (2x+1)(x-4)$ /1

P: $-8x^2$

S: $-7x$

F: $-8x, x$

(d) $f(x) = x^2 - 5x + 2$

$f(x+h) = (x+h)^2 - 5(x+h) + 2$

$= x^2 + 2xh + h^2 - 5x - 5h + 2$

$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{2xh + h^2 - 5h}{h}$

$= \lim_{h\to 0} \frac{h(2x+h-5)}{h}$

$= \lim_{h\to 0} (2x+h-5)$

$= 2x - 5$ /2

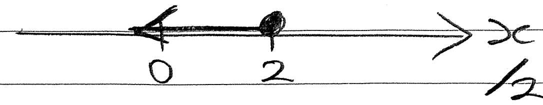
(e) $2(-3x+4) \geq 2x - 8$

$-6x + 8 \geq 2x - 8$

$16 \geq 8x$

$x \leq 2$

(e)



/2

(f) $f(x) = x^4 + 6x^3 - 2x^2 + x - 3$

$f'(x) = 4x^3 + 18x^2 - 4x + 1$

$f'(-2) = 4(-2)^3 + 18(-2)^2 - 4(-2) + 1$

$= -32 + 72 + 8 + 1$

$= 49$ /2

(g) $\left(\frac{a^{-2}}{b}\right)^{-3} \div \left(\frac{a}{b^2}\right)^4$

$= \frac{a^6}{b^{-3}} \div \frac{a^4}{b^8}$

$= \frac{a^6 b^3}{1} \times \frac{b^8}{a^4}$

$= a^2 b^{11}$

/2

General comments:

- Working should still be shown for 1 mark questions like (a) and (i)

- Fractions should be written with fraction bar horizontal

Eg $\frac{1}{\sqrt{2}}$ not $1/\sqrt{2}$

$\frac{7}{12}$ not $7/12$

Question 1 cont.

(h) $y = 3(4-5x)^6$

$\frac{dy}{dx} = 3 \times 6(4-5x)^5 \times -5$

$= -90(4-5x)^5$ 1/2

(i) $\Sigma P = 1$ for discrete prob. dist.

$\frac{1}{24} + \frac{1}{12} + \frac{1}{6} + \frac{1}{8} + k = 1$

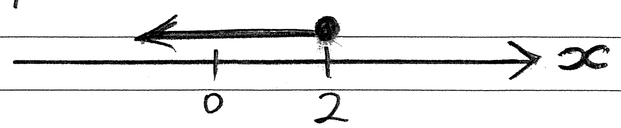
$\frac{5}{12} + k = 1$

$k = \frac{7}{12}$ 1/1

Comments:

- Setting out for (d) should include $\lim_{h \to 0}$ in the correct positions

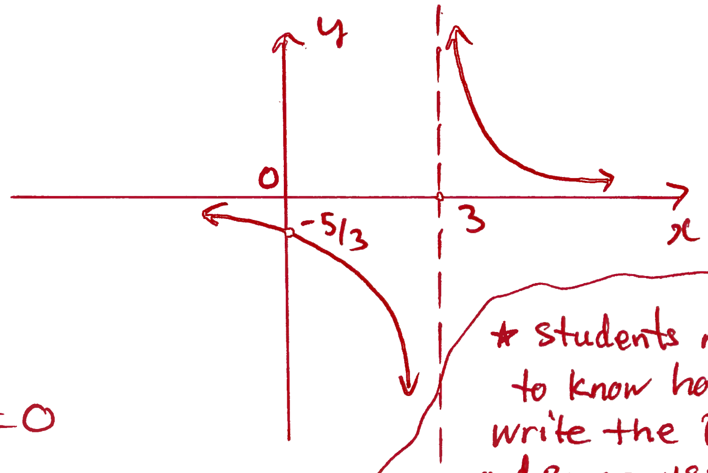
- Setting out for (e)
  - * Solution should be written $x \leq 2$, not left as $2 \geq x$
  - * Number line needs to include x! Also the closed circle should be Large enough to distinguish between an open or closed circle



- For part (f), no marks awarded for $f(-2)$. You need to find $f'(x)$ and then $f'(-2)$.

Q2

a) $y = \dfrac{5}{x-3}$



D: $(-\infty, 3) \cup (3, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

★ students need to know how to write the Domain and Range using interval notation. Many students wrote them incorrectly.

b) $3^{2x} - 7 \times 3^{x} + 12 = 0$

Let $u = 3^x$

$u^2 - 7u + 12 = 0$

$(u-3)(u-4) = 0$

$\left[\begin{array}{l} u = 3 \\ u = 4 \end{array}\right. \therefore \begin{array}{l} 3^x = 3 \\ 3^x = 4 \end{array} \therefore \left[\begin{array}{l} x = 1 \\ x = \dfrac{\ln 4}{\ln 3} \end{array}\right.$

c) $h(t) = 20 + 15t - 5t^2$

i) $t = 0 \longrightarrow h = 20$ (m)

ii) $h = 0 \therefore 20 + 15t - 5t^2 = 0$

OR $t^2 - 3t - 4 = 0$

$(t-4)(t+1) = 0$

$\left[\begin{array}{l} t = 4 \\ t = -1 \text{ (Reject)} \end{array}\right. \therefore \boxed{t = 4}$ sec

iii) $h = 30 \therefore 30 = 20 + 15t - 5t^2$

$t^2 - 3t + 2 = 0$

$(t-1)(t-2) = 0$

$\left[\begin{array}{l} t = 1 \text{ sec} \\ t = 2 \text{ sec} \end{array}\right.$ OR

$\boxed{Q2}$

c/ iv) when $h = 20$

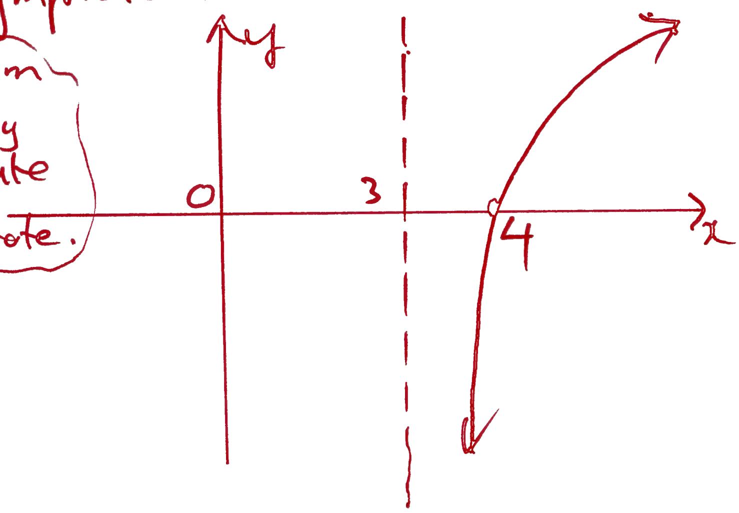
$$20 = 20 + 15t - 5t^2$$
$$t^2 - 3t = 0$$
$$t = 0$$
$$t = 3$$

The ball will be at the height of 20m again at $t = 3$ seconds.

d) $y = \log_e(x-3)$

• Vertical Asymptote : $x - 3 = 0 \quad \therefore x = 3$

* Sketching logarithm function, it is very important to indicate the vertical Asymptote.



e) $= \frac{2}{5} \times \frac{5}{9} = \frac{2}{9}$

f) $y = \dfrac{2x-3}{2-x}$

$$y' = \frac{2(2-x) + 1(2x-3)}{(2-x)^2}$$

$$y' = \frac{1}{(2-x)^2}$$

At $x = 1 \therefore y' = 1 = mT$

$$mN = \frac{-1}{mT} = \frac{-1}{1} = -1$$

Eq of the Normal at $A(1, -1)$

$$y + 1 = -1(x - 1)$$
$$y + 1 = -x + 1$$
$$\boxed{y = -x}$$

Question 3 – Solutions.

a) $$LHS = \cot\theta + \tan\theta$$
$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$
$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} \checkmark$$
$$= \frac{1}{\sin\theta \cdot \cos\theta}$$
$$= \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta} \checkmark$$
$$= \operatorname{cosec}\theta \,.\, \sec\theta$$
$$= RHS.$$

2.

* Students lost one mark if they did not set out their proof properly.

b) $$\sec\theta = \sqrt{2}$$
$$\frac{1}{\cos\theta} = \sqrt{2}$$
$$\therefore \cos\theta = \frac{1}{\sqrt{2}}$$
$$\theta = 45^\circ, (360^\circ - 45^\circ)$$
$$\theta = 45^\circ, 315^\circ \checkmark\checkmark$$

2.

* Students needed to consider the domain when giving their answers.

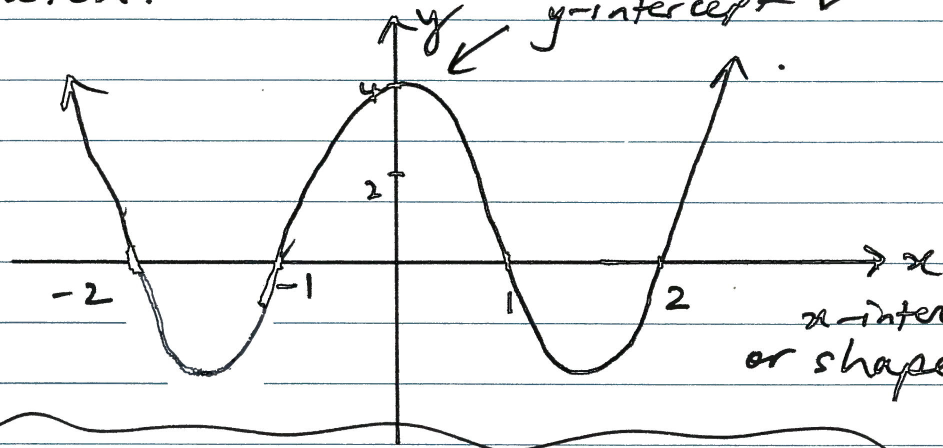
Question 3 - Solutions

c) $f(x) = (x^2-1)(x^2-4)$

$f(x) = (x-1)(x+1)(x-2)(x+2)$

i) Sketch:

y-intercept ✓



2

x-intercepts or shape ✓

* The y-intercept always needs to be labelled. Students lost a mark if it was not given/indicated.

ii) $f(x) > 0$ for:

$x < -2, \quad -1 < x < 1, \quad x > 2$ 1

* There was no loss of marks if students included the equality when giving their answer - but need to consider this in future

Question 3 – Solutions

d) i) $E(X) = 0\times0.1 + 1\times0.3 + 2\times0.3 + 3\times0.1 + 4\times0.2$

$= 0 + 0.3 + 0.6 + 0.3 + 0.8$

$\therefore \mu = 2$ ✓ ①

ii) $Var(X) = x^2 p(x) - \mu^2$

$= E(X^2) - \mu^2$

$= (0^2\times0.1 + 1^2\times0.3 + 2^2\times0.3 + 3^2\times0.1 + 4^2\times0.2) - 2^2$

$= 1.6$ ✓ ①

* 1 mark for each correct answer.
Some students used their calculator

Question 3 – Solutions.

e) $\log_2(x+4) + \log_2(x-1) - \log_2(x+2) = 2$

$$\log_2 \frac{(x+4)(x-1)}{(x+2)} = 2 \quad \checkmark$$

$$\frac{x^2+3x-4}{x+2} = 2^2$$

$$x^2+3x-4 = 4x+8$$

$$x^2-x-12 = 0$$

$$(x-4)(x+3) = 0 \quad \checkmark$$

either $x=4$ or $\begin{cases} x=-3 \\ \text{no solution}, x>1. \end{cases}$

$\therefore$ Only solution is $x=4$. $\checkmark$

* Students lost marks at the last step when they needed to consider whether the solutions worked

Question 3 - Solutions.

f) $1 = \cos x + \sin^2 x \qquad -\pi \le x \le \pi$

$(1-\cos^2 x) + \cos x - 1 = 0$

$\cos x - \cos^2 x = 0$

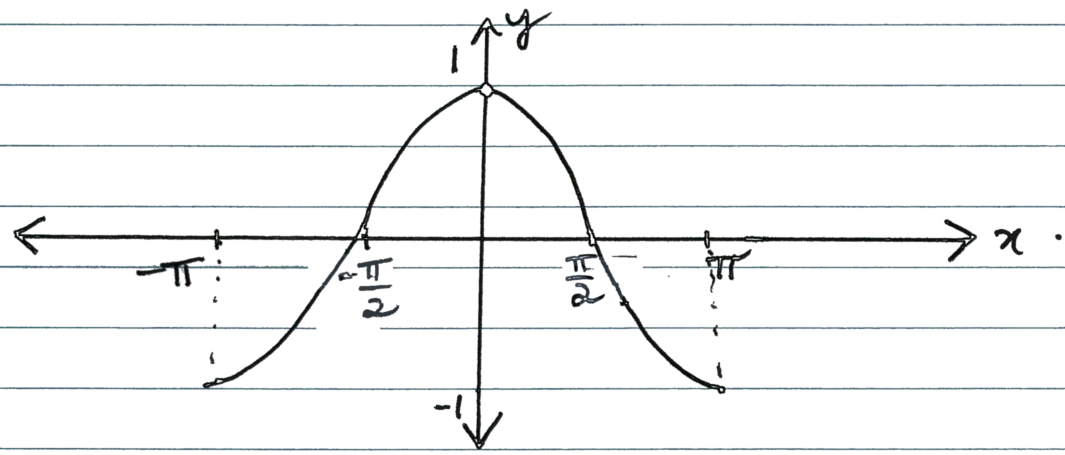
$\cos x(1 - \cos x) = 0$ ✓

$\therefore \cos x = 0 \quad$ or $\quad 1 - \cos x = 0$

$x = \pm\dfrac{\pi}{2}$ ✓ $\qquad \cos x = 1$

$x = 0$ ✓

* students needed to consider the domain for the correct answer



Question 4

(a) $y = x^3e^{-5x+2}$

Let $\quad u = x^3 \qquad v = e^{-5x+2}$

$\qquad u' = 3x^2 \qquad v' = -5e^{-5x+2}$

$y' = uv' + vu'$

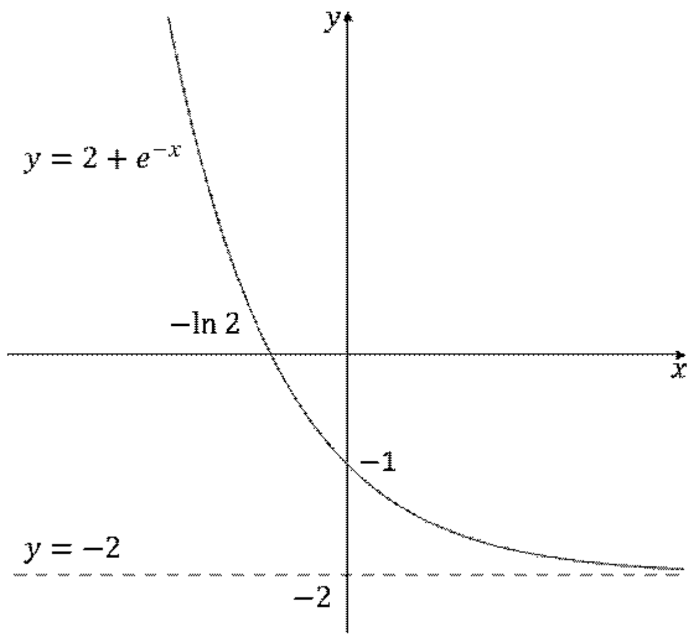
$= x^3(-5e^{-5x+2}) + e^{-5x+2}(3x^2)$

$= -5x^3e^{-5x+2} + 3x^2e^{-5x+2}$

$= x^2e^{-5x+2}(3 - 5x)$

1 mark was awarded for either finding both u' and v' correctly OR correctly using the product rule

(b)



1 mark was awarded finding and labelling two of the three main features (x-intercept, y-intercept, asymptote)

(c) Let L and T be the final positions of Lesley and Tracey respectively.

(i) By Pythagoras' Theorem

$LT^2 = 10^2 + 24^2$

$LT = \sqrt{10^2 + 24^2}$

$\quad = 26$ km

(ii) $\theta = \tan^{-1}\frac{10}{24}$

$= 23°$

∴The required bearing is $180° - 23° - 20° = 137°$

(iii) Let Lesley be x km west and Tracey be y km east of the initial position. Then

$\cos 70° = \frac{x}{24}$ and $\cos 20° = \frac{y}{10}$

$x = 24\cos 70°$ and $y = 10\cos 20°$

∴ The required distance is $x + y \approx 20$ km

(d) The shaded area is bounded by two minor arcs and two line segments of length 8 cm.

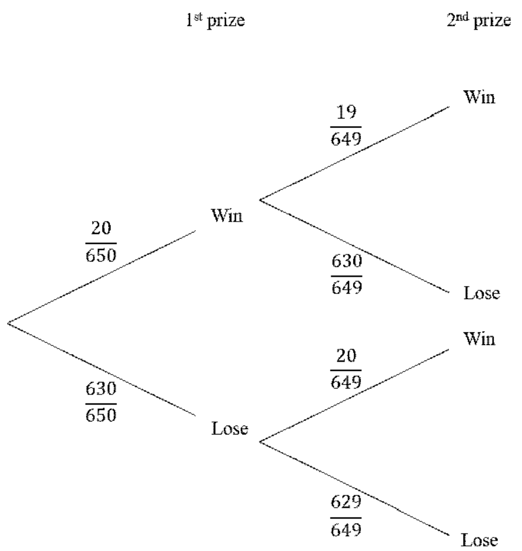
The larger circle has radius $8 + 8 = 16$ cm, and the smaller circle has radius 8 cm.

∴ The perimeter is

$$\frac{50}{360} \times 2\pi(16) + 8 + 8 + \frac{50}{360} \times 2\pi(8) = \frac{20\pi}{3} + 16 \text{ cm}$$

<u>Note</u>: We used the arc length formula $\ell = \frac{\theta}{360} \times 2\pi r$.

(e)



P(at least one prize)

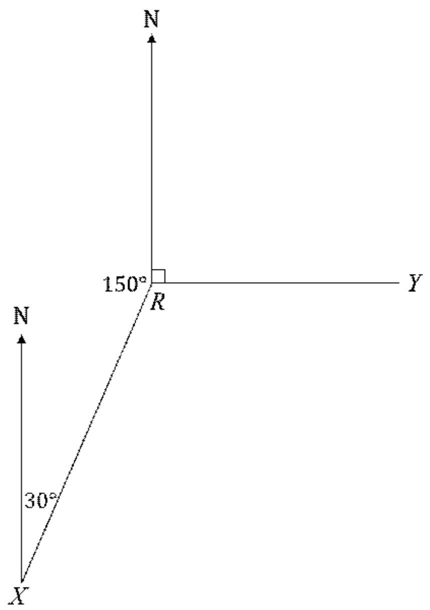
$= 1 - P$(neither first nor second prize)

$= 1 - P(1^{st}$ ticket loses$) \times P(2^{nd}$ ticket loses$)$

$= 1 - \frac{630}{650} \times \frac{629}{649}$

≈ 0.06 (2 d.p.)

(f) (i) By corresponding angles on parallel lines and angles at a point



$\angle XRY = 360° - 150° - 90°$

$= 120°$

Relevant geometrical reasoning was required for 1 mark, e.g. corresponding angles on parallel lines.

(ii) $\tan 28° = \frac{80}{XR}$

$\therefore XR = \frac{80}{\tan 28°}$

(iii) Similarly, $RY = \frac{80}{\tan 45°} = 80$

By the cosine rule in ΔXRY we have

$XY^2 = \left(\frac{80}{\tan 28°}\right)^2 + 80^2 - 2 \times \left(\frac{80}{\tan 28°}\right) \times 80 \times \cos 120°$

$= 41074.2944\ldots$

$\therefore XY = \sqrt{41074.2944\ldots}$

$\approx 203\ m$ (nearest metre)