Sydney Girls High School

MATHEMATICS

YEAR 11

2021 (online) Assessment Task 3

Reading time: 5 minutes

Writing time: 60 minutes

Total marks: 40

Topics: Functions, Calculus, Trigonometric functions, Exponential and Logarithmic Functions and Statistical Analysis (excluding Discrete probability distributions).

General Instructions:

- There are four (4) questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write using a black or blue pen.
- A correct answer without working will be awarded a maximum of 1 mark.

Question 1: (10 Marks)

a) Solve: i) $|7-6x|=5$ 2

ii) $\left(\frac{1}{3}\right)^{x+1}=27$ 2

b) Find the centre and radius of $x^2-2x+y^2+4y=1$. 2

c) If $\frac{\sqrt{6}+1}{\sqrt{3}+\sqrt{2}}=a\sqrt{3}+b\sqrt{2}$, find the values of a and b. 2

d) Find the equation of the tangent to the curve $y=x^2-3x$ at $x=1$. 2

Question 2: Start writing on a new page (10 Marks)

a) Find $P(X \mid Y)$ if $P(X \cap Y)=0.2$, $P(X)=0.3$ and $P(Y)=0.4$. 1

b) Differentiate the following:

i) $y=(5x-7)^5$ 1

ii) $y=\frac{3x-2\sqrt{x}}{\sqrt{x}}$ 2

iii) $y=\frac{e^{3x}-5}{x+1}$ 2

c) Given the points A (10,2) and B (2,8) find the equation of the perpendicular bisector of AB. 2

d) Solve $2\cos\theta=-1$ for $0\le\theta\le 2\pi$, giving exact solutions. 2

Question 3: Start writing on a new page **(10 Marks)**

a) Sketch the graph of $y = cosec\ x$ for $0° \leq x \leq 360°$. 1

b) If $\sin A = -\frac{1}{3}$ and $\tan A < 0$. Find $\sec A$. 2

c) Sketch the following, showing all the key features:

i) $y = -\sqrt{25 - x^2}$ 1

ii) $y = 2 - \frac{1}{x+1}$ 2

d) Angela guesses three questions in her multiple choice test, which has four options per question. Find the probability that Angela gets:

i) At least one correct answer 1

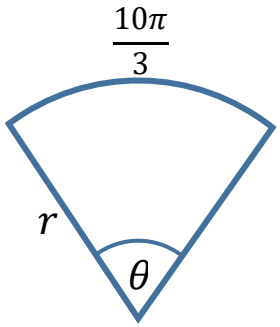
ii) Only one correct answer 1

e) Prove $\frac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$. 2

Question 4: Start writing on a new page (10 Marks)

a) The diagram shows a sector with radius r and angle θ where $0 < \theta \leq 2\pi$.

The arc length is $\frac{10\pi}{3}$.



i) Find the value of θ and hence, show that $r \geq \frac{5}{3}$. 2

ii) Calculate the area of the sector when $r = 4$. 2

b) Consider the function $f(x) = e^x$ and $g(x) = \ln(x-2)$.

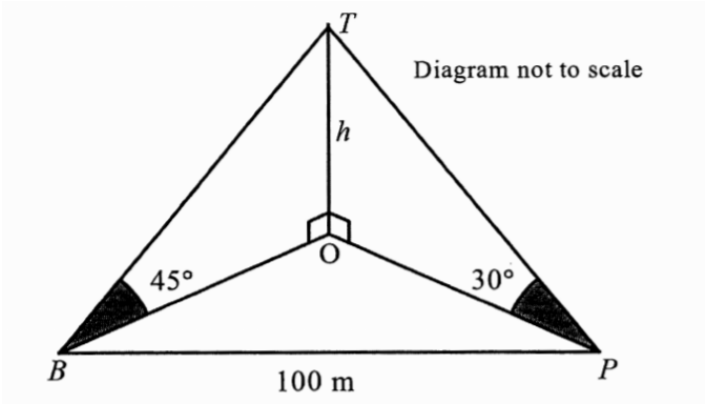
i) Find $f(g(x))$. 1

ii) Find the range of $f(g(x))$ in interval notation. 1

c) A surveyor stands at a point P, which is due east of the tower OT, of height h metres.

The angle of elevation of the top of the tower T from P is 30°. The surveyor then walks 100 metres to point B, which is on a bearing of 150° from the foot of tower 0.

The angle of elevation of the top of the tower from B is now 45°.



i) Express the length of OP in terms of h. 1

ii) Find the height of the tower.
Answer correct to 1 decimal place. 3

End of Paper

Question 1

(a)(i) $|7-6x|=5$

$7-6x=5$ or $-(7-6x)=5$

$6x=2$ $-7+6x=5$

$x=\frac{1}{3}$ ✓ $6x=12$

$x=2$ ✓

(ii) $\left(\frac{1}{3}\right)^{x+1}=27$

$3^{-x-1}=3^3$ ✓

$-x-1=3$

$x=-4$ ✓

(b) $x^2-2x+y^2+4y=1$

$(x-1)^2-1+(y+2)^2-4=1$ ✓

$(x-1)^2+(y+2)^2=6$

Centre $(1,-2)$ ✓

Radius $\sqrt{6}$

For getting at least one of the completed squares correct.

(c) $\frac{\sqrt{6}+1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{3\sqrt{2}-2\sqrt{3}+\sqrt{3}-\sqrt{2}}{3-2}$ ✓

$=2\sqrt{2}-\sqrt{3}$

$a=-1$ and $b=2$ ✓

(d) $y = x^2 - 3x$

$\frac{dy}{dx} = 2x - 3$ 

$m_T = -1$ at $x = 1$

$(1, -2)$

$y + 2 = -1(x - 1)$

$y = -x - 1$ 

Question 2

(a) $P(X|Y) = \dfrac{P(X \cap Y)}{P(Y)}$

$= \dfrac{0.2}{0.4}$

$= \dfrac{1}{2}$

(b)(i) $\dfrac{dy}{dx} = 5(5x-7)^4 \times 5$

$\dfrac{dy}{dx} = 25(5x-7)^4$

(ii) $y = 3\sqrt{x} - 2$

$\dfrac{dy}{dx} = \dfrac{3}{2}x^{-\frac{1}{2}}$

$\dfrac{dy}{dx} = \dfrac{3}{2\sqrt{x}}$

(iii) $\dfrac{dy}{dx} = \dfrac{3e^{3x}(x+1) - 1(e^{3x} - 5)}{(x+1)^2}$

$\dfrac{dy}{dx} = \dfrac{3xe^{3x} + 3e^{3x} - e^{3x} + 5}{(x+1)^2}$

$\dfrac{dy}{dx} = \dfrac{3xe^{3x} + 2e^{3x} + 5}{(x+1)^2}$

(c) $m_{AB} = \dfrac{8-2}{2-10}$

$m_{AB} = -\dfrac{3}{4}$

$m_{\perp} = \dfrac{4}{3}$

$MP_{AB} = (6, 5)$

$y - 5 = \dfrac{4}{3}(x-6)$

$3y - 15 = 4x - 24$

$4x - 3y - 9 = 0$

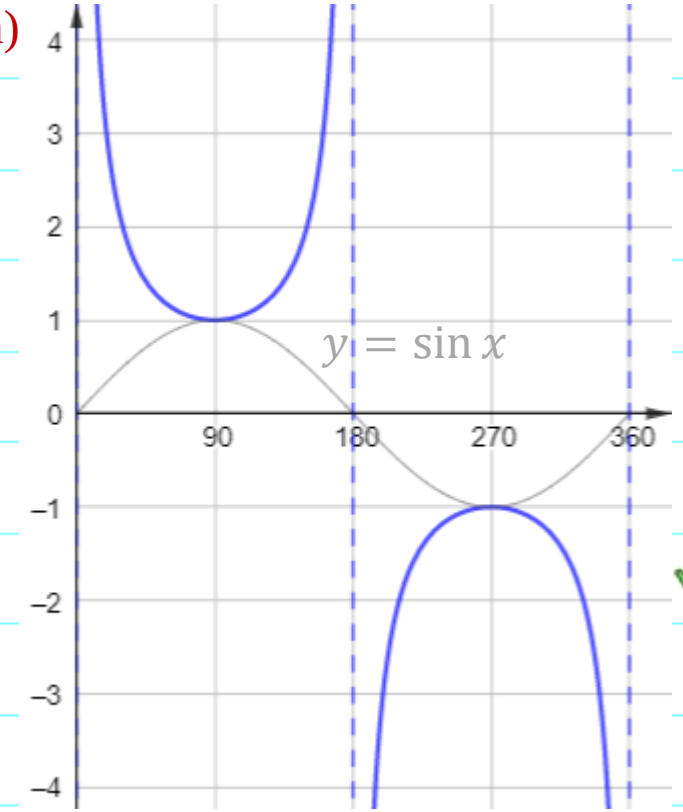
(d) $\cos\theta = -\dfrac{1}{2}$

$\theta = \pi \pm \dfrac{\pi}{3}$

$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

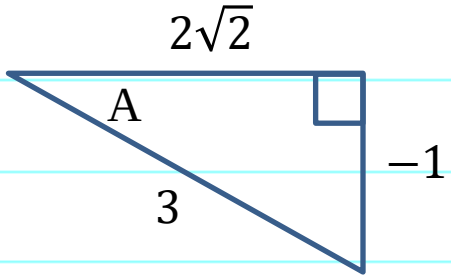
Question 3

(a)



(b) $\sin A = -\frac{1}{3}$ and $\tan A < 0$

So A is in the 4th quadrant.

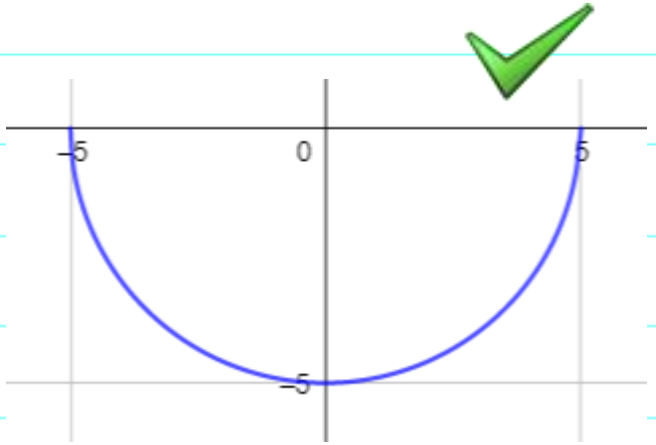


$$\sec A = \frac{1}{\cos A}$$

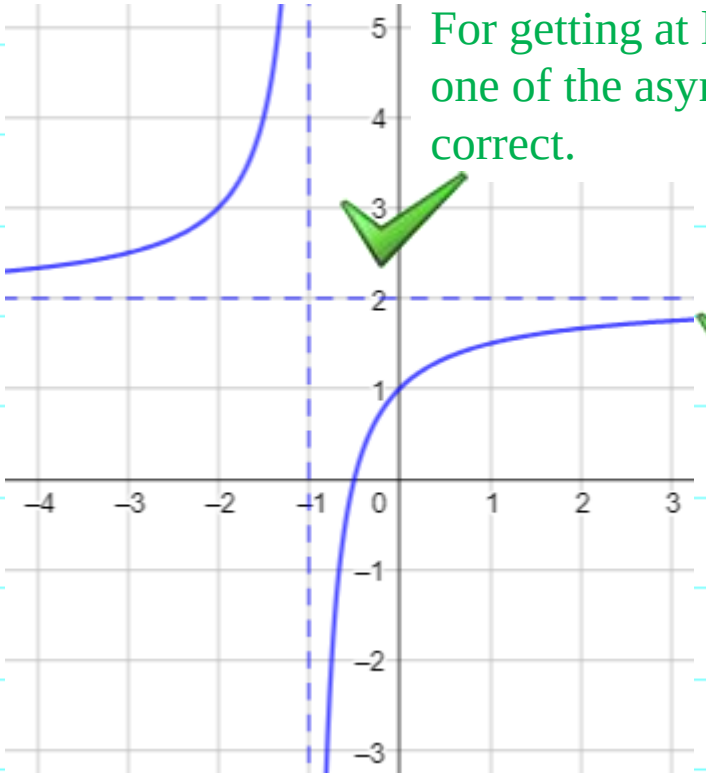
$$= \frac{3}{2\sqrt{2}}$$

$$= \frac{3\sqrt{2}}{4}$$

(c)(i)



(ii)



For getting at least one of the asymptotes correct.

(d)(i) $P(\text{at least one correct}) = 1 - P(\text{three incorrect})$

$$= 1 - \left(\frac{3}{4}\right)^3$$

$$= \frac{37}{64}$$

(ii) $P(\text{Only 1st correct}) + P(\text{Only 2nd correct}) + P(\text{Only 3rd correct})$

$$= \left(\frac{1}{4} \times \frac{3}{4} \times \frac{3}{4}\right) + \left(\frac{3}{4} \times \frac{1}{4} \times \frac{3}{4}\right) + \left(\frac{3}{4} \times \frac{1}{4} \times \frac{3}{4}\right) = 3 \times \frac{3^2}{4^3}$$

$$= \frac{27}{64}$$

(e) $LHS = \dfrac{\cos\theta}{1+\sin\theta} + \dfrac{\sin\theta}{\cos\theta}$

$$= \frac{\cos^2\theta + \sin\theta + \sin^2\theta}{(1+\sin\theta)\cos\theta}$$

$$= \frac{1+\sin\theta}{(1+\sin\theta)\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

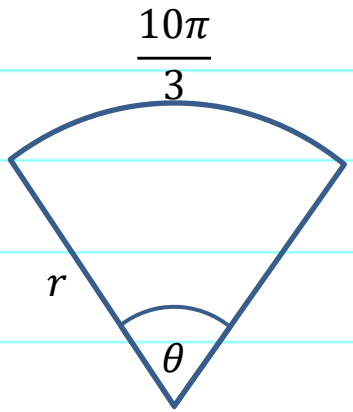
$$= RHS$$

Question 4

(a)(i) $l = r\theta$

$$\frac{10\pi}{3} = r\theta$$

$$\theta = \frac{10\pi}{3r}$$ ✓



Given that $\theta \leq 2\pi$

Thus $\frac{10\pi}{3r} \leq 2\pi$

$$\frac{10\pi}{6\pi} \leq r$$

$$r \geq \frac{5}{3}$$ ✓

(ii) Area $= \frac{1}{2}r^2\theta$ ✓

$$= \frac{1}{2} \times 4^2 \times \frac{10\pi}{3 \times 4}$$

$$= \frac{20\pi}{3}$$ ✓

(b)(i) $f(g(x)) = e^{\ln(x-2)}$ where $x - 2 > 0$

$= x - 2$ for $x > 2$ ✓

To get the mark the domain must be included.

(ii) $f(g(x)) > 0$

$(0, \infty)$ ✓

(c)(i) $\tan 30° = \frac{h}{OP}$

$OP = h\sqrt{3}$ ✓

(ii) $\tan 45° = \frac{h}{OB}$ ✓

$OB = h$ ✓

$100^2 = h^2 + 3h^2 - 2\sqrt{3}h^2 \cos 60°$ ✓

$100^2 = h^2(4 - \sqrt{3})$

$h = \frac{100}{\sqrt{4 - \sqrt{3}}} \approx 66.4\ m$ ✓