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CANDIDATE NUMBER

2022 Term III Annual Examination

Form V Mathematics Advanced

Monday 5th September 2022

8:40am

General Instructions

- Working time — 2 hours
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.
- A loose reference sheet is provided separate to this paper.

Total Marks: 80

Section I (8 marks) Questions 1 – 8

- This section is multiple-choice. Each question is worth 1 mark.
- Record your answers on the provided answer sheet.

Section II (72 marks) Questions 9 – 14

- Relevant mathematical reasoning and calculations are required.
- Answer the questions in this paper in the spaces provided.

Collection

- Write your candidate number on this page and on the multiple choice sheet.
- Place everything inside this question booklet.

Checklist

- Reference sheet
- Multiple-choice answer sheet
- Candidature: 180 pupils

Writer: KCCT

Section I

Questions in this section are multiple-choice.

Record the single best answer for each question on the provided answer sheet.

1. What is the angle of inclination of the line $y = -x + 2$?
 - (A) 45°
 - (B) 35°
 - (C) 135°
 - (D) 225°

2. What is the domain of the function $f(x) = \sqrt{x - 6}$?
 - (A) $x > 6$
 - (B) $x \geq 6$
 - (C) $x > -6$
 - (D) $x \leq -6$

3. The function $f(x) = (x + 2)^2$ is shifted 3 units to the right. Which of the following is the equation of the resulting function, $g(x)$?
 - (A) $g(x) = (x - 5)^2$
 - (B) $g(x) = (x - 1)^2$
 - (C) $g(x) = (x + 5)^2$
 - (D) $g(x) = (x + 1)^2$

4. What is the x -coordinate of the point on $y = e^{3x-1}$ where the tangent has a gradient of 3?
 - (A) -3
 - (B) $\frac{1}{3}$
 - (C) 1
 - (D) 3

5. What is the value of $\frac{\log_a 32}{\log_a 2}$?
- (A) $\log_a 30$
(B) $\log_a 16$
(C) 16
(D) 5
6. Given $P(A) = 0.6$, $P(B) = 0.4$ and $P(A \cup B) = 0.8$, what is the value of $P(A|B)$?
- (A) 0.2
(B) 0.5
(C) 0.6
(D) 0.8
7. Which of the following is a simplified form of $\frac{\sec(180^\circ - \theta)}{\tan(360^\circ - \theta)}$?
- (A) $-\sin \theta$
(B) $-\operatorname{cosec} \theta$
(C) $\sin \theta$
(D) $\operatorname{cosec} \theta$
8. Consider the following set of ordered pairs:
 $\{(3, 2), (5, 6), (2, 8), (4, 8)\}$
- What type of relationship does this set of ordered pairs represent?
- (A) one-to-one
(B) one-to-many
(C) many-to-one
(D) many-to-many

End of Section I

The paper continues in the next section

B L A N K P A G E

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CANDIDATE NUMBER

Section II

This section consists of long-answer questions.

Marks may be awarded for reasoning and calculations.

Marks may be lost for poor setting out or poor logic.

Record your answers in the space provided in this paper.

QUESTION NINE (12 marks)

Marks

(a) Evaluate $|| - 7| - 9|$.

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(b) Express $\frac{3}{2\sqrt{6}}$ with a rational denominator in simplest form.

1

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(c) Simplify $\sqrt{8} - \sqrt{2}$.

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(d) Fully factorise $25x^3y - x^3y^3$.

2

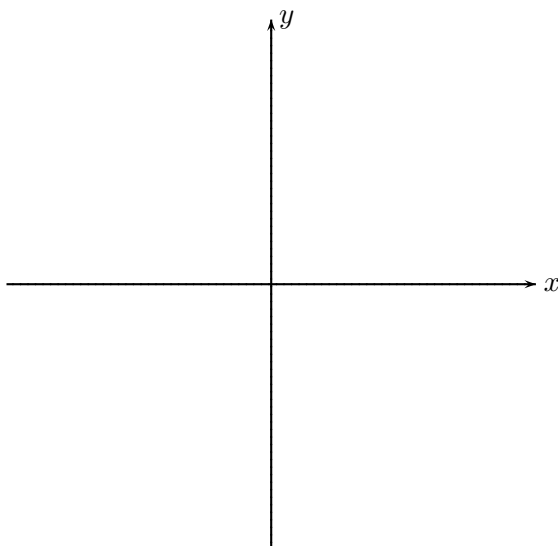
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- (e) (i) Sketch the graph of $y = |x + 2|$ showing all intercepts.

2



- (ii) Hence or otherwise solve $|x + 2| = 3$.

2

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- (f) Consider the functions $g(x) = 2x - 1$ and $h(x) = x^2 + 3$.

- (i) Find $g(2) - h(1)$.

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- (ii) Find $g(h(x))$.

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Extra writing space (Use this only to answer Question Nine)

If you use this space, clearly indicate which question you are answering.

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Extra writing space (Use this only to answer Question Nine)

If you use this space, clearly indicate which question you are answering.

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CANDIDATE NUMBER

QUESTION TEN (12 marks)

Marks

(a) Solve $\frac{32}{x} - x = 4$.

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(b) Differentiate the following with respect to x :

(i) $y = \frac{3}{x}$

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(ii) $y = (3 - 2x)^{10}$

1

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(c) Solve the following pair of simultaneous equations:

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$$4x + 3y = 1$$

$$3x - 2y = 5$$

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(d) By rationalising the denominator or otherwise, show that $\frac{2}{5 - \sqrt{2}} + \frac{2}{5 + \sqrt{2}}$ is rational.

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- (e) Differentiate $f(x) = x^2 + 1$ by first principles using the formula $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$. 3

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Extra writing space (Use this only to answer Question Ten)

If you use this space, clearly indicate which question you are answering.

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CANDIDATE NUMBER

QUESTION ELEVEN (12 marks)**Marks**

- (a) Determine whether the function $f(x) = \frac{x^3}{x^2 - 1}$ is odd, even or neither.

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- (b) Find the equation of the circle with centre $(12, -3)$ that passes through the origin.

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- (c) Express the following in simplest form:

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$$\frac{x^{-4} + 2x^{-1}}{x^{-5} + 2x^{-2}}$$

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- (d) Differentiate the following functions with respect to x . Give your answer in factorised form where possible.

(i) $y = e^{2x}(x + 3)$

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(ii) $y = \frac{x^3 + 1}{x + 2}$

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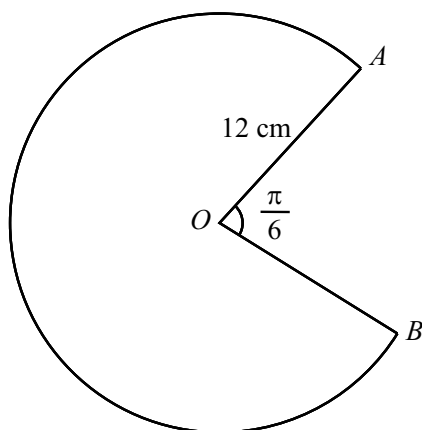
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- (e) The diagram below shows a sector with radius 12 cm. The acute angle $\angle AOB$ is $\frac{\pi}{6}$ radians.



- (i) Write down the size of the reflex angle AOB . Give your answer in terms of π .

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- (ii) Calculate the exact value of the perimeter of the sector.

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Extra writing space (Use this only to answer Question Eleven)

If you use this space, clearly indicate which question you are answering.

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CANDIDATE NUMBER

QUESTION TWELVE (12 marks)**Marks**

- (a) Alex (A) and Darren (D) are surveying a piece of land together. They set out from point P to set up position markers. Alex walked 240 m on a bearing of 248°T from P and sets his marker at A . Darren walked from P in a roughly south easterly direction but unfortunately sprained his ankle and decided to just set his marker there at the point D . Looking at his GPS system, he is 350 m away from camp and 400 m away from Alex.

- (i) Complete the diagram below showing all the information provided above.

1



- (ii) Calculate the bearing that Alex must travel to go to Darren's aid. Give your answer correct to the nearest minute.

3

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- (iii) Calculate the area enclosed by the points A , D and P . Give your answer correct to one decimal place.

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(b) Solve:

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$$\log_3 x - \log_3(x+2) + 1 = 0$$

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(c) Solve $2 \cos x + \sqrt{3} = 0$ for $0 < x < 2\pi$.

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(d) Find all possible values of m such that the line $y = mx$ is a tangent to the curve $y = x^2 + 1$.

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Extra writing space (Use this only to answer Question Twelve)

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If you use this space, clearly indicate which question you are answering.

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CANDIDATE NUMBER

QUESTION THIRTEEN (12 marks)**Marks**

- (a) News of a new video game always spreads exponentially at Grammar.

At 8:30 am, James, Alessandro and Dillon heard about the release of a new game “The Legend of Delta”. The number of Grammar boys N who have heard of this game can be modelled by the equation,

$$N = 3e^{kt}$$

where k is a constant and t is the time in minutes since 8:30 am.

- (i) Find the value of k , given that 160 boys had heard about “The Legend of Delta” by 12:30 pm. Give your answer as an exact value. 2

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- (ii) What time would it be when all 1300 boys at Grammar have heard of the game? Give your answer correct to the nearest minute. 1

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- (iii) Calculate the rate at which news of “The Legend of Delta” is spreading at 12:30 pm. 2

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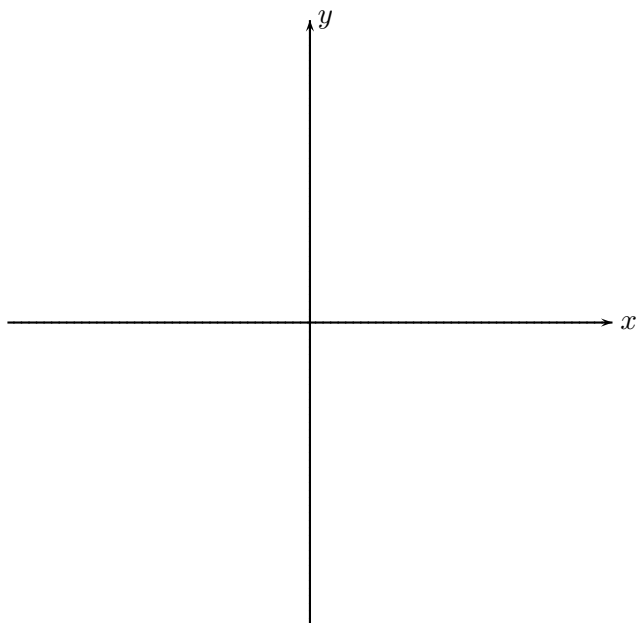
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- (b) Sketch the graph of $y = \log_2(x + 4) - 1$ showing all intercepts and significant features.

2



- (c) Show that $\frac{\sin \theta}{1 + \cot \theta} - \frac{\cos \theta}{1 + \tan \theta} = \sin \theta - \cos \theta$.

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(d) (i) Factorise $3 + 5x - 2x^2$.

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(ii) Hence, solve $3 + 5 \log_{10} x - 2 (\log_{10} x)^2 = 0$. Give your answers as exact values.

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Extra writing space (Use this only to answer Question Thirteen)

If you use this space, clearly indicate which question you are answering.

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CANDIDATE NUMBER

QUESTION FOURTEEN (12 marks)**Marks**

- (a) Dom is an excellent basketball player with an 86% success rate at scoring free throws. Each successful free throw is worth one point.

- (i) Dom gets fouled and is given two free throws. Calculate the probability that he will score at least one point. 2

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- (ii) Just before the final buzzer, Dom's team is losing by two points, so he attempts a shot worth three points. He is deliberately fouled by his opponent, so he misses the shot, but is awarded three free throw attempts. 2

What is the probability that he will score on exactly two of the three free throws?

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- (iii) Dom is known to have a 55% success rate at scoring three point shots. Justify whether it was a sensible move by the opposition to deliberately foul Dom on his last shot just before the buzzer. 2

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(b) Show that $f(x) = (k+2)x^2 + 4x + k + 2$ has no x -intercepts for all $k > 0$.

2

[illegible]

Extra writing space (Use this only to answer Question Fourteen)

If you use this space, clearly indicate which question you are answering.

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SYDNEY GRAMMAR SCHOOL



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CANDIDATE NUMBER

2022 Term III Annual Examination

Form V Mathematics Advanced

Monday 5th September 2022

- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A ☐ B ☐ C ☐ D ☐

Question Two

A ☐ B ☐ C ☐ D ☐

Question Three

A ☐ B ☐ C ☐ D ☐

Question Four

A ☐ B ☐ C ☐ D ☐

Question Five

A ☐ B ☐ C ☐ D ☐

Question Six

A ☐ B ☐ C ☐ D ☐

Question Seven

A ☐ B ☐ C ☐ D ☐

Question Eight

A ☐ B ☐ C ☐ D ☐



SOLUTIONS

2022 Term III Annual Examination

Form V Mathematics Advanced

Monday 5th September 2022

8:40am

General Instructions

- Working time — 2 hours
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.
- A loose reference sheet is provided separate to this paper.

Total Marks: 80

Section I (8 marks) Questions 1–8

- This section is multiple-choice. Each question is worth 1 mark.
- Record your answers on the provided answer sheet.

Section II (72 marks) Questions 9–14

- Relevant mathematical reasoning and calculations are required.
- Answer the questions in this paper in the spaces provided.

Collection

- Write your candidate number on this page and on the multiple choice sheet.
- Place everything inside this question booklet.

Checklist

- Reference sheet
- Multiple-choice answer sheet
- Candidature: 180 pupils

Writer: KCCT

Section I

Questions in this section are multiple-choice.

Record the single best answer for each question on the provided answer sheet.

1. What is the angle of inclination of the line $y = -x + 2$?

(A) 45°
(B) 35°
☒ (C) 135°
(D) 225°

2. What is the domain of the function $f(x) = \sqrt{x - 6}$?

(A) $x > 6$
☒ (B) $x \geq 6$
(C) $x > -6$
(D) $x \leq -6$

$$\begin{aligned} \uparrow x - 6 &\geq 0 \\ x &\geq 6 \end{aligned}$$

3. The function $f(x) = (x + 2)^2$ is shifted 3 units to the right. Which of the following is the equation of the resulting function, $g(x)$?

(A) $g(x) = (x - 5)^2$
☒ (B) $g(x) = (x - 1)^2$
(C) $g(x) = (x + 5)^2$
(D) $g(x) = (x + 1)^2$

4. What is the x -coordinate of the point on $y = e^{3x-1}$ where the tangent has a gradient of 3?

(A) -3
☒ (B) $\frac{1}{3}$
(C) 1
(D) 3

$$\begin{aligned} f'(x) &= 3e^{3x-1} = 3 \\ \Rightarrow 3x - 1 &= 0 \\ x &= \frac{1}{3} \end{aligned}$$

5. What is the value of $\frac{\log_a 32}{\log_a 2}$?

(A) $\log_a 30$

(B) $\log_a 16$

(C) 16

☒ (D) 5

$$\frac{\log_3 2^5}{\log_3 2} = 5$$

6. Given $P(A) = 0.6$, $P(B) = 0.4$ and $P(A \cup B) = 0.8$, what is the value of $P(A|B)$?

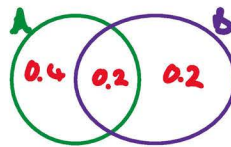
(A) 0.2

☒ (B) 0.5

(C) 0.6

(D) 0.8

$$P(A) = 0.6$$



$$P(B) = 0.4$$

$$P(A|B) = \frac{0.2}{0.4} = \frac{1}{2}$$

7. Which of the following is a simplified form of

$$\frac{\sec(180^\circ - \theta)}{\tan(360^\circ - \theta)}?$$

(A) $-\sin \theta$

(B) $-\operatorname{cosec} \theta$

(C) $\sin \theta$

☒ (D) $\operatorname{cosec} \theta$

$$\frac{1}{\csc(180-\theta)}$$

$$= \frac{1}{\csc \theta}$$

$$\frac{1}{\tan(360-\theta)}$$

$$= \frac{1}{\tan \theta} = \frac{\csc \theta}{\sin \theta}$$



8. Consider the following set of ordered pairs:

$$\{(3, 2), (5, 6), (2, 8), (4, 8)\}$$

What type of relationship does this set of ordered pairs represent?

(A) one-to-one

(B) one-to-many

☒ (C) many-to-one

(D) many-to-many

End of Section I

The paper continues in the next section

B L A N K P A G E

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CANDIDATE NUMBER

Section II

This section consists of long-answer questions.

Marks may be awarded for reasoning and calculations.

Marks may be lost for poor setting out or poor logic.

Record your answers in the space provided in this paper.

QUESTION NINE (12 marks)

Marks

(a) Evaluate $|-7-9|$.

1

$$= |-7-9|$$

$$= 2 - \textcircled{1}$$

(b) Express $\frac{3}{2\sqrt{6}}$ with a rational denominator in simplest form.

1

$$= \frac{3\sqrt{6}}{2 \cdot 6} = \frac{\sqrt{6}}{4} - \textcircled{1} \text{ simplify}$$

(c) Simplify $\sqrt{8} - \sqrt{2}$.

1

$$= 2\sqrt{2} - \sqrt{2}$$

$$= \sqrt{2} - \textcircled{1} \text{ Correct Solution}$$

(d) Fully factorise $25x^3y - x^3y^3$.

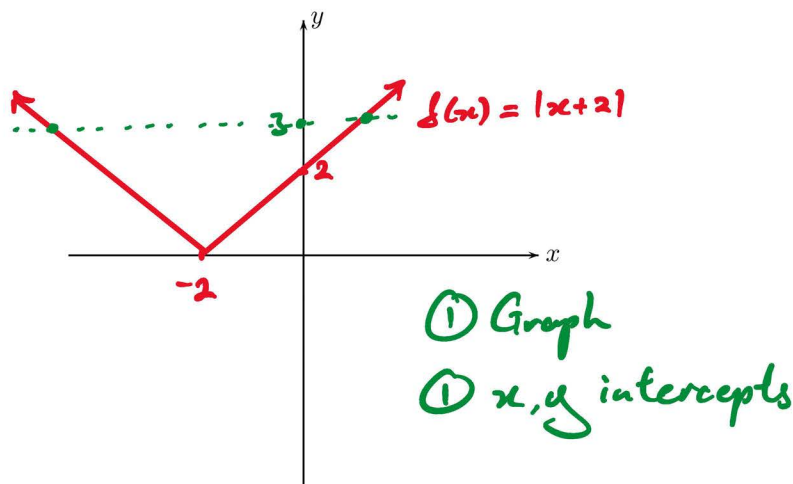
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$$= x^3y(25 - y^2) - \textcircled{1} \text{ Common factor}$$

$$= x^3y(5-y)(5+y) - \textcircled{1} \text{ Diff of 2 sqs.}$$

- (e) (i) Sketch the graph of
- $y = |x + 2|$
- showing all intercepts.

2



- (ii) Hence or otherwise solve
- $|x + 2| = 3$
- .

2

$$x + 2 = 3 \quad \text{or} \quad -x - 2 = 3$$

$$x = 1$$

$$-5 = x$$

① positive Soln'

① negative Soln'

- (f) Consider the functions
- $g(x) = 2x - 1$
- and
- $h(x) = x^2 + 3$
- .

- (i) Find
- $g(2) - h(1)$
- .

1

$$g(2) = 2(2) - 1 = 3 \quad h(1) = 1^2 + 3 = 4$$

$$g(2) - h(1) = 3 - 4 = -1 \quad \text{① Correct Solution}$$

- (ii) Find
- $g(h(x))$
- .

2

$$\begin{aligned} g(h(x)) &= 2(x^2 + 3) - 1 \quad \text{① Substitution} \\ &= 2x^2 + 6 - 1 \\ &= 2x^2 + 5 \quad \text{① Correct Solution} \end{aligned}$$

If you use this space, clearly indicate which question you are answering.

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If you use this space, clearly indicate which question you are answering.

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CANDIDATE NUMBER

QUESTION TEN (12 marks)

Marks

(a) Solve $\frac{32}{x} - x = 4$.

2

$$32 - x^2 = 4x$$

$$x^2 + 4x - 32 = 0$$

$$(x + 8)(x - 4) = 0$$

① Multiply x
and factorise

$$x = -8, +4$$

① Correct Sol'n

(b) Differentiate the following with respect to x :

(i) $y = \frac{3}{x} = 3x^{-1}$

1

$$y' = -3x^{-2} \text{ or } \frac{-3}{x^2}$$

① Correct Sol'n

(ii) $y = (3 - 2x)^{10}$

1

$$y' = 10(3 - 2x)^9 \times -2$$

$$= -20(3 - 2x)^9$$

① Correct Sol'n

(c) Solve the following pair of simultaneous equations:

3

$$4x + 3y = 1 \quad \times 2$$

$$3x - 2y = 5 \quad \times 3$$

$$\Rightarrow \begin{cases} 8x + 6y = 2 \\ 9x - 6y = 15 \end{cases} \quad \textcircled{1} \text{ progress}$$

$$\hline 17x = 17$$

$$x = 1 \quad \textcircled{1} \text{ found } x\text{-value}$$

$$\Rightarrow 4 + 3y = 1$$

$$3y = -3$$

$$y = -1 \quad \textcircled{1} \text{ found } y\text{-value}$$

(d) By rationalising the denominator or otherwise, show that $\frac{2}{5 - \sqrt{2}} + \frac{2}{5 + \sqrt{2}}$ is rational.

2

$$= \frac{10 + 2\sqrt{2} + 10 - 2\sqrt{2}}{25 - 2} \quad - \textcircled{1} \text{ Common Denominator}$$

$$= \frac{20}{23} \quad - \textcircled{1} \text{ Correct Solution}$$

(e) Differentiate $f(x) = x^2 + 1$ by first principles using the formula $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$. 3

$$f'(x) = \lim_{h \rightarrow 0} \frac{[(x+h)^2 + 1] - [x^2 + 1]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x^2} + 2xh + \cancel{h^2} + \cancel{1} - \cancel{x^2} - \cancel{1}}{h}$$

$$= \lim_{h \rightarrow 0} 2x + h$$

$$= 2x$$

① Correct Substitution utilising function notation

① Correctly simplified to $\lim_{h \rightarrow 0} 2x + h$

① Correct answer and usage of limit notation.

If you use this space, clearly indicate which question you are answering.

[illegible]

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CANDIDATE NUMBER

QUESTION ELEVEN (12 marks)

Marks

- (a) Determine whether the function $f(x) = \frac{x^3}{x^2 - 1}$ is odd, even or neither.

1

$$f(-x) = \frac{(-x)^3}{(-x)^2 - 1} = \frac{-x^3}{x^2 - 1} = -f(x)$$

$f(x)$ is odd

① Correct answer with working

- (b) Find the equation of the circle with centre (12, -3) that passes through the origin.

2

$$r^2 = 12^2 + 3^2$$

$$= 153$$

$$\Rightarrow (x-12)^2 + (y+3)^2 = 153$$

① Find radius OR correct use of centre in equation

① Correct equation

- (c) Express the following in simplest form:

2

$$\frac{x^{-4} + 2x^{-1}}{x^{-5} + 2x^{-2}}$$

$$= \frac{\frac{1}{x^4} + \frac{2}{x}}{\frac{1}{x^5} + \frac{2}{x^2}} \times \frac{x^5}{x^5} \quad \text{--- ① progress}$$

$$= \frac{x + 2x^4}{1 + 2x^3} = \frac{x(1 + 2x^3)}{1 + 2x^3} = x \quad \text{--- ① Correct Solution}$$

- (d) Differentiate the following functions with respect to x . Give your answer in factorised form where possible.

(i) $y = e^{2x}(x+3)$

2

$$\begin{array}{l} u = e^{2x} \quad u' = 2e^{2x} \\ v = (x+3) \quad v' = 1 \end{array} \quad \left. \vphantom{\begin{array}{l} u = e^{2x} \\ v = (x+3) \end{array}} \right\} \textcircled{1} \text{ progress}$$

$$\Rightarrow e^{2x} + 2e^{2x}(x+3) = e^{2x}[1+2(x+3)] \\ = e^{2x}(2x+7)$$

$\textcircled{1}$ Correct Sol'n
in factorised form

(ii) $y = \frac{x^3+1}{x+2}$ $\begin{array}{l} - u' = 3x^2 \\ - v' = 1 \end{array}$

2

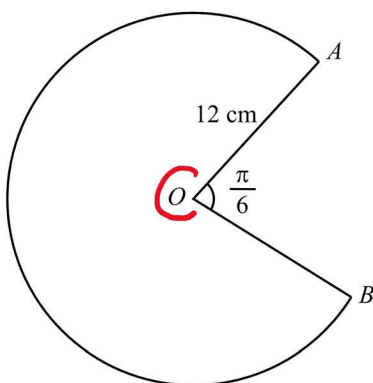
$$y' = \frac{3x^2(x+2) - (x^3+1)}{(x+2)^2} \quad \left. \vphantom{y' = \frac{3x^2(x+2) - (x^3+1)}{(x+2)^2}} \right\} \textcircled{1} \text{ progress}$$

Correct use of quotient rule

$$= \frac{3x^3 + 6x^2 - x^3 - 1}{(x+2)^2}$$

$$= \frac{2x^3 + 6x^2 - 1}{(x+2)^2} \quad \textcircled{1} \text{ Correct Sol'n}$$

- (e) The diagram below shows a sector with radius 12 cm. The acute angle $\angle AOB$ is $\frac{\pi}{6}$ radians.



- (i) Write down the size of the reflex angle AOB . Give your answer in terms of π .

1

$$2\pi - \frac{\pi}{6} = \frac{11\pi}{6} \text{ radians}$$

① Correct Soln

- (ii) Calculate the exact value of the perimeter of the sector.

2

$$\text{Arc Length} = 12 \times \frac{11\pi}{6}$$

$$= 22\pi \text{ cm} \quad \text{① Correct Arc Length}$$

$$\text{Perimeter} = 12 + 12 + 22\pi$$

$$= 24 + 22\pi \text{ cm} \quad \text{① Exact Value Answer}$$

If you use this space, clearly indicate which question you are answering.

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CANDIDATE NUMBER

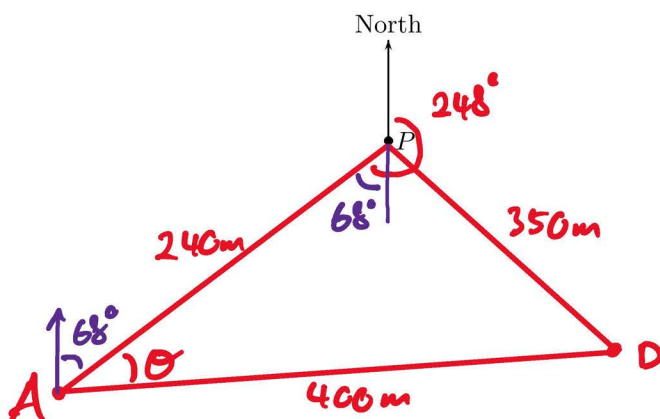
QUESTION TWELVE (12 marks)

Marks

- (a) Alex (A) and Darren (D) are surveying a piece of land together. They set out from point P to set up position markers. Alex walked 240 m on a bearing of 248° T from P and sets his marker at A. Darren walked from P in a roughly south easterly direction but unfortunately sprained his ankle and decided to just set his marker there at the point D. Looking at his GPS system, he is 350 m away from camp and 400 m away from Alex.

(i) Complete the diagram below showing all the information provided above.

1



① Labelled Diagram

- (ii) Calculate the bearing that Alex must travel to go to Darren's aid. Give your answer correct to the nearest minute.

3

$$\cos \theta = \frac{240^2 + 400^2 - 350^2}{2(240)(400)} \quad \text{① use cos rule}$$

$$\theta = \cos^{-1}\left(\frac{317}{640}\right) = 60^\circ 18' 35''$$

$$\Rightarrow \text{Bearing} = 68 + 60^\circ 19'$$

$$= 128^\circ 19'$$

① found bearing

① correct rounding

- (iii) Calculate the area enclosed by the points A, D and P. Give your answer correct to one decimal place.

2

$$\text{Area} = \frac{1}{2} \times 240 \times 400 \times \sin(60^\circ 19')$$

① use sine formula

$$\sim 41701.2 \text{ m}^2$$

$$\text{OR } \sim \underline{41698.3 \text{ m}^2} \quad \text{① correct rounding when keeping } \theta \text{ exact.}$$

(b) Solve:

2

$$\log_3 x - \log_3(x+2) + 1 = 0$$

$$\log_3 \left(\frac{x}{x+2} \right) = -1$$

$$\Rightarrow \frac{x}{x+2} = \frac{1}{3} \quad \textcircled{1} \text{ use log laws and simplify}$$

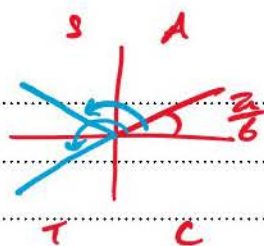
$$\Rightarrow 3x = x+2$$

$$\Rightarrow x = 1 \quad \textcircled{1} \text{ Correct Answer}$$

(c) Solve $2 \cos x + \sqrt{3} = 0$ for $0 < x < 2\pi$.

2

$$\cos x = \frac{-\sqrt{3}}{2}$$



$$x = \frac{5\pi}{6}, \frac{7\pi}{6}$$

① ①

(d) Find all possible values of m such that the line $y = mx$ is a tangent to the curve $y = x^2 + 1$.

2

$$\text{Tangent: } y = mx$$

$$\Rightarrow mx = x^2 + 1$$

$$\Rightarrow x^2 - mx + 1 = 0$$

$$\Delta = m^2 - 4 = 0$$

$$m = \pm 2$$

① Correct Solution

Alternative: Gradient of tangent $\Rightarrow y' = 2x$

At $x = x_0$: $m = 2x_0$ ① find derivative

$$\Rightarrow \text{On tangent: } y_0 = (2x_0)x_0 \quad \left. \begin{array}{l} \text{On Curve: } y_0 = x_0^2 + 1 \end{array} \right\} \begin{array}{l} 2x_0^2 = x_0^2 + 1 \\ \Rightarrow x_0^2 = 1 \end{array}$$

$$\Rightarrow x_0 = \pm 1 \quad \textcircled{1} \text{ Correct}$$

$$\Rightarrow m = \pm 2 \quad \text{Solution}$$

If you use this space, clearly indicate which question you are answering.

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If you use this space, clearly indicate which question you are answering.

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CANDIDATE NUMBER

QUESTION THIRTEEN (12 marks)

Marks

- (a) News of a new video game always spreads exponentially at Grammar.

At 8:30 am, James, Alessandro and Dillon heard about the release of a new game "The Legend of Delta". The number of Grammar boys N who have heard of this game can be modelled by the equation,

$$N = 3e^{kt}$$

where k is a constant and t is the time in minutes since 8:30 am.

- (i) Find the value of k , given that 160 boys had heard about "The Legend of Delta" by 12:30 pm. Give your answer as an exact value. 2

$$\text{At } t=240, N=3e^{240k}=160 \quad \textcircled{1}$$

$$\Rightarrow 240k = \ln\left(\frac{160}{3}\right)$$

$$k = \frac{1}{240} \ln\left(\frac{160}{3}\right) \quad \textcircled{1}$$

- (ii) What time would it be when all 1300 boys at Grammar have heard of the game? Give your answer correct to the nearest minute. 1

$$1300 = 3e^{0.0166t}$$

$$\Rightarrow 0.0166t = \ln\left(\frac{1300}{3}\right)$$

$$= 6.0715$$

$$t = 365.75 \sim 6 \text{ hr } 6 \text{ min}$$

$$\text{Approx. } 2:36 \text{ pm} \quad \textcircled{1} \text{ Correct time}$$

- (iii) Calculate the rate at which news of "The Legend of Delta" is spreading at 12:30 pm. 2

$$N = 3e^{kt} \Rightarrow \frac{dN}{dt} = k \cdot 3e^{kt} \quad \textcircled{1} \text{ find } \frac{dN}{dt}$$

$$= k \cdot N$$

$$\text{At } 12:30 \quad \frac{dN}{dt} = 0.0166 \times 160$$

$$= 2.66 \text{ student/min}$$

$$\textcircled{1} \text{ Correct Soln}$$

(b) Sketch the graph of $y = \log_2(x+4) - 1$ showing all intercepts and significant features.

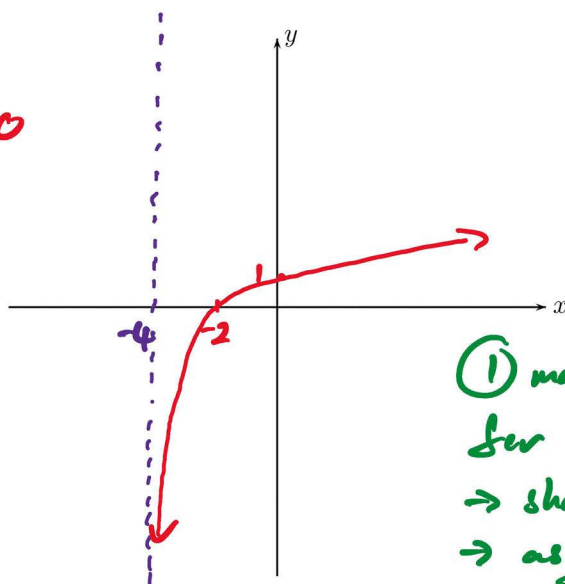
2

x-int:

$$\log_2(x+4) - 1 = 0$$

$$x+4 = 2$$

$$x = -2$$



y-int:

$$\begin{aligned} f(0) &= \log_2(4) - 1 \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

① mark is deducted for each mistake
 → shape
 → asymptote
 → x, y-int

(c) Show that $\frac{\sin \theta}{1 + \cot \theta} - \frac{\cos \theta}{1 + \tan \theta} = \sin \theta - \cos \theta$.

2

$$\text{LHS} = \frac{\sin \theta}{1 + \frac{\cos \theta}{\sin \theta}} - \frac{\cos \theta}{1 + \frac{\sin \theta}{\cos \theta}}$$

$$= \frac{\sin^2 \theta}{\sin \theta + \cos \theta} - \frac{\cos^2 \theta}{\cos \theta + \sin \theta}$$

① progress

$$= \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta + \cos \theta}$$

$$= \frac{(\cancel{\sin \theta + \cos \theta})(\sin \theta - \cos \theta)}{(\cancel{\sin \theta + \cos \theta})} = \text{RHS}$$

① Correct Solution

- (d) (i) Factorise
- $3 + 5x - 2x^2$
- .

1

$$\begin{array}{r} 3 \times -x = -x \\ 1 \times +2x = +6x \end{array}$$

$$\Rightarrow (3-x)(1+2x) \quad \textcircled{1} \text{ Correct Solu'}$$

- (ii) Hence, solve
- $3 + 5 \log_{10} x - 2 (\log_{10} x)^2 = 0$
- . Give your answers as exact values.

2

From (i) :

$$(3 - \log_{10} x)(1 + 2 \log_{10} x) = 0$$

$$\Rightarrow \log_{10} x = 3 \quad \text{or} \quad \log_{10} x = -\frac{1}{2}$$

$$\begin{aligned} x &= 10^3 \\ &= 1000 \end{aligned}$$

①

$$\begin{aligned} x &= 10^{-\frac{1}{2}} \\ &= \frac{1}{\sqrt{10}} \end{aligned}$$

①

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CANDIDATE NUMBER

QUESTION FOURTEEN (12 marks)

Marks

- (a) Dom is an excellent basketball player with an 86% success rate at scoring free throws. Each successful free throw is worth one point.

- (i) Dom gets fouled and is given two free throws. Calculate the probability that he will score at least one point. 2

$$P(\text{Score}) = 0.86 \quad P(\text{miss}) = 0.14$$

① Complementary

$$P(\text{at least 1}) = 1 - P(\text{miss both})$$

$$= 1 - 0.14^2$$

$$= 0.9804 \sim 98\% \quad \text{①}$$

- (ii) Just before the final buzzer, Dom's team is losing by two points, so he attempts a shot worth three points. He is deliberately fouled by his opponent, so he misses the shot, but is awarded three free throw attempts. 2

What is the probability that he will score on exactly two of the three free throws?

To draw: Dom needs to score 2 pts

This can happen 3 ways -

Miss Score Score / Score Miss Score / Score Score Miss

① Consider Cases

$$\Rightarrow P(2 \text{ pts}) = 3 \times (0.86)^2 \times (0.14)$$

$$= 0.31 \sim 31\% \quad \text{①}$$

- (iii) Dom is known to have a 55% success rate at scoring three point shots. Justify whether it was a sensible move by the opposition to deliberately foul Dom on his last shot just before the buzzer. 2

After fouling: Dom needs to score 3
free throws to win,

$$P(3 \text{ success}) = 0.86^3 = 0.636$$

$\sim 64\%$ - ① Calculate $p(\text{win})$

Since Dom's 3-point accuracy is 55%
and he needs to make it to win.

$\Rightarrow$ Fouling will increase his chance
of win which is not sensible.

① Correct reasoning

The paper continues on the next page

(b) Show that $f(x) = (k+2)x^2 + 4x + k+2$ has no x -intercepts for all $k > 0$.

2

$$a = (k+2) \quad b = 4 \quad c = (k+2)$$

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (4)^2 - 4(k+2)(k+2) \\ &= 16 - 4(k^2 + 4k + 4) \\ &= 16 - 4k^2 - 16k - 16 \\ &= -4k^2 - 16k \\ &= -4k(k+4)\end{aligned}$$

① Find
Discriminate

$$\text{Given } k > 0$$

$$\Delta = -4k(k+4) < 0$$

Therefore $\Delta < 0$, $\forall k > 0$

$\Rightarrow f(x)$ has no x -ints

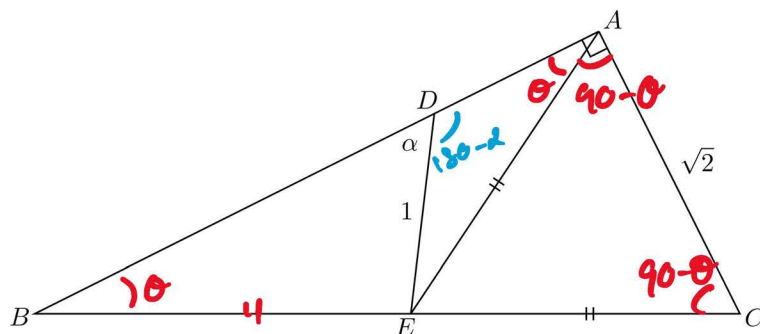
① logical conclusion

(c) The diagram below shows a right-angled triangle ABC where $AC = \sqrt{2}$ units long. 4

Point E lies on hypotenuse BC such that $AE = EC$.

Point D lies on AB such that $DE = 1$.

Calculate the size of $\angle BDE$, labelled α in the diagram.



Let $\angle DBE = \theta$, $\Rightarrow \angle ACB = 90 - \theta$

Since $\triangle ABC$ is a right angled triangle

$\angle EAC = 90 - \theta$ ($AE = EC$)

$\Rightarrow \angle EAD = 90 - (90 - \theta)$
 $= \theta$

Since $\angle EBA = \angle EAB = \theta \Rightarrow AE = BE$

① Show $AE = BE$

Consider $\triangle DEA$: $\angle EDA = 180 - \alpha$
 $\Rightarrow \frac{\sin(180 - \alpha)}{AE} = \frac{\sin \theta}{1}$

① Using Sine Rule

Using $\triangle ABC$: $\sin \theta = \frac{\sqrt{2}}{BC} = \frac{\sqrt{2}}{2AE}$

$\Rightarrow \frac{\sin(180 - \alpha)}{AE} = \frac{\sqrt{2}}{2AE}$

$\Rightarrow \sin \alpha = \frac{\sqrt{2}}{2} \Rightarrow \alpha = 45^\circ, 135^\circ$

① obtain both solutions

— END OF PAPER —