Sydney
Technical
High School

Name: ....

Maths Class: ....

Year 11
Mathematics Advanced

Preliminary HSC Course

Assessment 3

September, 2020

Time allowed: 120 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A reference sheet is provided

Section 1 Multiple Choice
Questions 1-10
10 Marks

Section II Questions 11-18
79 Marks

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Section 1 Multiple Choice (10 marks)

Colour the bubble of the correct answer on your multiple choice answer sheet in your answer booklet

1. What are the solutions of $(x-1)(2x+1)=0$?

   A. $x=1, x=\frac{1}{2}$

   B. $x=-1, x=\frac{1}{2}$

   C. $x=\pm 1, x=-\frac{1}{2}$

   D. *None of the above*

2. $\frac{d}{dx}(-4)=$

   A. -1

   B. 0

   C. $-4x$

   D. -4

3. If $x = 4y^2$, in which expression has y been correctly made the subject?

A. $y = \frac{\sqrt{x}}{2}$

B. $y = \pm\frac{\sqrt{x}}{2}$

C. $y = \sqrt{\frac{x}{2}}$

D. $y = \pm\frac{\sqrt{2x}}{2}$

4. If $27^2 = 3^x$, the value of x is

A. 4

B. 6

C. 8

D. 10

5. Which of the following is NOT a function?

A. $2x + y = 0$

B. $x = -3$

C. $y = 9$

D. $x^2 - 3y = 7$

6. Which of the following pairs are parallel lines?

A. $y = 2x - 5 \; and \; 3y - 6x + 8 = 0$

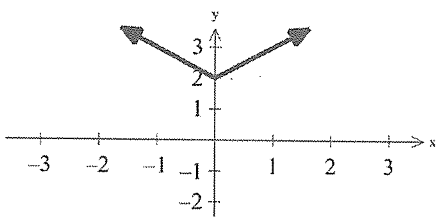
B. $x = 3 \; and \; y = 0$

C. $y = -3x + 8 \; and \; y = \frac{1}{3}x + 7$

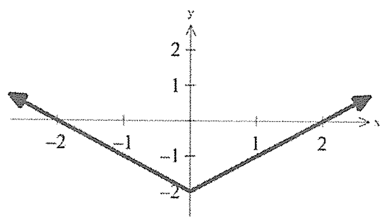
D. All of the above

7. The graph of $y = |x - 2|$ could be

A.



B.



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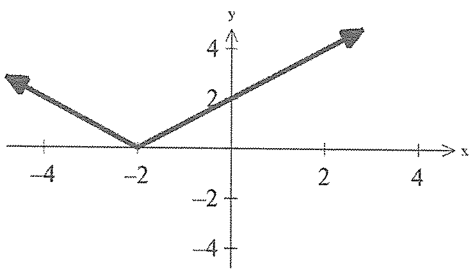
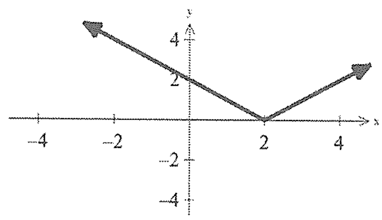


D.



8. What is the period of $y = 2tanx$?

A. $-\pi$

B. $\frac{\pi}{2}$

C. π

D. 2π

9. Which of the following always has a positive gradient?

A. $y = x^3$

B. $y = -2x + 9$

C. $y = x^2 - 5$

D. $\frac{3}{2}x + y = 0$

10. Given that $cos\theta = \frac{1}{2}$ $and\ \theta\ is\ acute, find\ the\ exact\ value\ of\ tan\theta.$

A. $tan\theta = \frac{1}{3}$

B. $tan\theta = \frac{1}{\sqrt{3}}$

C. $tan\theta = \sqrt{3}$

D. $tan\theta = -\sqrt{3}$

Section 2 (79 marks)

All answers are to be completed in your answer booklet. Show all necessary working

Question 11 (Start a New Page) **(10 marks)**

a) Fully simplify $x^2 - 5 + (2x - 3)^2$ **1**

b) Fully factorise $4x^2 - 10x - 24$ **2**

c) Differentiate the following with respect to x

i. $x^2 - 3x + 6$ **1**

ii. $-2x^{-3}$ **1**

d) Solve the following simultaneous equations. **2**

$x + 2y = 0$

$3x + 2y = 4$

e) Evaluate $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3}$ **1**

f) Find the number of roots of $2x^2 - 3x + 4 = 0$. **2**

End of Question 11

Question 12 **(Start a New Page)** **(10 marks)**

a) Differentiate the following with respect to x

i. $3x^{\frac{3}{2}}$ 1

ii. $(2x-1)^5$ 1

iii. $4x(x+2)^5$ 1

iv. $\frac{3x}{x-2}$ 1

b) Solve $|2x-1| = 5$ 2

c) Find the centre and radius of the circle $x^2 + y^2 - 2x + 6y = 13$ by completing the square. 2

d) Find the equation of the normal to the curve $y = -x^3 + x - 5$ at the point where $x = 1$. 2

End of Question 12

Question 13 (Start a New Page) **(10 marks)**

a) Find the midpoint of AB given $A\ (1,2)\ and\ B\ (-1,3)$. 1

b) Solve $\cot 2x = -1,\ \ 0° \leq x \leq 360°$. 3

c) Consider $y = -2^x + 1$

i. State the domain and range 2

ii. Find the x and y-intercepts. 1

iii. State the equation of the asymptote. 1

iv. Sketch the graph of $y = -2^x + 1$, showing x and y-intercepts and the asymptote. 1

v. State whether the function is odd, even or neither. 1

End of Question 13

Question 14 **(Start a New Page)** **(9 marks)**

a) Find, in general form, the equation of the line passing through $A\ (0, -1)$ and having angle of inclination of $60°$ with the positive direction of the x axis. **2**

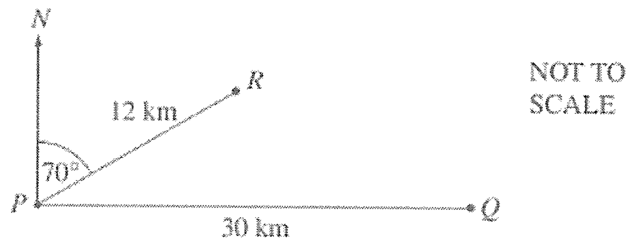
b) Find the exact value of:

i. $\cos 330°$ **1**

ii. $\tan(-210°)$ **1**

iii. $\operatorname{cosec}\left(\frac{5\pi}{6}\right)$ **1**

c)



The diagram shows a point P which is 30km due west of the point Q.

The point R is 12km from P and has a bearing from P of $070°$

i. Find the distance of R from Q, correct to 2 decimal places **2**

ii. Find the bearing of R from Q to the nearest degree **2**

End of Question 14

Question 15 (Start a New Page) **(11 marks)**

a) Solve $\sqrt{3}sin\theta - cos\theta = 0,\ 0 \leq \theta \leq 2\pi.$ 3

b) Given that $f(x) = 3x^2$

i. Find $f'(x)$ by first-principles. 2

ii. Find the point on $y = 3x^2$ where the tangent is horizontal. 2

c) Let $f(x) = \begin{cases} 2 - x^2, for\ x \leq 0 \\ 2 - x,\ for\ x > 0 \end{cases}$

i. Find $f(0)$. 1

ii. Test whether the graph is continuous at $x = 0$. 1

iii. Sketch the graph. (Do NOT find x intercepts) 2

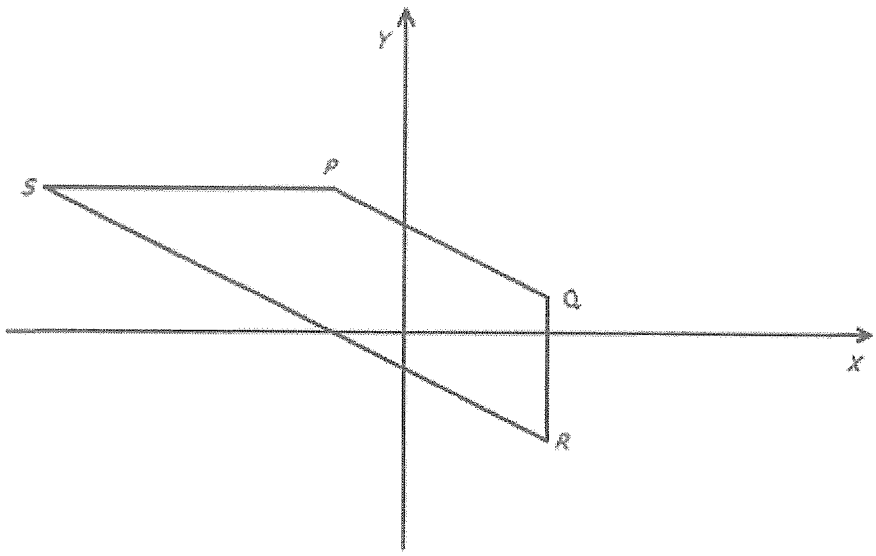
End of Question 15

Question 16 (Start a New Page) **(9 marks)**

a) Fully simplify $\dfrac{-a^2\, b^3}{a^{-1}\,(-b)^3\ c}$ **1**

b) Solve $3^{2x} + 3 \times 3^x - 4 = 0$ **3**

c) In the quadrilateral $PQRS$ the coordinates of the point P and Q are $(-2, 4)$ and $(4, 1)$ respectively. The equation of line SR is $x + 2y + 2 = 0$.



NOT TO SCALE

i. find the gradients of PQ and RS. Hence explain why the quadrilateral $PQRS$ is a trapezium. **2**

ii. Find the length of PQ in exact form. **2**

iii. The line QR is parallel to the y axis, find the coordinates of point R. **1**

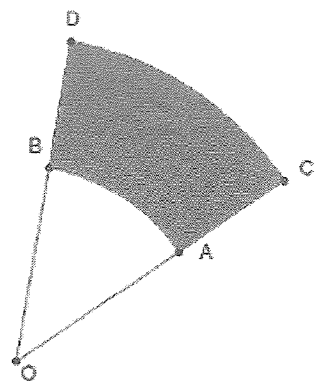
End of Question 16

Question 17 (Start a New Page) **(9 marks)**

a) Show that $f(x) = \frac{1}{x^3} - x^3$ is always decreasing. 2

b) In the diagram, the circular arcs AB and CD have centre O. It is given that $OB = r$,

$OD = 8\ units$ and $\angle BOA = \theta$ such that $\theta = \frac{2\pi r}{8}$.



Show that the area of the shaded region $ABDC$ is $A = 32\theta - \frac{8\theta^3}{\pi^2}$ 2

c) Consider $y = -cosx + 3$,

i. State the period 1

ii. State the amplitude 1

iii. Find the y intercept 1

iv. Sketch the graph of $y = -cosx + 3,\ 0 \le x \le 2\pi$ showing the features above. 2

End of Question 17

Question 18 (Start a New Page) **(11 marks)**

a) i. Show that $sin\theta\ cot\theta = cos\theta$ 1

ii. Hence solve $27cos\theta\ sin\theta\ cot\theta = sec\theta$ where $0° \leq \theta \leq 360°$ to the nearest degree. 2

b) A tank initially holds 3600 litres of water. The water drains from the bottom of the tank. The tank takes 60 minutes to empty.

A mathematical model predicts that the volume, V litres, of water that will remain in the tank after t minutes is given by

$$V = 3600\left(1 - \frac{t}{60}\right)^2 \text{ where } 0 \leq t \leq 60$$

i. What volume does the model predict will remain after ten minutes? 1

ii. At what rate does the model predict that the water will drain from the tank after twenty minutes? 2

iii. At what time does the model predict that the water will drain from the tank at its fastest rate? 2

c) Solve $(tanx - 1)(sinx - 1) = 0,\ 0 \leq x \leq 2\pi$ 3

End of examination

Section 1

1 D

2 B

4 $27^2 = 3^x$

$3^{3\times 2} = 3^x$

$x = 6$ B.

3 $x = 4y^2$

$y = \pm\sqrt{\frac{x}{4}} = \pm\frac{\sqrt{x}}{2}$

B

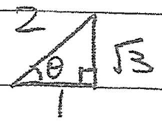
5 B

6 A

7 D

8 C

9 A $\frac{dy}{dx} = 3x^2 \geq 0$

10 

$\tan\theta = \sqrt{3}$

C

Section 2

11 (a) $x^2 - 5 + (2x-3)^2$

$= x^2 - 5 + 4x^2 - 12x + 9$

$= 5x^2 - 12x + 4$

11 (b) $4x^2 - 10x - 24$

$= 2(2x^2 - 5x - 12)$

$= 2(x-4)(2x+3)$

11(c)(i) $2x - 3$

11(c)(ii) $-2(-3)x^{-4}$

$= 6x^{-4}$ OR $\frac{6}{x^4}$

11(d) $x + 2y = 0 \dots ①$

$3x + 2y = 4 \dots ②$

① - ②: $-2x = -4$

$x = 2$

$y = -1$

(e) $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3}$

$= \lim_{x \to 3} \frac{(x-3)(x+1)}{x-3}$

$= 4$

(f) $\Delta = 9 - 4(4)(2)$

$= 9 - 32 < 0$

No zero.

12(a)(ii) $5(2x-1)^4 \cdot 2$

$= 10(2x-1)^4$

12(a)(iii) $4(x+2)^5 + 4x(5)(x+2)^4$

$= 4(x+2)^5 + 20x(x+2)^4$

12 (a) (iv) $= \dfrac{3(x-2) - 3x}{(x-2)^2} = \dfrac{-6}{(x-2)^2}$

12(a)(i) $3\left(\frac{3}{2}\right)x^{\frac{1}{2}}$

$= \frac{9}{2}x^{\frac{1}{2}}$

12(b) $|2x-1| = 5$

Case 1: $x \geq \frac{1}{2}$

$2x-1 = 5$

$2x = 6$

$x = 3$

Case 2: $x < \frac{1}{2}$

$2x-1 = -5$

$2x = -4$

$x = -2$

Answer : $x = 3, -2$

12(c) $x^2+y^2-2x+6y=13$

$(x^2-2x+1)+(y^2+6y+9)=13+1+9$

$(x-1)^2+(y+3)^2=23$

Centre: $(1,-3)$ Radius $=\sqrt{23}$ units

12(d) $y'=-3x^2+1$

$y'|_{x=1}=-2$

$m_{normal}=\frac{1}{2}$

At $x=1$, $y=-5$

$y+5=\frac{1}{2}(x-1)$

$y=\frac{1}{2}x-\frac{11}{2}$

13 (a) $\left(0, \frac{5}{2}\right)$

13 (b) $\cot 2x = -1$

$\tan 2x = -1$

$2x = 135°, 315°, 495°, 675°$

$x = 67.5°, 157.5°, 247.5°, 337.5°$

S	A
T	C

13(c) (i) Domain: All real x

Range: $y < 1$

(ii) At $x = 0$, $y = -2^0 + 1$

$y = 0$

y-intercept: 0

At $y = 0$, $0 = -2^x + 1$

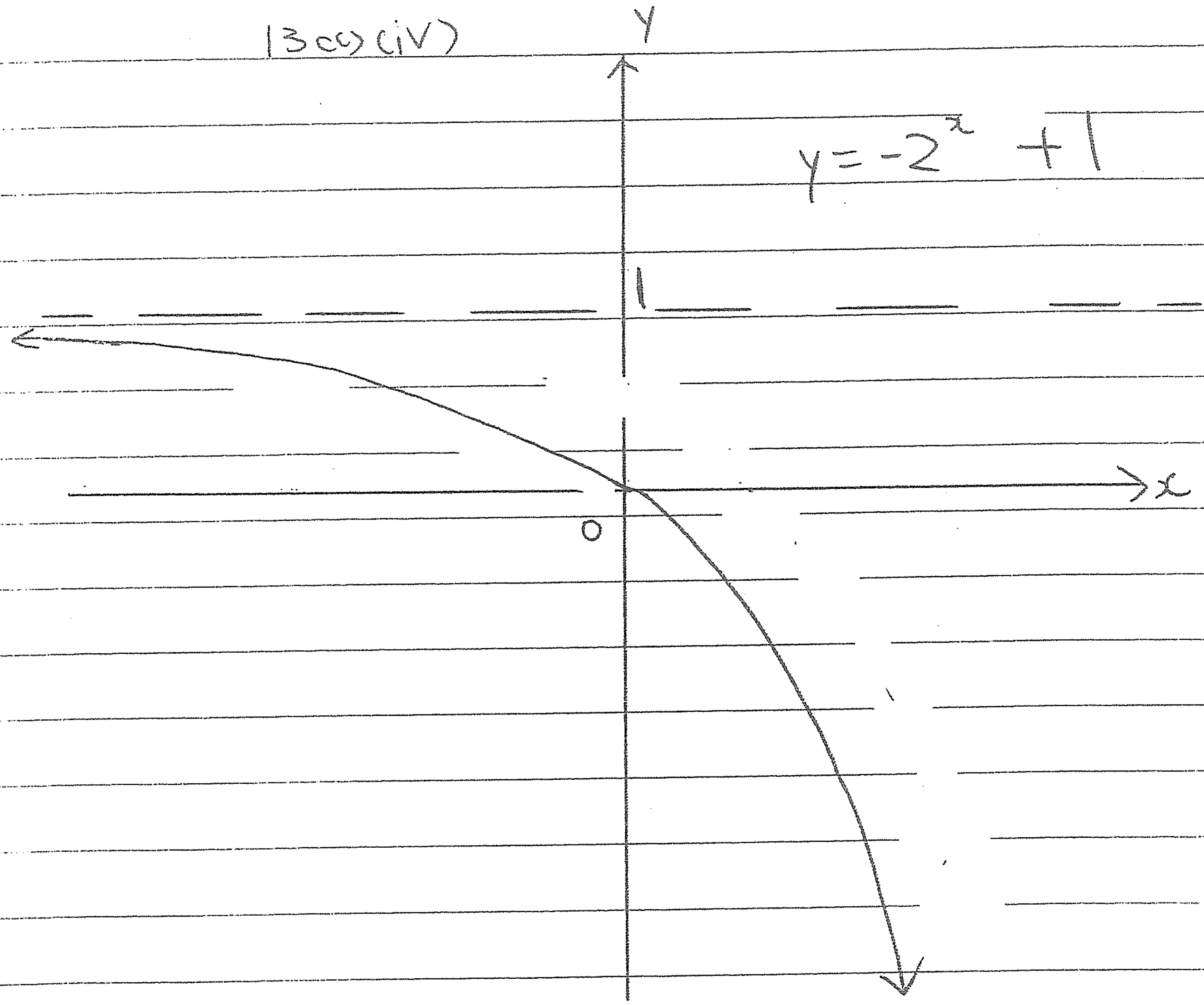
$-1 = -2^x$

$0 = x$

x-intercept: 0

13 (c) (iii) $y = 1$

13 (c) (iv)



13 c(V) Neither

14(a)

$$\tan 60^\circ = \sqrt{3}$$

$$y-(-1) = \sqrt{3}(x-0)$$

$$y+1 = \sqrt{3}x$$

$$\sqrt{3}x - y - 1 = 0$$

(b)(i)

$$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$$

$$\cos 330^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

(b)(ii) $\tan(-210^\circ)$

$$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$$

$$= -\tan 210^\circ$$

$$= -\tan 30^\circ$$

$$= -\frac{1}{\sqrt{3}}$$

(b)(iii) $\operatorname{cosec}\left(\frac{5\pi}{6}\right)$

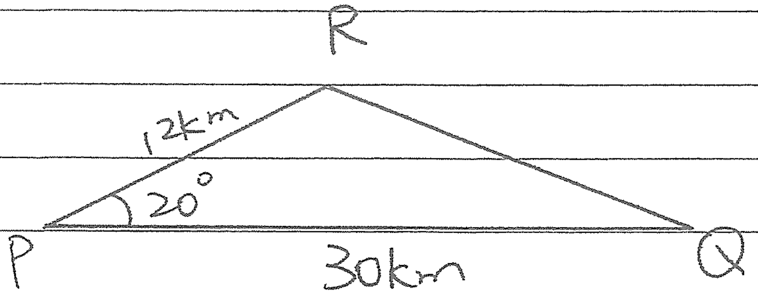
$$= \frac{1}{\sin\left(\frac{5\pi}{6}\right)}$$

$$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$$

$$= \frac{1}{\frac{1}{2}}$$

$$= 2$$

14 (c) (i)



$$RQ^2 = 12^2 + 30^2 - 2(12)(30)\cos 20^\circ$$

$$RQ = 19.16 \text{ km (2 decimal places)}$$

(ii) $\frac{RQ}{\sin 20^\circ} = \frac{12}{\sin \angle PQR}$

$\angle PQR = 12.36^\circ$

Bearing of R from Q is $282.36^\circ T$

15 (a)

$$\sqrt{3}\sin\theta - \cos\theta = 0$$

$$\sqrt{3}\sin\theta = \cos\theta$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ, 210^\circ$$
$$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$$

15 (b)(i) $f'(x) = \lim_{h\to0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to0} \frac{3(x+h)^2 - 3x^2}{h}$$

$$= \lim_{h\to0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$$

$$= \lim_{h\to0} \frac{6xh + 3h^2}{h}$$

$$= 6x$$

(b)(ii) $f'(x) = 0$

$$6x = 0$$

$$x = 0$$

At $x = 0$, $f(0) = 0$

The point is $(0, 0)$

15 (c) (i)

$f(0) = 2 - 0 = 2$

15 (c) (ii)

Sub $x=0$ into $2-x^2$

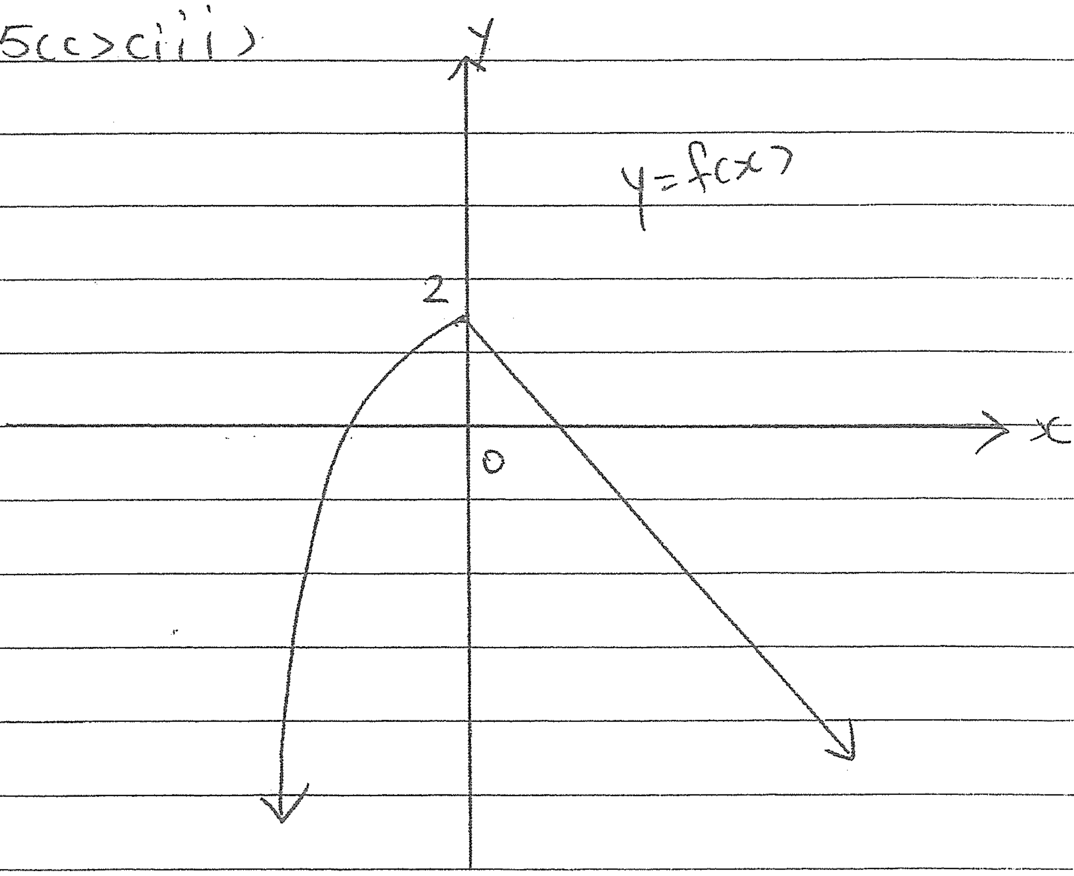
$2 - 0^2 = 2$

Sub. $x=0$ into $2-x$

$2 - 0 = 2$

So $f(x)$ is continuous at $x=0$.

15 (c) (iii)



16(a)

$$\frac{-a^2b^3}{a^{-1}\,(-b)^3\,c}$$

$$= \frac{-a^2\,b^3}{a^{-1}\,-b^3\,c}$$

$$= \frac{a^3}{c}$$

16(b)

$$3^{2x} + 3\cdot 3^x - 4 = 0$$

$$(3^x - 1)\,(3^x + 4) = 0$$

$$3^x = 1 \quad \text{or} \quad 3^x = -4 \text{ (rejected)}$$

$$x = 0$$

16(c)(i) $m_{PQ} = \frac{4-1}{-2-4} = -\frac{1}{2}$

$m_{RS} = -\frac{1}{2}$

There is a pair of parallel lines so PQRS is a trapezium.

16(c)(ii)

$PQ = \sqrt{(-2-4)^2 + (4-1)^2} = \sqrt{45}$

$= 3\sqrt{5}$ units

16(c)(iii)

Let a be the y-coordinate of R

So R is $(4, a)$

R is on the line $x + 2y + 2 = 0$

$4 + 2a + 2 = 0$

$a = -3$

So R is $(4, -3)$

17(a): $f'(x) = -3\left(\frac{1}{x^4}\right) - 3x^2$

$= -3\left(\frac{1}{x^4} + x^2\right)$

$\frac{1}{x^4} + x^2 \geq 0$

so $f'(x) \leq 0$

$f(x) = \frac{1}{x^3} - x^3$ is always decreasing

17 (b) Given $r = \frac{8\theta}{2\pi} = \frac{4\theta}{\pi}$,

$A = \frac{1}{2}\theta(8^2 - r^2)$

$= \frac{\theta}{2}\left(64 - \frac{16\theta^2}{\pi^2}\right)$

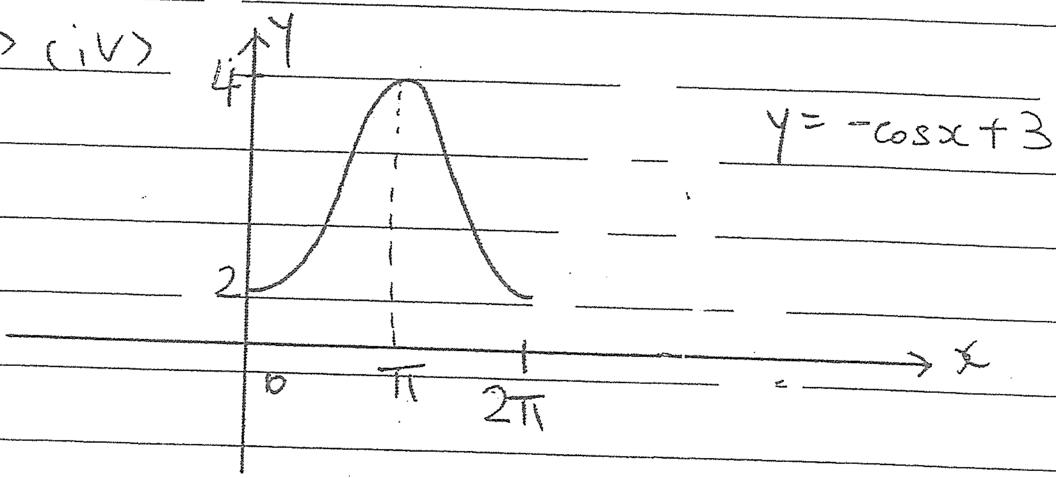
$= 32\theta - \frac{8\theta^3}{\pi^2}$

17(c)(i) 2π

(c)(ii) 1

(c)(iii) 2

(c)(iv)



18(a)(i)

$LHS = \cos\theta \tan\theta$

$= \cos\theta \left(\frac{\sin\theta}{\cos\theta}\right)$

$= \sin\theta$

$= RHS$

18(a)(ii) $27\cos\theta\sin\theta\cot\theta = \sec\theta$

$27\cos^2\theta = \sec\theta$

$\cos^3\theta = \frac{1}{27}$

$\cos\theta = \frac{1}{3}$

$\theta = 71^\circ, 289^\circ$

18(b)(i)

Sub. $t=10$,

$$V = 3600\left(1-\frac{10}{60}\right)^2$$
$$= 3600\left(1-\frac{1}{6}\right)^2$$
$$= 3600\left(\frac{25}{36}\right)$$
$$= 2500 \text{ L}$$

18(b)(ii)

$$\cancel{\frac{dV}{dt} = 3600(2)\left(1-\frac{t}{60}\right)\left(-\frac{1}{60}\right)}$$
$$= -120\left(1-\frac{t}{60}\right)$$
$$= -120+2t$$

Sub. $t=20$, $\frac{dV}{dt} = -80$

-80 L/minute

18(b)(iii)

$$\frac{dV}{dt} = -120+2t$$

Fastest rate means the most negative $\frac{dV}{dt}$.

$-120+2t$ is the most negative where $0 \le t \le 60$ when $t=0$.

18(c) $\tan x = 1$ or $\sin x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$ or $x = \frac{\pi}{2}$ (rejected)

We cannot have $\tan\frac{\pi}{2}$.

Year 11 2 unit Assessment 3 Markers Comments

Question 11

a) The binomial identity not used in the expansion. Algebraic expression not required to be factorised.
b) The quadratic expression was changed when divided by 2, instead of factorising by 2.
c) Subtracting directed numbers produced an error in the power of 'x' when applying the polynomial short-cut to differentiate.
d) Both x-value and y-value, are required for the complete solution.
e) The numerator is to be factorised to determine the value of the limit, as division by zero is not possible.
f) Recognise that if the discriminate (as opposed to the quadratic formula) is less than zero then, there are no real roots.

Question 12

a) Students from across the entire cohort are advised to make sure they are confident with the basic differentiation rules, as many unnecessary marks will be lost. In particular, students should be aware of when the product rule is needed.

b) This was generally well done. Some students put negatives in strange places, such as changing $2x + 1$ to $2x - 1$, and some discarded answers due to poor testing of values.

c) This was quite well done. Several students were stuck on completing the square, and others forgot to actually answer the question.

d) Students should practice this sort of question as it is very common. There were several students who seemed to be completely lost, not knowing how to proceed after differentiation. Otherwise, the most common error was using the gradient of the tangent, or not using the correct y-coordinate.

Question 13

b) Some students did not convert $0° \leq x \leq 360°$ into $0° \leq x \leq 720°$ so they got two answers only.

c) iii) the equation of the asymptote is $y = 1$. A lot of people wrote $y \neq 1$ which is incorrect.

c) iv) Graphs were poorly drawn. Some students did not use rulers, label axes. And some curves were poorly drawn with sharp corners.

Question 14

a) Too many don't understand what general form is-all to one side of = sign and coefficient of x must be positive!! Some out there don't know the formula m=tan(theta).

b) Bread and butter marks for advanced students, you must know how to do these.

c) Easiest method to use is the sine rule but first draw another compass through point Q. <PRQ needs to be added to 270 if giving a true bearing or subtracted from 90 if giving the other type.

Question 15

a) Too many forgot to leave their answers in radians

b) i See setting out of first principles question. Learning rule correctly helps too

ii Finding a point requires a x and y co-ordinate

c) ii Showing the test used to determine continuity is required

iii Too many forgot to label the y intercept

Question 16

a. Fully simplify means you must fully simplify all negatives and pro-numerals. Ok if you left in index form with negatives

b. Quadratic substitution was the best method. Simply substituting in a value is not enough to find all solutions to an equation. Also $3^x > 0$ so -4 could not be solved

c. i. Parallel Lines, trapezium – watch your spelling and watch your explanation

c. ii. Distance formula was used incorrectly

c. iii. Can't solve this using $m = \infty$, parallel to y-axis is not $m = 0$. See worked solutions.

Question 17

a) this should be an easy 2 marks. If in doubt differentiate, which would have scored you 1 mark. Any mention of the gradient or tangent always being negative would have scored the full marks.

Several students used limits. Limits is of no relevance in this question as the gradient needs to be investigated over its whole domain rather than just at the limits.

b) This question was surprisingly poorly done. Students need to become more familiar with their formula sheet. Students also need to recognise that this question and the formula are in radians and as soon as you incorrectly divide by 360 you have no opportunity for scoring any marks.

c) Even the most basic Advanced student must understand and correctly deduce the basic features of period and amplitude for trigonometric functions. Sadly a significant number of students (including Extension 1) seem to have no idea as to what these basic concepts are. Parts i., ii, and iii, were intended to guide students and help them with the sketch, many were led astray through incorrect basic answers.

Question 18

(a) (i) Identity proofs MUST be set out in the following way:

LHS = ...

= ....

= .... etc..

= RHS

(ii) "Hence" means use what you have just proved to do this next part

(b) Use words to state what you are doing:

e.g., ${dV}/_{dt}$ = ... Followed by

"Since ${dV}/_{dt}$ <0 the quantity is decreasing

(c) Why did some people expand this? Usually solving equations involves factorising, and this was already. Make sure you read the question carefully – in this case the answer was to be in radians

Since this WAS the last question, there was a trick in the final answer. See solutions