E-Math (HSC copy) Mathematics Yr 11 Task 4 Yearly 2005

TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

MATHEMATICS

(2 Unit / Extension 1)

YEAR 11 2005 YEARLY EXAMINATION

Assessment Task 4
Weighting: 40%

Time Allowed: *two and a half hours*
(plus 5 minutes reading time)

Outcomes referred to: P1, P2, P3, P4, P5, P6, P7, P8

Question	Topic	Mark	Max. Mark
1	Basic Arithmetic and the Calculator		10
2	Basic Algebra and Surds		10
3	Factorisation and Equations		10
4	Geometry		10
5	Functions		10
6	Trigonometry		10
7	Trigonometry		10
8	The Linear Function		10
9	Calculus		10
10	Calculus		10

Instructions:

1. Board Approved Calculators may be used.
2. Show all working.
3. Each question is 10 marks. Mark values are shown beside each part.
4. Each question must be answered on a separate page. Write your name and teacher on the top of each page.

QUESTION 1 (10 Marks)

a) Evaluate $\frac{45{\cdot}8 \times 7{\cdot}6}{22{\cdot}9}$ *(1 mark)*

b) Round off $0{\cdot}0276$ (to 2 significant figures) *(1 mark)*

c) Write $(3.1 \times 10^5)^2$ in scientific notation *(1 mark)*

d) Write $\frac{91}{208}$ as a percentage correct to 1 d.p. *(1 mark)*

e) Find 110% of $\$63{\cdot}74$ *(1 mark)*

f) What percentage is 35c of $\$1{\cdot}12$? *(2 marks)*

g) Express $\frac{8}{45}$ as a recurring decimal. *(1 mark)*

h) Express $0{\cdot}3\dot{3}\dot{7}$ as a common fraction. *(2 marks)*

QUESTION 2 (10 Marks) *Begin a new page*

a) Simplify $\sqrt[3]{125}$ *(1 mark)*

b) Simplify $(2\sqrt{7})^3$ *(2 marks)*

c) Write $(1-2\sqrt{7})^2$ in the form of $a + b\sqrt{x}$ *(2 marks)*

d) Rationalise the denominator of $\frac{\sqrt{7}+1}{\sqrt{7}}$ *(2 marks)*

e) Simplify $x^2y^{-3} \times x^{-4}y^0$ *(1 mark)*

f) Write $5\sqrt{5}$ in the form 5^n *(1 mark)*

g) Expand and simplify $(8r-5s)(8r+5s)$ *(1 mark)*

QUESTION 3 (10 Marks) *Begin a new page*

a) Factorise

i) $7n - 14nm$ *(1 mark)*

ii) $3x^2 + 11x + 6$ *(1 mark)*

iii) $a^3 + 8$ *(1 mark)*

iv) $25x^2 - 9$ *(1 mark)*

b) Simplify

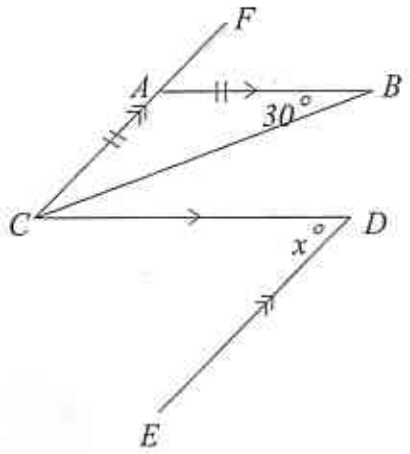
i) $\dfrac{3x^2 + 13x - 10}{x^2 + 5x}$ *(2 marks)*

ii) $\dfrac{a^2b}{cd} \div \dfrac{ab^2}{c^2d}$ *(2 marks)*

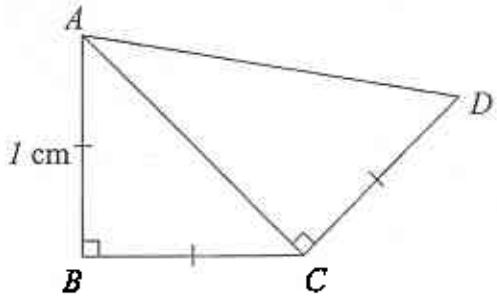
c) Solve $2x^2 - 3x + 1 = 0$ *(2 marks)*

QUESTION 4 (10 Marks) *Begin a new page*

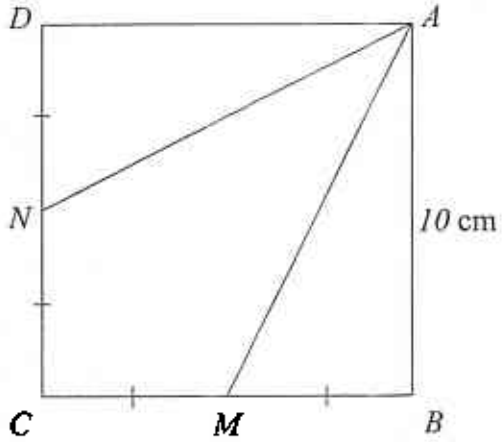
a) In the diagram given below, $AB \parallel CD$, $CA \parallel ED$ and $AB = AC$. Find the value of x, giving reasons. *(3 marks)*



b) Prove that $AD = \sqrt{3}$ cm. *(2 marks)*



c) $ABCD$ is a square of side length 10 cm. M is the midpoint of BC and N is the midpoint of CD.



i) Prove that $\Delta ABM \equiv \Delta ADN$. *(3 marks)*

ii) What type of quadrilateral is the figure $AMCN$? Give reasons. *(2 marks)*

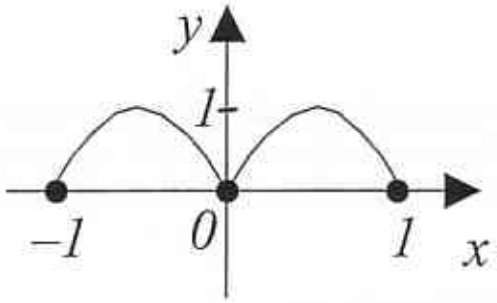
QUESTION 5 (10 Marks) *Begin a new page*

a) For the function $f(x) = 2x - 1$, find:

i) the value of $f(2)$ *(1 mark)*

ii) the value of x at which $f(x) = 0$ *(1 mark)*

b) Find the domain (D) and the range (R) of the function below. *(2 marks)*



c) State whether the function $y = \frac{1}{4-x} + \frac{1}{4+x}$ is even, odd or neither. Show working. *(2 marks)*

d) Sketch the function $y = 1 - x^2$ in the domain $-1 \leq x \leq 1$ *(2 marks)*

e) Shade the region(s) in the XY-plane indicated by the inequality $x^2 + y^2 < 1$ *(2 marks)*

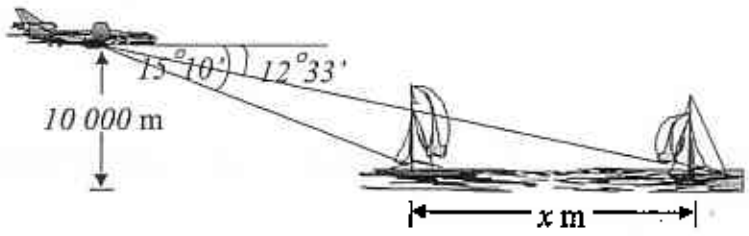
QUESTION 6 (10 Marks) *Begin a new page*

a) In which quadrants does θ lie if $\cos\theta$ is negative and $\tan\theta$ is negative? *(1 mark)*

b) Find the exact value of $\dfrac{\cos 45^\circ}{\tan 30^\circ}$ *(2 marks)*

c) Prove that $\dfrac{\cot\theta \cos ec\theta}{1+\cot^2\theta} = \cos\theta$ *(3 marks)*

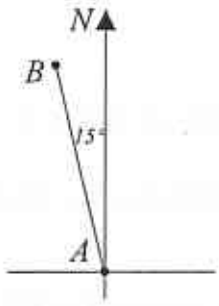
d) From a plane flying at a constant altitude of *10 000* m, the angle of depression of two boats is observed. The angle of depression of boat *A* is *15°10′* and at the same instant the angle of depression of boat *B* is *12°33′*. Find (to the nearest metre) the distance *x* between the two boats. *(2 marks)*



e) Find the exact value of $\tan(-150)^\circ$ *(2 marks)*

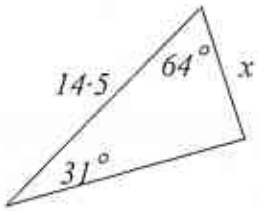
QUESTION 7 (10 Marks) *Begin a new page*

a) Write down the bearing of B from A. *(1 mark)*



b) Find, to the nearest degree, all possible values of θ if $\sin\theta = 0{\cdot}6018$ and $0 \leq \theta \leq 360°$ *(2 marks)*

c) Find x (to 1 decimal place). *(2 marks)*



d) In ΔABC, $\angle A = 36°$, $a = 11{\cdot}7$ and $b = 9{\cdot}6$. Find $\angle B$ (to the nearest degree). *(2 marks)*

e) A yacht sails due south of its home port for *175* nautical miles. It then turns and sails for *350* nautical miles on a course bearing *210°*. How far (in a straight line) is the yacht from its home port? Give your answer to the nearest nautical mile. *(3 marks)*

QUESTION 8 (10 Marks) *Begin a new page*

a) Sketch the graph of $y = -\frac{1}{2}$ *(1 mark)*

b) The points A and B have coordinates $(-3, 2)$ and $(4, 7)$, respectively. Find:

i) the gradient of the straight line passing through A and B *(1 mark)*

ii) the distance AB (correct to 1 decimal place) *(1 mark)*

c) Show that the lines $y = 3x + 1$ and $9x - 3y + 2 = 0$ are parallel. *(2 marks)*

d) Find the equation of the perpendicular bisector of the interval joining $(2, -5)$ and $(-4, 7)$. Express your answer in general form. *(3 marks)*

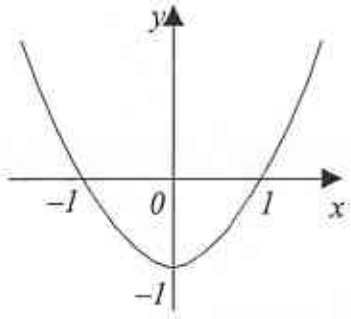
e) The straight line $3x - 4y + p = 0$ is 2 units from the point $(2, 5)$. Find the two possible values of p. *(2 marks)*

QUESTION 9 (10 Marks) *Begin a new page*

a) Find $\lim_{x \to -4} \frac{x^4 - 16}{x^2 - 4}$ *(2 marks)*

b) Using $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, find $f'(x)$ if $f(x) = 7x + 2$ *(2 marks)*

c) Sketch the gradient function $f'(x)$ of the curve below and give its equation. *(2 marks)*



d) If $f(x) = 12x^3$, find the value of $f'(2)$ *(2 marks)*

e) Find the gradient of the curve $y = 2x^2 + 3x - 1$ when $x = -2$ *(2 marks)*

QUESTION 10 (10 Marks) *Begin a new page*

a) Differentiate $12 + 7x^5$ with respect to x. *(1 mark)*

b) If $y = (2x-5)^3$, find $\frac{dy}{dx}$. *(2 marks)*

c) Find the equations of the tangents to the curve $y = x(x-1)$ at the points where the curve cuts the x axis. *(4 marks)*

d) Find the gradient of the normal to $y = x^2$ at the point (4, 16) *(1 mark)*

e) Differentiate $y = \frac{x}{x+1}$ with respect to x. *(2 marks)*

E Pratt (HOD (CAW)) Mathematics TASK 4 Yearly 2005

YEAR 11 YEARLY (MATHS (2U)) 2005

SOLUTIONS

Question (1)

a) $\frac{45.8 \times 7.6}{22.9} = 15.2$ ✓

b) $0.0276 = 0.028$ (2sf) ✓

c) $(3.1 \times 10^5)^2 = 9.61 \times 10^{10}$ ✓

d) $\frac{91}{208} \times 100\% = 43.75\%$
$= 43.8\%$ (1dp) ✓

e) $\frac{110}{100} \times \$63.74 = \$70.114$
$= \$70.11$ (2dp) ✓

f) $\frac{35c}{112c}$ ✓ $\times 100\% = 31.25\%$ ✓

g) $\frac{8}{45} = 0.17777\ldots$
$= 0.1\dot{7}$ ✓

h) let $x = 0.3\dot{3}\dot{7}$
$10x = 3.\dot{3}\dot{7}$ ✓
$1000x = 337.\dot{3}\dot{7}$

$1000x = 10x + 334$
$990x = 334$
$x = \frac{334}{990}$
$= \frac{167}{495}$ ✓

(10)

Question (2)

a) $\sqrt[3]{125} = 5$ ✓

b) $(2\sqrt{7})^3 = 2\sqrt{7} \times 2\sqrt{7} \times 2\sqrt{7}$
$= 8 \times 7 \times \sqrt{7}$
$= 56\sqrt{7}$ ✓ ✓

c) $(1-2\sqrt{7})^2 = (1-2\sqrt{7})(1-2\sqrt{7})$ ✓
$= 1 - 2\sqrt{7} - 2\sqrt{7} + 28$
$= 29 - 4\sqrt{7}$ ✓

d) $\frac{\sqrt{7}+1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{7+\sqrt{7}}{7}$ ✓ ✓

e) $x^2y^{-3} \times x^{-4}y^9 = x^{-2}y^6$ ✓

f) $5\sqrt{5} = 5 \times 5^{1/2}$
$= 5^{3/2}$ ✓

g) $(8r-5s)(8r+5s) = 64r^2 + 40rs - 40rs - 25s^2$
$= 64r^2 - 25s^2$ ✓

(10)

Question (3)

a) i) $7n - 14nm = 7n(1-2m)$ ✓

ii) $3x^2+11x+6 = (3x+2)(x+3)$ ✓

iii) $a^3+8 = (a+2)(a^2-2a+4)$ ✓

iv) $25x^2-9 = (5x-3)(5x+3)$ ✓

b) i) $\frac{3x^2+13x-10}{x^2+5x} = \frac{(3x-2)(x+5)}{x(x+5)}$ ✓

$= \frac{3x-2}{x}$ ✓

ii) $\frac{a^2b}{cd} \div \frac{ab^2}{c^2d} = \frac{a^2b}{cd} \times \frac{c^2d}{ab^2}$ ✓

$= \frac{ac}{b}$ ✓

c) $2x^2-3x+1=0$

$x = \frac{3 \pm \sqrt{9-4(2)(1)}}{2(2)}$

$= \frac{3 \pm \sqrt{1}}{4}$

$= \frac{3 \pm 1}{4}$

$= \frac{4}{4}, \frac{-2}{4}$

$= 1, -\frac{1}{2}$ ✓

✓

(10)

Question (4)

a) $\angle ACB = 30°$ (base $\angle$s of isos. $\triangle ABC$ are equal) ✓

$\angle BCD = 30°$ (alt $\angle$'s, AB // CD) ✓

$\angle CDE = x° = 60°$ (corresponding $\angle$s, CA // ED) ✓

b) $(AC)^2 = 1^2 + 1^2$ (pythag)

$(AC)^2 = 2$ ✓

$AC = \sqrt{2}$

$(AD)^2 = (AC)^2 + (DC)^2$

$= (\sqrt{2})^2 + (1)^2$

$= 2 + 1$

$(AD)^2 = 3$ ✓

$\therefore AD = \sqrt{3}$ as required.

c) i) In $\triangle$s ABM and ADN

i) $AB = AD = 10cm$ (sides of a square are equal) ✓

ii) $BM = DN = 5cm$ (N is the midpt of DC, M is the midpt of CB) ✓

iii) $\angle ADN = \angle ABM = 90°$ ($\angle$s in a square)

$\therefore \triangle ABM \equiv \triangle ADN$ (SAS) ✓

ii) AMCN is a kite ✓ as adjacent sides are equal.

✓ $AN = AM$ (matching sides on congr. $\triangle$s)

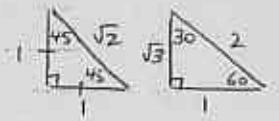
$NC = CM$ (N and M are midpts of DC and BC)

(10)

Question (6)

a) $\cos\theta < 0$ and $\tan\theta < 0$ in quadrant 2. ✓

b) $\dfrac{\cos 45}{\tan 30} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{3}}}$ ✓



$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{1}$

$= \dfrac{\sqrt{3}}{\sqrt{2}}$ ✓

c) LHS $= \dfrac{\cot\theta \operatorname{cosec}\theta}{1+\cot^2\theta}$

$= \dfrac{\cot\theta \operatorname{cosec}\theta}{\operatorname{cosec}^2\theta}$

$= \dfrac{\cot\theta}{\operatorname{cosec}\theta}$ ✓

$= \dfrac{\cos\theta}{\sin\theta} \div \dfrac{1}{\sin\theta}$ ✓

$= \dfrac{\cos\theta}{\sin\theta} \times \dfrac{\sin\theta}{1}$ ✓

$= \cos\theta$

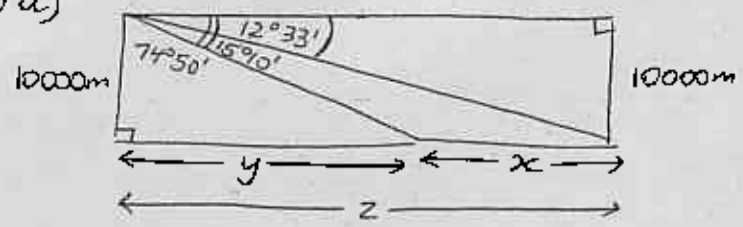
$= RHS.$

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

[redacted]

(6) d)



$\tan 77°27' = \dfrac{z}{10000}$

$z = 10000\tan 77°27'$ ✓

$\tan 74°50' = \dfrac{y}{10000}$

$y = 10000\tan 74°50'$

$x = z - y$

$= 10000\tan 77°27' - 10000\tan 74°50'$

$= 8030.605499\ldots$

$= 8031\text{m (nst m)}.$ ✓

e) $\tan(-150) = -\tan 150$

$= -\tan(180-30)$

$= --\tan 30$

$= +\dfrac{1}{\sqrt{3}}$

Question 5

a) $f(x) = 2x - 1$

i) $f(2) = 2(2) - 1$
$= 3$ ✓

ii) $0 = 2x - 1$
$1 = 2x$
$x = \frac{1}{2}$ ✓

b) $D: -1 \leq x \leq 1$ ✓
$R: 0 \leq y \leq 1$ ✓

c) $f(x) = \frac{1}{4-x} + \frac{1}{4+x}$

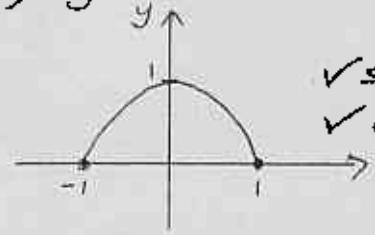
$f(-x) = \frac{1}{4-(-x)} + \frac{1}{4+(-x)}$

$= \frac{1}{4+x} + \frac{1}{4-x}$ ✓

$= f(x)$

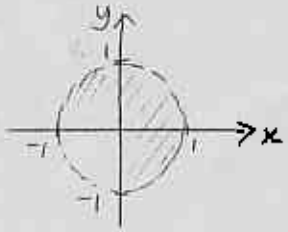
$f(x) = f(-x)$ ∴ even. ✓

d) $y = 1 - x^2$



✓ shape
✓ closed circles at $x = -1, 1$.

e)



✓ dotted circle
✓ shading.

(10)

Question 7

a) 345T or N15W ✓

b) $\sin\theta = 0.6018$

acute $\theta = \sin^{-1} 0.6018$
$= 36°59'$ ✓

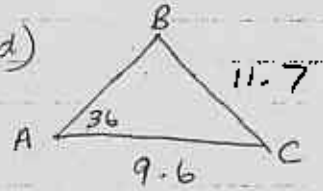
$\theta = 36°59'$ and $180° - 36°59'$
$= 36°59'$ and $143°0'$
$= 37°$ and $143°$ (nst degree) ✓

c) $\frac{x}{\sin 31} = \frac{14.5}{\sin 85}$ ✓

$x = \frac{14.5 \sin 31}{\sin 85}$

$= 7.496578832...$
$= 7.5$ (1dp) ✓

d)

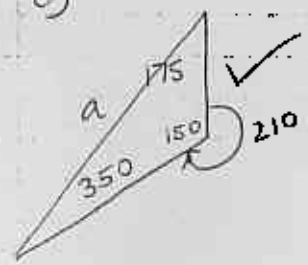


$\frac{\sin B}{9.6} = \frac{\sin 36}{11.7}$ ✓

$\sin B = \frac{9.6 \sin 36}{11.7}$

$B = 28°50'05.16''$ ✓
$= 29°$ (nst degree)

e)



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 175^2 + 350^2 - 2(175)(350)\cos 21$ ✓

$= 153125 - 122500\cos 210$ ✓

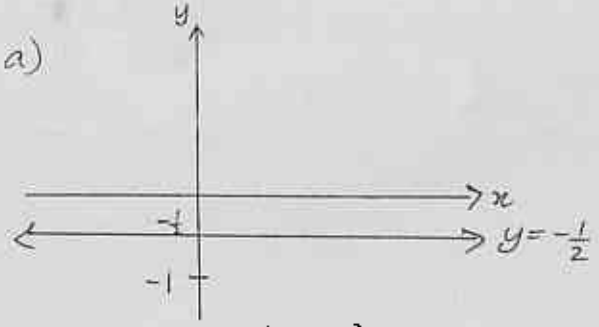
$= 259213.112$

$a = 509.1297595...$ ✓

$= 509$ nautical miles (nst n mile) (11)

Question (8)

a)



✓

b) $(-3, 2)$ $(4, 7)$

i) $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7-2}{4--3}$

$= \frac{5}{7}$ ✓

ii) $d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$

$= \sqrt{(7-2)^2 + (4--3)^2}$

$= \sqrt{25+144}$

$= \sqrt{169}$

$= 13$ ✓

c) parallel if gradients are same

$y = 3x + 1$ has gradient 3. ✓

$9x - 3y + 2 = 0$

$9x + 2 = 3y$ ✓

$3x + \frac{2}{3} = y$ has grad. 3 ∴ parallel.

(8)d) $(2, -5)$ $(-4, 7)$

midpt: $\left(\frac{2-4}{2}, \frac{-5+7}{2}\right)$

$= \left(\frac{-2}{2}, \frac{2}{2}\right)$

$= (-1, 1)$ ✓

gradient $= \frac{7--5}{-4-2}$

$= \frac{12}{-6}$

$= -2$ ✓

perp bisector will have gradient $\frac{1}{2}$ and go through midpoint.

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{2}(x - -1)$

$2y - 2 = x + 1$

$x - 2y + 3 = 0$. ✓

e) $3x - 4y + p = 0$... perp dist to $(2, 5)$ is 2

$2 = \frac{|3(2) - 4(5) + p|}{\sqrt{9+16}}$

$2 = \frac{|6 - 20 + p|}{\sqrt{25}}$

$2 = \frac{|-14+p|}{5}$

$10 = |-14+p|$

$10 = -14 + p$

$p = 24$ ✓

or

$10 = -(-14+p)$

$10 = 14 - p$

$p = 4$. ✓

Question (9.)

a) $\lim_{x \to -4} \frac{x^4 - 16}{x^2 - 4}$

$= \lim_{x \to -4} \frac{(x^2-4)(x^2+4)}{(x^2-4)}$ ✓

$= \lim_{x \to -4} x^2 + 4$

$= (-4)^2 + 4$

$= 16 + 4$

$= 20.$ ✓

b) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(x) = 7x + 2$

$f(x+h) = 7(x+h) + 2$

$= 7x + 7h + 2$

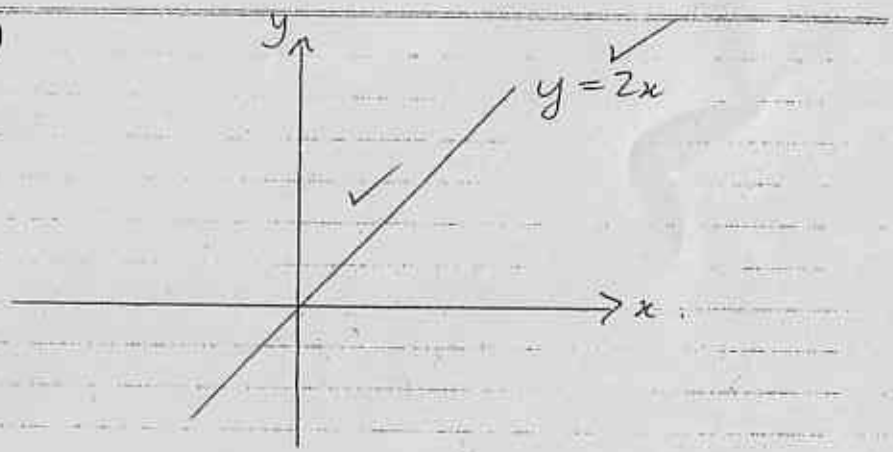
$\lim_{h \to 0} \frac{7x + 7h + 2 - (7x + 2)}{h}$ ✓

$= \lim_{h \to 0} \frac{7h}{h}$

$= \lim_{h \to 0} 7$

$= 7$ ✓

c)



✓

d) $f(x) = 12x^3$

$f'(x) = 36x^2$ ✓

$f'(2) = 36(2)^2$

$= 144$ ✓

e) $y = 2x^2 + 3x - 1$

$\frac{dy}{dx} = 4x + 3$ ✓

When $x = -2$

$\frac{dy}{dx} = 4(-2) + 3$

$= -5$ ✓

Question (10)

a) $y = 12 + 7x^5$

$y' = 35x^4$ ✓

b) $y = (2x-5)^3$

$y' = 3(2x-5)^2 \times 2$

$= 6(2x-5)^2$

✓ ✓

c) $y = x(x-1)$ cuts x axis at $x = 0$ $y = 0$

and $x = 1$ $y = 0$ ✓

$y = x^2 - x$

$y' = 2x - 1$

at $x = 0$ $y' = 2(0) - 1$

$= -1$ ✓

$x = 1$ $y' = 2(1) - 1$

$= 1$

tangent eqns:

$y - y_1 = m(x - x_1)$

$y - 0 = -1(x - 0)$

$y = -x$

$x + y = 0$ ✓

$y - y_1 = m(x - x_1)$

$y - 0 = 1(x - 1)$

$y = x - 1$

$x - y - 1 = 0$ ✓

d) $y = x^2$

$\frac{dy}{dx} = 2x$

at (4, 16) grad of tangent is 8

grad. of normal is $-\frac{1}{8}$ ✓

e) $y = \frac{x}{x+1}$ $\frac{dy}{dx} = \frac{(x+1) \times 1 - x \times 1}{(x+1)^2}$ ✓

$= \frac{1}{(x+1)^2}$ ✓

(10)