N Stathakis Mathematics Year 11 Task 4 Yearly 2006
(Hold Copy) PC TRINITY GRAMMAR SCHOOL Term 3
MATHEMATICS DEPARTMENT

YEAR 11
YEARLY EXAMINATION

August 2006
Assessment 4

Mathematics

Outcomes referred to: P1, P2, P3, P4, P5, P6, P7, P8

General Instructions

- Working time – $2\frac{1}{2}$ hours
- 5 minutes reading time.
- Write using blue or black pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Begin a NEW question on a NEW page.

Total marks – 100

- Attempt questions 1-10
- Weighting of 40%
- Each question is worth 10 marks

Question 1:

(a) Express $\frac{17}{45}$ as a recurring decimal **1**

(b) Evaluate correct to 2 decimal places $\sqrt{\frac{8.5-2.4}{3.2}}+(6.3)^3$. **2**

(c) Show that $0.3\dot{5}$ can be expressed as a common fraction by a method other than the use of a calculator. **2**

(d) Simplify $\sqrt{147}$. **1**

(e) Express $6\times\sqrt{\frac{1}{3}}$ as a complete surd. **1**

(f) Rationalise the denominator of $\frac{1}{\sqrt{5}+2}$. **2**

(g) Simplify $-45x^3y^2 \div 9x^3y^5$. **1**

Question 2:

(a) Simplify $3m^2 - 3mn - m + n$. 1

(b) Expand and simplify $a(a+1) - 3(2a+1)$. 1

(c) Expand and simplify $(5x-14)(2x+5)$ 1

(d) Factorise $36c^2 - 49d^2$ 1

(e) Factorise $\frac{a^3}{8} - 1$ 1

(f) Factorise $x^2 - 21x - 72$ 1

(g) (i) Factorise $6x^2 - 29x + 28$ 2

(ii) Solve $6x^2 - 29x + 28 = 0$ 2

Question 3:

(a) Simplify $\frac{x^2-7x+6}{x^2-36}$ **2**

(b) Express the following as an integer $5^3 \times 2^4 \times 10^{-3}$ **1**

(c) Solve for m : $(2m-5)^2 = 16$ **2**

(d) Graph on the number line the solution of: $\frac{3x}{5} - \frac{2x}{3} \rangle -2$ **3**

(e) Simplify $\frac{3a}{2b} - \frac{5a}{b}$ **2**

Question 4:

(a) Solve $(x+1)^2 = 4x$ **1**

(b) Solve: $\frac{5x-3}{2} = x+2$ **2**

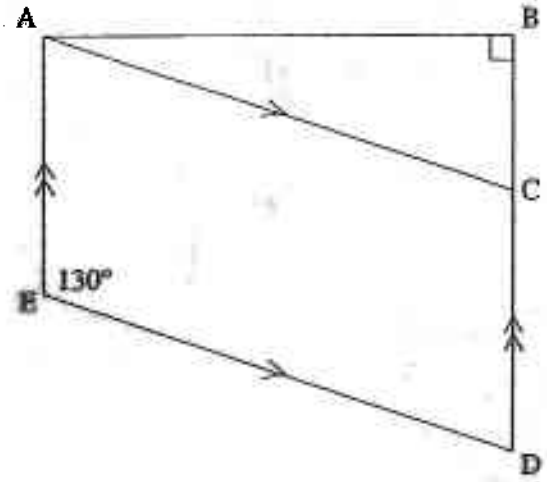
(c) Solve simultaneously $\begin{matrix} x+5y=16 \\ 2x-y=7 \end{matrix}$ by the use of the *elimination method* **3**

(d) Solve $|2x+5| < 3$ **3**

(e) Solve $22 \leq 5x-3 \leq 32$ **1**

Question 5:

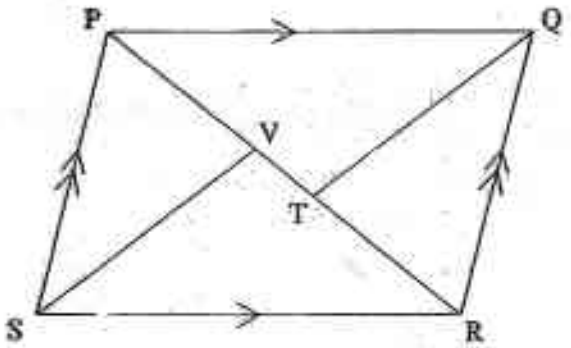
(a) In the diagram AE∥ BD, AC∥ ED, $\angle AED = 130°$ and $\angle ABC = 90°$.



NOT TO SCALE

(i) Copy this diagram onto your answer sheet.
(ii) Find the size of $\angle BAC$ giving reasons. **2**

(b)

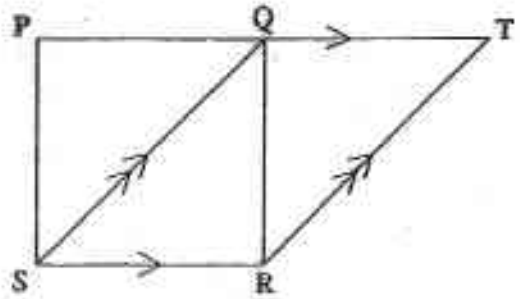


NOT TO SCALE

PQRS is a parallelogram. TQ bisects $\angle PQR$ and VS bisects $\angle PSR$.

(i) Copy this diagram onto your answer booklet.
(ii) State why $\angle PQR = \angle PSR$. **1**
(iii) Prove that ΔPVS and ΔRTQ are congruent. **3**
(iv) Hence find the length of TV if $PR = 20cm$ and $TR = 8cm$. **2**

(c) PQRS is a square with PQ=1unit. Find the perimeter of PTRS. **3**



NOT TO SCALE

Question 6:

(a) In the diagram the line L_1 has equation $3x-2y+3=0$ and the line L_2 passes through the point P(0,8) and Q(12, 0).

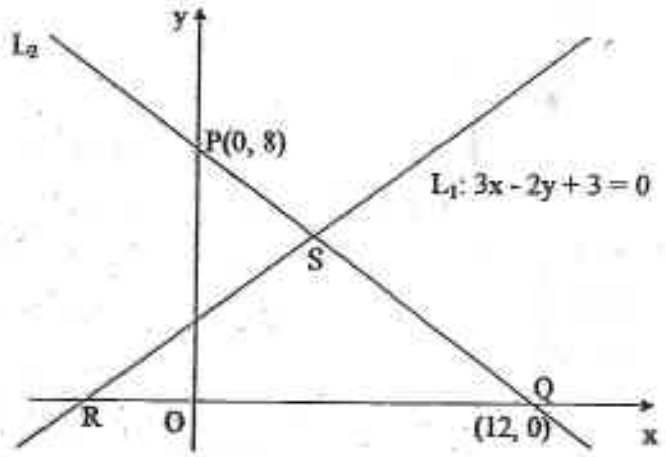


FIGURE NOT TO SCALE

(i) L_1 cuts the x-axis at R. Find the co-ordinates of R. **1**

(ii) Find the gradient of the line L_1. **1**

(iii) Show the line L_1 is perpendicular to the line L_2. **1**

(iv) Show the equation of the line L_2 is: $2x+3y-24=0$. **2**

(v) Find the coordinates of S, the point of intersection of the lines L_1 and L_2. **1**

(vi) Hence calculate the area of ΔQRS. **2**

(b) What is the domain and Range of $y=-\sqrt{9-x^2}$ **2**

Question 7:

(a) Sketch the curve $y = |2x| - 4$, showing all essential features. **3**

(b) Given:

$$f(x) = \begin{cases} x+1, & x \geq 1 \\ -1, & -1 < x < 1 \\ 1-x, & x \leq -2 \end{cases}$$

(i) Evaluate $f(1)$ **1**

(ii) Evaluate $f(\frac{1}{2})$ **1**

(c) Shade the common region(s) in the XY-plane indicated by the inequalities $x + 2y \geq 8$ and $y < 7$. **3**

(d) For $90° < A < 180°$ find A if $\sin A = \frac{\sqrt{3}}{2}$. **2**

Question 8:

(a) Find the exact value of: $\cot(-60^\circ)\operatorname{cosec}(240^\circ)$. **2**

(b) AB and CD are two vertical buildings with their bases A and C on horizontal ground. The height of AB is 30m. The angle of elevation of B as seen from C is 25° and the angle of elevation of D as seen from A is 40°.

(i) the horizontal distance between the buildings **2**

(ii) the height of CD. **2**

(iii) The angle of depression of B as seen from D. **1**

(c) A ship sailing in a direction $065^\circ T$ observes two lighthouses in a line due North. After travelling 4km, one of the lighthouses bears $285^\circ T$ and the other $315^\circ T$. Calculate the distance between the lighthouses. **3**

Question 9:

(a) (i) Find from first principles, the derivative of $f(x) = x^2 - 2$. **2**

(ii) Hence or otherwise, find $f'(x)$ at $x = -2$. **1**

(b) Find the derivative of the following equations.

(i) $y = 2x^3 + 3x$ **1**

(ii) $f(x) = \dfrac{2x^5 - x + 1}{x^3}$ **2**

(iii) $(x-3)\sqrt{x-3}$ **2**

(iv) $y = \dfrac{2x+7}{2x-3}$ **2**

Question 10:

(a) Find the equations of the tangent and normal to the parabola $y = 2x^2 - 5x + 1$ at the point on the parabola where the gradient is 3. **3**

(b) Find p if the roots of the equation $3x^2 + px - 12 = 0$ differ by 4. **2**

(c) Express $x^2 + 2x - 2$ in the form $ax(x+1) + bx^2 + c(x+1)$. **3**

(d) Find the equation of the family of parabolas having:

(i) the line $x = 2$ as axis and vertex (2,1). **1**

(ii) The line $x = -2$ as axis and focal length $\frac{1}{2}$ unit. **1**

SOLUTIONS

YEAR 11 (2006) Solutions (9) $-5y-3$ ✓

Question 1:

(a) $0.3\dot{7}$ ✓

b)

c) $x = 0.3555\cdots$

$10x = 3.5555$

$9x = 3.2$

$90x = 32$ ✓

$x = \frac{32}{90}$

$x = \frac{16}{45}$ ✓

d) $7\sqrt{3}$ ✓

e) $\sqrt{36 \times \frac{1}{3}}$

$= \sqrt{12}$ ✓

f) $\frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$

$= \frac{\sqrt{5}-2}{5-4}$ ✓

$= \sqrt{5}-2$ ✓

Question 2:

(a) $3m(m-n) - (m-n)$

$= (3m-1)(m-n)$ ✓

(b) $a^2 + a - 6a - 3$

$= a^2 - 5a - 3$ ✓

(c) $10x^2 - 28x + 25x - 70$

$= 10x^2 - 3x - 70$ ✓

(d) $(6c-7d)(6c+7d)$ ✓

(e) $\left(\frac{a}{2}-1\right)\left(\frac{a^2}{4}+\frac{a}{2}+1\right)$ ✓

(f) $(x+3)(x-24)$ ✓

9) (i) $(3x-4)(2x-7)$ ✓ ✓

(ii) $x = \frac{4}{3}$ ✓

$x = \frac{7}{2}$ ✓

Question 3:

a) Simplify

$$\frac{(x-1)(x-6)}{(x-6)(x+6)}$$ ✓✓

$$= \frac{x-1}{x+6}$$ ✓

b) $5^3 \times 2^4 \times (5\times 2)^{-3}$

$$= 5^3 \times 2^4 \times 5^{-3} \times 2^{-3}$$

$$= 5^0 \times 2^1$$ ✓

$$= 2$$

c) $2m - 5 = \pm 4$

$$2m = 5 \pm 4$$ ✓

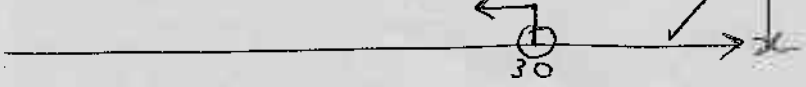
$$m = \frac{5}{2} \pm \frac{4}{2}$$ ✓

$$m = \frac{5}{2} \pm 2$$

d) $\frac{9x - 10x}{15} > -2$ ✓

$$-x > -30$$

$$x < 30$$ ✓✓



(e) $\frac{3a - 10a}{2b}$ ✓

$$= \frac{-7a}{2b}$$ ✓

Question 4:

(a) $x^2 + 2x + 1 = 4x$

$$x^2 - 2x + 1 = 0$$

$$(x-1)(x-1) = 0$$

$$x = 1$$ ✓

(b) $5x - 3 = 2x + 4$

$$3x = 7$$ ✓

$$x = \frac{7}{3}$$ ✓

(c) $2x + 10y = 32$ — (1)

$2x - y = 7$ — (2)

$$11y = 25$$ ✓

$$\boxed{y = \frac{25}{11}}$$

$$2x - \frac{25}{11} = 7$$ ✓

$$2x = 9\frac{3}{11}$$

$$\boxed{x = 4\frac{7}{11}}$$ ✓

(d) $2x + 5 < 3$

$2x < 2$ ✓

$\boxed{x < 1}$

$-2x - 5 < 3$

$-2x < 8$

$\boxed{x > -4}$ ✓

2) $25 \leq 5x \leq 35$

$5 \leq x \leq 7$

$x \leq 7$ ✓

$x \geq 5$. ✓

Question 5:

a)(ii) $\angle EDC = 50°$ — co-interior $\angle$'s in II lines

$\angle ACB = 50°$ ✓ — corresponding $\angle$'s in II lines

$\therefore$ In $\triangle ABC = \theta + 50 + 90$

$180° = \theta + 140$ ✓

$\angle BAC \therefore 40°$

(b)

(ii) opposite $\angle$'s in a parallelogram are equal.

$\therefore \angle PQR = \angle PSR$ ✓

(iii) $\angle SPR = \angle QRT$ — ✓
alt angles in II lines

$PS = QR$ — equal ✓
sides in a parallelogram

$\angle PSV = \angle RQT$ — ✓
opposite $\angle$'s equal that have been bisected.

$\therefore$ (AAS).

$\triangle PVS \equiv \triangle RTQ$.

(iv) $\therefore$ since

$PR = 20$ cm

and $PV = RT$ — ✓
congruent triangles.

$\therefore TV = 20 - 16$ ✓

$= 4$ cm.

(c) $PS = PQ = SR = 1$ unit. ✓

$RT = \sqrt{1^2 + 1^2}$

$RT = \sqrt{2}$ ✓

$\therefore QT = SR$ — opposite sides of a parallelogram are equal.

$\therefore P = 4 + \sqrt{2}$ units ✓

Question 6:

a) $3x - 2y + 3 = 0$

x-axis, let $y = 0$

$3x = -3$

$x = -1$

$\therefore R(-1, 0)$ ✓

i) Rearranging.

$2y = 3x + 3$

$y = \frac{3}{2}x + \frac{3}{2}$

$m = \frac{3}{2}$ ✓

iii) $L_2{}_m = -\frac{8}{12} = -\frac{2}{3}$

$\therefore -\frac{2}{3} \times \frac{3}{2} = -1$ ✓

$\therefore L_2 \perp L_1$

i) $y - 0 = -\frac{2}{3}(x - 12)$

$y = -\frac{2}{3}x + \frac{24}{3}$

$y = -\frac{2}{3}x + 8$ ✓

$3y = -2x + 24$

$2x + 3y - 24 = 0$ ✓

(v) $3x - 2y = -3$ — (1)

$2x + 3y = 24$ — (2)

$6x - 4y = -6$ — (1)

$6x + 9y = 72$ — (2)

$-13y = -78$

$\boxed{y = 6}$

$3x - 12 = -3$

$3x = 9$

$\boxed{x = 3}$

$S(3, 6)$ ✓

(vi) $A = \frac{1}{2}bh$ ✓

$A = \frac{1}{2} \times 13 \times 6$

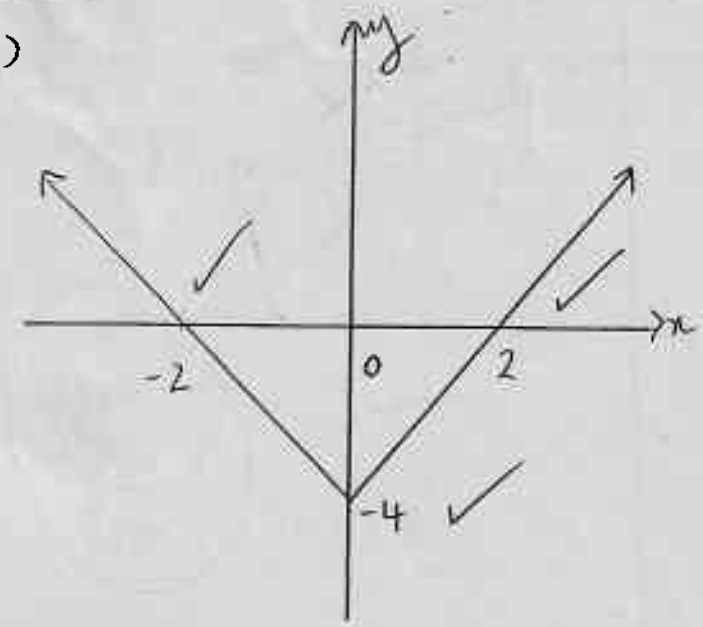
$\boxed{A = 39 \text{ units}^2}$ ✓

b) Domain: $-3 \leq x \leq 3$ ✓

Range: $-3 \leq y \leq 0$ ✓

Question 7:

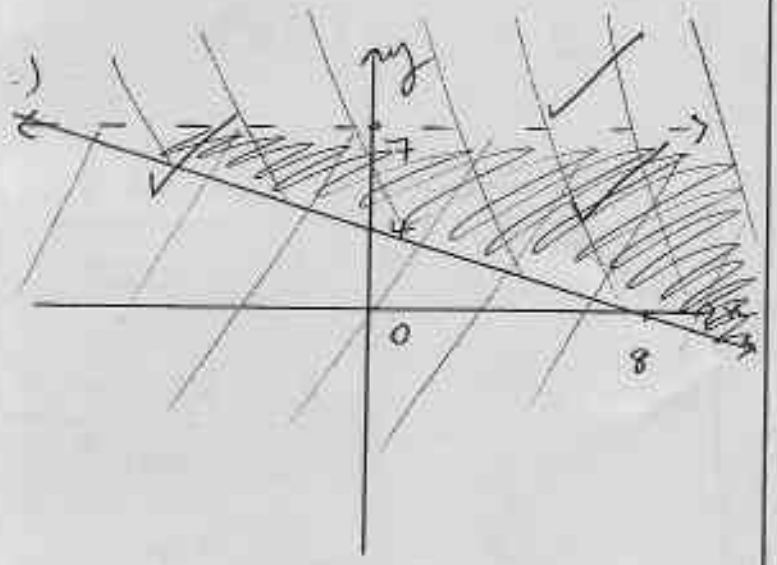
(a)



) (i) $f(1) = 2$ ✓

(ii) $f\left(\frac{1}{2}\right) = -1$ ✓

)



d)

$\sin 60° = \frac{\sqrt{3}}{2}$ ✓

$\therefore A = 120°$ ✓

Question 8:

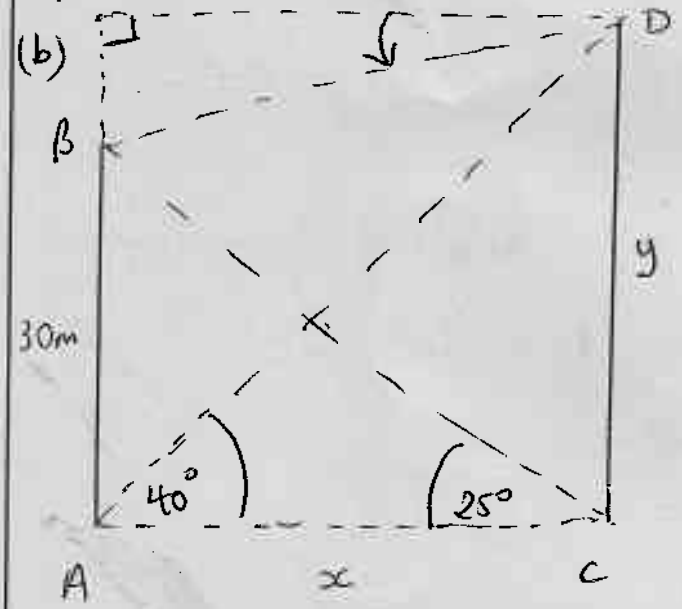
(a) $\frac{1}{\tan(-60°)} \times \frac{1}{\sin(240°)}$

$= \frac{1}{-\sqrt{3}} \times \frac{1}{-\sqrt{3}/2}$ ✓

$= -\frac{1}{\sqrt{3}} \times -\frac{2}{\sqrt{3}}$

$= \frac{2}{3}$ ✓

(b)



(i) $\frac{30}{x} = \tan 25°$

$x = \frac{30}{\tan 25°}$

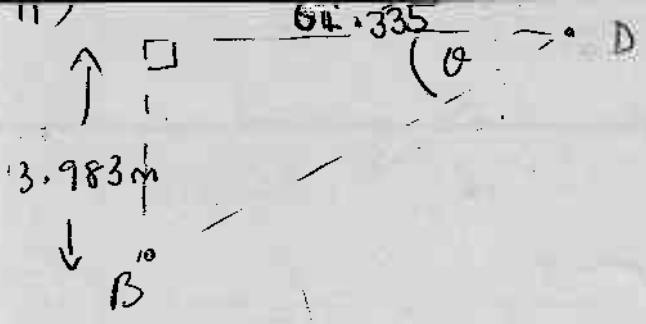
$x = 64.335\ m$

(ii) $\frac{y}{64.335} = \tan 40°$

$y = 64.335 \times \tan 40°$

$y = 53.983\ m$

ii)



$$\tan\theta = \frac{23.983}{64.335}$$

$$\theta = 20^\circ 27'$$

$$\theta = 20^\circ$$

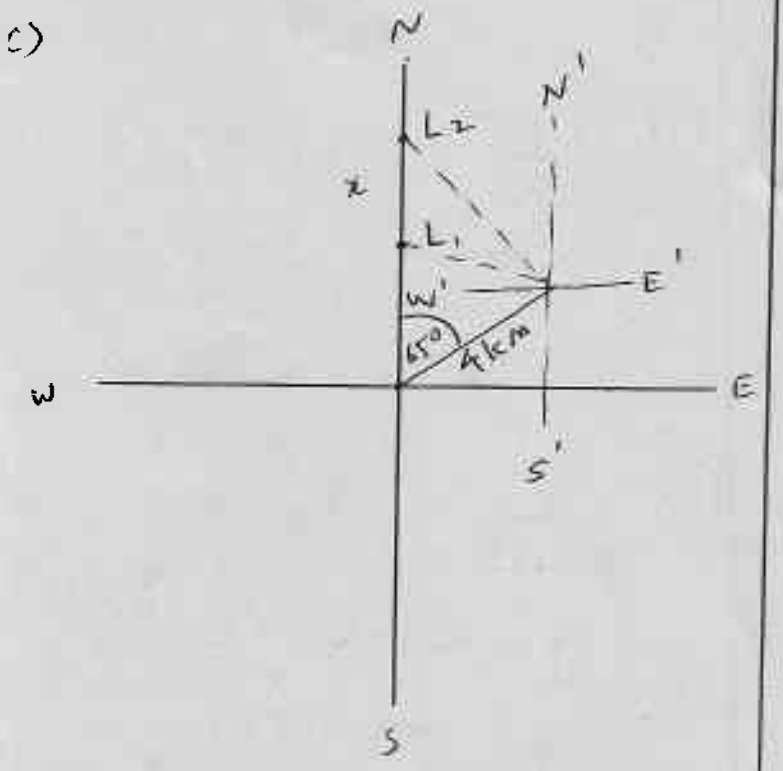
$\angle ADT = 40^\circ$ — alt $\angle$'s in ll lines

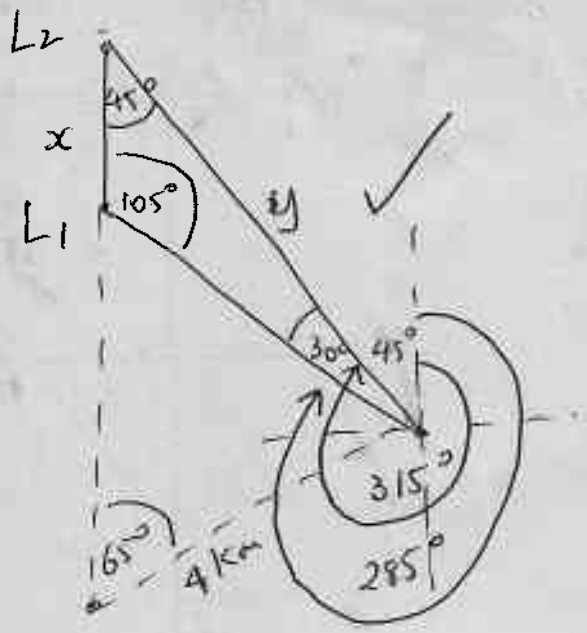
Angle of depression

$= 40^\circ - 20^\circ$

$\theta = 20^\circ$. ✓

c)





Part I : $\frac{y}{\sin 65^\circ} = \frac{4}{\sin 45^\circ}$

$$y = \frac{4 \sin 65^\circ}{\sin 45^\circ}$$ ✓

$$y = 5.1269.$$

$$\frac{x}{\sin 30^\circ} = \frac{5.1269}{\sin 105^\circ}$$

$$x = \frac{5.1269 \times \sin 30^\circ}{\sin 105^\circ}$$

$$x = 2.654$$ ✓

Question 9:

9) (i) $f(x+h) = x^2 + 2xh + h^2 - 2$

$f(x) = x^2 - 2$

$$m = \frac{f(x+h) - f(x)}{h}$$

$$m = \frac{(x^2 + 2xh + h^2 - 2) - (x^2 - 2)}{h}$$ ✓

$$m = \frac{2xh + h^2}{h}$$

$m = 2x + h$

$\therefore m = 2x + h$

$\lim h \to 0$

$m = 2x$ ✓

ii) $\therefore$ at $x = -2$

$m = -4$ ✓

b) (i) $\frac{dy}{dx} = 6x^2 + 3$ ✓

(ii) $f(x) = 2x^2 - \frac{1}{x^2} + \frac{1}{x^3}$ ✓

$f'(x) = 4x + 2x^{-3} - 3x^{-4}$

$f'(x) = 4x + \frac{2}{x^3} - \frac{3}{x^4}$ ✓

(iii) $\frac{d}{dx} = (x-3) \times \frac{1}{2}(x-3)^{-1/2}$ ✓

$+ \sqrt{x-3} \times 1$

$\frac{dy}{dx} = \frac{x-3}{2\sqrt{x-3}} + \sqrt{x-3}$ ✓

(iv) $\frac{dy}{dx} = \frac{(2x-3) \times 2 - (2x+7)2}{(2x-3)^2}$ ✓

$\frac{dy}{dx} = \frac{4x - 6 - 4x - 14}{(2x-3)^2}$

$\frac{dy}{dx} = \frac{-20}{(2x-3)^2}$ ✓

Question 10:

(a) $\frac{dy}{dx} = 4x - 5$

$3 = 4x - 5$

$8 = 4x$

$\boxed{2 = x}$ ✓ at $(2, -1)$

$m = 3$

tangent:

$y + 1 = 2(x - 2)$

$\boxed{y = 2x - 5}$ ✓

Normal:

$y + 1 = -\frac{1}{3}(x - 2)$

$y = -\frac{1}{3}x - \frac{1}{3}$ ✓

b) $\alpha - \beta = 4$

$\boxed{\alpha = \beta + 4}$

$\alpha + \beta = -\frac{p}{3}$

$\beta + 4 + \beta = -\frac{p}{3}$

$2\beta + 4 = -\frac{p}{3}$

$2\beta = -\frac{p}{3} - 4$ ✓

$\boxed{\beta = -\frac{p}{6} - 2}$

$\alpha\beta = -4$

$\beta(\beta + 4) = -4$

$\beta^2 + 4\beta + 4 = 0$

$(\beta + 2)(\beta + 2) = 0$

$\beta = -2$ ✓

$\therefore$

$-2 = -\frac{p}{6} - 2$

$0 = -p$ ✓

$0 = p$

(c)

$x^2 + 2x - 2 = ax(x+1) + bx^2 + c(x+1)$

RHS $ax^2 + ax + bx^2 + cx + c$

$(a+b)x^2 + (a+c)x + c$ ✓

$\therefore$ $a + b = 1$

$a + c = 2$

$\boxed{c = -2}$

$\therefore$ $\boxed{a = 4}$ ✓

$\boxed{b = -3}$

$\therefore$

$4x(x+1) - 3x^2 - 2(x+1)$ ✓

(d) (i) $(x-2) = \pm 4a(y-1)$ ✓

(ii) $(x+2)^2 = \pm 2(y-y_1)$ ✓