TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 11 YEARLY EXAMINATION

MATHEMATICS (2 UNIT)

Reading Time: 5 minutes
Length: two and half hours

Weighting: 40% towards final result

OUTCOMES Referred to: **P1, P2, P3, P4, P5, P6, P7, P8**

Instructions:

1. Attempt **ALL** questions
2. Show all necessary working
3. Each question is of equal value. Mark values are shown beside each part
4. **Begin** each question on a new sheet of paper. Ensure the *question number* is written at the beginning and your *name* and your *teacher's initials* **(PR/LP/AY/BL/ DK/MOD/SW)** is written on the top of each sheet of paper
5. If you use more than one sheet of paper for a question, make sure you staple them together ensuring you have 10 bundles to hand in at the end
6. If you don't attempt a question, write your name on a piece of paper and the question number and **hand it in**
7. Board approved calculators are allowed and will be reset prior to entering the examination hall. ***Students are reminded to reset their calculator back to Degrees Mode.***

Question 1 *(10 marks)*

(a) Evaluate $\dfrac{4.3\times10^{8}}{5.6\times10^{12}}$ giving your answer in scientific notation correct to 2 significant figures

(1 mark)

(b) Simplify: i. $\dfrac{3m}{3m+6}$ (1 mark)

ii. $\dfrac{y+7}{4}-\dfrac{y+3}{8}$ (2 marks)

(c) Simplify: $\sqrt{125}-5\sqrt{2}+2\sqrt{5}$ (2 marks)

(d) Solve: i. $2-3d=-7$ (1 mark)

ii. $|x-3|=5$ (2 marks)

(e) Expand and simplify:

$(3a-2b)^2$ (1 mark)

Question 2 *(10 marks)*

(a) Factorise completely:

i. $2a^2 - 6a$ (1 mark)

ii. $x^2 + 3x - 54$ (1 mark)

iii. $a^3 - a^2 + a - 1$ (2 marks)

iv. $8x^3 - 27$ (2 marks)

(b) Solve simultaneously:

$$x - y = -2$$
$$2x + 3y = 6$$ (2 marks)

(c) Solve: $x^2 - 4x > 0$ (2 marks)

Question 7 *(10 marks)*

(a) Given $\sin\theta = \frac{5}{8}$ and $\cos\theta < 0$ find the exact value of $\cos\theta$ and $\tan\theta$

(2 marks)

(b) Write down the exact value of:

i. $\cos 330^\circ$ (1 mark)

ii. $\sec 210^\circ$ (1 mark)

(c) Simplify:

i. $\cot\theta \sin\theta$ (1 mark)

ii. $\frac{\cos\theta}{\cos ec\theta - 1} + \frac{\cos\theta}{\cos ec\theta + 1}$ (3 marks)

iii. $\frac{\sin\theta}{\cos\theta \tan\theta}$ (2 marks)

Question 8 *(10 marks)*

(a) If $f(x)=\begin{cases}-1 \text{ for } -3\le x<0\\ +1 \text{ for } x=0\\ 0 \text{ for } x>0\end{cases}$

Sketch this function (1 mark)

(b) For the following graphs:

i. Sketch on separate diagrams showing their main features

ii. State the domain and range

α $y=|x-1|$ (3 marks)

β $x^2+y^2=9$ (2 marks)

χ $y=2^x$ (3 marks)

(c) State the largest possible domain for the function $f(x)=\sqrt{4-x^2}$ (1 mark)

Question 9 *(10 marks)*

(a) Simplify: $\dfrac{3^n \times 9^{n+1}}{27^n}$ (2 marks)

(b) Given that $5x^2 - 2x + 3 \equiv A(x-1) + B(x-1) + C$ for all values of x, find the value of A, B and C

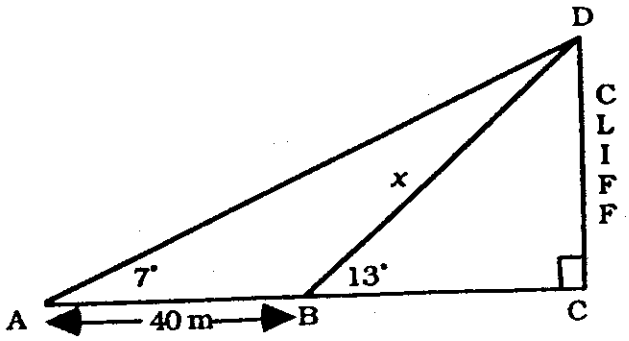
(2 marks)

(c) Find the centre and radius of the circle with equation $x^2 - 2x + y^2 + 6y - 6 = 0$ (4 marks)

(d) Simplify: $\dfrac{1+\dfrac{1}{\sqrt{2}}}{1-\dfrac{1}{\sqrt{2}}}$ (2 marks)

Question 10 *(10 marks)*

(a) The angle of elevation from B to the top of a cliff is 13^0. Katie walks 40 metres away from B and notes that the angle of elevation is now 7^0.



i. Find the value of x to 2 decimal places (2 marks)

ii. Find the height of the cliff to the nearest metre (2 marks)

(b) Simplify: $5-3\cos^2\theta-3\sin^2\theta$ (2 marks)

(c) If α and β are the roots of the equation $2x^2+6x+3=0$, find the value of:

i. $\alpha+\beta$ (1 mark)

ii. $\alpha\beta$ (1 mark)

iii. $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ (2 marks)

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Yr 11 Yearly Solutions 2007

1. a) $7.6785... \times 10^{-5} = 7.7 \times 10^{-5}$ (1)

b) i. $\frac{3m}{3(m+2)} = \frac{m}{m+2}$ (1)

ii. $\frac{y+7}{4} - \frac{y+3}{8} = \frac{2y+14-y-3}{8}$ (1)

$= \frac{y+11}{8}$ (1)

c) $\sqrt{125} - 5\sqrt{2} + 2\sqrt{5} = 5\sqrt{5} - 5\sqrt{2} + 2\sqrt{5}$ (1) for $5\sqrt{5}$

$= 7\sqrt{5} - 5\sqrt{2}$ (1)

d) i. $2 - 3d = -7$

$-3d = -9$

$d = 3$ (1)

ii. $|x-3| = 5$

$x - 3 = 5$ $\quad$ $-x + 3 = 5$

$x = 8$ (1) $\quad$ $-x = 2$

$x = -2$ (1)

e) $(3a - 2b)^2 = 9a^2 - 12ab + 4b^2$ (1)

2. a) i. $2a^2 - 6a = 2a(a-3)$ (1)

ii. $x^2 + 3x - 54 = (x+9)(x-6)$ (1)

iii. $a^3 - a^2 + a - 1 = a^2(a-1) + (a-1)$ (1)

$= (a-1)(a^2+1)$ (1)

iv. $8x^3 - 27 = (2x-3)(4x^2 + 6x + 9)$

b) $x - y = -2 \quad \ldots 1$

$2x + 3y = 6 \quad \ldots 2$

$x = y - 2$

subs $x = y - 2$ into $\ldots 2$

$2(y-2) + 3y = 6$

$2y - 4 + 3y = 6$

$5y = 10$

$y = 2$

1 for method

1 for answer

$x = 2 - 2$ $\therefore (0, 2)$

$= 0$

c) $x^2 - 4x > 0$

$x(x-4) > 0$ 1 for factorising

$x < 0, \quad x > 4$ 1 for both answers

3. a) $\angle YRV = \angle XTV = 90°$ (b is rectangle)
$\angle RYV = \angle YVT$ ($RY \parallel VT$, alternate $\angle$s)
$\angle VXT = \angle YVR$ ($VR \parallel TS$, alternate $\angle$s)

$\therefore \triangle RYV \parallel\parallel \triangle TVX$ (equiangular)

(1) for 'equiangular' reason

(1) for each reasoning

4 total

b) $f(x) = x^2 - 5x + 6$ (1) for correct substitution

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 5(x+h) + 6 - (x^2 - 5x + 6)}{h}$$

1 mark

$$= \frac{x^2 + 2xh + h^2 - 5x - 5h + 6 - x^2 + 5x - 6}{h}$$

$$= \frac{2xh + h^2 - 5h}{h}$$

$$= 2x + h - 5$$ (1) for answer

c) $2x^2 + (1-k)x + k - 3 = 0$

1 mark for $\alpha, 2\alpha$
1 mark for sum/product
1 mark for answer

roots are α, 2α

$\alpha + 2\alpha = \frac{-b}{a}$

$= \frac{-(1-k)}{2}$

$3\alpha = \frac{-1+k}{2}$

$\alpha = \frac{-1+k}{6}$

$\alpha \times 2\alpha = \frac{c}{a}$

$2\alpha^2 = \frac{k-3}{2}$

$2\left(\frac{k-1}{6}\right)^2 = \frac{k-3}{2}$

$\frac{k^2 - 2k + 1}{36} = \frac{k-3}{4}$

$4k^2 - 8k + 4 = 36k - 108$

$4k^2 - 44k + 112 = 0$

$k^2 - 11k + 28 = 0$

$(k-7)(k-4) = 0$

$\therefore k = 4, 7$

4. a) i. $y = 4x^3 - x^2 + 8$

$y' = 12x^2 - 2x$ (1)

ii. $y = (x^2 - 1)^{\frac{1}{2}}$

$y' = \frac{1}{2}(x^2-1)^{-\frac{1}{2}} \times 2x$ (1)

$= \frac{x}{\sqrt{x^2-1}}$ (1)

iii. $y = \frac{x^2 + x}{x}$

$= x + 1$ (1)

$y' = 1$ (1)

iv. $y = \frac{x+5}{x+2}$

$u = x+5 \qquad v = x+2$

$u' = 1 \qquad v' = 1$

$y' = \frac{vu' - uv'}{v^2}$

$= \frac{x+2 - (x+5)}{(x+2)^2}$

$= \frac{x+2-x-5}{(x+2)^2}$

$= \frac{-3}{(x+2)^2}$

1 for method
1 for answer

v. $y = 2x^4(x+1)^3$

$u = 2x^4 \qquad v = (x+1)^3$

$u' = 8x^3 \qquad v' = 3(x+1)^2$

$y' = 8x^3(x+1)^3 + 2x^4 \times 3(x+1)^2$

$= 2x^3(x+1)^2[4(x+1) + 3x]$

$= 2x^3(x+1)^2(7x+4)$

1 method
1 subs
1 simplified answer

5. a) $\frac{a^3 b^{-12}}{a^3 b^{-2}} = a^5 b^{-10}$

$= \frac{a^5}{b^{10}}$

} 1 for either answer

b) $x = 4.23434...$

$10x = 42.3434...$

$100x = 423.434...$

$1000x = 4234.34...$

1 method

$990x = 4192$

$x = \frac{4192}{990}$

$= 4\frac{116}{495}$

1 answer

c) $y = 3x^2 + 2$ when $x = 1$, $y = 5$

$y' = 6x$

$S'(1) = 6$

1 for y value
1 for m
1 for eqn

$y - y_1 = m(x - x_1)$

$y - 5 = 6(x - 1)$

$y - 5 = 6x - 6$

$y = 6x - 1$

d) a. i. B, C

ii. Between B and C

B. $0 < \text{grad} < 7$

2 if full range
1 if only integers given

6. a)

b) $M = \left(\frac{-1+1}{2}, \frac{0+6}{2}\right)$ 1 for substitution, 1 for answer

$= (0, 3)$

c) $m_{AM} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0-3}{-1-0}$

$= 3$

$y - 3 = 3(x - 0)$

$y = 3x + 3$

d) $3x + y - 9 = 0$

$y = 0$

$3x = 9$

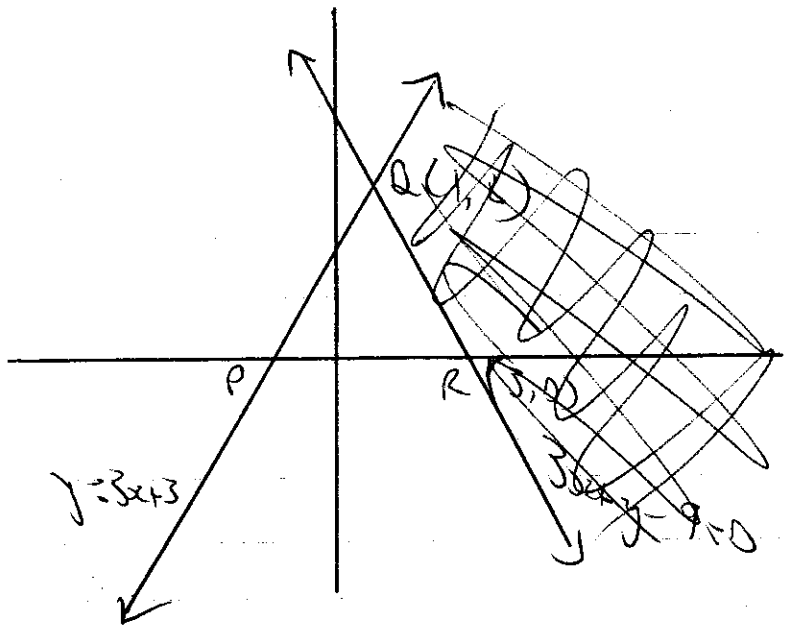
$x = 3$ $\therefore (3, 0)$

e) $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ 1 for substitution

$= \frac{|3x-1 + 1x0 - 9|}{\sqrt{3^2 + 1^2}}$

$= \frac{|-12|}{\sqrt{10}}$

$= \frac{12}{\sqrt{10}}$



$y \leq 3x+3$

$3x+y-9 \geq 0$

1 for shading
1 for testing

Test (0,0)
$0 \leq 3$ ✓
$-9 \geq 0$ ×

Test (4,0)
$0 \leq 15$ ✓
$3(4)-9 \geq 0$
$3 \geq 0$ ✓

7. a) 

$\cos\theta = -\frac{\sqrt{39}}{8}$ 1 for sign

$\tan\theta = -\frac{5}{\sqrt{39}}$ 1 for values

b) i. $\cos 330 = \cos 30$

$= \frac{\sqrt{3}}{2}$ (1)

ii. $\sec 210 = \frac{1}{\cos 210}$

$= \frac{1}{-\cos 30}$

$= -\frac{2}{\sqrt{3}}$ (1)

c) i. $\cot\theta \sin\theta = \frac{\cos\theta}{\sin\theta} \times \sin\theta$

$= \cos\theta$ (1)

ii. $\frac{\cos\theta}{\cos\sec\theta - 1} + \frac{\cos\theta}{\cos\sec\theta + 1} = \frac{\cos\theta(\cos\sec\theta + 1) + \cos\theta(\cos\sec\theta - 1)}{(\cos\sec^2\theta - 1)}$

(1) for fraction

$= \frac{\cot\theta + \cos\theta + \cot\theta - \cos\theta}{\cos\sec^2\theta - 1}$

$= \frac{2\cot\theta}{\cos\sec^2\theta - 1}$ (1)

$= \frac{2\cot\theta}{\cot^2\theta}$

$= \frac{2}{\cot\theta}$

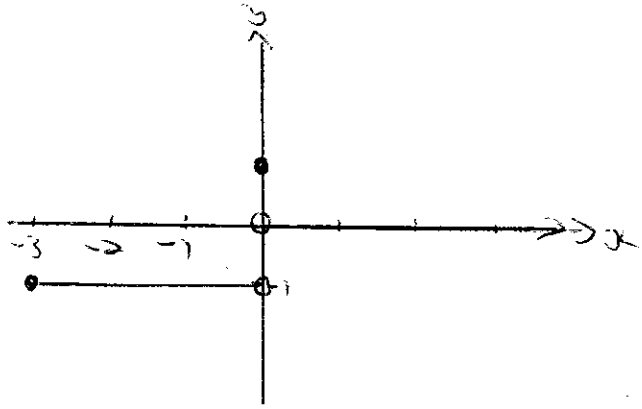
$= 2\tan\theta$ or $\frac{2}{\cot\theta}$ (1)

iii. $\frac{\sin\theta}{\cos\theta\tan\theta} = \frac{\sin\theta}{\cos\theta \times \frac{\sin\theta}{\cos\theta}}$ (1)

$= \frac{\sin\theta}{\sin\theta}$

$= 1$ (1)

8. a)



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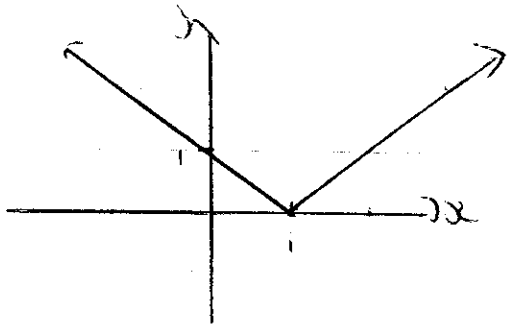
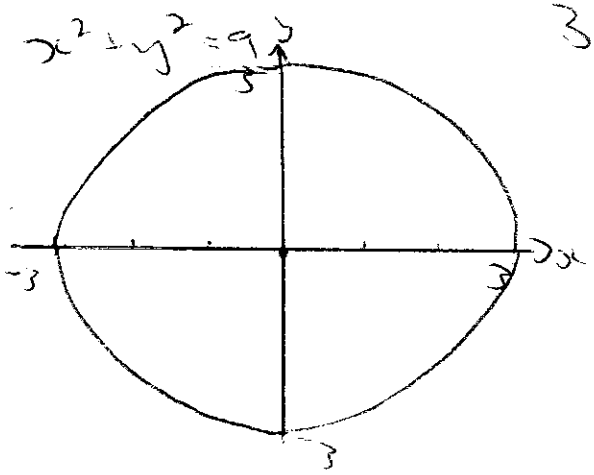
b) α. $y = |x-1|$



D: all real values of x
R: $y \geq 0$

1 mark for graph
1 each for D & R
3 total

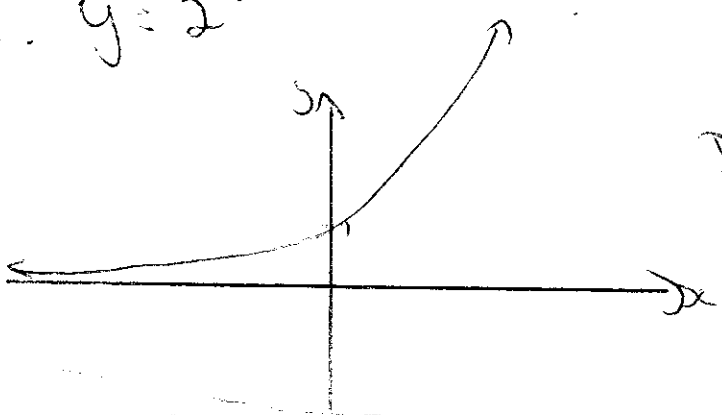
β. $x^2 + y^2 = 9$



$D: -3 \leq x \leq 3$
$R: -3 \leq y \leq 3$

1 mark for diag
1 mark for D & R
2 total

χ. $y = 2^x$



D: all real values of x
$R: y > 0$

1 mark for graph
1 mark each for D & R
3 total

c) $D: -2 < x < 2$ (1)

9. a) $\frac{3^n \times 9^{n+1}}{27^n} = \frac{3^n \times (3^2)^{n+1}}{3^{3n}}$ — 1 for method

$= \frac{3^n \times 3^{2n+2}}{3^{3n}}$

$= \frac{3^{3n+2}}{3^{3n}}$

$= 3^2$

$= 9$ } 1 mark

b) $5x^2 - 2x + 3 \equiv A(x-1)^2 + B(x-1) + C$

RHS: $A(x^2 - 2x + 1) + Bx - B + C$

$= Ax^2 - 2Ax + A + Bx - B + C$

$= Ax^2 + x(B - 2A) + A - B + C$

2 total

$\therefore A = 5$

$B - 2A = -2$

$A - B + C = 3$

$B - 2A = -2$ $\quad$ $A - B + C = 3$

$B - 2(5) = -2$ $\quad$ $5 - 8 + C = 3$

$B = 8$ $\quad$ $C = 6$

c) $x^2 - 2x + y^2 + 6y - 6 = 0$

$x^2 - 2x + 1 + y^2 + 6y + 9 = 6 + 1 + 9$ (1)

$(x-1)^2 + (y+3)^2 = 16$ (1)

centre: $(1, -3)$ (1)

radius: 4 (1)

4 total

d) $\dfrac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}} = 1+\frac{1}{\sqrt{2}} \div \left(1-\frac{1}{\sqrt{2}}\right)$

$= \dfrac{\sqrt{2}+1}{\sqrt{2}} \div \left(\dfrac{\sqrt{2}-1}{\sqrt{2}}\right)$ } (1)

$= \dfrac{\sqrt{2}+1}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}-1}$ 2 total

$= \dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ (1)

10. a) i. $\angle DBA = 167°$

$\therefore \angle BDA = 6°$

$$\frac{40}{\sin 6} = \frac{x}{\sin 7}$$ ① for substitution

$x \sin 6 = 40 \sin 7$

$x = \frac{40 \sin 7}{\sin 6}$

$= 46.63585\ldots$ — rounding error

$= 46.64\ m$ ① for answer

ii. $\sin 13 = \frac{diff}{46.64}$ ①

$diff = 10.49\ldots$

$= 10\ m$ ①

b) $5 - 3\cos^2\theta - 3\sin^2\theta$

$= 5 - 3\cos^2\theta - 3(1 - \cos^2\theta)$ ① for trig identity

$= 5 - 3\cos^2\theta - 3 + 3\cos^2\theta$

$= 2$ ①

c) $2x^2 + 6x + 3 = 0$

i. $\alpha + \beta = \frac{-b}{a}$

$= \frac{-6}{2}$

$= -3$ ①

ii. $\alpha\beta = \frac{c}{a}$

$= \frac{3}{2}$ ①

iii. $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$ ①

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{(-3)^2 - 2\left(\frac{3}{2}\right)}{\frac{3}{2}} = 4$ ①