TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 11 2008 ASSESSMENT TASK 4

MATHEMATICS
YEARLY EXAMINATION

Tuesday, 26th August, 2008
8:30 am to 11:00 am

Time Allowed – *Two and a half hours plus 5 minutes reading*

WEIGHTING 40% towards final assessment

Outcomes referred to: P1, P2, P3, P4, P5, P6, P7.

INSTRUCTIONS:

1. Attempt ALL questions.
2. Show all necessary working in every question.
3. Begin each question on a separate piece of paper.
4. There are 10 questions worth 12 marks each. Total marks 120.
5. Marks for individual part questions are shown beside each part.
6. Non-programmable silent Board of Studies approved calculators are permitted.
7. Write your name, class and your teacher's name on the front of each answer page.

Question 1 (12 marks)

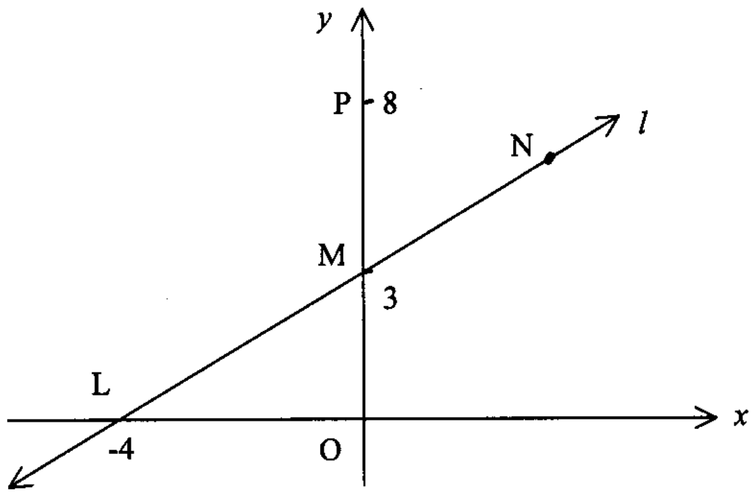
Start each new question on a new page. Write your name and your teacher's name at the top of the page

		Marks
a)	Find the value of $13^{-1.2}$ correct to two significant figures.	1
b)	Factorise $2x^2+3x-2$.	2
c)	Rationalise the denominator of $\dfrac{4}{\sqrt{5}+2}$.	2
d)	Write 0.000 581 in scientific notation.	1
e)	Simplify $3\sqrt{32}$.	2
f)	The price of a car increased by 4%. If the car now costs \$15 080, how much did it cost previously?	2
g)	If $A=P(1+\frac{r}{100})^n$, find P when $A=352$, $r=12$ and $n=5$.	2

Question 2 (12 marks)

Start each new question on a new page. Write your name and your teacher's name at the top of the page.

Marks



The line l cuts the x axis at L(-4,0) and y axis at M(0,3) as shown. N is a point on the line l, and P is the point (0,8).

Copy the diagram into your Writing Booklet.

a) Find the equation of the line l. **2**

b) Show that the point (16,15) lies on the line l. **1**

c) By considering the lengths of ML and MP, show that ΔLMP is isosceles. **2**

d) Calculate the gradient of the line PL. **1**

e) M is the midpoint of the interval LN. Find the coordinates of the point N. **2**

f) Show that $\angle NPL$ is a right angle. **2**

g) Find the equation of the circle that passes through the points N, P, and L. **2**

Question 3 (12 marks)

Start each new question on a new page. Write your name and your teacher's name at the top of the page.

	Marks
a) Differentiate $y = 5x^2 - 3x$ from first principles.	3
b) Differentiate:	
(i) $7x^6 + \dfrac{2}{x^3}$	2
(ii) $6x^2\sqrt{x}$	2
(iii) $\dfrac{(2x-9)^3}{5x+1}$	2
c) Find the equation of the tangent to the curve $y = 2x^3 - 7$ at the point where $x = 1$.	3

Question 4 (12 marks)

Start each new question on a new page. Write your name and your teacher's name at the top of the page.

Marks

a) Simplify

(i) $6a - 3b - a + 7b$ **1**

(ii) $(-4t^2)^3$ **1**

b) Factorise fully:

(i) $x^2 - 2x - 24$ **2**

(ii) $5y - 15 + xy - 3x$ **1**

c) Expand and simplify: $(4x-3)^2$ **2**

d) Simplify the following: $\frac{x-1}{2} - \frac{x+4}{3}$ **2**

e) Simplify: $\frac{5m+10}{m^2 - m - 2} \div \frac{m^2 - 4}{3m+3}$ **3**

Question 5 (12 marks)

Start each new question on a new page. Write your name and your teacher's name at the top of the page.

Marks

a) Solve the following:

(i) $\dfrac{x+3}{7}+\dfrac{x-1}{3}=4$ **2**

(iii) $2y^2+y-3=0$ **2**

b) Solve the following and graph your solution on a number line. **2**

$$|3x+5|>22$$

c) Solve the following giving your answer correct to 2 decimal places. **2**

$$x^2 + 2x - 7 = 0$$

d) Solve the following pairs of simultaneous equations. **2**

(i)
$$\begin{aligned} 5x - y &= 19 \\ 2x + 5y &= -14 \end{aligned}$$

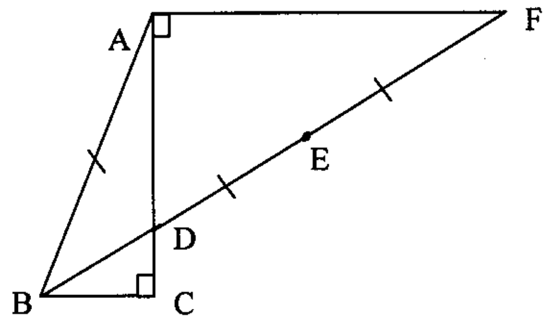
(ii) **2**
$$\begin{aligned} x - y - 7 &= 0 \\ xy &= -12 \end{aligned}$$

Question 6 (12 marks)

Start each new question on a new page. Write your name and your teacher's name at the top of the page.

Marks

a)



In the diagram, $AC \perp BC$, $AC \perp AF$ and $AB = DE = EF$.

Copy or trace the diagram on to your answer page.

(i) Show that $\angle DBC = \angle DFA$. **2**

(ii) On your diagram, mark the point G on the line AF such that $EG \parallel AC$. Show that $\Delta AGE \equiv \Delta FGE$. **2**

(iii) Prove that $\angle ABD = 2\angle DBC$. **2**

b) A stick 1 metre long placed vertically on the ground casts a shadow 60 cm long, and at the same time a building casts a shadow 3 m long. What is the height of the building? **2**

c) The height of a cone is twice its radius. Find the volume of the cone in terms of r. **2**

d) To cover a rectangular bench top, 400 tiles, each 10 cm by 5 cm are used. If square tiles of length 4 cm are used, how many would be needed? **2**

Question 7 (12 marks)

Start each new question on a new page. Write your name and your teacher's name at the top of the page.

	Marks
a) If $f(x) = x^3 - 6x^2 + 5$ find $f(-2)$.	1
b) Given $f(x) = x^2 - 4$, find values of x when $f(x) = 32$.	1
c) Show whether $f(x) = 4x - x^3$ is an odd or even function.	2
d) Sketch the region which satisfies the following conditions, $x \geq -3$, $y < 2x + 1$ and $x^2 + y^2 \geq 9$, showing x and y intercepts and asymptotes.	3
e) State the domain and range of the curves below:	
(i) $y = \dfrac{1}{x+3}$	1
(ii) $y = \sqrt{4 - x^2}$	1
f) Sketch the curve $y = x^2 - 3x - 4$ showing the axis of symmetry, the x and y intercepts and the vertex.	3

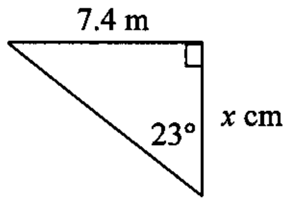
Question 8 (12 marks)

Start each new question on a new page. Write your name and your teacher's name at the to

Marks

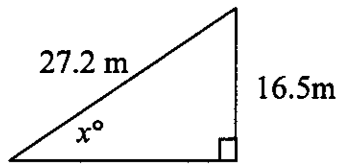
a) Find the value of x in the following diagrams, correct to the nearest whole number.

i)



FIGURES NOT TO SCALE

ii)



4

b) (i) Find θ to the nearest minute if $\sin\theta = 0.82$. 1

(ii) If $\cos\theta = \frac{2}{7}$, find the exact ratio of $\tan\theta$. 2

c) The angle of depression from the top of an 18 m high building to a person on the ground below is 47° 28'. How far out from the building is the person, to 1 decimal place. 2

d) A plane flies south from Sydney for 590 km and turns and flies west for 360 km.

(i) Draw a diagram for this information. 1

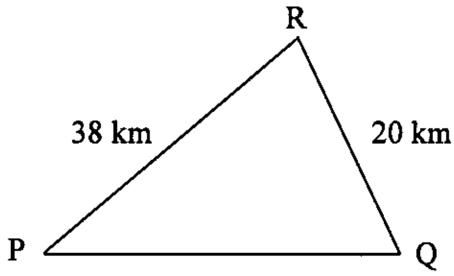
(iii) What is its bearing from Sydney to the nearest degree? 2

Question 9 (12 marks)

Start each new question on a new page. Write your name and your teacher's name at the top.

Marks

a) In the diagram, the point Q is due east of P. The point R is 38km from P and 20km from Q. The bearing of R from Q is 325°.

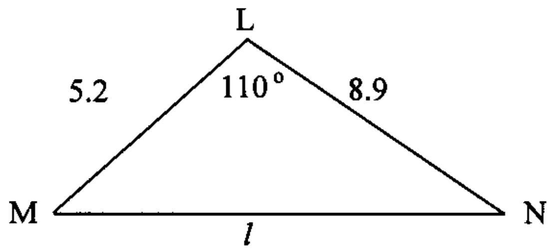


(i) What is the size of ∠PQR? **2**

(ii) What is the bearing of R from P? **1**

b) Find the exact value of $5\sin 30^0 \sec 30^0$ and rationalise the denominator. **2**

c)



In the diagram, LMN is a triangle where LM = 5.2 metres, LN = 8.9 metres and angle MLN = 110°.

(i) Find the value of l correct to 1 decimal place. **2**

(ii) Calculate the area of triangle of triangle LMN. **1**

d) Simplify $\dfrac{\tan\theta\sec\theta}{1+\tan^2\theta}$ **2**

e) Solve the equation $2\sin\theta - 1 = 0$ for $0^0 \le \theta \le 360^0$. **2**

Question 10 (12 marks)

Start each new question on a new page. Write your name and your teacher's name at the top of the page.

Marks

a) If α and β are the roots of the equation $x^2+5x-8=0$, find the value of :

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha-2)(\beta-2)$ **2**

(iv) $\alpha^2+\beta^2$ **2**

b) (i) Find the discriminant of the quadratic equation x^2-2x+3. **1**

(ii) What does your answer in part (i) indicate about the roots of the quadratic x^2-2x+3? **1**

c) Solve $4x-x^2\geq 0$. **2**

d) Find the values of the constants A and B such that

$$20x-17\equiv A(x-4)-B(5x+1)$$ **2**

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

YEAR 11 MATHEMATICS YEARLY SOLUTIONS

Question 1

a) $13^{-1.2} = 0.046$ 1

b) $(2x-1)(x+2)$ 2

c) $\frac{4}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{4\sqrt{5}-8}{5-4} = 4\sqrt{5}-8$ 2

d) 5.81×10^{4} 1

e) $3\sqrt{32} = 3 \times \sqrt{16} \times \sqrt{2}$

$= 3 \times 4 \times \sqrt{2}$

$= \underline{12\sqrt{2}}$ 2

f) $1.04x = 15080$

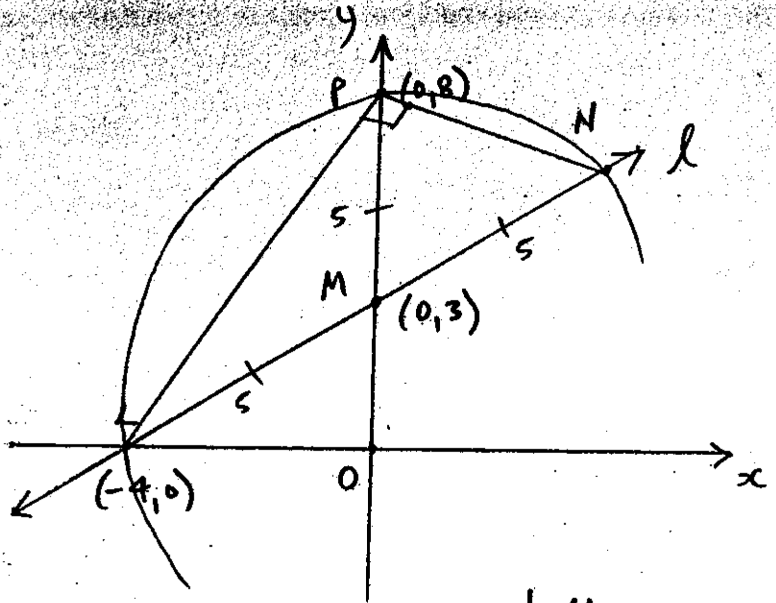
$x = 15080 \div 1.04$

$= \underline{\$14500}$ 2

g) $352 = P(1+0.12)^5$

$P = \frac{352}{1.12^5} = 199.73$ (2 dp) 2

Question 2 – Solutions (DK)



(a) $m_l = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\overset{x_1\ y_1}{(-4,0)}\ \overset{x_2\ y_2}{(0,3)}$ $\quad = \dfrac{3-0}{0-(-4)}$

(Alternative: two point formula) $\quad = \dfrac{3}{4}$. ①

y-intercept: $b = 3$

$\therefore$ equation: $y = \frac{3}{4}x + 3$. ①

(b) Subst. $(16,15)$ into $y = \frac{3}{4}x + 3$.

$RHS = \frac{3}{4}(16) + 3$
$= 12 + 3$
$= 15$
$= LHS$
$= y$.

$\therefore (16,15)$ lies on the line l. ①

(c) $ML = \sqrt{(0-(-4))^2 + (3-0)^2} = \sqrt{16+9} = 5$. ①

$MP = 8 - 3 = 5$

Since $ML = MP = 5$ then ΔLMP is isosceles. ①

(d) $m_{PL} = \dfrac{8-0}{0-(-4)} = \dfrac{8}{4} = 2$. ①

(e) $L\overset{x_1\ y_1}{(-4,0)}\quad N(x_2, y_2)$

$M\left(\dfrac{-4+x_2}{2}, \dfrac{0+y_2}{2}\right) = M(0,3)$

$\therefore \dfrac{-4+x_2}{2} = 0$ and $\dfrac{0+y_2}{2} = 3$

$-4 + x_2 = 0 \qquad y_2 = 6$

$x_2 = 4$.

$\therefore N(4,6)$ ②

(f) From (d), $m_{PL} = 2$. (Alternative: Pythagoras' Thm)

$\overset{x_1\ y_1}{P(0,8)}\ \overset{x_2\ y_2}{N(4,6)}$ $\quad m_{PN} = \dfrac{6-8}{4-0} = \dfrac{-2}{4} = -\dfrac{1}{2}$. ①

Now, $m_{PL} \times m_{PN} = 2 \times -\frac{1}{2}$
$= -1$ ①

$\therefore LP \perp PN$

& $\angle NPL = 90°$.

(g) Centre: $(0,3)$
radius $= ML = MN = MP = 5$ } ①

$\therefore$ equation: $x^2 + (y-3)^2 = 25$ ①

Question 3

) $f(x) = 5x^2 - 3x$

$f(x+h) = 5(x+h)^2 - 3(x+h)$

$= 5(x^2 + 2xh + h^2) - 3x - 3h$

$= 5x^2 + 10xh + 5h^2 - 3x - 3h$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{5x^2 + 10xh + 5h^2 - 3x - 3h - 5x^2 + 3x}{h}$$

$$= \lim_{h \to 0} \frac{10xh + 5h^2 - 3h}{h}$$

$$= \lim_{h \to 0} 10x - 5h - 3$$

3

$$= \underline{10x - 3}$$

2

b) (i) $42x^5 - 6x^{-4} = 42x^5 - \frac{6}{x^4}$

2

(ii) $6x^2 \times x^{\frac{1}{2}} = 6x^{\frac{5}{2}}$ $\quad \frac{d}{dx}(6x^{\frac{5}{2}}) = \underline{\underline{15x^{3/2}}}$

(iii) $$\frac{(5x+1) \times 3 \times 2(2x-9)^2 - (2x-9)^3 \times 5}{(5x+1)^2}$$

$$= \frac{(2x-9)^2 [30x + 6 - 5(2x-9)]}{(5x+1)^2}$$

$$= \frac{(2x-9)^2 (30x + 6 - 10x + 45)}{(5x+1)^2}$$

$$= \frac{(2x-9)^2 (20x + 51)}{(5x+1)^2}$$

2

c) $y = 2x^3 - 7$, $y_1 = 2(1)^3 - 7 = 2 - 7 = -5$

$\frac{dy}{dx} = 6x^2 = 6(1)^2 = 6$

$y - y_1 = m(x - x_1)$

$y - (-5) = 6(x - 1)$

$y + 5 = 6x + 6$

$\underline{y = 6x + 1}$ or $\underline{6x - y + 1 = 0}$ 3

Question 4

a) (i) $5a + 4b$ 1

(ii) $-64t^6$ 1

b) (i) $(x-6)(x+4)$ 2

(ii) $5(y-3) + x(y-3) = (5+x)(y-3)$ 1

c) $(4x-3)^2 = 16x^2 - 24x + 9$ 2

d)
$$\frac{x-1}{2} - \frac{x+4}{3} = \frac{3(x-1)}{6} - \frac{2(x+4)}{6}$$
$$= \frac{3x-3-2x-8}{6}$$
$$= \frac{x-11}{6}$$
2

e)
$$\frac{5m+10}{m^2-m-2} \div \frac{x^2-4}{3m+3} = \frac{5(m+2)}{(m-2)(m+1)} \times \frac{3(m+1)}{(m+2)(m-2)}$$
$$= \frac{5}{(m-2)^2}$$
3

Question 5

a) (i) $\frac{x+3}{7} + \frac{x-1}{3} = 4$

$3x+9 + 7x-7 = 84$ (1)

$10x + 2 = 84$

$10x = 82$

$x = 8\frac{1}{5}$ (1)

(ii) $2y^2 - 2y + 3y - 3 = 0$

$2y(y-1) + 3(y-1) = 0$ (1)

$(2y+3)(y-1) = 0$

$2y+3=0,\ y-2=0$

$2y = -3$

$y = -\frac{3}{2}$ or $y=1$ (1)

b) $|3x+5| > 22$

$3x+5 > 22$

$3x > 17$

$x > \frac{17}{3}$

$x > 5\frac{2}{3}$ (1)

$-(3x+5) > 22$

$-3x-5 > 22$

$-3x > 27$

$x < -\frac{27}{3}$

$x < -9$ (1)

c) $x = \frac{-2 \pm \sqrt{2^2 - 4\times1\times{-7}}}{2\times1}$

$= \frac{-2 \pm \sqrt{4 + 28}}{2}$

$= \frac{-2 \pm \sqrt{32}}{2}$ (1)

$= 1.83$ or -3.83 (1)

d) (i) $5x - y = 19$ (1)

$2x + 5y = -14$ (2)

(1)×5 $25x - 5y = 95$ (3)

(2)+(3) $27x = 81$ 1

$x = 3$

Sub $x = 3$ into (1)

$5(3) - y = 19$

$15 - y = 19$

$-y = 4$

$y = {}^{-}4$ 1

$\therefore x = 3, y = {}^{-}4$

(ii) $x - y - 7 = 0$ (1)

$xy = -12$ (2)

From (1) $y = x - 7$ (3)

sub $y = x - 7$ into (2)

$x(x-7) = -12$

$x^2 - 7x + 12 = 0$

$(x-4)(x-3) = 0$

$x = 3, 4$

If $x = 3, y = 3 - 7 = -4$

If $x = 4, y = 4 - 7 = -3$

$\therefore x = 3, y = {}^{-}4$ and $x = 4, y = {}^{-}3$

1 1

Question 6

or proof with angle sum of Δ

a) (i) $\angle BCD = \angle FAC = 90^\circ$ (given)

$\therefore AF \parallel BC$ (alternate angles equal only on $\parallel$)

$\therefore \angle DBC = \angle DFA$ (alternate angles in parallel lines)

(ii) $\angle AGE = 90^\circ$ (co-int with $\angle GAD$, $AD \parallel GE$)

$\angle EGF = 90^\circ$ ($\angle$ on straight line)

$\therefore \angle AGE = \angle EGF$ (both 90°)

Since E is midpoint of DF, G must be midpoint of AF (~~equal~~ intercept theorem)

$\therefore AG = GF$

$GE = GE$ (common)

$\therefore \Delta AGE \equiv \Delta FGE$ (SAS)

(1) two reasons

(1) third reason & congruence

(iii) Let $\angle DBC = \alpha$

$\angle AFE = \alpha$ from (i)

$\angle EAF = \angle AFE$ (corresponding angles $\Delta AGE \equiv \Delta FGE$)

$= \alpha$

$\angle AED = 2\alpha$ (ext angle of Δ equals sum of int opp angles)

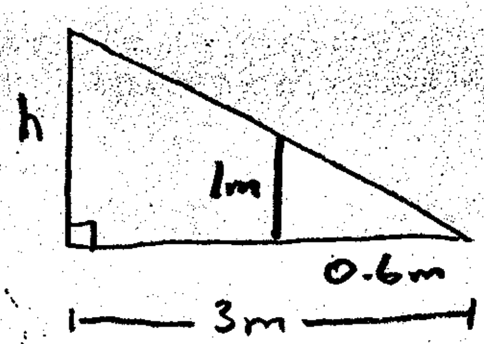
Now, $AE = EF$ (corresponding sides $\Delta AGE \equiv \Delta FGE$)

$\therefore AB = AE$ both equal to EF

$\angle ABD = \angle AED$ (base angles of isosceles)

$= 2\alpha = 2\angle DBC$

b)



$$\frac{h}{3} = \frac{1}{0.6}$$

$$h = \frac{1}{0.6} \times 3$$

$$= \underline{5m}$$

c) $\frac{1}{3}\pi r^2 h.$

$$V = \pi \times r^2 \times 2r \times \frac{1}{3}$$

$$= \underline{2\pi r^3} \times \frac{1}{3} = \frac{2}{3}\pi r^3$$

(cone volume $= \frac{1}{3}\pi r^2 h.$)

d) Total area $= (10 \times 5) \times 400$

$= 20\,000 \text{ cm}^2$

N° of tiles $= 20\,000 \div (4 \times 4)$

$= \underline{1250}$

Question 7

a) $f(-2) = (-2)^3 - 6(-2)^2 + 5 = \underline{-27}$ 1

b) $x^2 - 4 = 32$

$x^2 = 36$

$\underline{x = \pm 6}$ 1

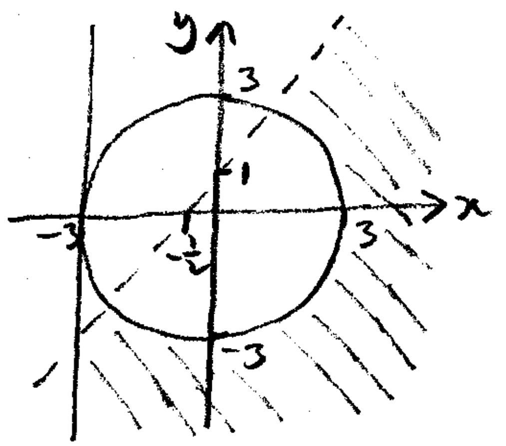
c) $f(-x) = 4(-x) - (-x)^3$

$= -4x + x^3$

$= -f(x)$

$\therefore$ $\underline{f(x) \text{ is odd}}$ 2

d)



3

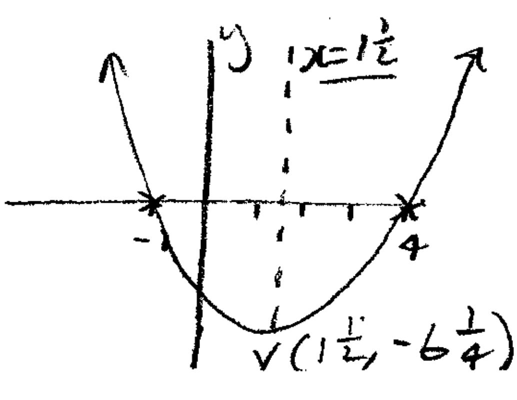
e) (i) Domain : all real x, $x \neq -3$ 1

Range : all real y, $y \neq 0$

(ii) Domain : $-2 \leq x \leq 2$ 1

Range : $0 \leq y \leq 2$

f)



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Question 8

a) (i) $\tan 23 = \dfrac{7.4}{x}$

$x = 7.4 \div \tan 23$

$= \underline{17\text{m}}$ (2)

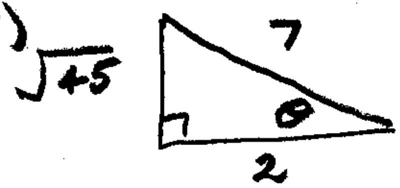
(ii) $\sin x = \dfrac{16.5}{27.2}$

$x = \underline{37^\circ}$ (2)

b) (i) $\sin\theta = 0.82$

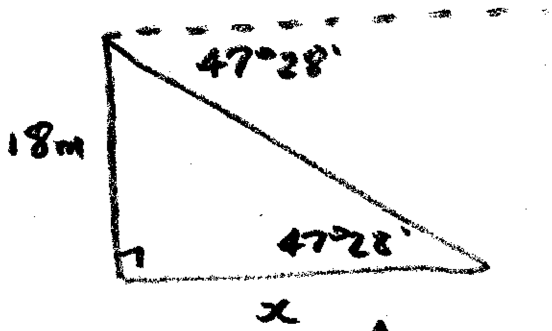
$\angle\theta = \underline{55^\circ 5'}$ (1)

(ii)



$\underline{\tan\theta = \dfrac{\sqrt{45}}{2}}$ (2)

c)

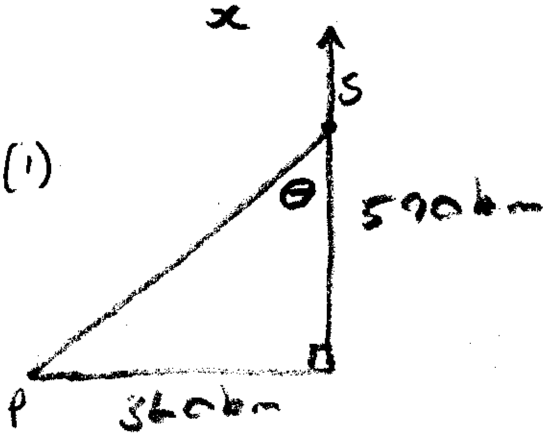


$\tan 47^\circ 28' = \dfrac{18}{x}$

$x = 18 \div \tan 47^\circ 28'$

$= \underline{16.5\text{m}}$ (2)

d) (1)



(1)

(ii) $\tan\theta = \dfrac{360}{590}$

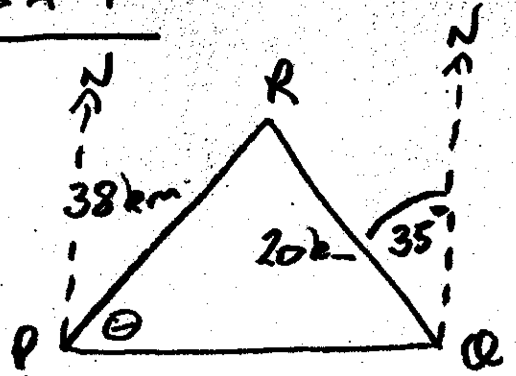
$\theta = 31^\circ 23'$

Bearing $= 180 + 31^\circ 23'$

$= \underline{211^\circ 23'}$ (2)

Question 9

a)



(i) $\angle PQR = 90 - 35 = 55°$ (2)

(ii) $\frac{\sin \theta}{20} = \frac{\sin 55}{38}$

$\sin \theta = \frac{\sin 55}{38} \times 20$

$\angle \theta = 25°32'$

$\therefore$ Bearing is $90 - 25°32' = 064°28'$ (1)

b) $5 \sin 30 \sec 30 = 5 \times \frac{1}{2} \times \frac{1}{\frac{\sqrt{3}}{2}}$

$= \frac{5}{2} \times \frac{2}{\sqrt{3}}$ (M1)

$= \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{5\sqrt{3}}{3}$ (1)

c) (i) $l^2 = 5.2^2 + 8.9^2 - 2 \times 5.2 \times 8.9 \times \cos 110$ (M1)

$= 11.7\,m$ (1)

(ii) $A = \frac{1}{2} \times 5.2 \times 8.9 \times \sin 110$

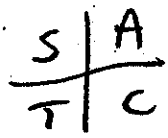
$= 21.7\,m^2$ (1)

d) $\dfrac{\tan\theta\sec\theta}{1+\tan^2\theta} = \dfrac{\tan\theta\sec\theta}{\sec^2\theta}$

$= \dfrac{\tan\theta}{\sec\theta}$ M1

$= \dfrac{\sin\theta}{\cos\theta} \div \dfrac{1}{\cos\theta}$

$= \underline{\sin\theta}$ 1



e) $2\sin\theta - 1 = 0$

$\sin\theta = \frac{1}{2}$

$\theta = 30°, 180-30$

$= \underline{30°, 150°}$ 1 1

Question 10

a) $x^2 + 5x - 8 = 0$

(i) $\alpha + \beta = \frac{-b}{a} = -5$ 1

(ii) $\alpha\beta = \frac{c}{a} = -8$ 1

(iii) $\alpha\beta - 2\alpha - 2\beta + 4 = \alpha\beta - 2(\alpha + \beta) + 4$

$= -8 - 2 \times -5 + 4$

$= -8 + 10 + 4$

$= \underline{6}$ 2

(iv) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-5)^2 - 2 \times -8$

$= 25 + 16$

$= \underline{41}$ 2

b) (i) $\Delta = (-2)^2 - 4 \times 1 \times 3$

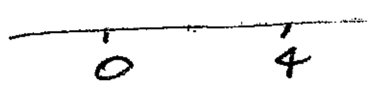
$= 4 - 12$ 1

$= \underline{-8}$

(ii) No real roots since $\Delta < 0$ 1

c) $4x - x^2 \geq 0$

$x(4 - x) \geq 0$ 2



$\underline{0 \leq x \leq 4}$

d) $20x-17 \equiv Ax-4A-5Bx-B$

$\equiv (A-5B)x-(4A+B)$

$A-5B=20$ ①

$4A+B=17$ ②

$4\times$① $4A-20B=80$ ③

③$-$② $-21B=63$

$\underline{B=-3}$

$A+15=20$

$\underline{A=5}$

2