TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 11 2009 ASSESSMENT TASK 4

MATHEMATICS/MATHEMATICS EXTENSION 1

YEARLY EXAMINATION

Monday 31st August, 2009
8:30 am to 11:00am

Time Allowed – *Two and a half hours plus 5 minutes reading time*

WEIGHTING 40% towards final assessment

Outcomes referred to: **P2, P3, P4, P5, P6, P7, P8**

INSTRUCTIONS:
1. Attempt **ALL** questions.
2. Show all necessary working in every question.
3. Begin each question on a separate piece of paper.
4. There are 10 questions worth 10 marks each. Total marks 100.
5. Marks for individual part questions are shown beside each part.
6. Non- programmable silent Board of studies approved calculators are permitted.
7. Write your name, class and your teacher's name on the front of each answer page.

Question 1 (10 marks)
Start each new question on a new page. Write your name and your teacher's name at the top of the page.

		Marks
(a)	Evaluate $\frac{1}{15+5\times3}$, correct to three significant figures.	2
(b)	Simplify $\sqrt{32}+4\sqrt{8}$	1
(c)	Solve for x: $2^x=\frac{1}{2}$	1
(d)	Rationalise the denominator : $\frac{2}{3-\sqrt{5}}$	2
(e)	Solve $\lvert x-3\rvert<4$	2
(f)	Factorise: $9-16t^2$	2

Question 2 (10 marks)

Start each new question on a new page. Write your name and your teacher's name at the top of the page.

		Marks
(a)	Solve: $\frac{x}{2}+\frac{x}{3}=1$	2
(b)	Simplify: $\frac{x}{x^2-9}-\frac{3}{x-3}$	2

(c) The function $f(x)$ is defined as:

$$f(x) = \begin{Bmatrix} 2x & -4 \le x < 0 \\ 9 - x^2 & 0 \le x \le 3 \end{Bmatrix}$$

(i)	Find the value of $f(0)$.	1
(ii)	Sketch $y = f(x)$	2
(iii)	State the range of $y = f(x)$	1

(d) Sketch on separate diagrams, showing all essential features:

(i)	$y = \lvert x \rvert + 2$	1
(ii)	$(x-2)^2 + y^2 = 9$	1

Question 3 (10 marks)

Start each new question on a new page. Write your name and your teacher's name at the top of the page.

Marks

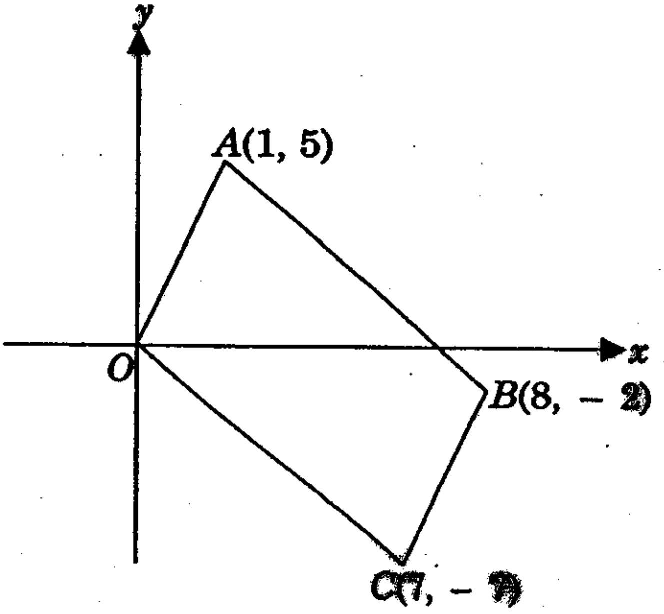


Diagram is NOT drawn to scale

In the diagram, OABC is a quadrilateral where the coordinates of O, A, B and C are (0,0), (1,5), (8,-2) and (7,-7) respectively.

(a) Find the midpoint of the interval joining AC. 1

(b) Find the gradient of AB. 1

(c) Show that the equation of the line containing AB is $x + y = 6$. 1

(d) Find the exact length of AB. 2

(e) Show that OC is parallel to AB. 1

(f) Explain why OABC is a parallelogram. 1

(g) Find the perpendicular distance from O to AB. Leave your answer in simplest surd form. 2

(h) Find the area of parallelogram OABC. 1

Question 4 (10 marks)

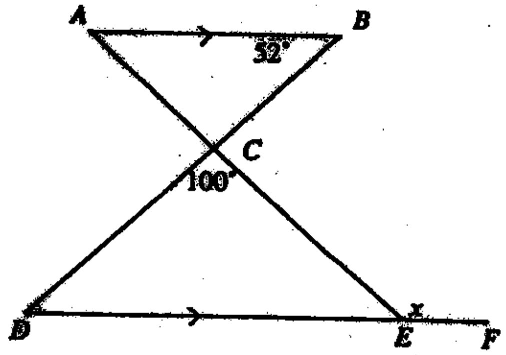
Start each new question on a new page. Write your name and your teacher's name at the top of the page.

Marks

(a) Solve simultaneously: $3x - y = 5$

$5x + 3y = -8$ 3

(b)



AB ∥ DF, $\angle ABC = 52^\circ$, $\angle DCE = 100^\circ$.

(i) Copy the diagram onto your answer page.

(ii) Find x° giving reasons. 2

(c) Evaluate: $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$ 2

(d) For the parabola $x^2 + 6x - 8y + 9 = 0$, find the:

(i) coordinates of the vertex. 1

(ii) focal length 1

(iii) coordinates of the focus 1

Question 5 (10 marks)

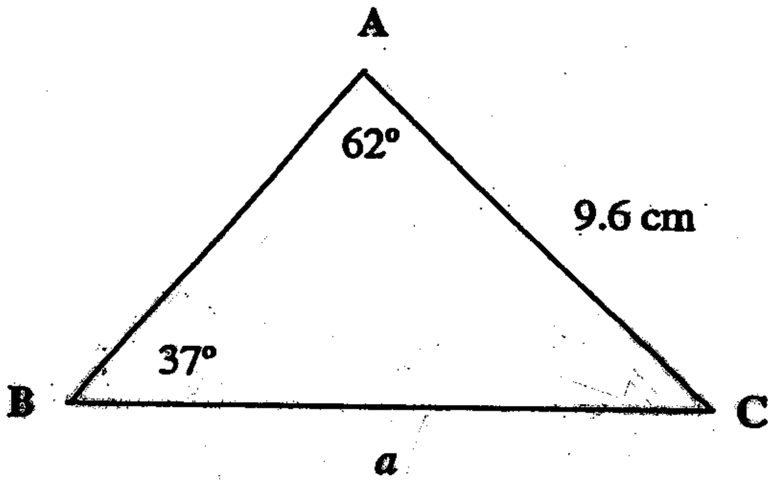
Start each new question on a new page. Write your name and your teacher's name at the top of the page.

Marks

(a) Write down the values of θ, $0 \leq \theta \leq 360^\circ$, for which:

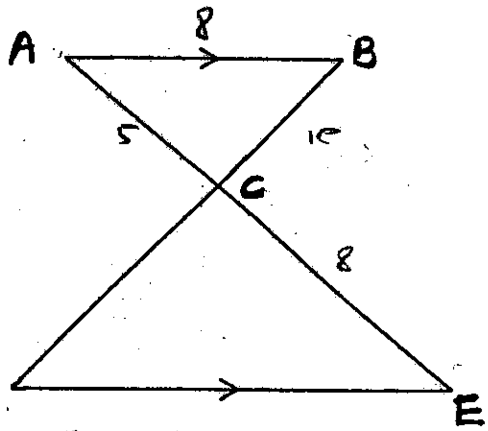
$$2\cos\theta = -1$$ 2

(b)



In the given triangle, find the length of side a correct to one decimal place. 2

(c) In the diagram, AB | | DE.



(i) Prove that ΔABC is similar to ΔEDC. 3

(ii) If $AC = 5cm, AB = 8cm, BC = 10cm$ $CE = 8cm$, find the length CD. 2

(d) Evaluate the exact value of: $\sec(-45^\circ)$ 1

Question 6 (10 marks)

Start each new question on a new page. Write your name and your teacher's name at the top of the page.

Marks

(a) Find the coordinates of the vertex of $y = x^2 + 4x - 12$ **2**

(b) Express $7x^2 - 5x + 4$ in the form $Ax(x+1) + B(x+1) + C$ **3**

(c) For what values of k does $2x^2 - 5x + k = 0$ have real roots? **2**

(d) Given that α and β are the roots of the quadratic equation $2x^2 - 3x - 5 = 0$, find:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha + 1)(\beta + 1)$ **1**

Question 7 (10 marks)

Start each new question on a new page. Write your name and your teacher's name at the top of the page.

Marks

(a) Indicate clearly on a diagram the region where $y > x + 2$ and $y \geq 3 - x^2$ hold simultaneously. 2

(b) From a point P, point Q bears 065° and is $20m$ away. Another point R, is $28m$ from P on a bearing of 287°.

(i) Show this information on a diagram. 1

(ii) Find the distance (to the nearest metre from Q to R). 3

(c) Determine whether the function $f(x) = \frac{x^2-1}{x}$ is even, odd or neither. 2

(d) Factorise fully: $ab - 5b + 20 - 4a$ 2

Question 8 (10 marks)

Start each new question on a new page. Write your name and your teacher's name at the top of the page.

Marks

(a) Differentiate with respect to x:

(i) $5x^2 - 3x + 1$ 1

(ii) $f(x) = (3x - 1)^{-2}$ 2

(iii) $x(x + 5)^4$ 2

(iv) $y = \frac{x}{(x+3)^2}$ 2

(b) Find the equation of the tangent to the curve $y = 3 - x^4$ at the point where $x = -1$. 3

Question 9 (10 marks)

Start each new question on a new page. Write your name and your teacher's name at the top of the page.

Marks

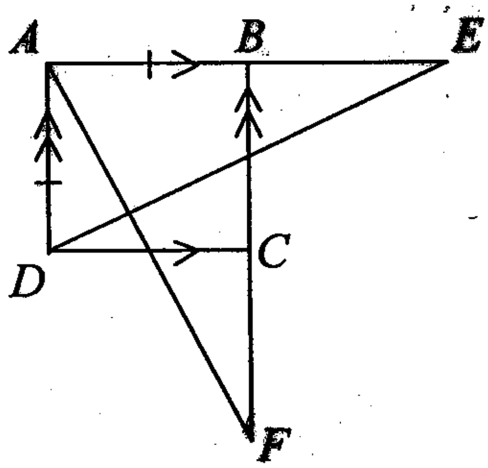
(a) The equation $4x^2 - px + q = 0$ has one root three times the other. If the smaller is α, show that:

(i) $p = 16\alpha$ 2

(ii) $q = 12\alpha^2$ 2

(iii) $3p^2 = 64q$ 2

(b) In diagram $ABCD$ is a square. AB is produced to E so that $AB = BE$ and BC is produced to F so that $BC = CF$.



(i) Copy the diagram onto your answer page.

(ii) Prove $\Delta AED \equiv \Delta BFA$ 3

(iii) Hence prove that $\angle AED = \angle BFA$ 1

Turn over the page for Question 10

Question 10 (10 marks)

Start each new question on a new page. Write your name and your teacher's name at the top of the page.

Marks

(a) Solve: $4^x - 12(2^x) + 32 = 0$ **3**

(b) Find all the values of x, $0 \leq x \leq 360^\circ$, for which: $sin^2 x = \frac{3}{4}$ **3**

(c) (i) The sum of the radii of two circles is $100cm$. If one of the circles has a radius xcm, show that the sum of the areas of the two circles is given by

$$A = 2\pi(x^2 - 100x + 5000)$$ **3**

(ii) For what values of x is this area a minimum? **1**

End of Examination

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SOLUTIONS

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Question 1

(a) $0.0\dot{3}$ 1

$= 0.0333$ 1 (3 sig figs).

(b) $4\sqrt{2} + 8\sqrt{2}$

$= 12\sqrt{2}$ 1

(c) $2^x = 2^{-1}$

$x = -1$ 1

(d) $\dfrac{2}{3-\sqrt{5}} \times \dfrac{3+\sqrt{5}}{3+\sqrt{5}}$ 1

$= \dfrac{2(3+\sqrt{5})}{9-5}$

$= \dfrac{6+2\sqrt{5}}{4}$ ← up to here was also accepted.

$= \dfrac{3+\sqrt{5}}{2}$ 1

(e) $x - 3 < 4$ OR $-x + 3 < 4$

$x < 7$ 1 $\quad -x < 1$

$x > -1$ 1

(f) $(3 - 4t)(3 + 4t)$
1 1

QUESTION 2

(a) $\dfrac{3x + 2x}{6} = 1$ 1

$5x = 6$

$x = \dfrac{6}{5}$ or $1\frac{1}{5}$ 1

(b) $\dfrac{x}{(x-3)(x+3)} - \dfrac{3}{x-3}$ 1

$= \dfrac{x - 3(x+3)}{(x-3)(x+3)}$

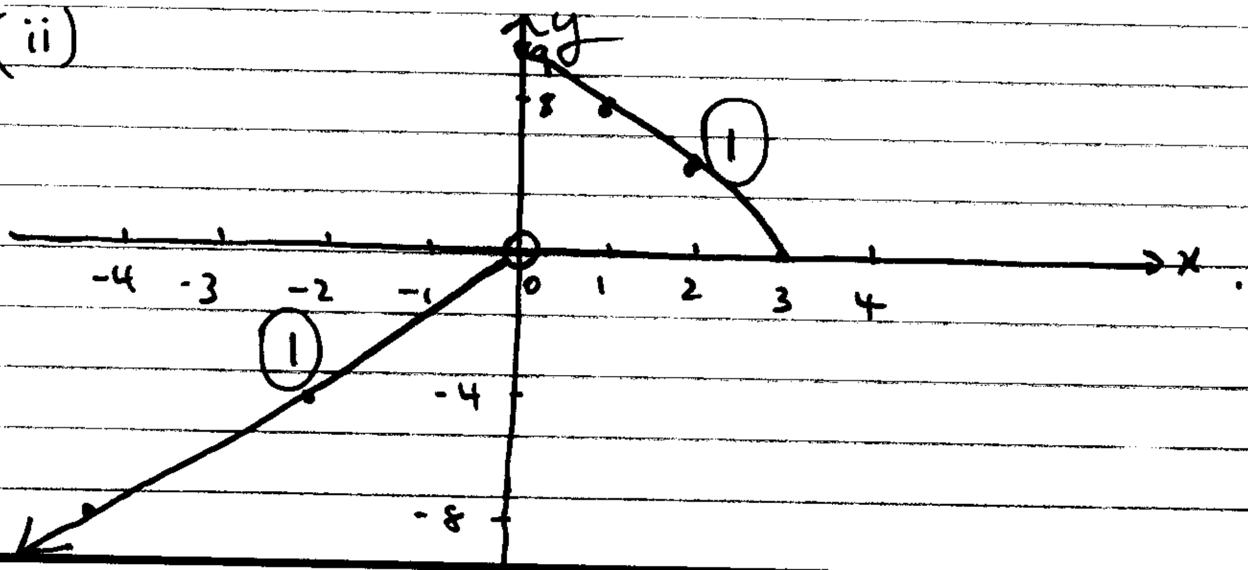
$= \dfrac{x - 3x - 9}{(x-3)(x+3)}$

$= \dfrac{-2x - 9}{(x-3)(x+3)}$ 1

(c) (i) $f(0) = 9 - (0)^2$

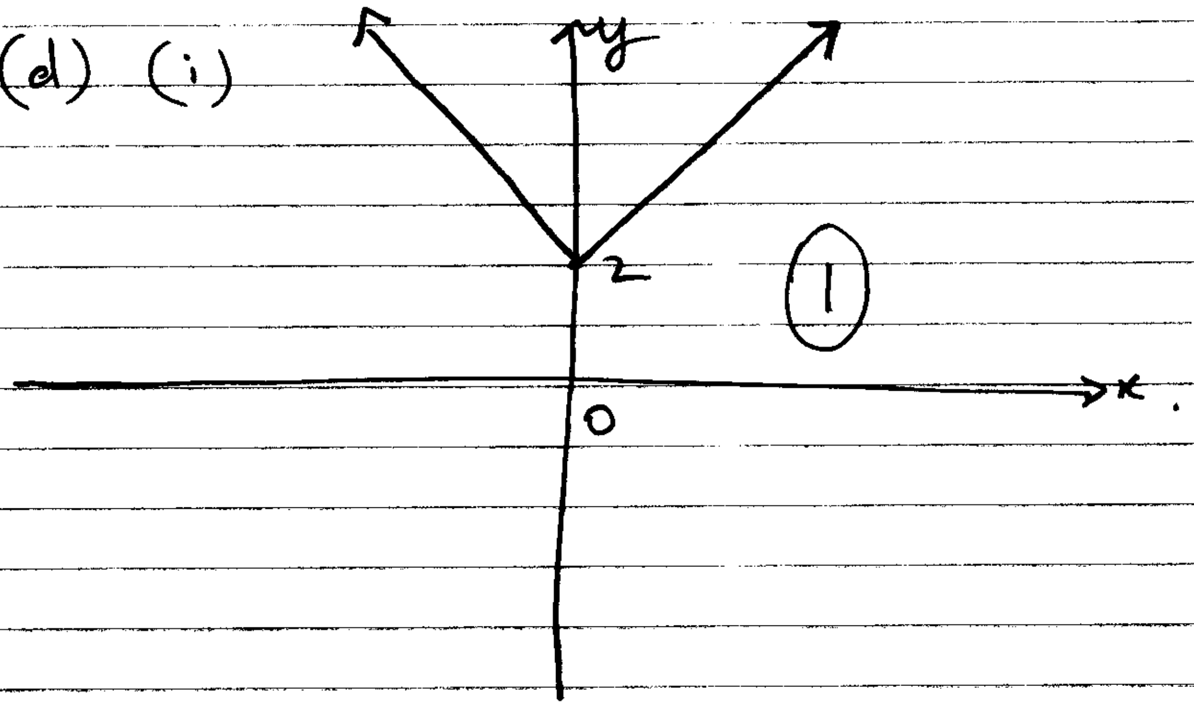
$= 9$ 1

(ii)



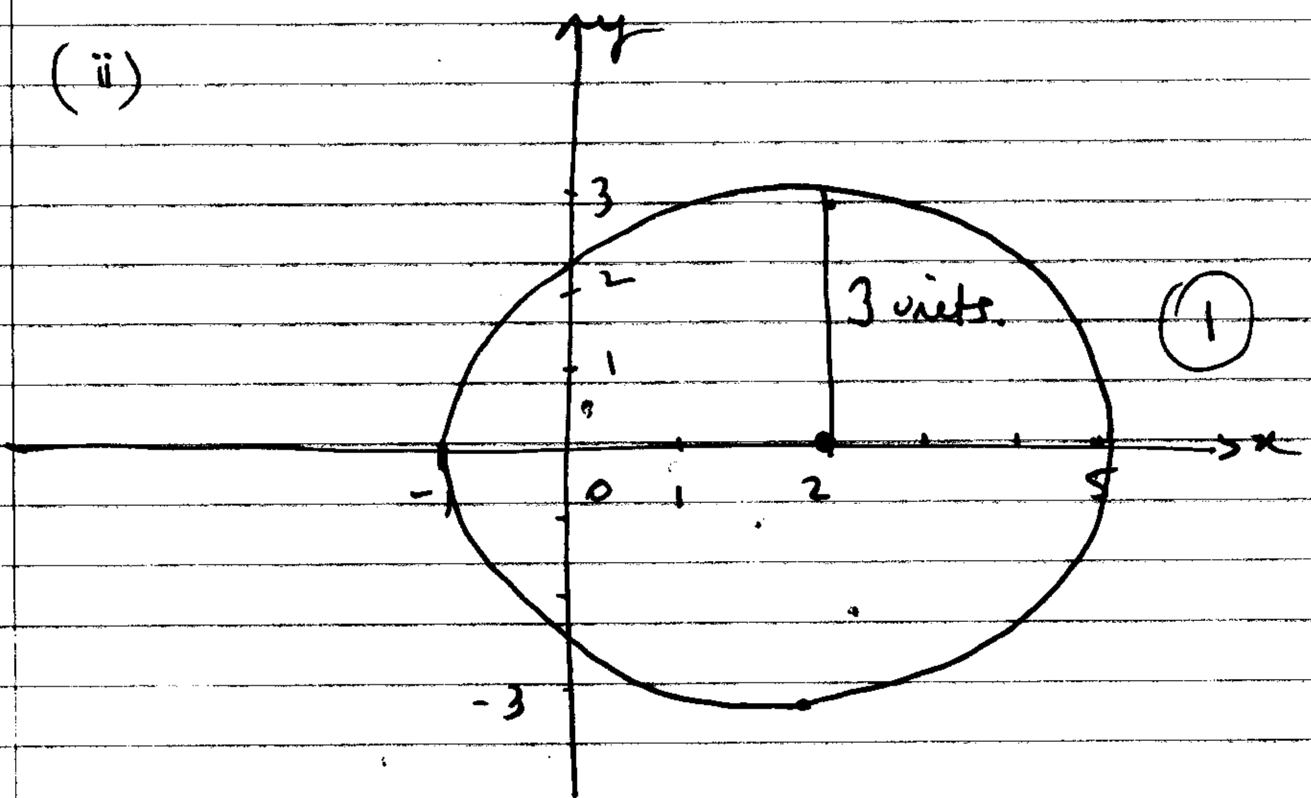
(iii) $R: \{-8 \leq y \leq 9\}$ 1

(d) (i)



1

(ii)



1

QUESTION 3

(a) $M(x, y) = \left(\frac{1+7}{2}, \frac{5-7}{2}\right)$

$= (4, -1)$ 1

(b) $m = \frac{-2-5}{8-1}$

$m = \frac{-7}{7}$

$m_{AB} = -1$ 1

(c) $y - y_1 = m(x - x_1)$ $m = -1, (1, 5)$

$y - 5 = -1(x - 1)$

$y - 5 = -x + 1$ 1

$x + y = 6$

(d) $d_{AB} = \sqrt{(8-1)^2 + (-2-5)^2}$ 1

$= \sqrt{7^2 + 7^2}$

$= \sqrt{98}$ accepted.

$d_{AB} = 7\sqrt{2}$ 1 (exact length)!

(e) $m_{OC} = \frac{-7-0}{7-0}$

$m_{OC} = -1 = m_{AB}$ from (b). 1

(f) OABC is a parallelogram because sides AB ∥ OC and $d_{AB} = 7\sqrt{2}$

$$= \sqrt{(7-0)^2 + (-7-0)^2}$$

$$= \sqrt{98} \quad 1$$

$$d_{OC} = 7\sqrt{2}$$

(g) $$d = \frac{|0\times1 + 0\times1 - 6|}{\sqrt{1^2 + 1^2}} \quad 1 \qquad O(0,0)$$

$$x + y = 6$$

$$d = \frac{|-6|}{\sqrt{2}}$$

$$d = \frac{6}{\sqrt{2}} \text{ or } \frac{6\sqrt{2}}{2} = 3\sqrt{2} \text{ units.} \quad 1$$

(h) Area = base × perpendicular height.

$$\text{Area} = 7\sqrt{2} \times 3\sqrt{2}$$

$$= 21 \times 2$$

$$\text{Area} = 42 \text{ unit}^2 \quad 1$$

QUESTION 4

(a)
$$9x - 3y = 15 \quad \text{——— } (1)$$

1

$$5x + 3y = -8 \quad \text{——— } (2)$$

(1) + (2)

$$14x = 7$$
$$x = \tfrac{1}{2} \quad 1$$

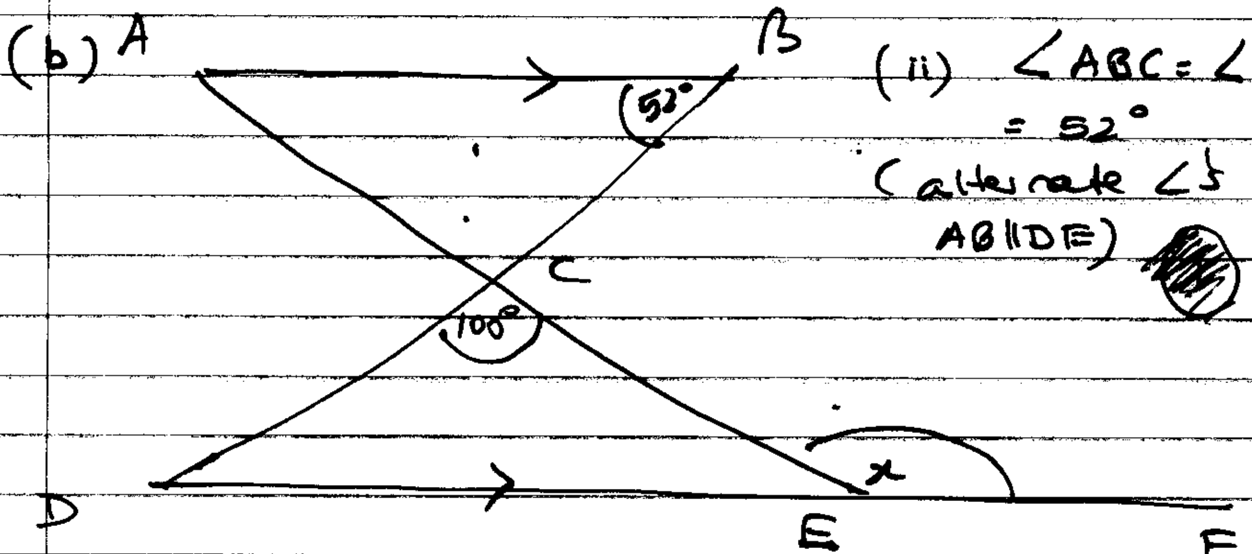
* if the was a mistake from the elimation process, (−1), but then carried through.

sub x into (1)

$$9\left(\tfrac{1}{2}\right) - 3y = 15$$
$$\frac{9}{2} - 3y = 15$$
$$-3y = \frac{21}{2}$$
$$y = -3\tfrac{1}{2} \quad 1$$

$$\left(\tfrac{1}{2}, -3\tfrac{1}{2}\right)$$

(b)



(ii) $\angle ABC = \angle EDC$
$= 52°$
(alternate ∠'s in AB ∥ DE)

In $\triangle CDE$, angle sum of $\triangle = 180°$

$\therefore \quad 100° + 52° + \angle CED = 180°$

$\angle CED = 28°$. 1

$\therefore \quad \angle CEF = x° = 180° - 28°$
$= 152°$ 1

straight angle.

(c) $\displaystyle\lim_{x \to 2} \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{(x-2)}}$ 1

$\displaystyle\lim_{x \to 2} (x^2 + 2x + 4)$

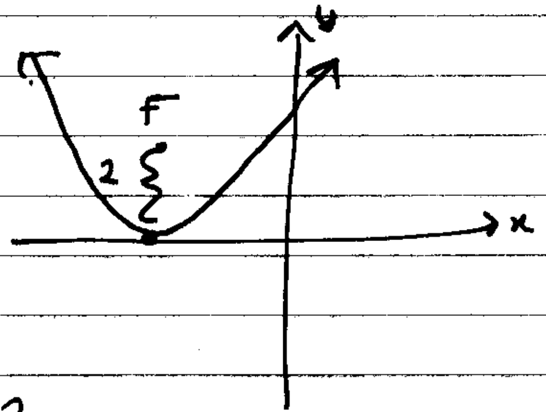
$= 2^2 + 2(2) + 4$

$= 12$ 1

(d) $x^2 + 6x = 8y - 9$

$(x+3)^2 - 9 = 8y - 9$

$(x+3)^2 = 8y$



$\therefore \quad V(-3, 0) \quad a = 2$

(i) $V(-3, 0)$ 1 (iii) $F(-3, 2)$ 1

(ii) $a = 2$ 1 * if mistake with Vertex, but the rest carried through correctly (-1)

QUESTION 5

(a) $\cos\theta = -\frac{1}{2}$

✓ S	A
✓ T	C

$\cos 60^\circ = \frac{1}{2}$

$\therefore \quad \theta = 120^\circ, 240^\circ.$

1 1

(b) $$\frac{a}{\sin 62^\circ} = \frac{9.6}{\sin 37^\circ}.$$

$$a = \frac{9.6 \times \sin 62^\circ}{\sin 37^\circ}. \quad 1$$

$a = 14.1$ cm (1 dec. pl)

1

(c) (i) $\angle ACB = \angle ECD$ —— vertically opp $\angle$'s are equal.

1

$\angle BAC = \angle CED$ —— alternate $\angle$'s in $\parallel$ lines. are equal.

1

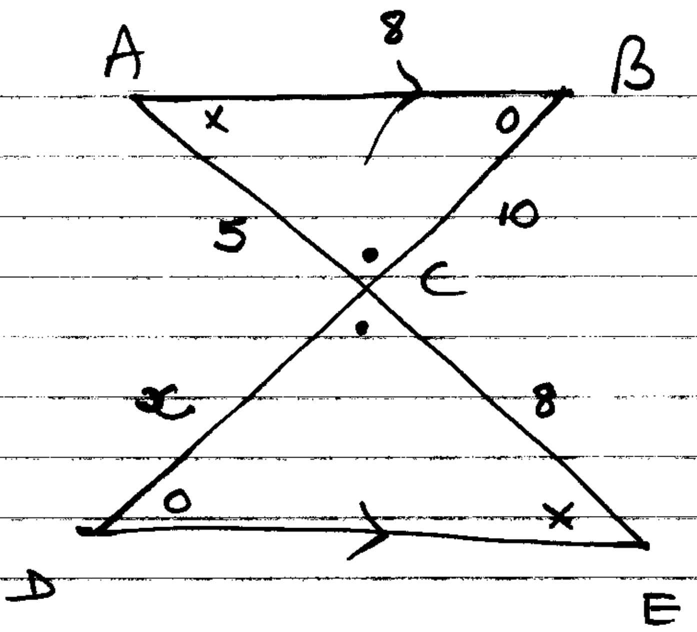
$\angle ABC = \angle EDC$ —— alternate $\angle$'s in $\parallel$ lines are equal.

1

$\therefore$ equiangular. $\triangle ABC \parallel\!| \triangle EDC$

(ii)

(ii)



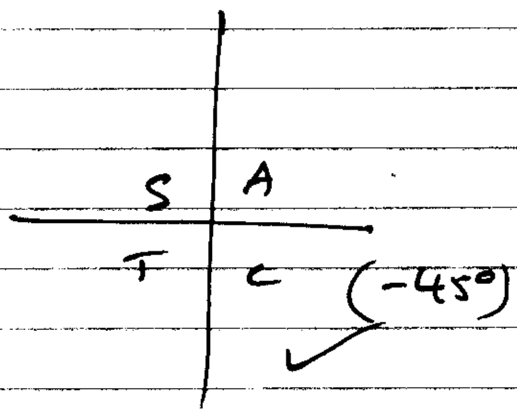
Since $\triangle ABC \;|||\; \triangle EDC$ has been proved.

$$\frac{5}{8} = \frac{10}{x}$$ (1) corresponding sides are in the same ratio.

$$5x = 80$$

$$x = 16 \text{ cm.}$$ (1)

(d) $$\frac{1}{\cos(-45^\circ)}$$



$$= \frac{1}{(+)\frac{1}{\sqrt{2}}}$$

$$= \sqrt{2}$$ (1)

QUESTION 6

(a) $x = \frac{-b}{2a}$ (axis of symmetry)

$x = \frac{-4}{2(1)}$

$x = -2$ 1

$\therefore$ sub $x = -2$ into y.

$y = (-2)^2 + 4(-2) - 12$

$y = 4 - 8 - 12$ 1

$y = -16$

$\therefore V(-2, -16)$

(b) $7x^2 - 5x + 4 \equiv Ax^2 + Ax + Bx + B + C$

$\equiv Ax^2 + (A+B)x + (B+C)$

$\therefore$ equating coefficients.

$\boxed{A = 7}$ 1

$A + B = -5$

$7 + B = -5$

$\boxed{B = -12}$ 1

$B + C = 4$

$-12 + C = 4$

$\boxed{C = 16}$ 1

$\therefore 7x^2 - 5x + 4$

$\equiv 7x(x+1) - 12(x+1) + 1$

(students need to express in the above form).

(c) $\Delta \geqslant 0$ for real roots

$b^2 - 4ac \geqslant 0$ (1)

$(-5)^2 - 4(2)(k) \geqslant 0$

$25 - 8k \geqslant 0$

$25 \geqslant 8k$

$\frac{25}{8} \geqslant k$

$\therefore k \leqslant 3\frac{1}{8}$ (1)

(d) (i) $\alpha + \beta = -\frac{b}{a}$

$= \frac{3}{2}$ (1)

(ii) $\alpha\beta = \frac{c}{a}$

$= -5/2$ (1)

(iii) $(\alpha + 1)(\beta + 1)$

$= \alpha\beta + \beta + \alpha + 1$

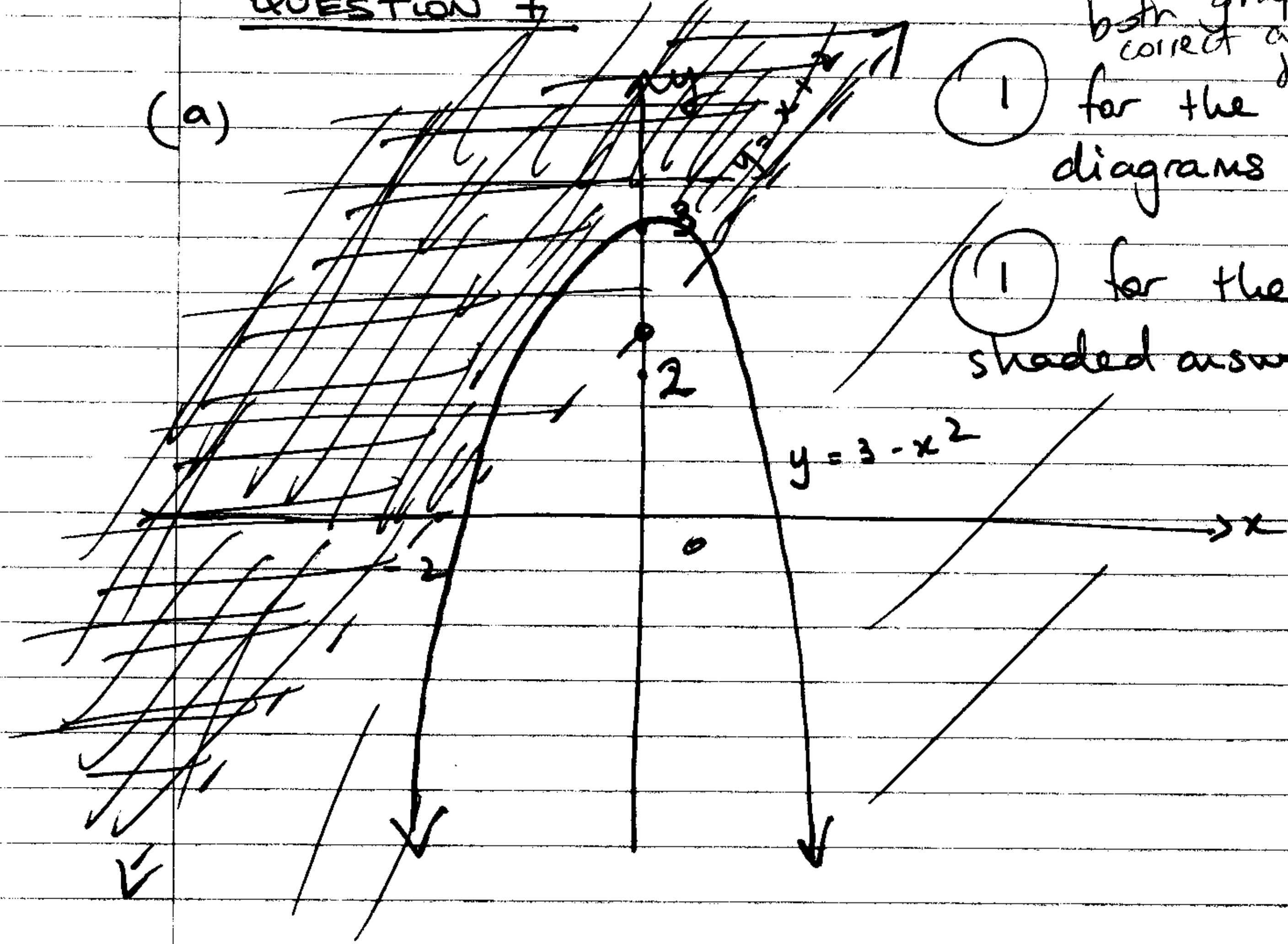
$= \alpha\beta + (\alpha + \beta) + 1$

$= -5/2 + 3/2 + 1$ (1)

$= 0$

QUESTION 7

(a)

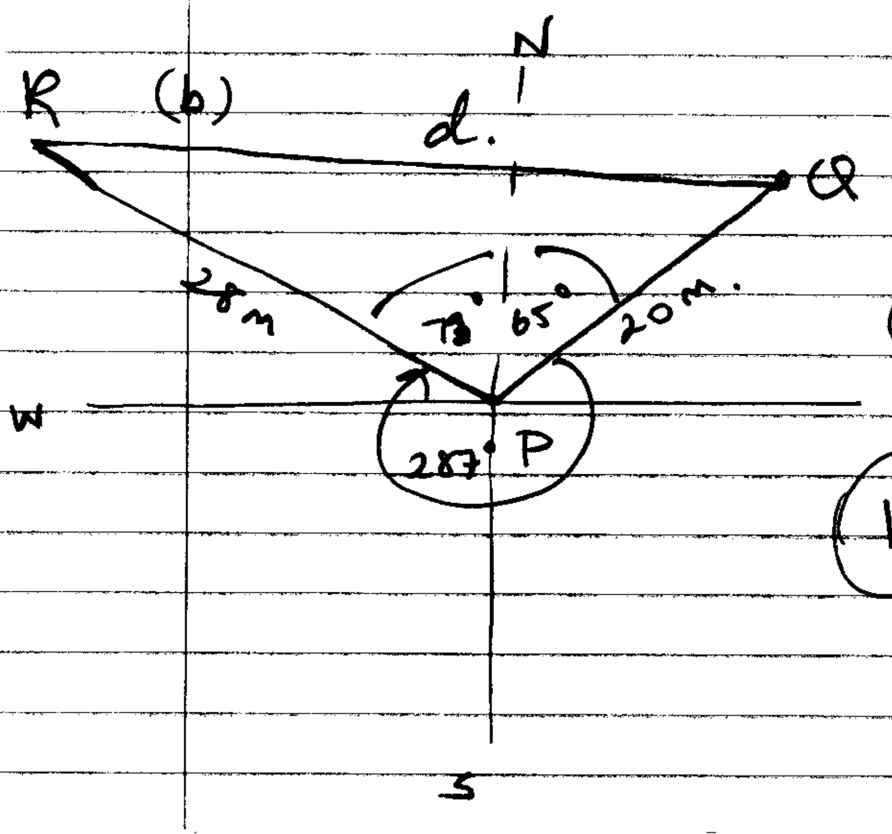


both graphs correct and dotted

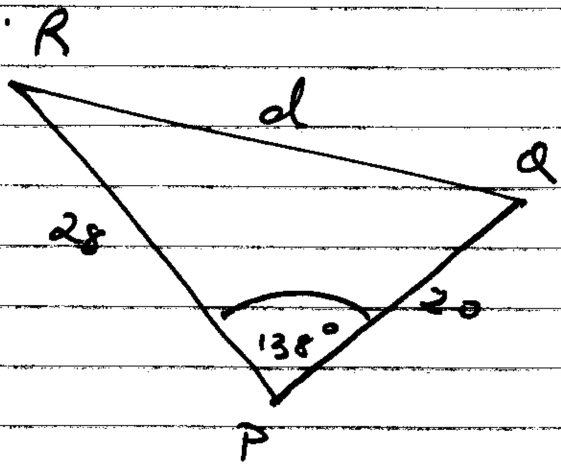
1 for the solid diagrams

1 for the shaded answer.

(b)



(i)



1

(ii) $d_{QR}^2 = 28^2 + 20^2 - 2(28)(20)\cos 138^\circ$ (1)

if incorrect can still get 2
if included 1

$d_{QR}^2 = 2016.32$ (1)

$d_{QR} = 45\text{ m}.$ (1)

(c) $f(x) = \dfrac{x^2 - 1}{x}$

$$f(-x) = \frac{(-x)^2 - 1}{-x}$$

$$= \frac{x^2 - 1}{-x}$$

$$= \frac{-(x^2 - 1)}{x}$$

$$f(-x) = \frac{-x^2 + 1}{x}.$$

$\therefore$ for even $f(x) = f(-x)$

$$\frac{x^2 - 1}{x} \neq \frac{-x^2 + 1}{x}.$$

for odd $f(-x) = -f(x)$

$$\frac{-x^2 + 1}{x} = -\left(\frac{x^2 - 1}{x}\right) \quad (1)$$

$$= \frac{-x^2 + 1}{x} \quad (1)$$

$\therefore \quad f(-x) = -f(x)$

odd function.

(d) $b(a-5) + 4(5-a)$

$= b(a-5) - 4(a-5)$ 1

$= (b-4)(a-5)$ 1

QUESTION 8

(a) (i) $\dfrac{d}{dx} = 10x - 3 \qquad 1$

(ii) $f'(x) = -2(3x-1)^{-3} \times (3) \quad 1 \quad 1$

$= -6(3x-1)^{-3}$

OR

$f'(x) = \dfrac{-6}{(3x-1)^3}$

(iii) $\dfrac{d}{dx} = x \times 4(x+5)^3 + 1 \times (x+5)^4 \quad 1 \quad 1$

$= 4x(x+5)^3 + (x+5)^4$

$= (x+5)^3\left[4x + (x+5)\right]$

$= (x+5)^3(5x+5).$

(iv) $\dfrac{dy}{dx} = \dfrac{(x+3)^2 \times 1 - x\left[2(x+3)^1\right]}{(x+3)^4} \quad 1$

$= \dfrac{(x+3)^2 - 2x(x+3)}{(x+3)^4} \quad 1$

$$= \frac{x^2 + 6x + 9 - 2x^2 - 6x}{(x+3)^4}$$

$$\frac{dy}{dx} = \frac{-x^2 + 9}{(x+3)^4}$$

(b) $\frac{dy}{dx} = -4x^3$ 1

$\frac{dy}{dx}$ at $x = -1$

$m = -4(-1)^3$

$m = 4$ 1

$\therefore$ when $x = -1$, $y = 3 - (-1)^4$

$y = 3 - 1$

$(-1, 2)$ $m = 4$ $y = 2$

$y - 2 = 4(x - -1)$

$y - 2 = 4x + 4$

1

$y = 4x + 6$ or $4x - y + 6 = 0$.

QUESTION 9

(a) (i) let α, 3α.

$$\alpha + \beta = \frac{-b}{a}$$

$$\alpha + 3\alpha = \frac{--p}{4} \quad 1$$

$$4\alpha = \frac{p}{4}$$

$$\cancel{\alpha = \frac{p}{16}}$$

$$16\alpha = p \quad 1$$

(ii) $$\alpha\beta = \frac{c}{a}$$

$$\alpha \times (3\alpha) = \frac{q}{4} \quad 1$$

$$3\alpha^2 = \frac{q}{4}$$

$$12\alpha^2 = q \quad 1$$

(iii) if from (i) $\alpha = \dfrac{p}{16}$

sub α into (ii)

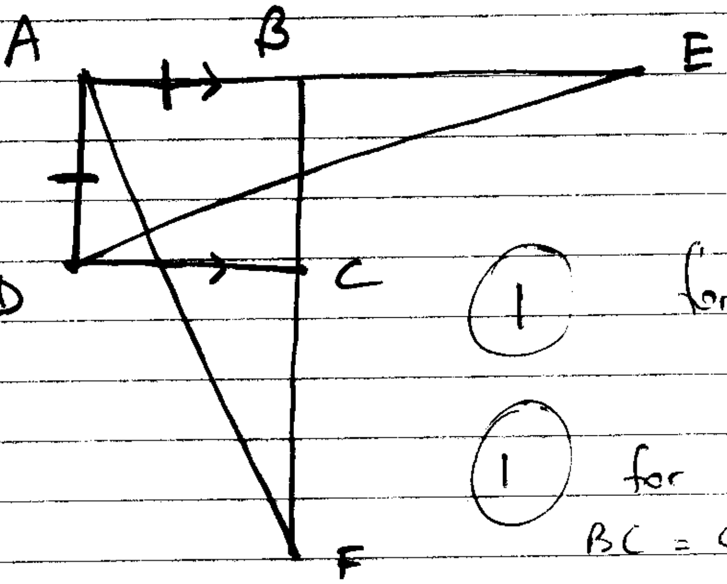
$$12\left(\frac{p}{16}\right)^2 = q \quad 1$$

$$12 \times \left(\frac{p^2}{256}\right) = q$$

$$\frac{3p^2}{64} = q$$

$$3p^2 = 64q \quad 1$$

(b)



1 for $\angle 90°$ and $AD = AB$.

1 for stating $AB = BE$ and $BC = CF$

(ii) $AD = AB$ — given (sides of a square are =).

$\angle EAD = \angle FBA = 90°$ — interior angles of a square are $90°$.

Since $AB = BE$ and $BC = CF$ and $AB = AD$, $\therefore AE = BF$

1

$\therefore \triangle AED \equiv \triangle BFA$ (SAS)

(iii) Since $\Delta AED \equiv \Delta BFA$,

$\angle AED = \angle BFA$, corresponding angles in congruent Δ's. 1

QUESTION 10

(a) $(2^2)^x - 12(2^x) + 32 = 0$

$(2^x)^2 - 12(2^x) + 32 = 0$

$\therefore$ let $m = 2^x$ 1

$m^2 - 12m + 32 = 0$

$(m-4)(m-8) = 0$ 1

$\therefore$ $m = 4$ or $m = 8$

since $m = 2^x$.

$2^x = 4$ $\qquad$ $2^x = 8$

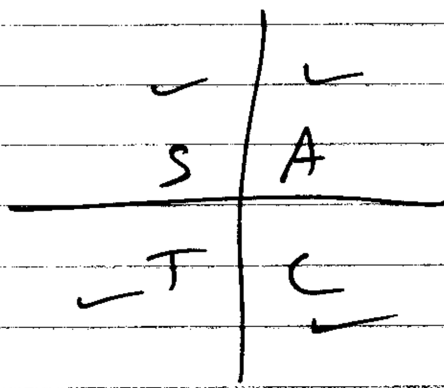
$2^x = 2^2$ 1 $\qquad$ $2^x = 2^3$

$\boxed{x = 2}$ or $\boxed{x = 3}$.

(b). $\sin x = \pm\sqrt{3/4}$

$\sin x = \pm\dfrac{\sqrt{3}}{2}$

1



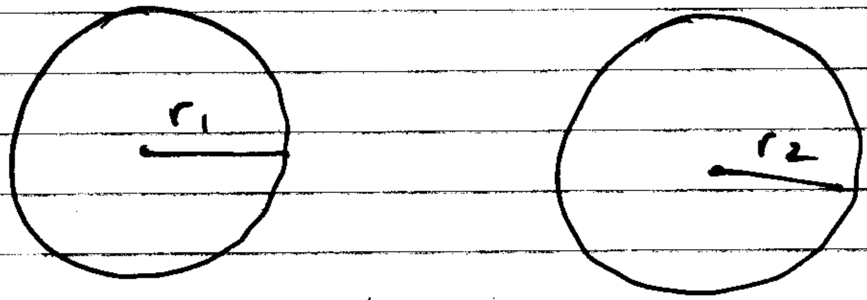
$$\sin \cancel{45^\circ}\ 60^\circ = \frac{\sqrt{3}}{\sqrt{2}}$$

$$\therefore \quad x = \cancel{45^\circ}, \cancel{135^\circ}, \cancel{225^\circ}, \cancel{315^\circ}.$$

1 1

$60^\circ, 120^\circ, 240^\circ, 300^\circ$

(c)(i)



let $r_1 + r_2 = 100\,cm$

if $r_1 = x$

$\therefore \quad x + r_2 = 100$

$r_2 = 100 - x\ cm.$ 1

$A_1 = \pi x^2$

$A_2 = \pi(100-x)^2 = \pi(10\,000 - 200x + x^2)$

$\therefore \quad A = A_1 + A_2$

$A = \pi x^2 + 10\,000\pi - 200\pi x + \pi x^2$ 1

$A = 2\pi x^2 - 200\pi x + 10\,000\pi$

$A = 2\pi(x^2 - 100x + 5000)$ 1

(ii) $\dfrac{dA}{dx} = 2\pi\left(2x - 100\right)$

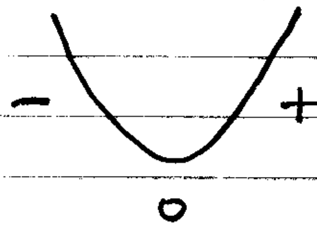
let $\dfrac{dA}{dx} = 0$

$0 = 2\pi\left(2x - 100\right)$

$50 = x$

1

x	40	50	60
$\frac{dA}{dx}$	$-$	0	+



$\therefore$ for $x = 50$, this area is a minimum.

* to get the 1 mark, must test stationary pt.