TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 11 – 2010 YEARLY EXAMINATIONS

MATHEMATICS

ASSESSMENT TASK 4
WEIGHTING 40%

Monday 23rd August 2010
8:30 a.m. – 11.00 a.m.

OUTCOMES REFERRED TO: P2, P3, P4, P5, P6, P7

General Instructions

- Reading time – **5 minutes.**
- Working time – $2\frac{1}{2}$ **hours.**
- Write using blue or black pen.
- Board-approved calculators may be used.
- All necessary working should be shown in every question.
- **Begin** each question on a **new page**.
- Write your **name, class** and your **teacher's name** on the front of each answer sheet of paper.

Total marks – 100

- Attempt Questions 1-10.
- All questions are of equal value.
- Mark values are shown at the side of each question part.

Question 1. (10 marks) **Start each question a new page.**
Write your name and your teacher's name at the top of each page.

(a) Evaluate $\dfrac{(2.1)^3}{\sqrt{9.5-2.8}}$ correct to:

(i) 2 decimal places. 1

(ii) 2 significant figures. 1

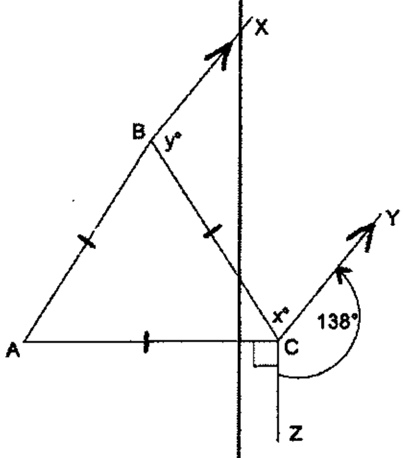
(b) An importer buys 1800 toy dolls for \$6240 and sells them at a profit of 50%. Calculate the selling price of each doll. 1

(c) Factorise completely $12a^2 + 6a^3$ 1

(d) Simplify $2\sqrt{20} - \sqrt{45}$ 2

(e) Expand and simplify $(3y-2)(y+1)$. 2

(f) Find the values of x and y. 2



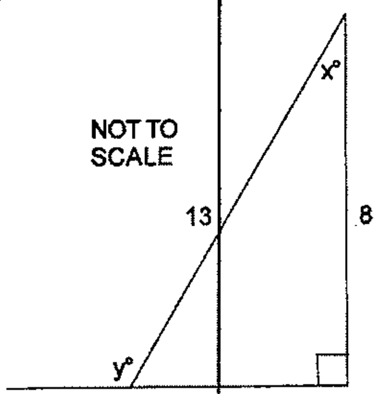
Question 2. (10 marks) **Start each question a new page.**

Write your name and your teacher's name at the top of each page.

(a) If $f(x) = 2x + |x|$, find the value of $f(-2)$. **1**

(b) Express $0.4\dot{2}$ as a fraction in its simplest form. **2**

(c) Find the values of x and y to the nearest degree. **2**



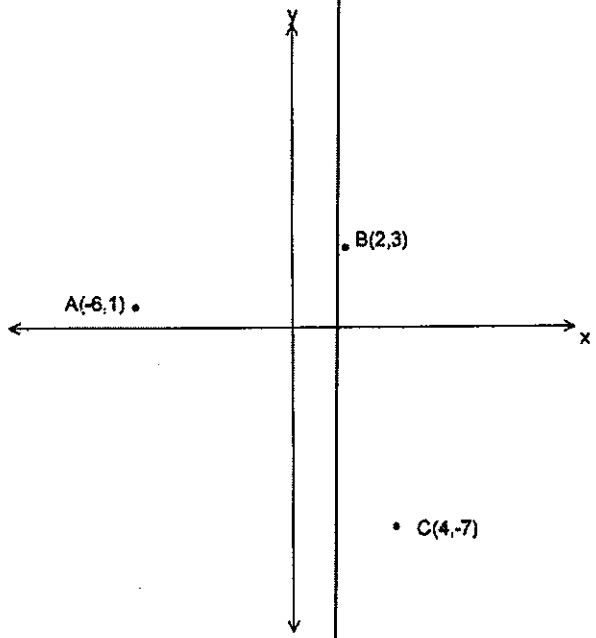
(d) Express $\dfrac{(3k-4)}{2} - \dfrac{(k-2)}{5}$ as a single fraction in its simplest form. **2**

(e) Solve the pair of simultaneous equations. **3**

$$5m - 3n = 21$$
$$3m + n = 7$$

Question 3. (10 marks) Start each question a new page.
Write your name and your teacher's name at the top of each page.

In the diagram, the points A(-6, 1), B(2, 3), and C(4, -7) are the vertices of a triangle.



(a) Find the exact length of AB in simplest surd form. **2**

(b) Find the gradient of AB. **1**

(c) Show that the equation of the line AB is $x-4y+10=0$. **2**

(d) Find the equation of the line passing through the point C that is parallel to the y-axis. **1**

(e) Find the perpendicular distance from C to the line AB. Leave your answer in exact form. **2**

(f) Hence, or otherwise find the area of triangle ABC. **1**

(g) Find the point D such that $ABCD$ is a parallelogram. **1**

Question 4. (10 marks) **Start each question a new page.**

Write your name and your teacher's name at the top of each page.

(a) Simplify $tan\theta\ cosec\theta$. **1**

(b) State the domain of the function $y = \dfrac{1}{x-2}$. **1**

(c) State the range of the function $y = 4 - x^2$. **1**

(d) Rationise the denominator of $\dfrac{6}{\sqrt{3}+1}$. **2**

(e) Solve $4 - 3x < 6(x-1)$ **2**

(f) Indicate on a diagram using shading, the region for which the following inequalities hold simultaneously:

$y \le x^2 - 1$ and $x^2 + y^2 < 4$ **3**

Question 5. (10 marks) Start each question a new page.
Write your name and your teacher's name at the top of each page.

(a) Differentiate from first principles $f(x) = 3x^2 - 2x + 5$. **3**

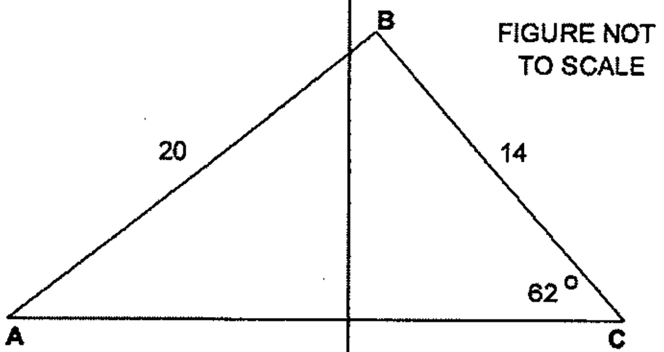
(b) Differentiate with respect to x:

(i) $2x^5 - 4x^2 + 3$ **1**

(ii) $(4x+5)^4$ **1**

(iii) $\dfrac{3x^2+1}{x}$ **1**

(c)



(i) Find the size of $\angle BAC$ to the nearest degree. **2**

(ii) Find the area of ΔABC. **2**

Question 6. (10 marks) **Start each question a new page.**

Write your name and your teacher's name at the top of each page.

(a) If α and β are the roots of the equation $2x^2 - 3x - 6 = 0$, find the value of:

(i) $2\alpha + 2\beta$. **1**

(ii) $\alpha^2\beta^2$. **1**

(iii) $\alpha^2 + \beta^2$. **2**

(b) Find the exact value of $\tan 240^\circ \times \sin 135^\circ$ **2**

(c) Simplify $2 - 2\sin^2\theta$. **1**

(d) Solve the equation: $2\cos\theta - 1 = 0$ *for* $0^\circ \leq \theta \leq 360^\circ$. **2**

(e) Sketch the graph for $y = \sin x$ *for* $0^\circ \leq x \leq 360^\circ$. **1**

Question 7. (10 marks) **Start each question a new page.**

Write your name and your teacher's name at the top of each page.

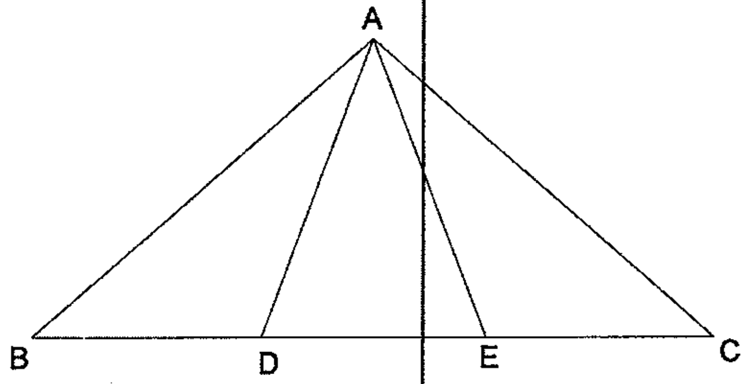
(a) Factorise $5a^2 + a - 18$. **2**

(b) Find the values of x for which $|2x-1| \geq 5$. **2**

(c) (i) Sketch the graphs of $y = 3 - x$ and $y = |x| - 1$. **2**

(ii) Write down the coordinates of all, if any, points of intersection of $y = 3 - x$ and $y = |x| - 1$. **1**

(d) Given $AB = AC$ and $\angle BAD = \angle CAE$.



(i) Prove that ΔBAD and ΔCAE are congruent. **2**

(ii) Hence or otherwise, prove that ΔDAE is isosceles. **1**

Question 8. (10 marks) **Start each question a new page.**
Write your name and your teacher's name at the top of each page.

(a) Show whether the function $f(x)=x^3-x$ is odd, even or neither. **2**

(b) Find the values of k for which the quadratic equation $x^2-(k+2)x+4=0$ has no real roots. **2**

(c) Simplify $\dfrac{3x^2+7x+2}{x^2+4x+4} \div \dfrac{9x^2-1}{3x^2-3xy-x+y}$ **3**

(d) Find the values of A, B and C for the identity

$3x^2-16x+17 \equiv A(x-2)^2+B(x-2)+C-5$ **3**

Question 9. (10 marks) **Start each question a new page.**
Write your name and your teacher's name at the top of each page.

(a) Two trees on a street have heights of 9 m and 3 m respectively. At midday the shorter tree casts a shadow of 5 m. How far apart are the trees? **2**

(b) Solve $\dfrac{x+4}{7}=\dfrac{x+5}{4}$ **2**

(c) Solve the pair of simultaneous equations.

$$2x-y=5$$
$$y=x^2-4x$$
3

(d) Evaluate $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2+2x-8}$. **3**

Question 10. (10 marks) Start each question a new page.
Write your name and your teacher's name at the top of each page.

(a) Sketch the function $f(x)=\begin{Bmatrix} x^2 \text{ if } x<0 \\ 3x \text{ if } 0\le x<2 \\ 6 \text{ if } x\ge 2 \end{Bmatrix}$. **3**

(b) Find the equation of the tangent to the curve $y=\dfrac{x+4}{x+1}$
at the point where $x = 2$. **3**

(c) A helicopter travels from base station S on a bearing of $235°$ for 58 km to Outpost A. It then travels 135 km on a bearing of $340°$ to outpost B.

(i) How far is outpost B from base station S correct to 1 decimal place? **2**

(ii) Find the bearing of outpost B from base station S to the nearest degree. **2**

End of paper

YR 11 - TGS MATHEMATICS (2U) AUGUST 2010

Q1. a) i) 3.577836439
3.58 (1)

ii) 3.6 (1)

b) $\frac{6240 \times 1.5}{1800} = \$5.20$ (1)

c) $6a^2(2 + a)$ (1)

d) $4\sqrt{5} - 3\sqrt{5}$ (M1)
$= \sqrt{5}$ (1)

e) $3y^2 + 3y - 2y - 2$ (M1)
$= 3y^2 + y - 2$ (1)

f) $x = 72°$ (1)

$y = 108°$ (1)

Q2. a) $f(-2) = 2(-2) + |-2|$

$= -4 + 2$

$= -2$ (1)

b) $10x = 4.222\ldots$

$x = 0.422\ldots$

$9x = 3.8$ (M1)

$x = \frac{38}{90} = \frac{19}{45}$ (1)

c) $\cos x = \frac{8}{13}$

$x = \cos^{-1}\left(\frac{8}{13}\right)$

$= 52^\circ$ (1)

$\therefore y = 142^\circ$ (1)

d) $\frac{5(3k-4)}{10} - \frac{2(k-2)}{10}$

$= \frac{15k - 20 - 2k + 4}{10}$ (M1)

$= \frac{13k - 16}{10}$ (1)

e) $5m - 3n = 21$

$9m + 3n = 21$ (M1)

$14m = 42$

$m = 3$ (1)

$n = 7 - 3(3)$

$= 7 - 9$

$n = -2$ (1)

Q3. a) $AB = \sqrt{(2+6)^2 + (3-1)^2}$ (M1)

$= \sqrt{68}$

$= 2\sqrt{17}$ (1)

b) $m_{AB} = \dfrac{3-1}{2+6}$

$= \dfrac{1}{4}$ (1)

c) $y - 3 = \dfrac{1}{4}(x-2)$ (M1)

$4y - 12 = x - 2$

$x - 4y + 10 = 0$ (1)

d) $x = 4$ (1)

e) $d = \dfrac{|(1)(4) - 4(-7) + 10|}{\sqrt{1+16}}$ (M1)

$= \dfrac{42}{\sqrt{17}}$ OR $\dfrac{42\sqrt{17}}{17}$ (1)

f) $A = \dfrac{1}{2} \cdot 2\sqrt{17} \times \dfrac{42}{\sqrt{17}}$

$= 42$ units^2 (1)

g) $D(-4, -9)$ (1)

Q4. a) $\frac{\cancel{\sin\theta}}{\cos\theta} \cdot \frac{1}{\cancel{\sin\theta}}$

$= \sec\theta$ (1)

b) Domain: all real x, $x \neq 2$ (1)

c) Range: all real x, $y \leq 4$ (1)

d) $\frac{6}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$

$= \frac{6(\sqrt{3}-1)}{2}$ (M1)

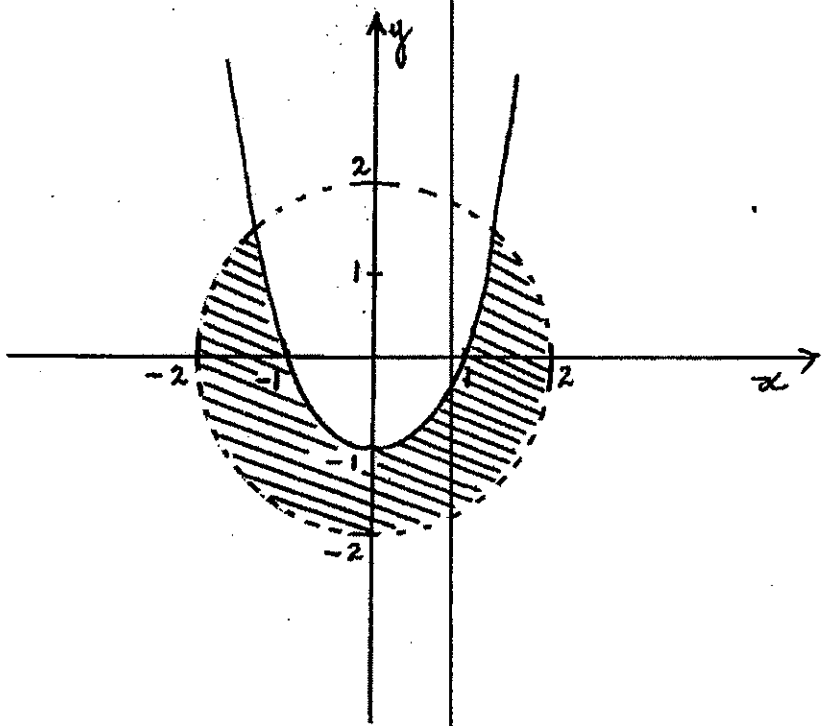
$= 3(\sqrt{3}-1)$ (1) OR $3\sqrt{3}-3$

e) $4 - 3x < 6x - 6$

$-9x < -10$

$x > \frac{10}{9}$

f)



Q5. a) $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \dfrac{3(x^2+2xh+h^2) - 2(x+h) + 5 - (3x^2 - 2x + 5)}{h}$ (M1)

$= \lim_{h \to 0} \dfrac{\cancel{3x^2}+6xh+3h^2 - \cancel{2x} - 2h \cancel{+5} - \cancel{3x^2} + \cancel{2x} \cancel{-5}}{h}$ (M1)

$= \lim_{h \to 0} \dfrac{h(6x+3h-2)}{h}$

$= 6x - 2$ (1)

b) i) $10x^4 - 8x$ (1)

ii) $4(4x+5)^3(4)$
$= 16(4x+5)^3$ (1)

iii) $\dfrac{d}{dx}\left[3x + x^{-1}\right]$
$= 3 - x^{-2}$ (1)
$= 3 - \dfrac{1}{x^2}$

c) i) $\dfrac{\sin A}{14} = \dfrac{\sin 62^\circ}{20}$
$A = \sin^{-1}\left[\dfrac{14 \sin 62^\circ}{20}\right]$ (M1)
$A = 38^\circ$ (1)

ii) $\therefore \angle B = 180 - (62 + 38)$
$= 80^\circ$ (1)

$A = \frac{1}{2}(20)(14)(\sin 80^\circ)$
$= 137.8730854$
$\doteqdot 138$ units^2

Q6. a) $\alpha + \beta = \frac{3}{2}$ $\qquad$ $\alpha\beta = -3$

i) $2(\alpha + \beta)$

$= 2\left(\frac{3}{2}\right)$

$= 3$ (1)

ii) $(\alpha\beta)^2$

$= (-3)^2$

$= 9$ (1)

iii) $\alpha^2 + \beta^2$

$= (\alpha + \beta)^2 - 2\alpha\beta$ ← (M1)

$= \left(\frac{3}{2}\right)^2 - 2(-3)$

$= 8\frac{1}{4}$ (1)

b) $\tan 240^\circ \times \sin 135^\circ$

$= \sqrt{3} \times \frac{1}{\sqrt{2}}$ (M1)

$= \frac{\sqrt{3}}{\sqrt{2}}$ (1) OR $\frac{\sqrt{6}}{2}$

c) $2(1 - \sin^2\theta)$

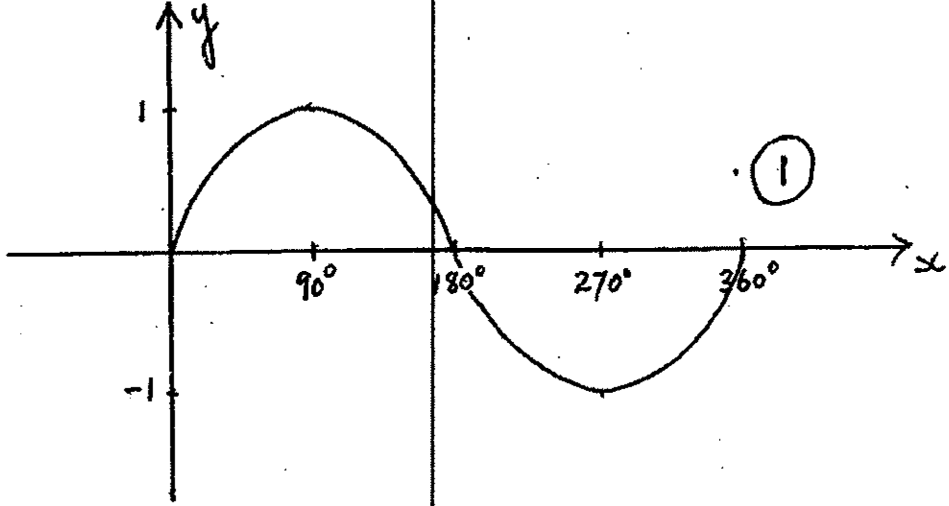
$= 2\cos^2\theta$ (1)

d) $\cos\theta = \frac{1}{2}$

$\theta = 60^\circ, \ 300^\circ$

(1) (1)

e)



(1)

Q7. a) $5a^2 + a - 18$

$= (5a - 9)(a + 2)$ (1) (1)

b) $2x - 1 \geq 5$

$2x \geq 6$

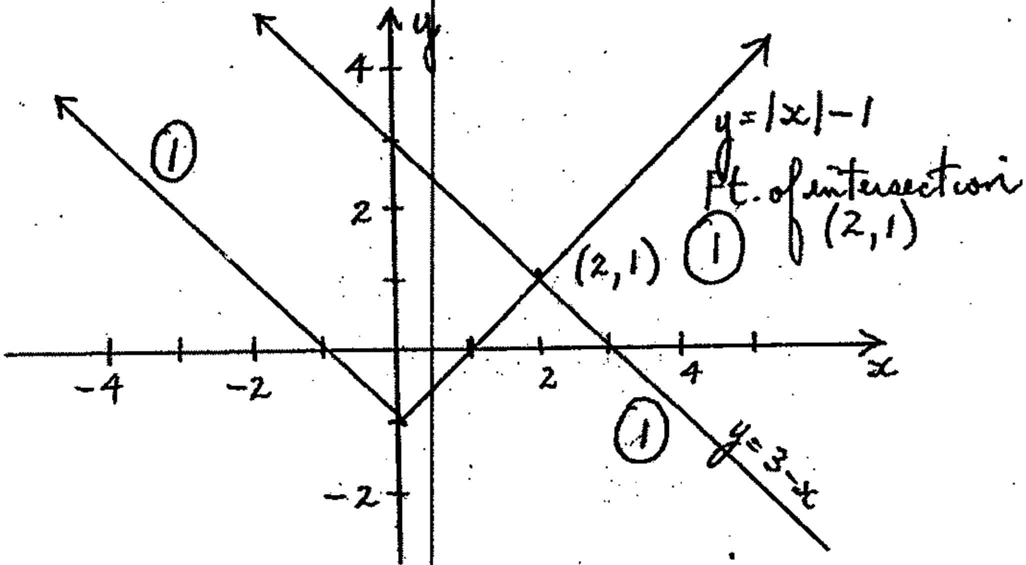
$x \geq 3$ (1)

$2x - 1 \leq -5$

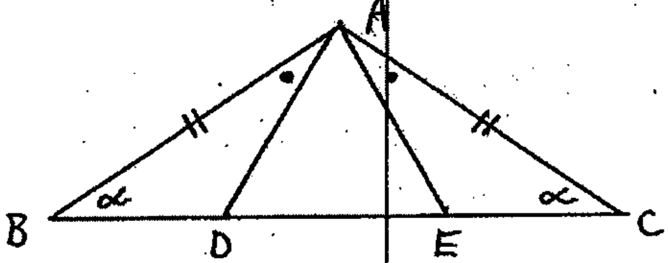
$2x \leq -4$

$x \leq -2$ (1)

c)



d)



i) $AB = AC$ Given

$\angle B = \angle C$ Base ∠'s isos. Δ =. (M1)

$\angle BAD = \angle CAE$ Given

$\Delta BAD \equiv \Delta CAE$ A.A.S (1)

ii) $AD = AE$ Corr. sides ≡ Δ's =. (1)

∴ ΔDAE is isosceles

Q8.

a) $f(x) = x^3 - x$

$f(-x) = -x^3 + x$ (M1)

$= -(x^3 - x)$

$\therefore$ since $f(-x) = -f(x)$ function is odd (1)

b) $b^2 - 4ac < 0$

$(k+2)^2 - 4(1)(4) < 0$

$k^2 + 4k + 4 - 16 < 0$ (M1)

$k^2 + 4k - 12 < 0$

$(k+6)(k-2) < 0$

$\therefore -6 < k < 2$ (1)

c) $$\frac{(\cancel{3x+1})(\cancel{x+2})}{(x+2)(\cancel{x+2})} \text{ (M1)} \times \frac{(\cancel{3x-1})(x-y)}{(\cancel{3x+1})(\cancel{3x-1})} \text{ (M1)}$$

$$= \frac{x-y}{x+2} \quad (1)$$

d) $3x^2 - 16x + 17 \equiv A(x^2 - 4x + 4) + Bx - 2B + C - 5$

$\equiv Ax^2 - 4Ax + 4A + Bx - 2B + C - 5$

$\equiv Ax^2 + (B - 4A)x + 4A - 2B + C - 5$

$\therefore A = 3$ (1)

$B - 4A = -16$

$B - 12 = -16$

$B = -4$ (1)

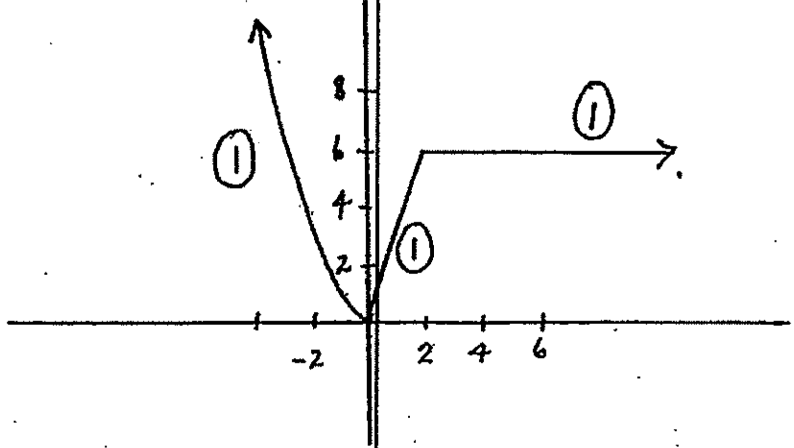
$4A - 2B + C - 5 = 17$

$12 + 8 + C - 5 = 17$

$C = 2$ (1)

Q10.

a)



b) $\frac{dy}{dx} = \frac{(x+1)(1) - (x+4)(1)}{(x+1)^2}$

$= \frac{x+1-x-4}{(x+1)^2}$

$= \frac{-3}{(x+1)^2}$ (M1)

when $x=2$,

$m_T = \frac{-3}{9}$ $\quad y = \frac{6}{3}$

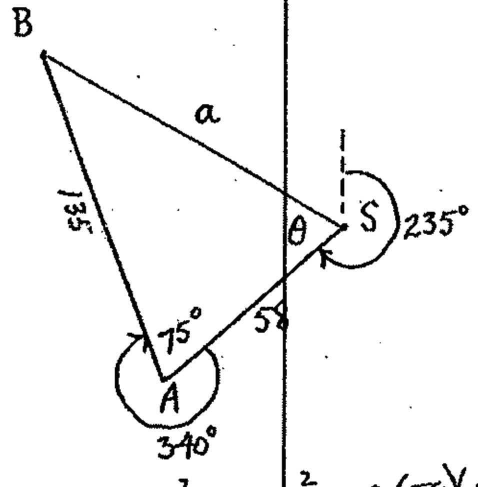
$= -\frac{1}{3}$ (M1) $\quad = 2$ $\quad (2, 2)$

$y - 2 = -\frac{1}{3}(x-2)$

$3y - 6 = -x + 2$

$x + 3y - 8 = 0$ (1)

c) i)



$a^2 = 58^2 + 135^2 - 2(58)(135)(\cos 75°)$ (M1)

$a = 132.4$ km (1)

ii) $\frac{\sin \theta}{135} = \frac{\sin 75°}{132.4}$

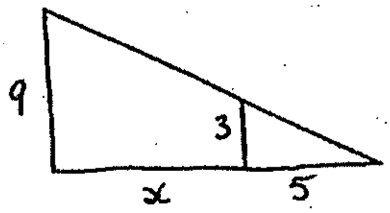
$\theta = \sin^{-1}\left[\frac{135 \sin 75°}{132.4}\right]$

$\theta = 80°$ (1)

Bearing is $235 + 80$

$= 315°$ (1)

Q9. a)



$$\frac{3}{9} = \frac{5}{x+5} \quad \text{M1}$$

$$3x + 15 = 45$$
$$3x = 30$$
$$x = 10 \quad 1$$

b)

$$4(x+4) = 7(x+5) \quad \text{M1}$$
$$4x + 16 = 7x + 35$$
$$-19 = 3x$$
$$x = -\frac{19}{3} \quad 1$$

c)

$$y = 2x - 5$$
$$y = x^2 - 4x$$
$$\therefore x^2 - 4x = 2x - 5 \quad \text{M1}$$
$$x^2 - 6x + 5 = 0$$
$$(x-5)(x-1) = 0$$
$$x = 5 \qquad x = 1$$
$$y = 5 \quad 1 \qquad y = -3 \quad 1$$

d)

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{(x-2)}(x+4)} \quad \text{M1}$$

$$= \lim_{x \to 2} \frac{x^2 + 2x + 4}{x+4} \quad \text{M1}$$

$$= \frac{12}{6}$$

$$= 2 \quad 1$$