TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 11 – 2011 YEARLY EXAMINATION

MATHEMATICS

ASSESSMENT TASK 4
WEIGHTING 40%

Monday 22nd August 2011
8.45 a.m. – 11.15 a.m.

OUTCOMES REFERRED TO: P2, P3, P4, P5, P6, P7, P8.

General Instructions

- Reading time – **5 minutes**.
- Working time – **$2\frac{1}{2}$ hours**.
- Write using blue or black pen.
- Board-approved calculators may be used.
- All necessary working should be shown in every question.
- **Begin** each question on a **new page**.
- Write your **name, class** and your **teacher's name** on the front of each answer sheet of paper.

Total marks – 100

- Attempt Questions 1-10.
- All questions are of equal value.
- Mark values are shown at the side of each question part.

Question 1. (10 marks) **Start each question a new page.**

Write your name and your teacher's name at the top of each page.

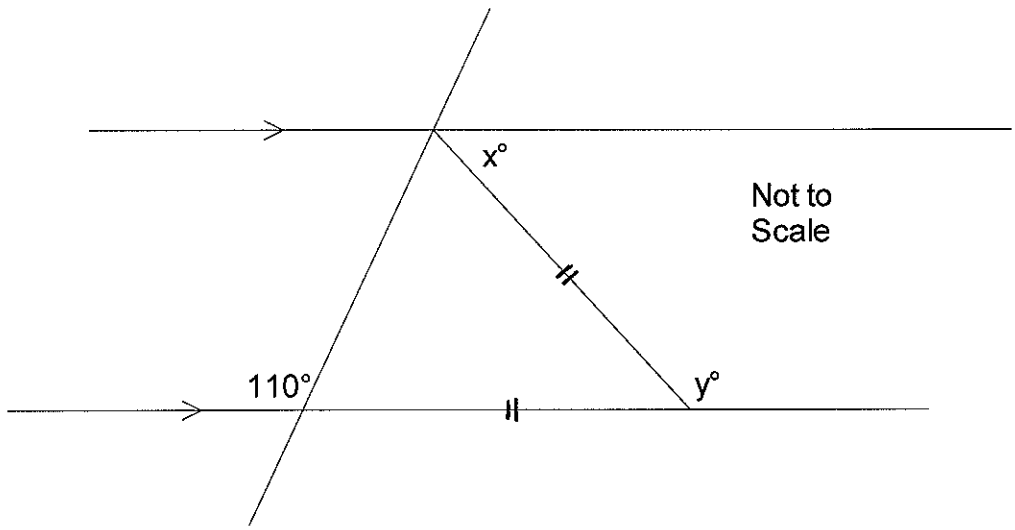
(a) Evaluate $\dfrac{\sqrt{\pi^2+9}}{2.1}$ correct to:

(i) 2 decimal places. **1**

(ii) 2 significant figures. **1**

(b) Factorise $a^2+13a-30$. **2**

(c) Find the values of x and y. **2**



(d) The marshal paid \$139.50 for a meal at a restaurant. This included a 12.5% tip. What was the cost of the meal without the tip? **2**

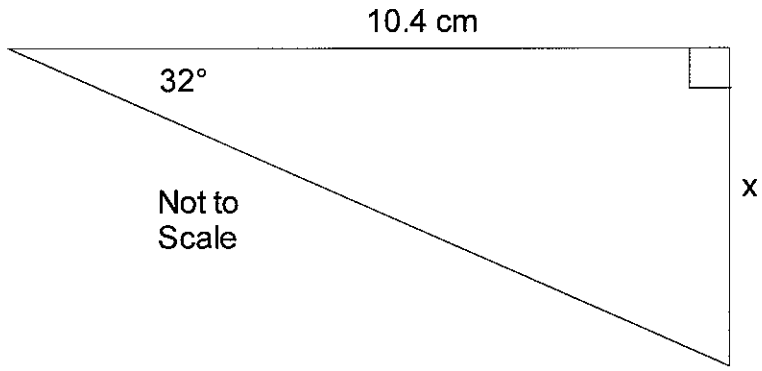
(e) Expand and simplify $(2\sqrt{3}-1)(\sqrt{3}+4)$. **2**

Question 2. (10 marks) Start each question a new page.
Write your name and your teacher's name at the top of each page.

(a) State the equation of the horizontal line passing through (-5, 1). **1**

(b) Solve $|2x+1| \leq 7$. **2**

(c) Find the value of x correct to 1 decimal place. **2**



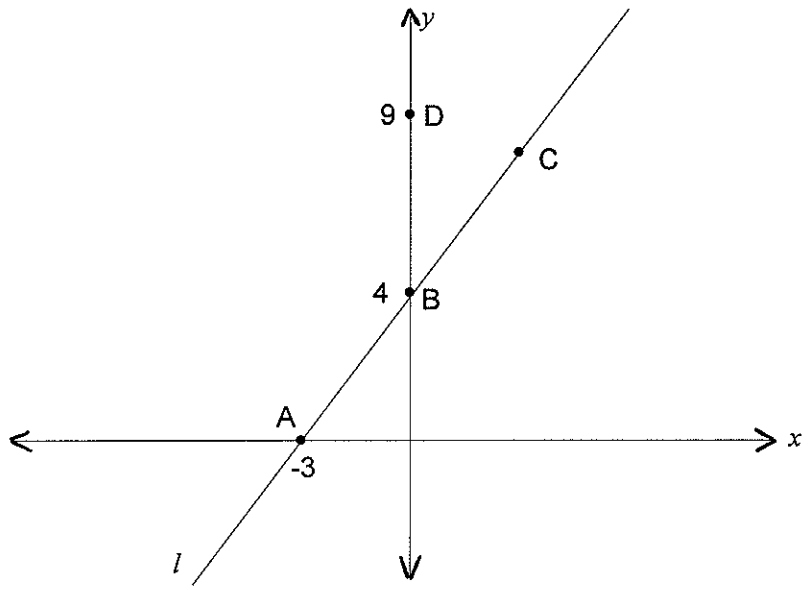
(d) Solve the following pair of simultaneous equations. **3**

$$3x - 2y = 10$$
$$y = 2 - 2x$$

(e) Find the integers a and b such that $\dfrac{1}{2-\sqrt{3}} = a + b\sqrt{3}$. **2**

Question 3. (10 marks) Start each question a new page.
Write your name and your teacher's name at the top of each page.

In the diagram below the line l cuts the x-axis at A(-3, 0) and the y-axis at B(0, 4).
The point D has the coordinates (0, 9).



(a) Find the gradient of the line l. **1**

(b) Hence, show that the equation of the line l is $4x-3y+12=0$. **1**

(c) The point C lies on l and B is the midpoint of the interval AC.
Find the coordinates of C. **2**

(d) Find the length of AD. **2**

(e) Show that AD is perpendicular to DC. **2**

(f) Find the equation of the line parallel to DC that passes through A. **2**

Question 4. (10 marks) **Start each question a new page.**
Write your name and your teacher's name at the top of each page.

(a) Find the exact value of $\sec 30^\circ$. **1**

(b) State the domain of the function $y = \sqrt{x-8}$. **1**

(c) State the range of the function $y = |x| + 2$. **1**

(d) Solve $\dfrac{x-5}{x+2} = \dfrac{3}{2}$. **2**

(e) Simplify $\dfrac{x^3 - 27}{x^2 - 9}$. **2**

(f) Indicate on a diagram using shading, the region for which the following inequalities hold simultaneously:

$y \leq 1 - x^2$ and $y > x - 1$. **3**

Question 5. (10 marks) **Start each question a new page.**
Write your name and your teacher's name at the top of each page.

(a) Solve the equation $\tan\theta + 1 = 0$ *for* $0^\circ \le \theta \le 360^\circ$. **2**

(b) Differentiate with respect to x:

(i) $5x^3 + 2x - 6$. **1**

(ii) $\dfrac{6x-1}{3x+2}$. **2**

(iii) $4\sqrt{x} - \dfrac{1}{x^2}$. **2**

(c) Differentiate from first principles $f(x) = 4x^2 - 3x$. **3**

Question 6. (10 marks) **Start each question a new page.**

Write your name and your teacher's name at the top of each page.

(a) If α and β are the roots of the equation $2x^2 + x - 4 = 0$, find the value of:

(i) $\alpha\beta + \alpha + \beta$. **2**

(ii) $\frac{1}{\alpha} + \frac{1}{\beta}$. **2**

(iii) $\alpha^2 + \beta^2$. **2**

(b) Simplify $\sqrt{1 - \cos^2 \theta}$. **1**

(c) Sketch on separate diagrams, the graphs of each of the following:

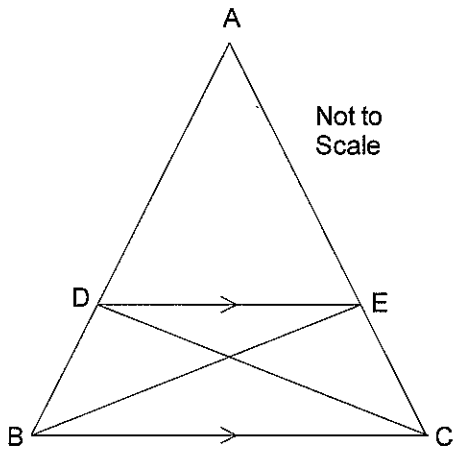
(i) $y = (x+2)^2$. **1**

(ii) $x^2 + (y-2)^2 = 4$. **2**

Question 7. (10 marks) Start each question a new page.
Write your name and your teacher's name at the top of each page.

(a) Evaluate $\lim_{x \to -1} \dfrac{4x+4}{x^2+5x+4}$. **2**

(b) In the diagram, ABC is an isosceles triangle where AB = AC and DE is parallel to BC.



Copy or trace your diagram on to your paper.

(i) Show that ADE is an isosceles triangle. **2**

(ii) Show that DB = EC. **2**

(iii) Show that triangles DBC and ECB are congruent. **2**

(c) Factorise fully $12e^2 - 14e - 6$. **2**

Question 8. (10 marks) Start each question a new page.
Write your name and your teacher's name at the top of each page.

(a) Show whether the function $f(x) = x^2 + x$ is odd, even or neither. **2**

(b) Find the values of k for which the expression $kx^2 - 2kx + 3$ is positive definite. **2**

(c) Sketch the function $f(x) = \begin{cases} 2 & \text{if } x < -2 \\ -x & \text{if } -2 \le x < 0 \\ x^2 & \text{if } x \ge 0 \end{cases}$. **3**

(d) Express the following with a rational denominator in its simplest form.

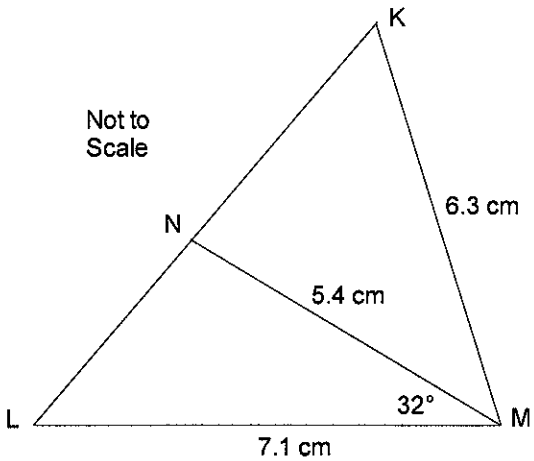
$$\frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}}$$ **3**

Question 9. (10 marks) Start each question a new page.
Write your name and your teacher's name at the top of each page.

(a) Sketch the graph of $y = \cos x + 1$ *for* $0^\circ \le x \le 360^\circ$. **2**

(b) Prove $\left(\sec^2\theta - 1\right)\cos^2\theta = \sin^2\theta$. **2**

(c) In the figure below, KM = 6.3 cm, LM = 7.1 cm, NM = 5.4 cm and $\angle NML = 32^\circ$.



Copy or trace the diagram on to your paper.

(i) Find the size of $\angle LNM$. **2**

(ii) Find the length of NK. **3**

(iii) Find the area of ΔKLM. **1**

Question 10. (10 marks) Start each question a new page.
Write your name and your teacher's name at the top of each page.

(a) Solve the following pair of simultaneous equations.

$$4a + 3b = 3$$
$$3a - 2b = -19$$

3

(b) Solve $\left(x+\frac{1}{x}\right)^2 - 6\left(x+\frac{1}{x}\right)+8=0$. **3**

(c) Find the equation of the normal to the curve $y=(2x+1)^2$ at the point where x = -1. **4**

End of paper

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Q1. 2.0685531329

a) (i) 2.07 (1)

(ii) 2.1 (1)

b) $a^2 + 13a - 30$

$= (a + 15)(a - 2)$ (1) (1)

c) $x = 40°$ (1)

$y = 140°$ (1)

d) $1.125x = 139.50$

$x = \frac{139.50}{1.125}$ (M1)

$= \#124$ (1)

e) $(2\sqrt{3} - 1)(\sqrt{3} + 4)$

$= 2(3) + 8\sqrt{3} - \sqrt{3} - 4$ (M1)

$= 2 + 7\sqrt{3}$ (1)

Q2.

a) $y = 1$ (1)

b) $-7 \leq 2x + 1 \leq 7$ (M1)

$-8 \leq 2x \leq 6$

$-4 \leq x \leq 3$ (1)

c) $\tan 32^\circ = \frac{x}{10.4}$ (M1)

$x = 10.4 \times \tan 32^\circ$

$= 6.5$ cm (1 d.p.) (1)

d) $3x - 2(2 - 2x) = 10$ (M1)

$3x - 4 + 4x = 10$

$7x = 14$

$x = 2$ (1)

$y = 2 - 2(2)$

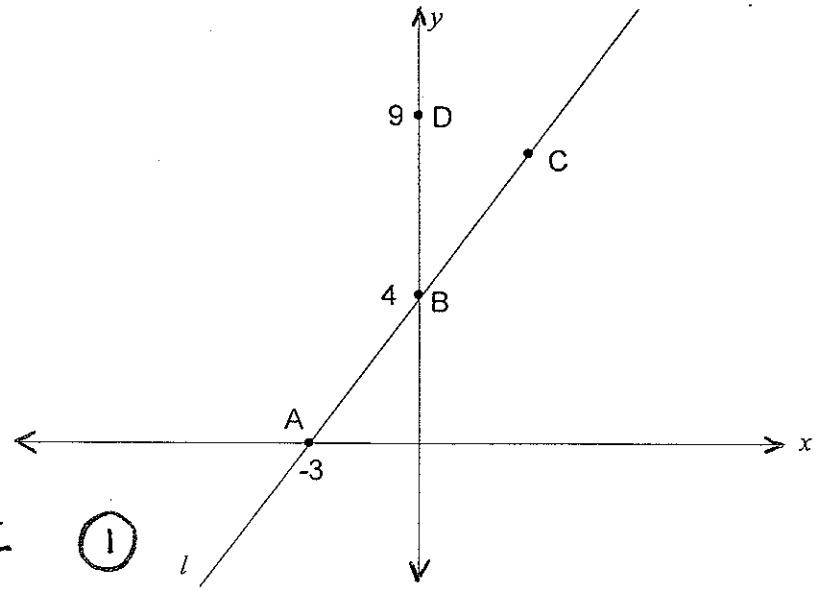
$= -2$ (1)

e) $\frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$

$= \frac{2 + \sqrt{3}}{4 - 3}$

$= 2 + \sqrt{3}$ where $a = 2$ and $b = 1$ (1) (1)

Q3.



a) $m_{AC} = \frac{4}{3}$ (1)

b) $y - 4 = \frac{4}{3}(x - 0)$ (M1)
$3y - 12 = 4x$
$\therefore 4x - 3y + 12 = 0$

c) $\frac{x + (-3)}{2} = 0$ $\quad \frac{y + 0}{2} = 4$
$\therefore x = 3$ (1) $\quad y = 8$ (1)

d) $AD = \sqrt{(-3-0)^2 + (0-9)^2}$ (M1)
$= \sqrt{9 + 81}$
$= \sqrt{90}$ OR $3\sqrt{10}$ (1)

e) $m_{AD} = \frac{9}{3}$ $\quad m_{DC} = \frac{8-9}{3-0}$ (M1)
$= 3$ $\quad = -\frac{1}{3}$
$AD \perp DC$ since $3 \times (-\frac{1}{3}) = -1$ (1)

f) $y - 0 = -\frac{1}{3}(x - (-3))$ (M1)
$y - 0 = -\frac{1}{3}(x + 3)$
$y = -\frac{1}{3}x - 1$ (1)

Q4.

a) $\sec 30^\circ = \frac{2}{\sqrt{3}}$ OR $\frac{2\sqrt{3}}{3}$ (1)

b) Domain: $x \in \mathbb{R}, x \geq 8$ (1)

c) Range: $y \in \mathbb{R}, y \geq 2$ (1)

d) $\frac{x-5}{x+2} = \frac{3}{2}$

$2x - 10 = 3x + 6$ (M1)

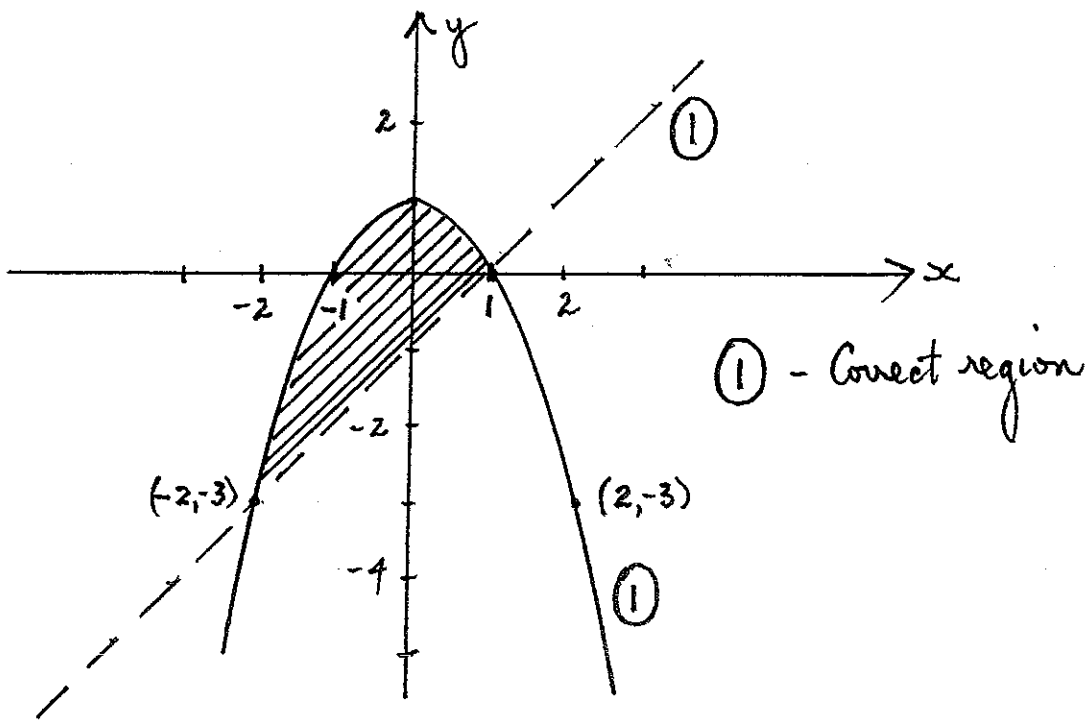
$-16 = x$

$x = -16$ (1)

e) $\frac{(x-3)(x^2+3x+9)}{(x-3)(x+3)}$ (M1)

$= \frac{x^2+3x+9}{x+3}$ (1)

f)



(1)

(1) - Correct region

(1)

Q5

a) $\tan\theta = -1$

$\therefore \theta = 135°, 315°$ (1) (1)

b) i) $\frac{d}{dx}(5x^3 + 2x - 6)$

$= 15x^2 + 2$ (1)

ii) $y' = \frac{(3x+2)(6) - (6x-1)(3)}{(3x+2)^2}$ (M1)

$= \frac{18x + 12 - 18x + 3}{(3x+2)^2}$

$= \frac{15}{(3x+2)^2}$ (1)

iii) $f(x) = 4x^{\frac{1}{2}} - x^{-2}$

$f'(x) = 2x^{-\frac{1}{2}} + 2x^{-3}$ (1) (1)

c) $f(x) = 4x^2 - 3x$

$f(x+h) = 4(x^2 + 2xh + h^2) - 3(x+h)$

$= 4x^2 + 8xh + 4h^2 - 3x - 3h$

$f'(x) = \lim_{h\to 0} \frac{4x^2 + 8xh + 4h^2 - 3x - 3h - (4x^2 - 3x)}{h}$ (M1)

$= \lim_{h\to 0} \frac{4x^2 + 8xh + 4h^2 - 3x - 3h - 4x^2 + 3x}{h}$

$= \lim_{h\to 0} \frac{h(8x + 4h - 3)}{h}$ (M1)

$= \lim_{h\to 0} 8x + 4h - 3$

$= 8x - 3$ (1)

Q6

a) $\alpha + \beta = -\frac{1}{2}$ $\quad \alpha\beta = \frac{-4}{2} = -2$

i) $\alpha\beta + (\alpha + \beta)$
$= -2 - \frac{1}{2}$ (M1)
$= -2\frac{1}{2}$ (1)

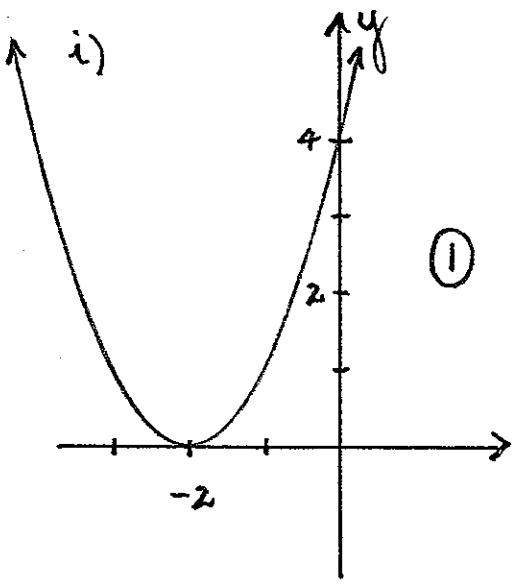
ii) $\frac{1}{\alpha} + \frac{1}{\beta}$
$= \frac{\beta + \alpha}{\alpha\beta}$ (M1)
$= \frac{-\frac{1}{2}}{-2}$
$= \frac{1}{4}$ (1)

iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$= (-\frac{1}{2})^2 - 2(-2)$ (M1)
$= 4\frac{1}{4}$ (1)

b) $\sqrt{1 - \cos^2\theta}$
$= \sqrt{\sin^2\theta}$
$= \sin\theta$ (1)

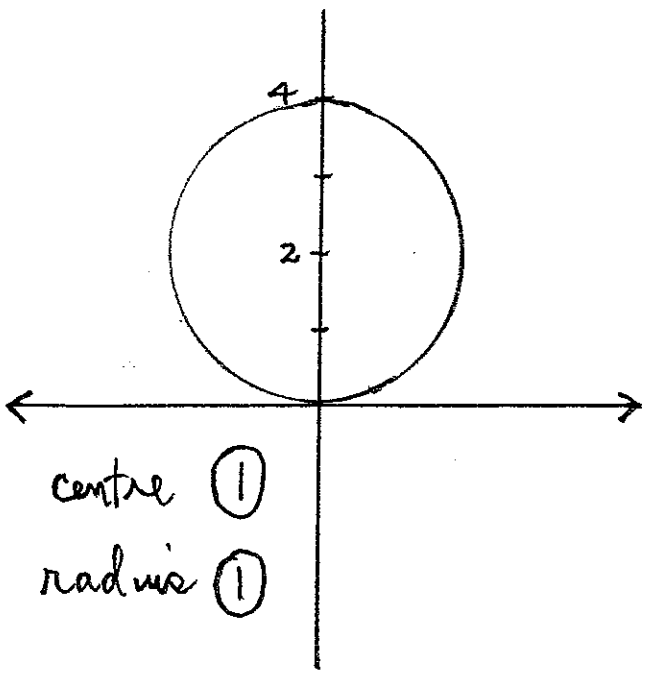
c)

i)



(1)

ii)



centre (1)

radius (1)

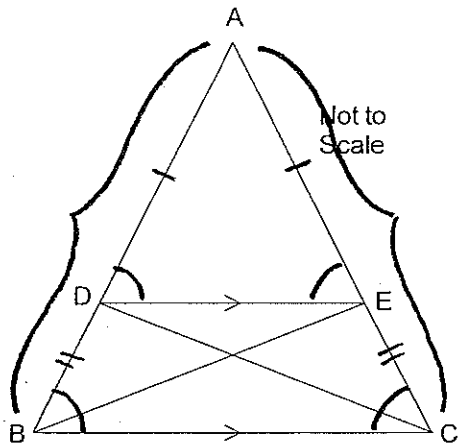
Q7.

a) $\lim_{x \to -1} \frac{4(x+1)}{(x+1)(x+4)}$ (M1)

$= \lim_{x \to -1} \frac{4}{x+4}$

$= \frac{4}{3}$ (1)

b)



(absolutely important to indicate 'equality' and/or direct reference to parallel lines.)

i) $\angle ABC = \angle ACB$ — $\triangle ABC$ is isosceles

$\angle ADE = \angle ABC$, $\angle AED = \angle ACB$ — Corr. $\angle$'s $=$. ($DE \parallel BC$) (1)

$\therefore \angle ADE = \angle AED$ — Equal $\angle$'s are equal (1)

$\therefore \triangle ADE$ is isosceles

any other valid approach but must use correct reasons, that apply to that method.

ii) $AB = AC$ — Given

$AD = AE$ — $\triangle ADE$ is isosceles (1)

$AB - AD = AC - AE$ — Equals subtracted from equals are $=$. (1)

$\therefore DB = EC$

iii) $DB = EC$, $\angle ABC = \angle ACB$ — Proven above

$BC = BC$ — BC is a common side (1)

$\triangle DBC \equiv \triangle ECB$ — S.A.S (1)

c) $2(6e^2 - 7e - 3)$ (1)

$= 2(3e + 1)(2e - 3)$ (1)

(must be completely factorised for last mark)

Q8.

a) $f(x) = x^2 + x$

$f(-x) = (-x)^2 + (-x)$ (M1)

$= x^2 - x$

$f(-x) \neq f(x)$

$f(-x) \neq -f(x)$ $\therefore$ $f(x)$ is neither odd or even (1)

b) $\Delta < 0$ and $k > 0$ (M1)

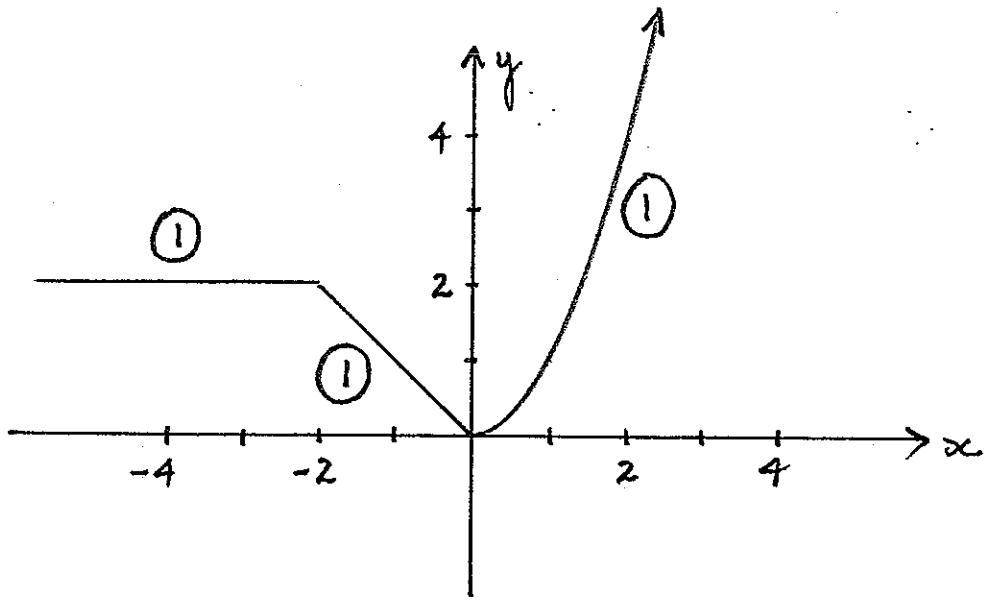
$(-2k)^2 - 4(k)(3) < 0$

$4k^2 - 12k < 0$

$4k(k-3) < 0$

$0 < k < 3$ (1)

c)



d) $\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$

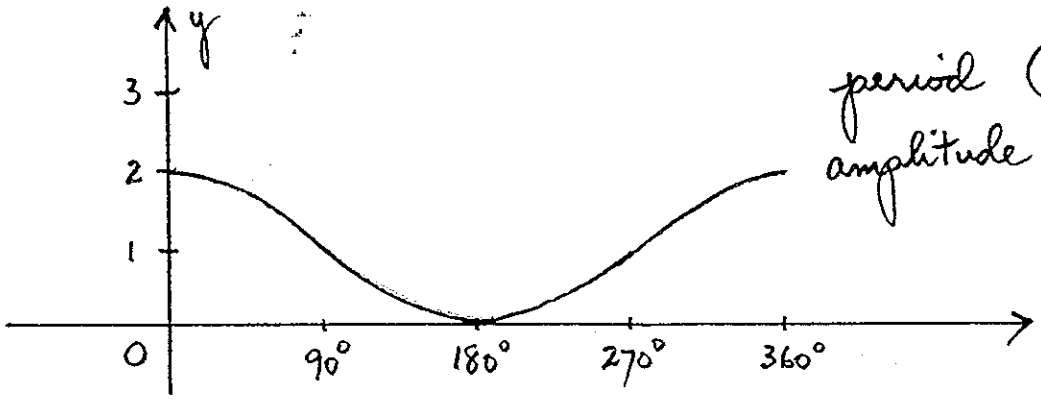
$= \frac{18 + 12\sqrt{6} + 12}{18 - 12}$ (M1)

$= \frac{30 + 12\sqrt{6}}{6}$ (M1)

$= 5 + 2\sqrt{6}$ (1)

Q9.

a)



period (1)

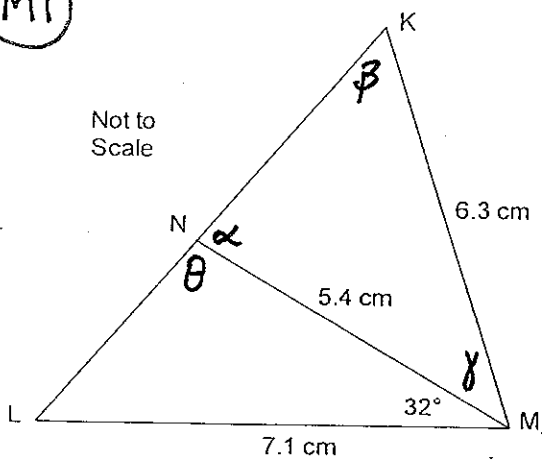
amplitude (1)

b) $(\sec^2\theta - 1)\cos^2\theta$

$= \tan^2\theta \cos^2\theta$ (M1)

$= \frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta$

$= \sin^2\theta$ (1)



c) i)

$LN^2 = 5.4^2 + 7.1^2 - 2(5.4)(7.1)\cos 32°$

$= 14.54167199$

$LN = 3.8$ cm (1 d.p) (M1)

ii) $\frac{\sin \angle LNM}{7.1} = \frac{\sin 32°}{3.813354427}$

$\angle LNM = \sin^{-1}\left(\frac{7.1 \times \sin 32°}{3.813354427}\right)$

$\theta = 80°38'$ OR $80.6°$ (1)

ii) $\therefore \alpha = 180 - 80.6$

$= 99.4°$ (1)

$\frac{\sin 99.4}{6.3} = \frac{\sin \angle NKM}{5.4}$

$\beta = \sin^{-1}\left[\frac{5.4(\sin 99.4)}{6.3}\right]$

$= 57.7°$ (1)

$\angle NMK = 180 - (57.7 + 99.4)$

$\gamma = 22.9°$

$\frac{NK}{\sin 22.9°} = \frac{6.3}{\sin 99.4°}$ (1)

$NK = \frac{6.3(\sin 22.9°)}{\sin 99.4} = 2.5$ cm

iii) $A = \frac{1}{2}(7.1)(6.3)\sin(32° + 22.9°)$

$= 18.3$ cm^2 (1)

$\cos\theta = \frac{3.8133...^2 + 5.4^2 - 7.1^2}{2(3.8133...)(5.4)}$

$= -0.162858514$

$\therefore \theta = 99°22'$ OR $99.4°$ (1)

ii) $\therefore \alpha = 180 - 99.4$

$= 80.6°$

$\frac{\sin 80.6}{6.3} = \frac{\sin \angle NKM}{5.4}$

$\beta = \sin^{-1}\left[\frac{5.4(\sin 80.6°)}{6.3}\right]$

$= 57.7°$ (1)

$\angle NMK = 180 - (80.6 + 57.7)$

$\gamma = 41.7°$

$\frac{NK}{\sin 41.7°} = \frac{6.3}{\sin 80.6}$ (1)

$NK = \frac{6.3(\sin 41.7)}{\sin 80.6} = 4.2$ cm

$A = \frac{1}{2}(7.1)(6.3)\sin(32 + 41.7)$

$= 21.5$ cm^2 (1)

Q10.

a) $4a + 3b = 3 \quad \cdots ①$
$3a - 2b = -19 \quad \cdots ②$
$8a + 6b = 6$
$9a - 6b = -57$ (M1)

$17a = -51$
$a = -3$ (1)

$4(-3) + 3b = 3$
$3b = 15$
$b = 5$ (1)

b) let $w = x + \frac{1}{x}$
$w^2 - 6w + 8 = 0$
$(w-4)(w-2) = 0$ (M1)

$x + \frac{1}{x} - 4 = 0$
$x^2 - 4x + 1 = 0$
$x = \frac{-(-4) \pm \sqrt{16 - 4(1)(1)}}{2(1)}$
$= \frac{4 \pm 2\sqrt{3}}{2}$
$= 2 \pm \sqrt{3}$ (1)

$x + \frac{1}{x} - 2 = 0$
$x^2 - 2x + 1 = 0$
$(x-1)(x-1) = 0$
$x = 1$ (1)

c) $y = (2x+1)^2$
$\frac{dy}{dx} = 4(2x+1)$ (M1)
when $x = -1$, $y = (-2+1)^2 = 1$ $(-1, 1)$ (1)

$m = \frac{dy}{dx} = 4(-1)$
$= -4$
$\perp m = \frac{1}{4}$ (1)

$y - 1 = \frac{1}{4}(x+1)$
$y = \frac{1}{4}x + \frac{5}{4}$ OR $x - 4y + 5 = 0$ (1)