Name: …….

Class Teacher: …….

*Do **NOT** write solutions on this question paper. Any working on the question paper will **NOT** be marked.*

TRINITY GRAMMAR SCHOOL

Mathematics Department

2012

YEARLY EXAMINATION

PRELIMINARY ASSESSMENT TASK 4

Year 11

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2½ hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- Show all necessary working in Questions 11 – 16
- Write your Name **and** Class Teacher on each of the writing booklet(s) submitted
- **ASSESSMENT WEIGHTING:** 40%

Total marks – 100

Section I Pages 3 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 12

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours 15 minutes for this section

Blank page

Section I **10 marks**

- Circle the correct response on the answer sheet provided
- Each question is worth 1 mark

---

1 Use your calculator to evaluate $\dfrac{112.8}{16.1\times 2.93}$ correct to 3 significant figures

(A) 2.39 (B) 2.391 (C) 20.5 (D) 20.528

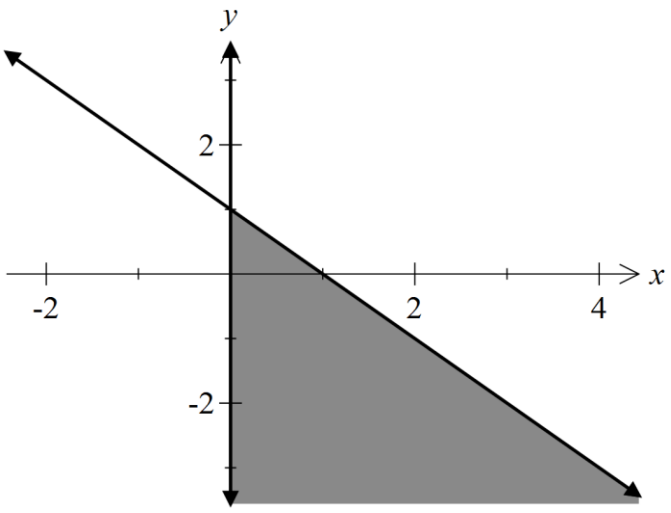
2 Simplify $\dfrac{a^6\times(a^8)^2}{a^2}$

(A) a^8 (B) a^{11} (C) a^{14} (D) a^{20}

3 Make y the subject of the equation $2y=1-xy$

(A) $y=\dfrac{1-x}{3}$ (B) $y=\dfrac{1}{2+x}$ (C) $y=\dfrac{1-xy}{2}$ (D) $y=\dfrac{1-2y}{x}$

4

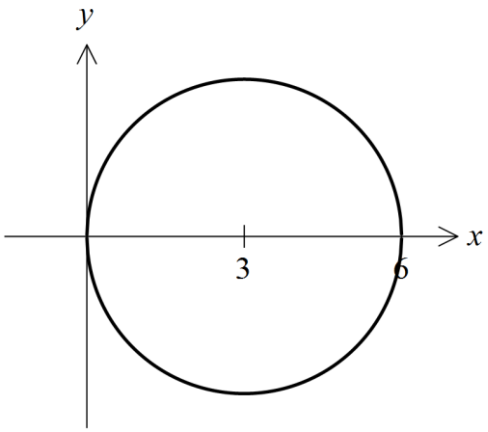


Which pair of inequalities best describes the shaded region in the above diagram

(A) $x+y\le 1$ and $x\ge 0$ (B) $x+y\le 1$ and $y\ge 0$

(C) $x+y\ge 1$ and $x\ge 0$ (D) $x+y\ge 1$ and $y\ge 0$

5



The equation of the circle drawn above is

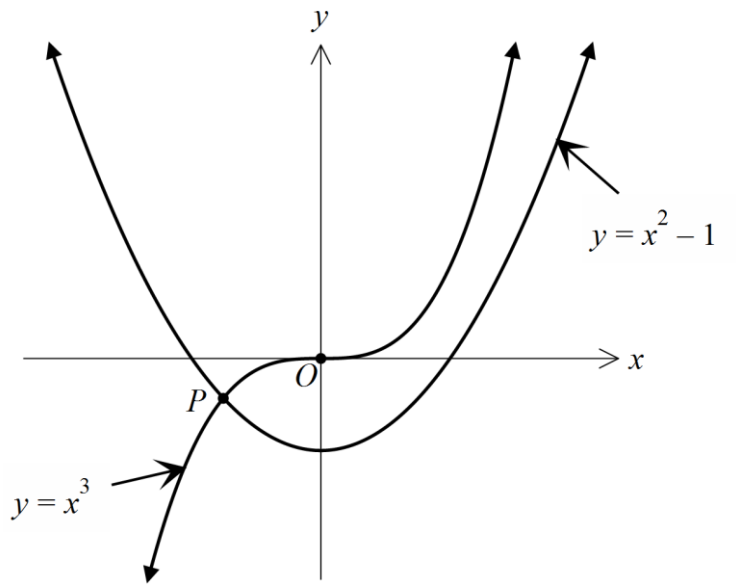
(A) $(x+3)^2+y^2=9$

(B) $(x+3)^2+y^2=36$

(C) $(x-3)^2+y^2=9$

(D) $(x-3)^2+y^2=36$

6



The graphs of $y=x^3$ and $y=x^2-1$ intersect at P.
Which equation gives the x co-ordinate of P?

(A) $x^2-x^3+1=0$ (B) $x^3-x^2+1=0$

(C) $x^3+x^2-1=0$ (D) $x^3+x^2+1=0$

7 Factorise $y^2-(x+3)^2$

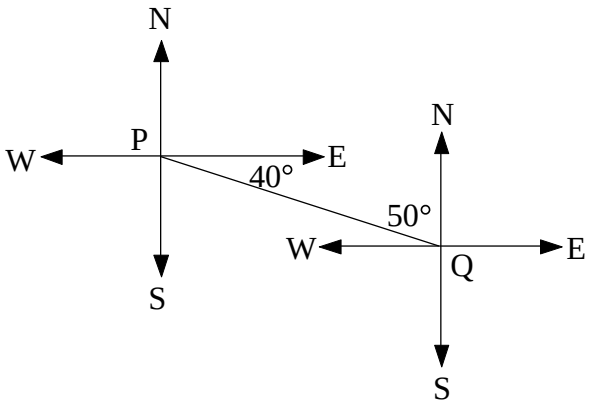
(A) $(y-x+3)(x+y+3)$

(B) $(y-x-3)(x+y+3)$

(C) $(y-x+3)^2$

(D) $(y-x-3)^2$

8



The bearing of P from Q is

(A) 040° (B) 050° (C) 130° (D) 310°

9 Given $180° < q < 270°$, which of the following is correct?

(A) $\sin\theta > 0$ and $\tan\theta > 0$

(B) $\sin\theta > 0$ and $\tan\theta < 0$

(C) $\sin\theta < 0$ and $\tan\theta > 0$

(D) $\sin\theta < 0$ and $\tan\theta < 0$

10 The expression $\dfrac{b+\sqrt{2b}}{\sqrt{b}}$ =

(A) $\sqrt{2}+b$ (B) $\sqrt{3b}$ (C) $3\sqrt{b}$ (D) $\sqrt{2}+\sqrt{b}$

End of Section I

Section II 90 marks

- Begin each question in a new writing booklet.
- Write your name **and** your class teacher's name on each answer booklet used
- Show all necessary working

Question 11 (15 marks)

(a) Fully factorise $x^4 - 27x$. **2**

(b) Solve $|3x-4|<8$. **2**

(c) Graph the solution to $x^2 - 11x + 24 > 0$ on a real number line. **2**

(d) Simplify $\dfrac{4}{m^2-4} - \dfrac{1}{m-2}$. **3**

(e) Solve simultaneously $\begin{cases} 3a-4b=11 \\ 2a+3b=-4 \end{cases}$. **3**

(f) Find the values of a and b such that $\dfrac{\sqrt{5}}{\sqrt{5}-2} = a+\sqrt{b}$. **3**

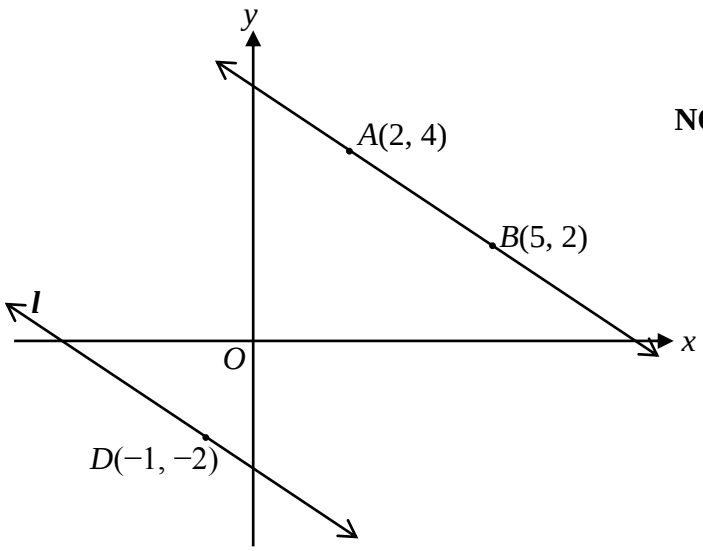
Question 12 (15 marks) **Begin a NEW answer booklet**

(a) (i) Sketch the graph of $y = \dfrac{1}{x-3}$, showing its main features. **2**

(ii) State the domain for the function $y = \dfrac{1}{x-3}$. **1**

(b) Given $f(x) = x^{-2} + \dfrac{2}{x}$, simplify $f\left(\dfrac{1}{a}\right)$. **2**

(c) Consider the diagram below with points $A(2, 4)$ and $B(5, 2)$.



NOT TO SCALE

(i) Find the gradient of the line through AB. **2**

(ii) Show that the equation of the line $\boldsymbol{l}$ parallel to the line AB and passing through $D(-1, -2)$ is $2x + 3y + 8 = 0$. **2**

(iii) Show the distance OA is $2\sqrt{5}$ units. **2**

(iv) Find the equation of the circle with centre at the origin and passing through A. **1**

(v) The radius AO of the circle in part (iv) is produced to C on the circumference. Find the co-ordinates of the point C. **2**

(vi) State the perpendicular distance from the point B to the line $\boldsymbol{l}$. **1**

Question 13 (15 marks) **Begin a NEW answer booklet**

(a) Solve the following equations in the domain $0° \le \theta \le 360°$.

(i) $\sin\theta = \frac{1}{2}$ **2**

(ii) $\cos 2\theta = 0$ **2**

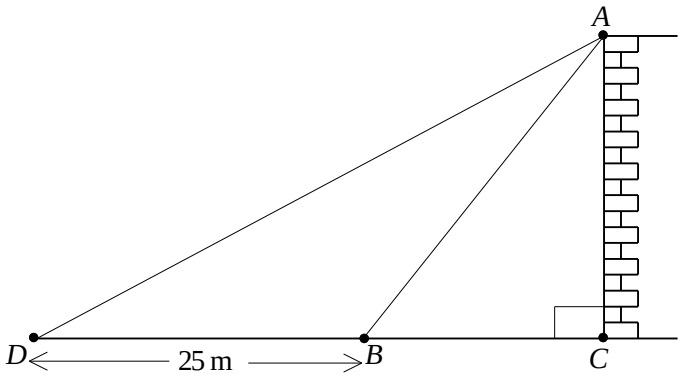
(iii) $\tan\theta = -\sqrt{3}$ and $\sin\theta < 0$ **2**

(b) Simplify $\frac{\sin^2\theta}{1-\cos\theta} - \frac{\sin^2\theta}{1+\cos\theta}$ **2**

(c) (i) Show that $f(x) = x\sqrt{4-x^2}$ is an odd function. **2**

(ii) Hence, or otherwise, if $f\ x-1\ = \sqrt{3}$, state the value of $f\ 1-x\ $. **1**

(d)



NOT TO SCALE

A person at *D* observes the angle of elevation of the top of a building *A* to be 28^o.

The person walks from *D*, 25m towards the building across a horizontal plane to a point *B* and finds the angle of elevation now to be 32^o.

Copy the diagram into your writing booklet and label all the given information.

(i) Explain why $\angle DAB = 4°$. **1**

(ii) Using the Sine Rule, find correct to one decimal place, the length *BA*. **2**

(iii) Find the height of the building, correct to one decimal place. **1**

Question 14 (15 marks) **Begin a NEW answer booklet**

(a) Find real constants *A*, *B* and *C* such that $2x^2 + x + 1 \equiv Ax\ (x-1) + B\ (x-1) + C$. **3**

(b) The roots of the equation $2x^2 - 7x + 12 = 0$ are α and β. Find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **2**

(iv) $\alpha^2\beta + \beta^2\alpha$ **2**

(v) $\alpha^2 + \beta^2$ **2**

(c)(i) Write down in simplest form, the discriminant
$(4k+1)\ x^2 - 2\ (k+1)\ x - (1-2k) = 0$. **2**

Hence, find the value of *k* such that the roots of $(k-1)\ x^2 + (k+1)\ x - 2 = 0$ are equal.

(ii) Find the value of *k* in the equation $(k-1)\ x^2 + (k+1)\ x - 2 = 0$ so that one root is the reciprocal of the other. **2**

(**Hint:** Let the roots be α and $\dfrac{1}{\alpha}$)

Question 15 (15 marks) **Begin a NEW answer booklet**

(a) A function is defined as $f(x) = \begin{cases} 1-x & x < 0 \\ x^2+1 & x \geq 0 \end{cases}$

(i) Find the value of $f(-2) + f(3)$. **2**

(ii) Sketch the graph of $y = f\ x\ $. **2**

(b) If $y = 5x^4 + \frac{3}{x} + 2\sqrt{x}$, find $\frac{dy}{dx}$. **3**

(c) Use the quotient rule to find $\frac{dy}{dx}$ if $y = \frac{x^2+1}{3-x}$. **2**

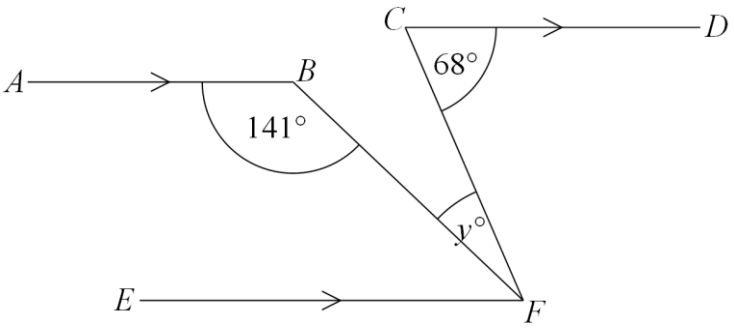
(d) If $f(x) = (2x-5)^4$ find the value of x such that $f'\ x\ = 0$. **2**

(e) Find the equation of the tangent to the curve $y = x^3 + 5x - 4$ at the point where $x = 1$. Leave your answer in the form $y = mx + b$. **2**

(f) Find the value of $\lim_{x \to 3} \left(\frac{x^2 - 9}{x-3} \right)$. **2**

Question 16 (15 marks) **Begin a NEW answer booklet**

(a)

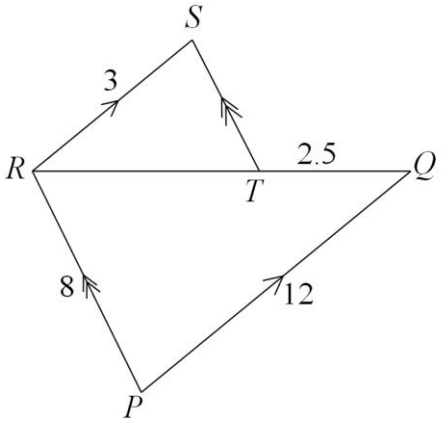


NOT TO SCALE

Copy this diagram into your writing booklet.

Using the information on the diagram above, find the value of y, giving reasons for your answer. **3**

(b)



NOT TO SCALE
(all measurements are in cm)

Copy this diagram into your writing booklet.

In the diagram above $RS \parallel PQ$ and $ST \parallel RP$.

(i) Prove that ΔRST is similar to ΔQRP. **3**

(ii) Find the length of ST, giving a reason for your answer. **2**

(c) Find the exact value of $\sin 150° \sec(-150°)$. **2**

(Question 16 continues on the following page)

Question 16 continued

(d) Sketch the graph of the parabola $x^2 - 6x + 9 + 12y = 0$, showing **and** stating the coordinates of its vertex, focus and the equation of the directrix. **3**

(e) The point $P(x, y)$ is equidistant from the lines $x = 2$ and $y = -4$. **2**
Find the equation of the locus of P.

End of Paper

TRINITY GRAMMAR SCHOOL
2012, Year 11 Mathematics
Yearly Examination, Preliminary Assessment Task 4
SECTION I
ANSWER SHEET

Name: …….

Class Teacher: …….

Be sure to write your answers for **Section I** on this answer sheet. After you have selected an answer, **CIRCLE** the correct answer. To change an answer, erase your previous mark completely, and then record your new answer. ***Mark only one answer for each question.***

1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
7	A	B	C	D
8	A	B	C	D
9	A	B	C	D
10	A	B	C	D

TRINITY GRAMMAR SCHOOL
2012, Year 11 Mathematics
Yearly Examination, Preliminary Assessment Task 4
SECTION I
ANSWER SHEET

Name: Solutions

Class Teacher: .....

Be sure to write your answers for **Section I** on this answer sheet. After you have selected an answer, **CIRCLE** the correct answer. To change an answer, erase your previous mark completely, and then record your new answer. ***Mark only one answer for each question.***

1	(A)	B	C	D
2	A	B	C	(D)
3	A	(B)	C	D
4	(A)	B	C	D
5	A	B	(C)	D
6	A	(B)	C	D
7	A	(B)	C	D
8	A	B	C	(D)
9	A	B	(C)	D
10	A	B	C	(D)

SECTION II

Question 1:

(a) $x(x^3 - 27)$ (A1)

$= x(x-3)(x^2 + 3x + 9)$ (A1)

(b) $3x - 4 < 8$ OR $-3x + 4 < 8$

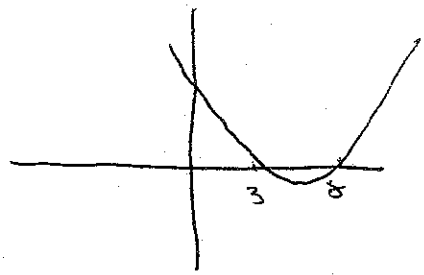
$3x < 12$ $-3x < 4$

$x < 4$ (A1) $x > -\frac{4}{3}$ (A1)

(c) $(x - 8)(x - 3) > 0$

1 mark / 2 marks for JUST factorising (A1)

$x > 8$ or $x < 3$ (A1)



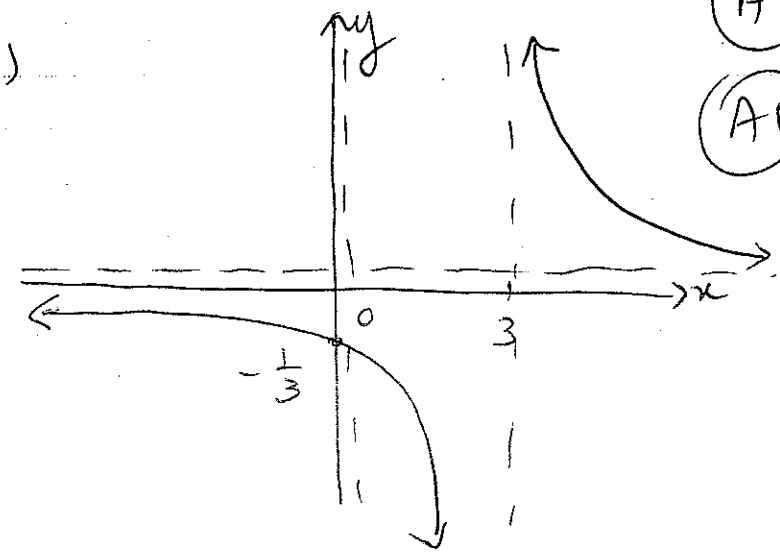
(d) (A1) $\frac{4}{(m-2)(m+2)} - \frac{1}{m-2}$

$= \frac{4 - (m+2)}{(m-2)(m+2)}$ (A1)

$= \frac{2 - m}{(m-2)(x+2)}$ (A1)

Question 12:

(a) (i)



A1 shape

A1 Asymptotes.

(ii) $D = \{x \mid x \neq 3,\ x \in \mathbb{R}\}$ A1

(b) $f\left(\frac{1}{a}\right) = \frac{1}{\left(\frac{1}{a}\right)^2} + \frac{2}{\frac{1}{a}}$ M1

$f\left(\frac{1}{a}\right) = a^2 + 2a$ A1

(c) (i) $m = \frac{4-2}{2-5}$ M1

$m = \frac{2}{-3}$ A1

(ii) $y + 2 = -\frac{2}{3}(x+1)$ M1

$3y + 6 = -2x - 2$

$2x + 3y + 8 = 0$ M1

(e) $6a - 8b = 22$ — (1)

$6a + 9b = -12$ — (2)

(1) − (2)

$-17b = 34$

$b = -2$ (M1)

$3a - 4(-2) = 11$

$3a = 3$

$a = 1$

$\left\{ \begin{array}{l} a = 1 \\ b = -2 \end{array} \right.$ (A1) (A1)

(f) $\dfrac{\sqrt{5}(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)}$ (M1)

$= \dfrac{5 + 2\sqrt{5}}{5 - 4}$

$5 + 2\sqrt{5} = a + \sqrt{b}$

$a = 5$ (A1)

$b = 20$ (A1)

(iii) $d = \sqrt{(2-0)^2 + (4-0)^2}$

$d = \sqrt{4 + 16}$ (M1)

$d = \sqrt{20}$

$d = 2\sqrt{5}$ units (A1)

(iv) $x^2 + y^2 = (2\sqrt{5})^2$

$x^2 + y^2 = 20.$ (A1)

(v) $(0,0) = \left(\frac{x+2}{2}, \frac{y+4}{2}\right)$ (M1)

$\frac{x+2}{2} = 0$ | $\frac{y+4}{2} = 0$

$x = -2$ | $y = -4$

$C(-2,-4)$ (A1)

(vi) $d = \frac{|5(2) + 3(2) + 8|}{\sqrt{2^2 + 3^2}}$ $B(5,2)$

$d = \frac{|10 + 6 + 8|}{\sqrt{13}}$ $\frac{24}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{24\sqrt{13}}{13}.$

$d = \frac{24}{\sqrt{13}}$ units. (A1)

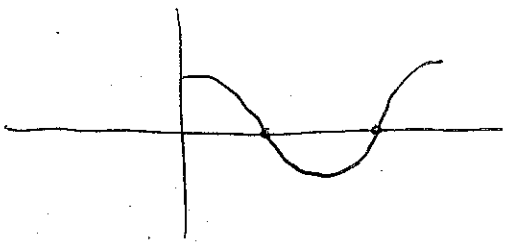
Question 13

(a) (i) $\sin 30^\circ = \frac{1}{2}$ ✓ ✓

$\theta = 30^\circ, 150^\circ$

A1 A1

(ii) $2\theta = 90^\circ, 270^\circ, 480^\circ, 630^\circ$ $\quad 0^\circ \leq 2\theta \leq 720^\circ$

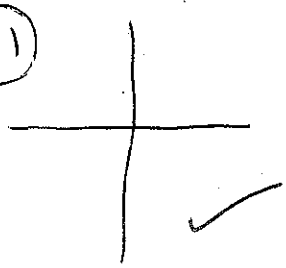


$\therefore \theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ.$

A1 A1

(iii) $\tan 60^\circ = \sqrt{3}$ M1

$\therefore \theta = 300^\circ.$ A1



(b) $$\frac{\sin^2\theta(1+\cos\theta) - \sin^2\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{2\sin^2\theta\cos\theta}{1-\cos^2\theta}$$ M1

$$= \frac{2\sin^2\theta\cos\theta}{\sin^2\theta}$$

$$= 2\cos\theta.$$ A1

(c) (i) If $f(x)$ is odd, $f(-x) = -f(x)$

$$f(-x) = -x\sqrt{4-(-x)^2} \quad (M1)$$

$$= -x\sqrt{4-x^2}$$

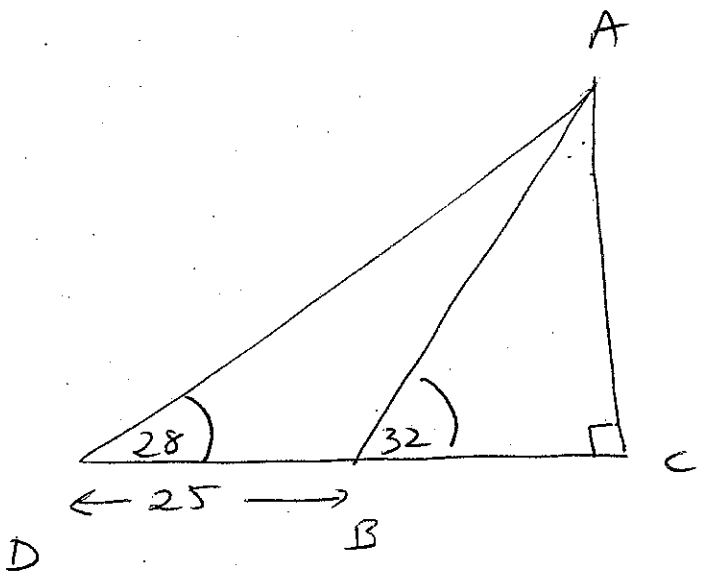
$$= -\left(x\sqrt{4-x^2}\right) \quad (M1)$$

$$= -f(x)$$

$\therefore$ $f(x)$ is an odd function.

(ii) $\therefore$ $f(1-x) = -\sqrt{3}$ (A1)

(d)



Since $\angle ABC = 32°$

$\therefore \angle DBA = 180° - 32°$ * a variation is acceptable

$= 148°$

(A1)

$\therefore$ $\angle$ sum of $\triangle DAB$

$28° + 148° + \angle DAB = 180°$

$\angle DAB = 180° - 176° = 4°$

(ii) $\frac{BA}{\sin 28^\circ} = \frac{25}{\sin 40^\circ}$ (M1)

$BA = \frac{25 \sin 28^\circ}{\sin 40^\circ}$

$BA = 18.26$

$BA = 18.3\,m$ (A1)

(iii) $\frac{h}{18.26} = \sin 32^\circ.$

$h = 18.26 \times \sin 32^\circ.$

$h = 9.68$

$h = 9.7\,M$ (A1)

Question 14:

(a) $RHS = Ax^2 - Ax + Bx - B + C$

$= Ax^2 + (B + A)x + (C - B)$

$\boxed{2 = A}$ (A1)

$B + A = 1$

$\boxed{B = -1}$ (A1)

$1 = C - B$

$\boxed{0 = C}$ (A1)

(b) (i) $\alpha + \beta = \frac{7}{2}$ (A1)

(ii) $\alpha\beta = \frac{12}{2}$

$= 6$ (A1)

(iii) $\frac{\beta + \alpha}{\alpha\beta}$ (M1) $= \frac{7/2}{6}$

$= \frac{7}{12}$ (A1)

(iv) $\alpha\beta(\alpha + \beta) = 6\left(\frac{7}{2}\right)$

(M1) $= 21$ (A1)

(v) $(\alpha + \beta)^2 - 2\alpha\beta$

$= \left(\frac{7}{2}\right)^2 - 2(6)$ (M1)

$= \frac{49}{4} - 12$ (A1)

$= \frac{1}{4}$

(c) (i) $\Delta = [-2(k+1)]^2 - 4(4k+1)(1-2k)$ M1

$\Delta = 4(k^2+2k+1) - 4(4k+1-8k^2-2k)$

$\Delta = 4k^2 + 8k + 4 - 16k - 4 + 32k^2 + 8k$

$\Delta = 36k^2$ A1

(ii) $\alpha + \beta = \frac{-(k+1)}{(k-1)}$

$\alpha\beta = \frac{-2}{k-1}$

$\alpha + \frac{1}{\alpha} = \frac{-(k+1)}{k-1}$

$\alpha \times \frac{1}{\alpha} = \frac{-2}{k-1}$ M1

$1 = -\frac{2}{k-1}$

$k-1 = -2$

$\boxed{k = -1}$ A1

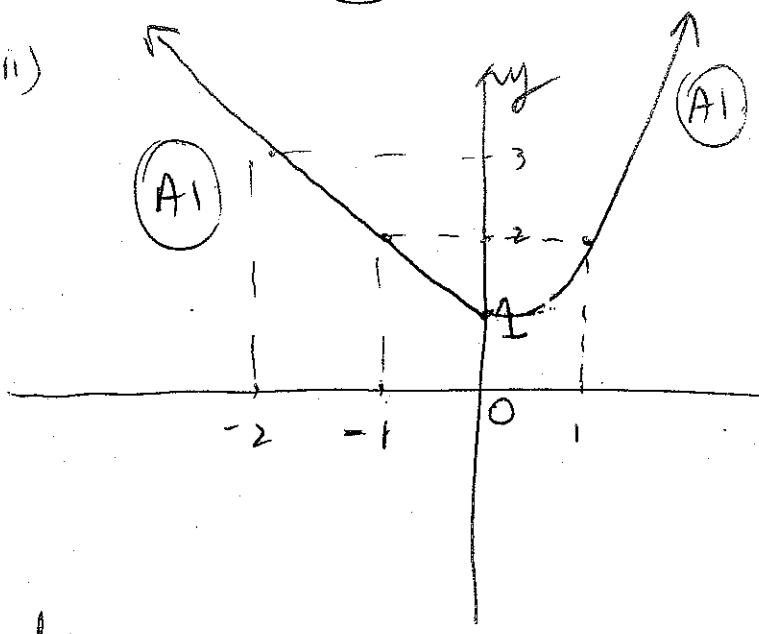
Question 15:

(a) (i) $[1-(-2)] + [9+1]$ (M1)

$= 3 + 10$

$= 13$ (A1)

(ii)



marks are given iff (0,1) is showing and shapes are accurate.

(b) $\frac{dy}{dx} = 20x^3 - 3x^{-2} + x^{-\frac{1}{2}}$

$\frac{dy}{dx} = 20x^3 - \frac{3}{x^2} + \frac{1}{\sqrt{x}}$

(A1) (A1) (A1)

(c) $\frac{dy}{dx} = \frac{(3-x)\cdot(2x) - (x^2+1)(-1)}{(3-x)^2}$ (M1)

$= \frac{6x - 2x^2 + x^2 + 1}{(3-x)^2}$

$= \frac{6x + 1 - x^2}{(3-x)^2}$ (A1)

(d) $f'(x) = 4(2x-5)^3 \times 2$

$f'(x) = 8(2x-5)^3$ (M1)

$\therefore \quad 0 = 8(2x-5)^3$

$0 = 2x - 5$

$5 = 2x$

$x = 5/2$ (A1)

(e) $\frac{dy}{dx} = 3x^2 + 5x$ $\qquad x = 1, \; y = 2$

$m = 8$ (A1)

$y - 2 = 8(x-1)$

$y - 2 = 8x - 8$

$y = 8x - 6.$ (A1)

(f) $\lim_{x \to 3} \frac{(x-3)(x+3)}{x-3}$ (M1)

$= \lim_{x \to 3} x + 3$

$= 6.$ (A1)

Question 16:

(a) $\angle EFB = 180^\circ - 141^\circ$ — co-interior $\angle$'s in ll lines AB ll EF are supplementary

$\angle EFB = 39^\circ$ (M1)

$\angle EFC = \angle EFB + \angle BFC = 68^\circ$ — alt $\angle$'s in ll lines CD ll EF are (=)

(M1)

$\therefore \quad 68^\circ = 39^\circ + y^\circ$

$29^\circ = y^\circ$ (A1)

Aim: To Prove $\triangle RST \equiv \triangle QRP$.

(b)(i) Proof: (M2)

$\angle STR = \angle PRS$ — alt $\angle$'s in ll lines ST ll RP are (=).

$\angle SRT = \angle PQR$ — alt $\angle$'s in ll lines RS ll PQ are (=)

$\therefore \quad \triangle RST \equiv \triangle QRP$

(equiangular) (A1)

(ii) $\frac{ST}{8} \times \frac{3}{12}$ | $\frac{ST}{RP} = \frac{RS}{PQ}$ (M1)

$12\,ST = 24$

$ST = 2$ cm. (A1)

(c) $\sin 150^\circ \sec(-150^\circ)$

$\left(\frac{1}{2}\right) \times \frac{1}{-\sqrt{3}/2}$ M1

$= \frac{1}{2} \times -\frac{2}{\sqrt{3}}$

$= -\frac{1}{\sqrt{3}}$ A1

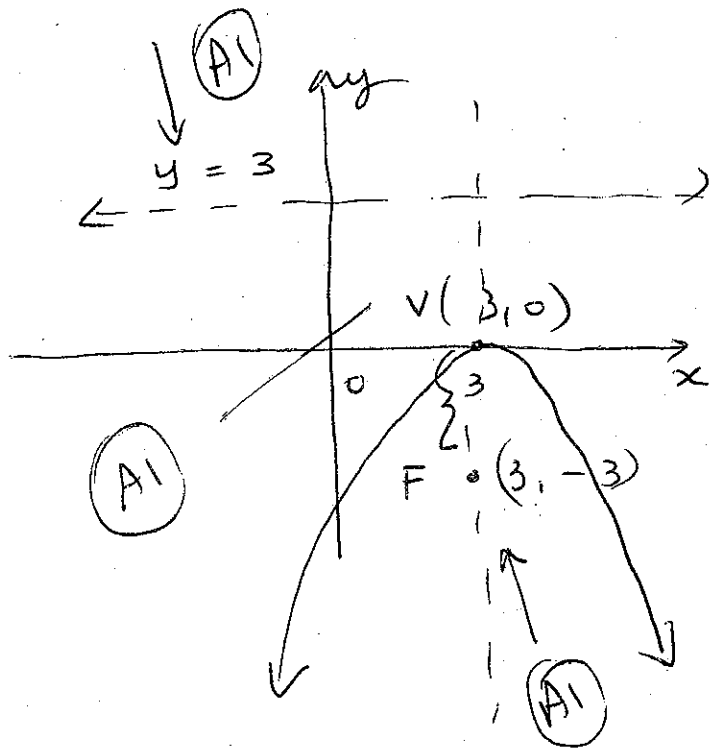
(d) $x^2 - 6x + 9 = -12y$

$(x-3)^2 = -12y$

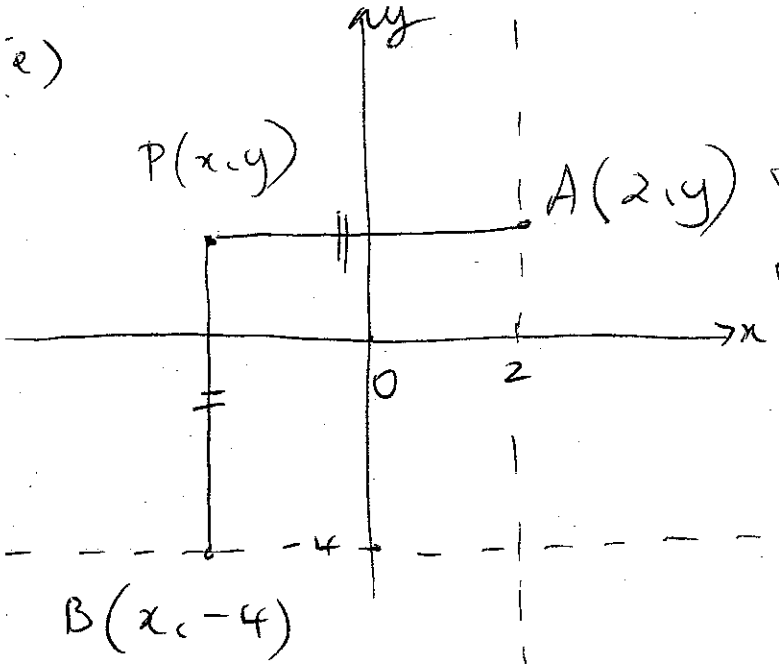
$V(3,0)$ $\quad a = 3$

$F(3,-3)$

$y = 3.$



(e)



$PA = PB$

$\sqrt{(x-2)^2 + (y-y)^2} = \sqrt{(x-x)^2 + (y+4)^2}$

$\sqrt{(x-2)^2} = \sqrt{(y+4)^2}$

$\pm(x-2) = y + 4$

A1 $y = x - 6$ OR

A1 $y = -x - 2$