



Trinity Grammar School

Mathematics Department

2014
YEARLY EXAMINATION

PRELIMINARY ASSESSMENT TASK 4

Year 11

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2.5 hours
- Write using black or blue pen
- Only approved calculators for this course are allowed in this task
- Show all necessary working
- Write your Board of Studies Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- Write your Board of Studies Student
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.
- **Preliminary Assessment Weighting:**
40%

Board of Studies Student Number
(Year 12 only)

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Class Teacher:

.....

Name:

(Year 11 only)

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Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Monday, 25 August 2014

Total marks – 100

Section I

10 marks

- Attempt all Questions
- Allow about **15** minutes for this section

Section II

90 marks

- Attempt all Questions
- Allow about **2 hours 15** minutes for this section

Section I 10 marks

- Attempt Questions 1 – 10
 - Allow about 15 minutes for this section
 - Shade the correct response on the multiple-choice answer sheet for Questions 1 – 10
 - Each question is worth 1 mark
-

1 What is 38 900 000 in scientific notation?

- (A) 0.389×10^8
- (B) 38.9×10^6
- (C) 3.89×10^7
- (D) 3.89×10^{-7}

2 Which of the following is equal to $\frac{1}{\sqrt{17} + \sqrt{3}}$?

- (A) $\frac{\sqrt{17} - \sqrt{3}}{14}$
- (B) $\frac{14}{\sqrt{17} - \sqrt{3}}$
- (C) $\frac{\sqrt{17} - \sqrt{3}}{6}$
- (D) $\frac{\sqrt{17} + \sqrt{3}}{14}$

- 3 What are the solutions of $\sin x = \frac{\sqrt{3}}{2}$, for $0^\circ \leq x \leq 360^\circ$?
- (A) 60° or 240°
- (B) 30° or 150°
- (C) 30° or 210°
- (D) 60° or 120°
- 4 What is the equation of a circle with centre (2, -1) and radius 3?
- (A) $(x-2)^2 + (y+1)^2 = 3$
- (B) $(x+2)^2 + (y-1)^2 = 3$
- (C) $(x-2)^2 + (y+1)^2 = 9$
- (D) $(x+2)^2 + (y-1)^2 = 9$
- 5 What is the value of $\lim_{x \rightarrow 3} \left(\frac{x^3 - 27}{x - 3} \right)$?
- (A) 27
- (B) 0
- (C) 9
- (D) undefined

6 What is the vertex of the parabola $y=2x^2-8x+1$?

- (A) $(4,1)$
- (B) $(2,-7)$
- (C) $(-2,25)$
- (D) $(-7,2)$

7 What is the value of $f'(2)$ given $f(x)=5x^3$?

- (A) 120
- (B) 32
- (C) 40
- (D) 60

8 What are the solutions of $|x+3|=8$?

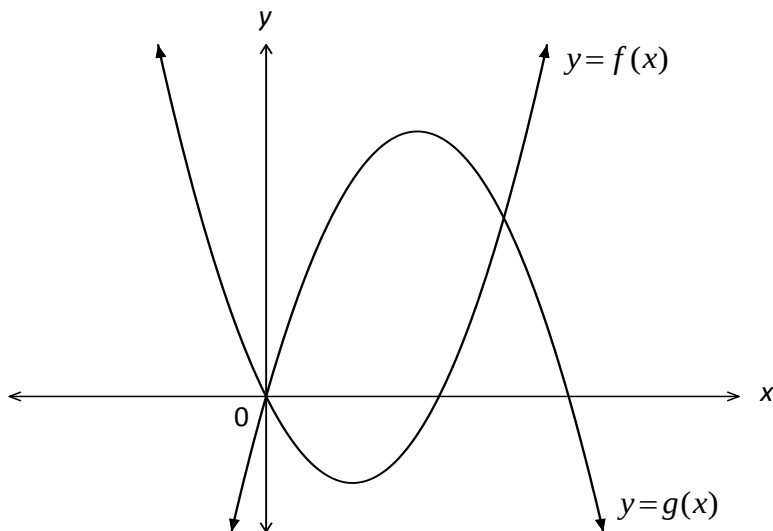
- (A) $x = 5$
- (B) $x = 5$ or $x = 11$
- (C) $x = -5$
- (D) $x = 5$ or $x = -11$

- 9 The sum of the interior angles of a regular polygon is 1440° .

How many sides does the polygon have?

- (A) 6
- (B) 10
- (C) 12
- (D) 8

- 10 The diagram below shows the graphs $y = f(x)$ and $y = g(x)$.



For solving $f(x) = g(x)$, which of the following statements is true about the discriminant (Δ) ?

- (A) $\Delta > 0$
- (B) $\Delta = 0$
- (C) $\Delta < 0$
- (D) $\Delta \leq 0$

End of Section I
Section II commences on the next page

Section II 90 marks

- Attempt Questions 11 – 17
 - Allow about 2 hours and 15 minutes for this section
 - Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
 - In Questions 11 – 17, your responses should include all relevant mathematical reasoning and/or calculations.
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Question 11 (12 marks)

Use a SEPARATE writing booklet

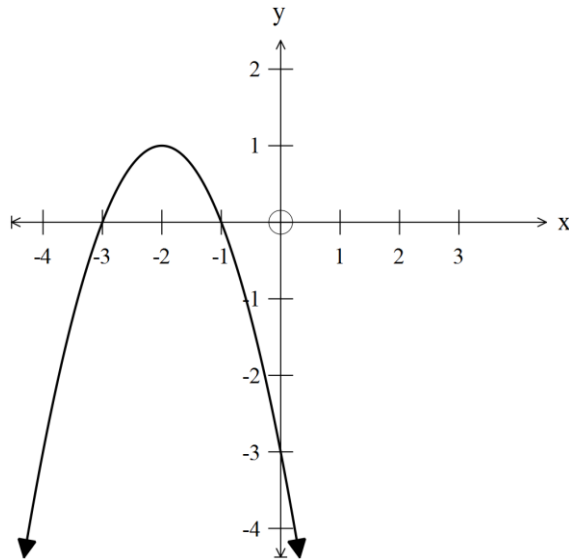
- (a) Evaluate, to 2 significant figures $\frac{(2.4)^2}{\sqrt{2\pi-5}}$ **2**
- (b) Simplify $\frac{x^2 - x + xy - y}{x + y}$ **3**
- (c) Let α and β be the solutions of $2x^2 + 4x - 1 = 0$.
Find:
- (i) $\alpha + \beta$ **1**
- (ii) $\alpha\beta$ **1**
- (iii) $\alpha^2 + \beta^2$ **2**
- (d) If $3x^2 + x - 4 \equiv Ax(x+1) + Bx^2 + C(x+1)$, find the values of A , B and C . **3**

Question 12 (13 marks)

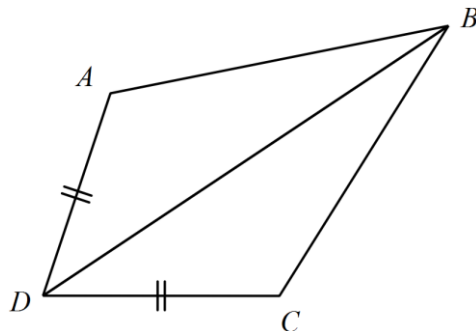
Use a SEPARATE writing booklet

(a) Solve $x^2 + 2x - 5 = 0$, leaving the answer in simplified surd form. 3

(b) Write down the equation of the parabola shown below. 3



(c) In the diagram shown below $AD = DC$ and BD bisects $\angle ADC$.



(i) Prove that $\triangle ADB \equiv \triangle CDB$. 3

(ii) Show that $AB = CB$. 1

(d) Find the equation of the tangent to the curve $y = 2x^3 + 5x + 3$ at the point $(1, 10)$. 3

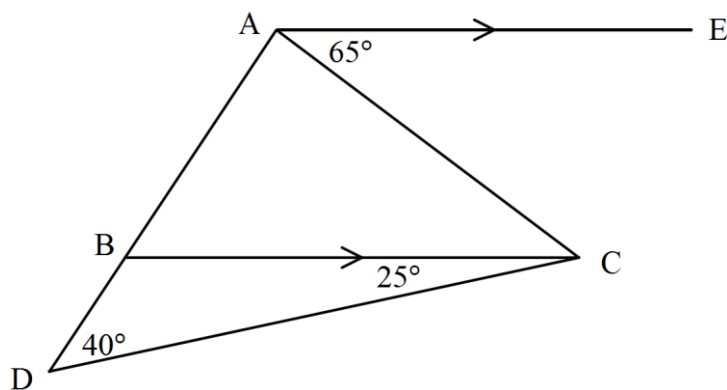
Question 13 (13 marks)

Use a SEPARATE writing booklet

- (a) In the diagram below, AE is parallel to BC . 2

$$\angle BDC = 40^\circ, \angle BCD = 25^\circ \text{ and } \angle EAC = 65^\circ.$$

Prove that $\triangle ABC$ is isosceles giving reasons.



NOT TO SCALE

- (b) Differentiate:

(i) $y = \frac{2}{x}$ 2

(ii) $f(x) = 3x(x-2)^4$, giving the answer in simplified form. 3

(iii) $g(x) = \frac{x}{x-1}$ 2

- (c) (i) Sketch the graph of $f(x)$, showing all the important features.

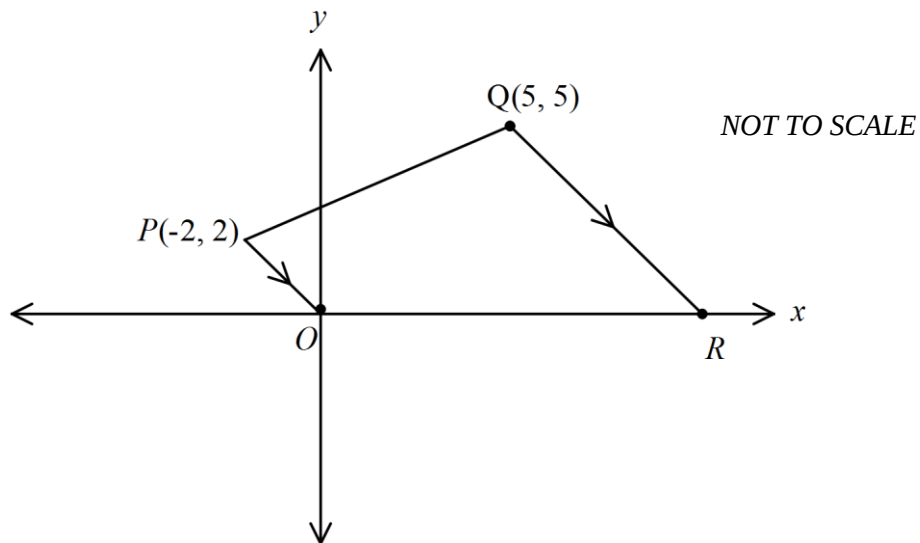
$$f(x) = \begin{cases} 3^x & \text{for } x \geq 0 \\ 2-x^2 & \text{for } x < 0 \end{cases} \quad \text{3}$$

- (ii) For which values of x is $y = f(x)$ not differentiable? 1

Question 14 (13 marks)

Use a SEPARATE writing booklet

In the diagram $OPQR$ is a trapezium with OP parallel to RQ . P and Q are the points $(-2, 2)$ and $(5, 5)$ respectively. Point O is the origin and the line RQ meets the x -axis at R .



- (a) Write down the midpoint of PQ . 1
- (b) Write down the gradient of OP . 1
- (c) Show that the equation of the line RQ is $x + y - 10 = 0$. 2
- (d) Find the x - coordinate of point R . 1
- (e) Find the perpendicular distance from point O to the line RQ . 2
Give the answer in exact form.
- (f) Find the length of QR , giving the answer in simplified exact form. 3
- (g) Hence or otherwise, find the area of trapezium $ORQP$. 3

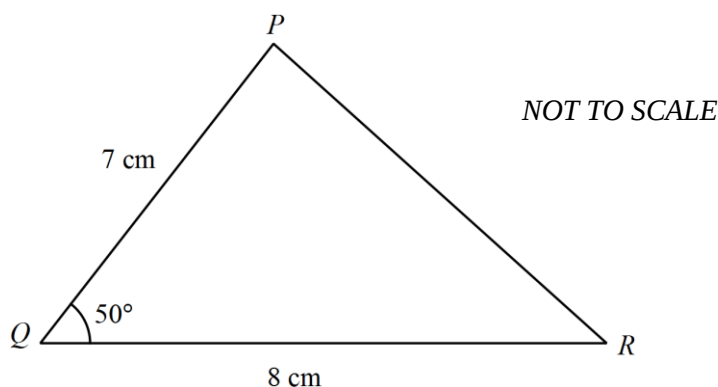
Question 15 (13 marks)

Use a SEPARATE writing booklet

(a) Solve $5x - x^2 < 0$ 2

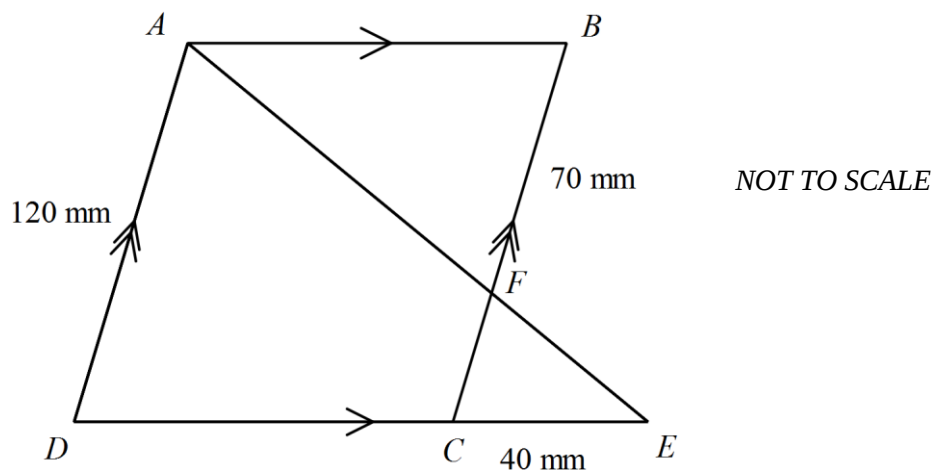
(b) Find the coordinates of the point at which the curve $y = x^3 - x^2$ has a tangent with a gradient of 1. 3

(c) (i) Find the length of PR , giving your answer correct to 1 decimal place. 2



(ii) Find the area of $\triangle PQR$, correct to the nearest cm^2 . 1

(d) $ABCD$ is a parallelogram with DC produced to E . $BF = 70 \text{ mm}$, $CE = 40 \text{ mm}$ and $AD = 120 \text{ mm}$.



(i) Prove that $\triangle ABF$ is similar to $\triangle ECF$ 2

(ii) Find the length of AB . 3

Question 16 (13 marks)

Use a SEPARATE writing booklet

- (a) Find the angle the line $\sqrt{3}x + y - 2 = 0$ makes with the positive direction of the x -axis. **3**
- (b) (i) State the domain and range of the curve $y = \sqrt{25 - x^2}$. **2**
- (ii) Find the gradient function of the curve $y = \sqrt{25 - x^2}$. **2**
- (iii) Find the equation of the normal to the curve at the point $x = 3$. **3**
- (c) Prove $\frac{1 - \tan^2 x}{1 + \tan^2 x} = \cos^2 x - \sin^2 x$. **3**

Question 17 (13 marks)

Use a SEPARATE writing booklet

- (a) For what values of k does the equation $(k-2)x^2 + x + 4 = 0$ have real roots. **3**
- (b) Using $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$, differentiate $f(x) = x^2 + 6x$. **3**
- (c) Solve $\cot^2 \theta = 3 \cot \theta$, for $0^\circ \leq \theta \leq 360^\circ$. **3**
- (d) For what values of a , will $ax^2 + 5x + a$ be positive definite. **4**

End of Section II
End of Examination

Year 11 Mathematics Assessment Task 4 - Solutions

SECTION I

1. 3.89×10^7 (C)

2. $\frac{1}{\sqrt{17} + \sqrt{3}} \times \frac{\sqrt{17} - \sqrt{3}}{\sqrt{17} - \sqrt{3}}$

$= \frac{\sqrt{17} - \sqrt{3}}{17 - 3}$

$= \frac{\sqrt{17} - \sqrt{3}}{14}$ (A)

3. $x = 60^\circ \text{ or } 120^\circ$ (D)

4. $(x-2)^2 + (y+1)^2 = 9$ (C)

5. $\lim_{x \rightarrow 3} \frac{(x-3)(x^2+6x+9)}{x-3}$

$= 27$ (A)

6. $x = \frac{8}{4} = 2$

$y = -7$ $V(2, -7)$ (B)

7. $f'(x) = 15x^2$

$f'(2) = 60$ (D)

8. $x + 3 = 8$
 $x = 5$

$x + 3 = -8$
 $x = -11$ (D)

9. $180(n-2) = 1440$

$n - 2 = 8$ (B)
 $n = 10$

10. (A)

SECTION II

Question 11

a) $5.084845882 \div 5.1 \approx 5.1$ (2)

b) $\frac{x(x-1)+y(x-1)}{x+y}$ ✓

$= \frac{(x+y)(x-1)}{x+y}$ ✓

$= x-1$ ✓

(3)

c) i) $\alpha + \beta = -\frac{b}{a} = -\frac{4}{2} = -2$ (1) ✓

ii) $\alpha\beta = \frac{c}{a} = -\frac{1}{2}$ ✓ (1)

iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ ✓
 $= (-2)^2 - 2(-\frac{1}{2})$
 $= 4 + 1$ (2)
 $= 5$ ✓

d)

$3x^2 + x - 4 \equiv Ax^2 + Ax + Bx^2 + Cx + C$
 $\equiv (A+B)x^2 + (A+C)x + C$

$C = -4$

$A + C = 1$

$A + B = 3$ ✓

$A - 4 = 1$

$5 + B = 3$

$A = 5$ ✓

$B = -2$ ✓

(3)

Question 12

a) $x^2 + 2x - 5 = 0$

$$x = \frac{-2 \pm \sqrt{4 - 4 \times 1 \times -5}}{2} \quad \checkmark$$

$$= \frac{-2 \pm \sqrt{24}}{2} \quad \checkmark$$

$$= -1 \pm \sqrt{6} \quad \checkmark \quad (3)$$

b) $y = a(x+2)^2 + 1$ $\checkmark \checkmark$
 sub (0, -3) eqn and correct sub

$$\therefore -3 = a(0+2)^2 + 1$$

$$-3 = 4a + 1$$

$$4a = -4$$

$$a = -1 \quad \checkmark$$

$$\therefore y = -(x+2)^2 + 1$$

OR

$$y = a(x+3)(x+1) \quad \checkmark \checkmark$$

sub (0, -3)

$$\therefore -3 = a(0+3)(0+1)$$

$$-3 = 3a$$

$$a = -1 \quad \checkmark$$

$$\therefore y = -(x+3)(x+1) \quad (3)$$

c)

i) In $\triangle ADB$ and $\triangle CDB$

- $AD = DC$ (given) $\checkmark$
- $\angle ADB = \angle CDB$ (given) $\checkmark$ for a
- DB is a common side $\checkmark$ 3 point

$\therefore \triangle ADB \equiv \triangle CDB$ (SAS) $\checkmark$

(3)

ii) $AB = CB$

(corresponding sides $\checkmark$
 in congruent triangles
 are equal) (1)

d) $y' = 6x^2 + 5 \quad \checkmark$

• when $x = 1$

$$m_{\text{tangent}} = 11 \quad \checkmark$$

$$\therefore y - 10 = 11(x - 1)$$

$$y - 10 = 11x - 11$$

$$y = 11x - 1 \quad \checkmark$$

(3)

Question 13

a) $\angle ACB = 65^\circ$ (alternate angles, $AE \parallel BC$) ✓

$\angle ABC = 40^\circ + 25^\circ$
 $= 65^\circ$ (exterior angle of a triangle) ✓

$\therefore \triangle ABC$ is isosceles since base angles are equal. (2)

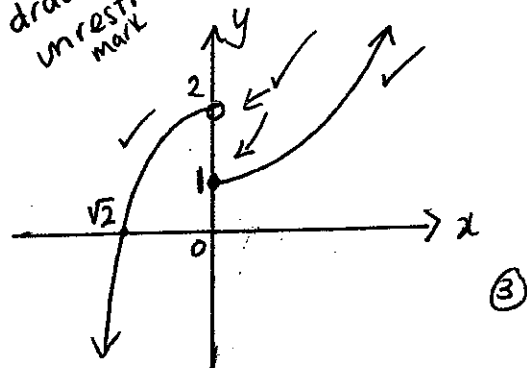
b) i) $y = 2x^{-1}$ ✓

$y' = -2x^{-2}$ ✓
 $= \frac{-2}{x^2}$ (2)

ii) $f'(x) = 3(x-2)^4 + 3x \cdot 4(x-2)^3$ ✓ product rule
 $= 3(x-2)^4 + 12x(x-2)^3$
 $= 3(x-2)^3(x-2 + 4x)$
 $= 3(x-2)^3(5x-2)$ (3)

iii) $g'(x) = \frac{(x-1) \cdot 1 - x \cdot 1}{(x-1)^2}$ ✓ quotient rule
 $= \frac{x-1-x}{(x-1)^2}$
 $= \frac{-1}{(x-1)^2}$ ✓ (2)

c) if they draw both unrestricted - 1 mark



ii) Not differentiable at $x=0$. ✓ (1)

Question 14

a) Midpoint_{PQ} $\left(\frac{-2+5}{2}, \frac{2+5}{2}\right)$
 $= \left(\frac{3}{2}, \frac{7}{2}\right) \checkmark$

①

b) $m_{op} = \frac{2-0}{-2-0}$
 $= -1 \checkmark$ ①

c) $m_{QR} = -1 \checkmark$
 $y-5 = -1(x-5)$
 $y-5 = -x+5$
 $x+y-10=0 \checkmark$ ②

d) let $y=0$
 $x+0-10=0 \checkmark$ ①
 $x=10$

e) $d = \frac{|0+0-10|}{\sqrt{1+1}} \checkmark$ ②
 $= \frac{10}{\sqrt{2}} \checkmark$

f) $d_{QR} = \sqrt{(5-10)^2 + (5-0)^2} \checkmark$
 $= \sqrt{25 + 25}$

$= \sqrt{50} \checkmark$

$= 5\sqrt{2} \checkmark$ ③

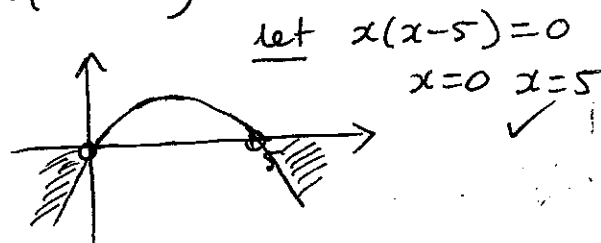
g) $d_{op} = \sqrt{(-2-0)^2 + (2-0)^2}$
 $= \sqrt{8} \checkmark$
 $= 2\sqrt{2}$

✓ area of trap

$A = \frac{1}{2} \times \frac{10}{\sqrt{2}} \times (5\sqrt{2} + 2\sqrt{2})$
 $= \frac{5}{\sqrt{2}} \times 7\sqrt{2}$
 $= 35 \text{ units}^2 \checkmark$

Question 15

a) $x(5-x) < 0$



$\therefore x < 0$ or $x > 5$ ✓ (2)

b) $3x^2 - 2x = 1$ ✓

$3x^2 - 2x - 1 = 0$ $3x \times -1$

$(3x+1)(x-1) = 0$ $x \times -1$

$x = -\frac{1}{3}$ $x = 1$ ✓ x -values

$y = -\frac{4}{27}$ $y = 0$ ✓ y -values

$(-\frac{1}{3}, -\frac{4}{27})$ $(1, 0)$ (3)

c) i) $PR^2 = 7^2 + 8^2 - 2 \times 7 \times 8 \times \cos 50^\circ$ ✓

$PR = \sqrt{7^2 + 8^2 - 2 \times 7 \times 8 \times \cos 50^\circ}$

$= 6.403732327$ ✓

≈ 6.4 (2)

ii) $A = \frac{1}{2} \times 7 \times 8 \times \sin 50^\circ$

$= 21.4492$ ✓

$= 21 \text{ cm}^2$

d)

i) In $\triangle ABF$ and $\triangle ECF$

$\bullet \angle ABC = \angle BCE$ ✓
 $\bullet \angle BAE = \angle AEC$ } alt angles, $AB \parallel DE$

$\therefore \triangle ABF \parallel \triangle ECF$ ✓
 (equiangular) (2)

ii) $\frac{BF}{CF} = \frac{AB}{CF}$ ✓ (matching sides are in the same ratio)

$\frac{70}{50} = \frac{AB}{40}$ ✓ correctly sub.

$2800 = 50AB$

$AB = 56$ ✓ (3)

Question 16

a) $y = -\sqrt{3}x + 2$

$m = -\sqrt{3}$ ✓

$\tan \theta = -\sqrt{3}$ ✓

$\theta = 180^\circ - 60^\circ$

$= 120^\circ$ ✓

(3)

b) i) Domain: $-5 \leq x \leq 5$ ✓

Range: $0 \leq y \leq 5$ ✓ (2)

ii) $y = (25 - x^2)^{1/2}$ ✓

$\frac{dy}{dx} = \frac{1}{2}(25 - x^2)^{-1/2} \cdot -2x$ ✓

$= -x(25 - x^2)^{-1/2}$

$= \frac{-x}{\sqrt{25 - x^2}}$

(2)

iii) • when $x=3$ $y=4$

• $m_{\text{tangent}} = -\frac{3}{4}$ ✓

• $\therefore m_{\text{normal}} = \frac{4}{3}$ ✓

• $y - 4 = \frac{4}{3}(x - 3)$

$3y - 12 = 4x - 12$

$4x - 3y = 0$ ✓

(3)

c)

LHS = $\frac{1 - \tan^2 x}{1 + \tan^2 x}$

(3)

$= \frac{1 - \tan^2 x}{\sec^2 x}$ ✓

$= \cos^2 x - \tan^2 x \cos^2 x$ ✓

$= \cos^2 x - \frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x$ ✓

$= \cos^2 x - \sin^2 x$

$= \text{RHS}$

13

Question 17

a) $\Delta \geq 0$ ✓
 $1 - 4 \cdot 4(K-2) \geq 0$
 $1 - 16K + 32 \geq 0$
 $33 - 16K \geq 0$ ✓
 $-16K \geq -33$ (3)
 $K \leq \frac{33}{16}$ ✓

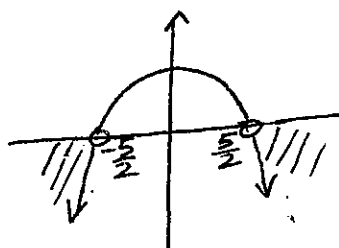
b) ✓
 $f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 6(x+h) - (x^2 + 6x)}{h}$
 $= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 6x + 6h - x^2 - 6x}{h}$
 $= \lim_{h \rightarrow 0} \frac{2xh + h^2 + 6h}{h}$ ✓
 $= \lim_{h \rightarrow 0} (2x + h + 6)$ ✓
 $= 2x + 6$ ✓

c) $\cot \theta (\cot \theta - 3) = 0$ ✓

$\cot \theta = 0$	$\cot \theta = 3$
$\frac{1}{\tan \theta} = 0$	$\frac{1}{\tan \theta} = 3$
$\tan \theta = \frac{1}{0}$	$\tan \theta = \frac{1}{3}$

tan θ is undefined when $\theta = 90^\circ, 270^\circ$ ✓
 $\theta = 18^\circ 26', 198^\circ 26'$ ✓

d) $a > 0$ to be concave up
 $\Delta < 0$ no real roots
 $25 - 4 \cdot a \cdot a < 0$
 $25 - 4a^2 < 0$ ✓
 $(5-2a)(5+2a) < 0$
let $(5-2a)(5+2a) = 0$
 $a = \frac{5}{2} \quad a = -\frac{5}{2}$ ✓



$\therefore a < -\frac{5}{2}$ or $a > \frac{5}{2}$ ✓
but since $a > 0$
then $a > \frac{5}{2}$ ✓ (4)

Year 11 Mathematics Assessment Task 4 - Solutions
