Trinity Grammar School

Mathematics Department

2016

YEARLY EXAMINATION

PRELIMINARY ASSESSMENT
TASK 4

Year 11

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2.5 hours
- Write using black or blue pen
- BOSTES approved calculators for this course may be used
- A Reference Sheet for this course is supplied
- In Questions **11-16**, show all relevant mathematical reasoning and/or calculations
- Write your BOSTES Student Number (Year 12 HSC) or Name (Year 10 or 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.
- **Preliminary Assessment Weighting:** 40%

BOSTES Student Number
(Year 12 only)

Class Teacher:

……..…

Name:
(Year 10 or 11 only)

……...

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Tuesday, 30 August 2016

Total marks – 100

Section I

10 marks

- Attempt all Questions
- Allow about **15** minutes for this section

Section II

90 marks

- Attempt all Questions
- Allow about **2** hours and 15 minutes for this section

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Section I 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Shade the correct response on the multiple-choice answer sheet for Questions 1 – 10
- Each question is worth 1 mark

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1 What is $pqx^2 - p^3q$ fully factorised?

(A) $pq(x+p)(x-p)$

(B) $p^2(x+q)(x-q)$

(C) $pq(x^2-p^2)$

(D) $p^2(x^2-q^2)$

2 What is the derivative of the expression $6x^4+9x^2-12x+9$?

(A) $24x^3+18x-12$

(B) $4x^3+2x-12$

(C) $24x^3+18x^2-12$

(D) $24x^3+18x-3$

3 If $f(x) = x^2 - 2x$, what is the expression for $f(x+2) - f(2)$?

(A) $x^2 + 2x$

(B) $x^2 + 6x$

(C) $x^2 + 2x - 8$

(D) $x^2 + 6x - 16$

4 In solving the equation $y-\frac{2y+1}{4}=5$, which of the following is correct?

(A) y is an integer

(B) y is irrational

(C) $y<0$

(D) $y>0$

5 What is the domain and range of the function $f(x)=\sqrt{1-x}$?

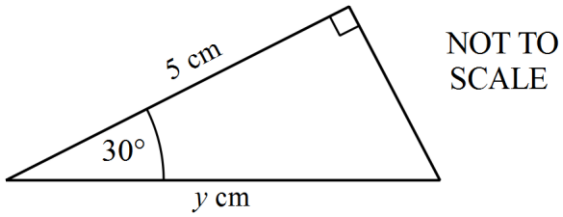
(A) Domain: $x \leq 1$, Range: $y \leq 0$

(B) Domain: $x \leq 1$, Range: $y \geq 0$

(C) Domain: $x \geq 1$, Range: $y \leq 0$

(D) Domain: $x \geq 1$, Range: $y \geq 0$

6 What is the exact value of y in the diagram below?



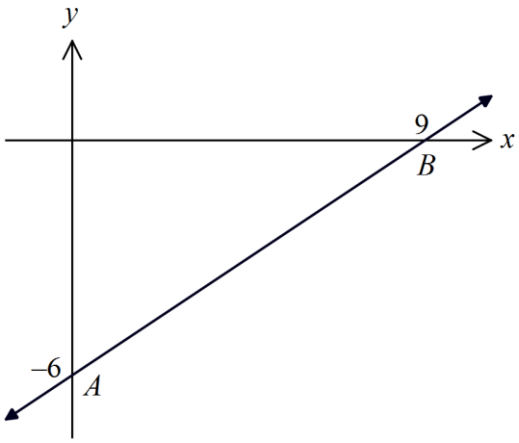
(A) $y=\frac{5\sqrt{3}}{2}$

(B) $y=\frac{10\sqrt{3}}{3}$

(C) $y=5\sqrt{3}$

(D) $y=10$

7 What is the equation of the line AB shown on the number plane?



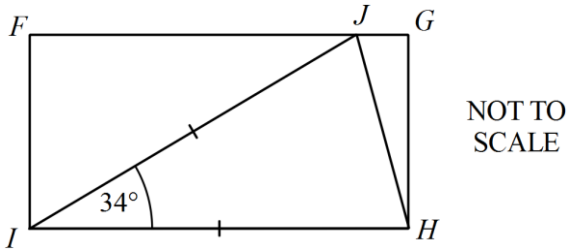
(A) $y = \frac{3x}{2} - 6$

(B) $y = \frac{3x}{2} + 9$

(C) $y = \frac{2x}{3} - 6$

(D) $y = \frac{2x}{3} + 9$

8 In the diagram, $FGHI$ is a rectangle. J is a point on FG, such that $IJ = IH$.
What is the size of $\angle GJH$?



(A) 34°

(B) 56°

(C) 73°

(D) 74°

9 What are the x coordinates of the points of intersection of the parabola $y = x^2 + 6x - 3$ and the straight line $y = 2x + 9$?

(A) $x = -4, 8$

(B) $x = -6, 2$

(C) $x = 3, 2$

(D) $x = -2, 4$

10 If $\cos\alpha = \frac{a}{b}$, where α is an acute angle, what is the value of $\cos(180° + \alpha)$?

(A) $\frac{\sqrt{b^2 - a^2}}{b}$

(B) $\frac{a}{b}$

(C) $-\frac{a}{b}$

(D) $-\frac{b}{a}$

End of Section I
Section II commences on the next page

Section II 90 marks

- Attempt Questions 11-16
- Allow about 2 hours and 15 minutes for this section
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
- In Questions 11 – 16, your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Calculate the value of $\dfrac{6.7 + 1.5 \times 0.31}{\sqrt{3.125 \times 5}}$ correct to two decimal places. **1**

(b) Express $0.\dot{3}\dot{5}$ as a fraction in its simplest form. Show all working. **2**

(c) Expand and simplify: $(3a - 2b)(3a + 2b) - (3a - 2b)$ **2**

(d) Simplify: $\dfrac{2x^2 + 11x - 6}{2x^2 + 3x - 2}$ **2**

(e) Find the exact solutions of the equation: $2x^2 - 4x - 3 = 0$ **2**

(f) Solve: $|3m - 5| = 6$ **2**

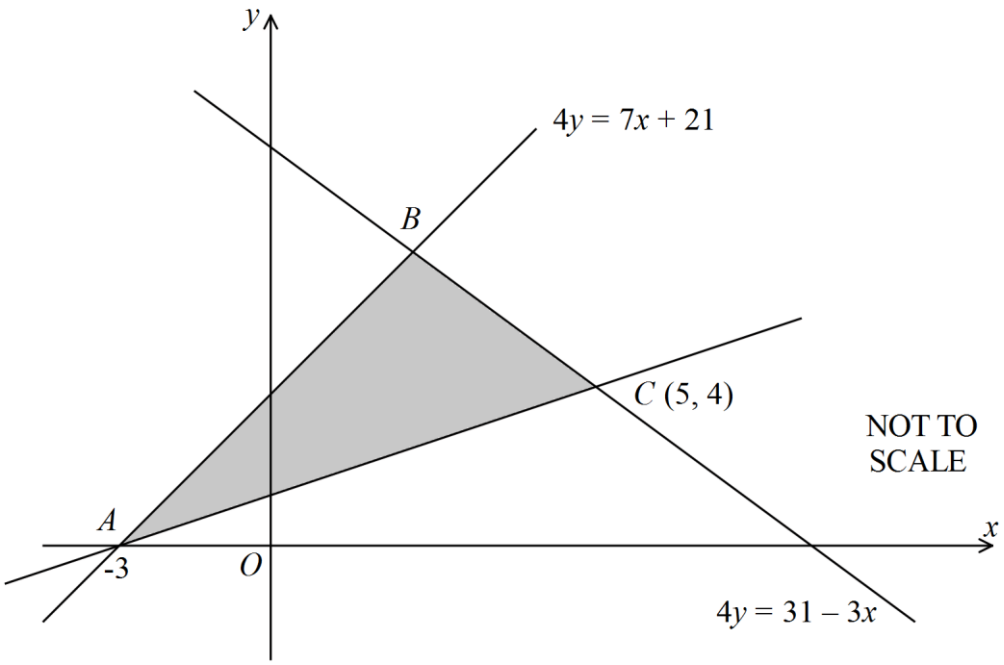
(g) Rationalise the denominator of $\dfrac{\sqrt{2}}{1 - 3\sqrt{2}}$. **2**

(h) Solve: $-1 \leq 2p - 1 < 3$ **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) In the diagram the lines $4y = 7x + 21$ and $4y = 31 - 3x$ intersect at B. A has the coordinates (-3, 0) and C has the coordinates (5, 4).



Copy or trace the diagram into your answer booklet.

(i) Find the angle that the line BC makes with the positive direction of the x-axis. Give your answer to the nearest degree. **2**

(ii) Show that the line AC has the equation $2y = x + 3$. **2**

(iii) Show that the coordinates of B are (1, 7). **2**

(iv) Find the shortest distance from B to the line AC. Give an exact length. **2**

(v) Show that the distance from A to C is $4\sqrt{5}$ units. **1**

(vi) Find the area of ΔABC. **1**

Question 12 continues on the next page.

(b) ABC is a right angled triangle in which $AC \perp BC$. If $AB = \sqrt{1 + x^2}$, $BC = \sqrt{1 - x^2}$ and $AC = \sqrt{x}$, find the value(s) of x. **2**

(c) Find the exact values of:

(i) $\tan 30°$ **1**

(ii) $\cos 135°$ **1**

(iii) $\sin(-120°)$ **1**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

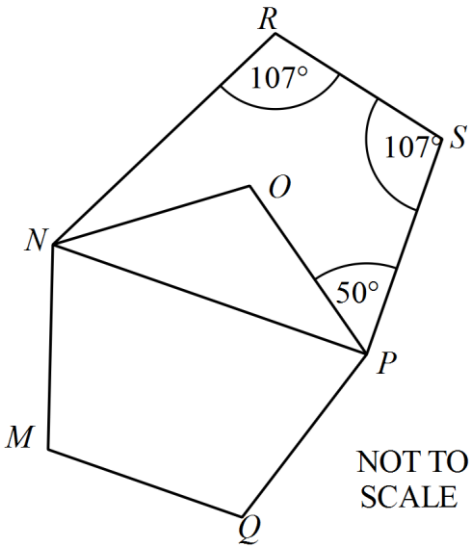
(a) Solve $5^3 \times 25^k = 1$ for k. **2**

(b) Fully factorise:

(i) $x^3 - 8$ **2**

(ii) $3x^2 + 7xy + 2y^2$ **2**

(c) The polygon $MNOPQ$ is a regular pentagon. $\angle OPS = 50°$ and $\angle NRS = \angle RSP = 107°$, as shown in the diagram.



(i) Show, giving reasons, that $\angle ONP = 36°$. **2**

(ii) Giving reasons to support your calculations, find the size of $\angle ONR$ **1**

(d) Differentiate:

(i) $x^4 - 3x^2 + 2x$ **1**

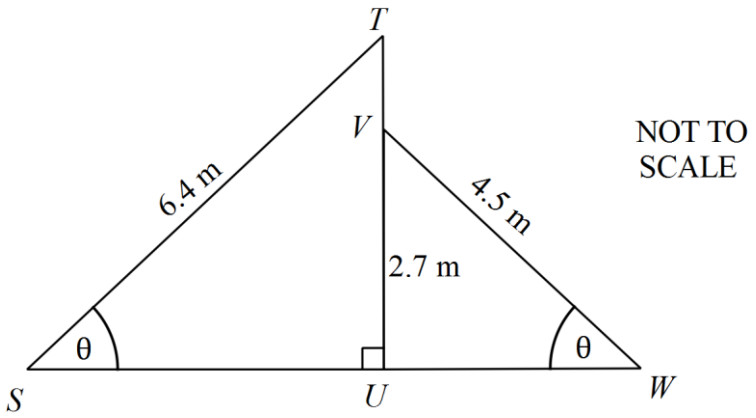
(ii) $x\sqrt{x}$ **2**

(iii) $x^3(x^2 - 5)^7$ **3**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) In the diagram below, TU is perpendicular to SW and $\angle TSU = \angle VWU$. **3**
Given that ST = 6.4 m, VU = 2.7 m and VW = 4.5 m, find the length of TV.



(b) State the domain and range of $(x-2)^2 + (y+3)^2 = 25$. **2**

(c) If $\tan\theta = -\frac{2}{3}$ and $\sin\theta > 0$, find the exact values of:

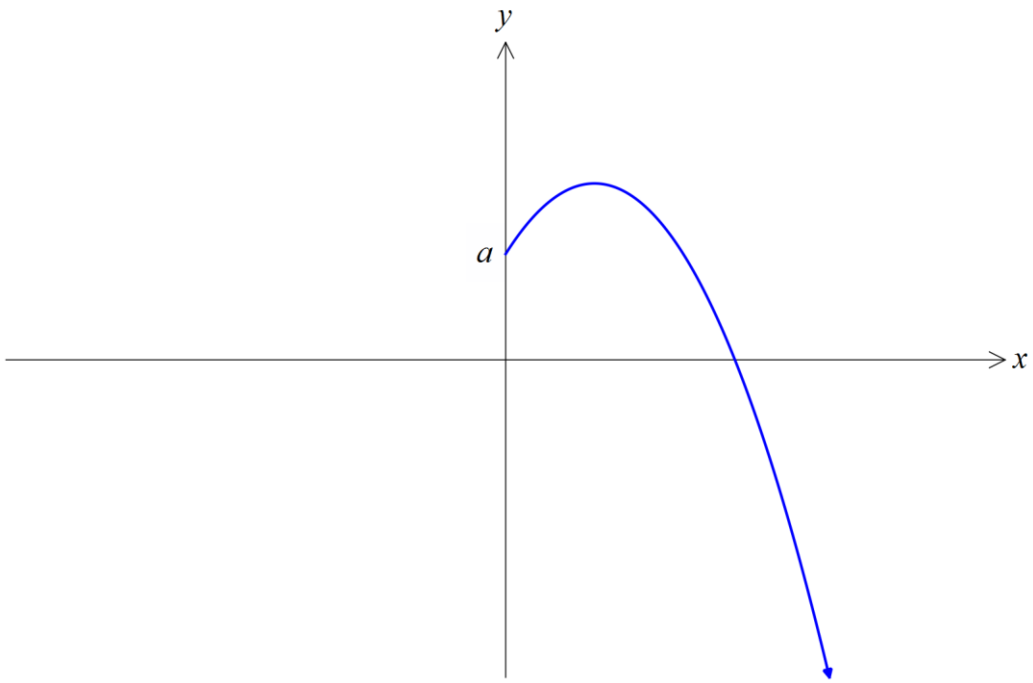
(i) $\cos\theta$ **2**

(ii) $\operatorname{cosec}\theta$ **2**

(d) Find the equation of the tangent to the curve $y = x^2 + 5x - 2$ at the point (1, 4). **3**

Question 14 continues on the next page.

(e) Below is the graph of $f(x)$ for the domain $x \geq 0$.



Copy or trace the graph into your answer booklet *three* times and complete the graph for $x < 0$ to ensure $f(x)$ is:

(i) an even function. **1**

(ii) an odd function. **1**

(iii) is a function that is neither even nor odd. **1**

End of Question 14

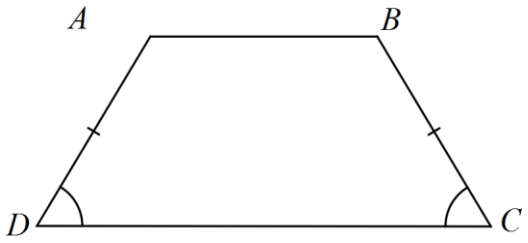
Question 15 (15 marks) Use a SEPARATE writing booklet

(a) There are three towns, *A*, *B* and *C*. From town *C*, the bearing of *A* is 068° and the bearing of *B* is 134°. Town *B* is 160 km from *C* and 250 km from *A*.

(i) Draw a diagram representing this information. **1**

(ii) Calculate the distance from A to C. Give your answer to the nearest meter. **3**

(b) In the diagram below, ABCD is a quadrilateral. $\angle ADC = \angle BCD$ and AD = BC.



NOT TO SCALE

Copy or trace the diagram into your answer booklet, marking the given information.

(i) Prove that $\Delta DAC \equiv \Delta CBD$, giving reasons. **2**

(ii) Hence, prove that $\angle ADB = \angle ACB$. **2**

(c) For the function $f(x) = 3x - \frac{1}{x}$, find the coordinates where the gradient is 7. **3**

(d) If

$$f(x) = \begin{cases} x^2 & x \geq 2 \\ 3x + 4 & x < 2 \end{cases}$$

(i) Find the value of $f(2) + f(0)$. **2**

(ii) Sketch the function. **2**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet

(a) From first principles, differentiate $f(x) = x^2 - 3x$ **3**

(b) Solve the equation $\cos^2 x - \sin^2 x = \frac{1}{2}$ for $0° \leq x \leq 360°$. **3**

(c) Sketch $y = \frac{1}{x-3} + 2$ showing all intercepts and asymptotes. **2**

(d) The equation $(x - 3y + 5) + k(x + 2y) = 0$ represents a family of straight lines through a fixed point P.

(i) For what value of k is one of the lines parallel to the straight line $x + y = 2$? **1**

(ii) For what values of k does one of the lines pass through the centre of the circle $x^2 + y^2 - 10y + 21 = 0$? **2**

(iii) Find the coordinates of P. **2**

(e) Find $\lim_{x \to -5} \frac{-5 + \sqrt{20 - x}}{5 + x}$ **2**

End of Question 16
End of Section II
End of Examination

Trinity Grammar School
Mathematics Department
2016
Year 11 Mathematics
YEARLY EXAMINATION

SOLUTIONS

Preliminary Assessment Task 4

- Write your answers for **Section I** on this answer sheet.
- ***Shade completely only ONE answer for each question.***
- To change an answer, erase your previous mark completely and then record your new answer.

1	●	B	C	D
2	●	B	C	D
3	●	B	C	D
4	A	B	C	●
5	A	●	C	D
6	A	●	C	D
7	A	B	●	D
8	A	B	●	D
9	A	●	C	D
10	A	B	●	D

Question 11 (15 marks)

(a) 1.81 1 mark

(b) $n = 0.353535\ldots$
$100n = 35.3535353\ldots$ 1 mark
$99n = 35$
$n = \frac{35}{99}$ 1 mark

(c) $(3a - 2b)(3a + 2b) - (3a - 2b)$
$= 9a^2 - 4b^2 - 3a + 2b$
1 mark each expansion

(d) $\frac{2x^2 + 11x - 6}{2x^2 + 3x - 2}$
$= \frac{(2x - 1)(x + 6)}{(2x - 1)(x + 2)}$ 1 mark factorisation
$= \frac{x + 6}{x + 2}$ 1 mark simplification

(e) $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$ 1 mark subst
$= \frac{4 \pm \sqrt{40}}{4}$ 1 mark
$= \frac{2(2 \pm \sqrt{10})}{4\ 2}$
$= \frac{2 \pm \sqrt{10}}{2}$

(f) $3m - 5 = 6$
$3m = 11$
$m = \frac{11}{3}$ 1 mark

OR

$3m - 5 = -6$
$3m = -1$
$m = -\frac{1}{3}$ 1 mark

(g) $\frac{\sqrt{2}}{1 - 3\sqrt{2}} \times \frac{1 + 3\sqrt{2}}{1 + 3\sqrt{2}}$ 1 mark for conjugate
$= \frac{\sqrt{2} + 6 \times 2}{1 - 9 \times 2}$
$= \frac{\sqrt{2} + 12}{-17}$ 1 mark
$= \frac{-(\sqrt{2} + 12)}{17}$

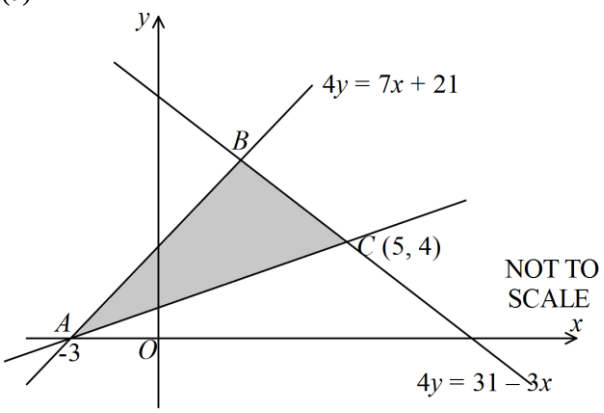
(h) $-1 \leq 2p - 1 < 3$
$0 \leq 2p < 4$ 1 mark
$0 \leq p < 2$ 1 mark for both answers.

Must be written as a single inequality.

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet

(a)



(i) $m_{BC} = -\frac{3}{4}$ 1 mark

$\tan\theta = -\frac{3}{4}$

$\theta = 143°$ 1 mark

(ii) $m_{AC} = \frac{4-0}{5-(-3)}$

$= \frac{4}{8}$ 1 mark

$= \frac{1}{2}$

$y - 0 = \frac{1}{2}(x - (-3))$ 1 mark

$2y = x + 3$

(iii) $4y = 31 - 3x$ ①

$4y = 7x + 21$ ②

subtract ① from ②

$0 = 10x - 10$

$10x = 10$ 1 mark

$x = 1$

$4y = 7(1) + 21$

$4y = 28$ 1 mark

$y = 7$

$\therefore B(1, 7)$

(iv) $d = \frac{|(1) - 2(7) + 3|}{\sqrt{(1)^2 + (-2)^2}}$ 1 mark

$= \frac{|10|}{\sqrt{5}}$ 1 mark

$= 2\sqrt{5}$ units

(v)

$AC = \sqrt{(5 - -3)^2 + (4 - 0)^2}$ 1 mark

$= \sqrt{80}$

$= \sqrt{16 \times 5}$

$= 4\sqrt{5}$

(vi) $A = \frac{1}{2} \times 2\sqrt{5} \times 4\sqrt{5}$ 1 mark

$= 20\ units^2$

(b) $\left(\sqrt{1 + x^2}\right)^2 = \left(\sqrt{1 - x^2}\right)^2 + (\sqrt{x})^2$

$1 + x^2 = 1 - x^2 + x$ 1 mark

$2x^2 - x = 0$

$x(2x - 1) = 0$

$x = 0$ or $x = \frac{1}{2}$

$x = 0$ is not a solution.

$\therefore$ $x = \frac{1}{2}$ 1 mark

Must eliminate $x = 0$ as a solution.

(c) Find the exact values of:

(i) $\tan 30° = \frac{1}{\sqrt{3}}$ 1 mark

(ii) $\cos 135°$

$= \cos(180 - 45)$

$= -\cos 45$

$= -\frac{1}{\sqrt{2}}$ 1 mark

(iii) $\sin(-120°)$

$= -\sin 60$

$= \frac{-\sqrt{3}}{2}$ 1 mark

End of Question 12

Question 13 (15 marks)

(a) $5^3 \times 25^k = 1$

$5^3 \times (5^2)^k = 1$ 1 mark

$5^3 \times 5^{2k} = 1$

$5^{2k+3} = 5^0$

$\therefore \quad 2k + 3 = 0$

$k = -\frac{3}{2}$ 1 mark

(b)

(i) $(x-2)(x^2+2x+4)$ 2 marks.
1 mark each bracket

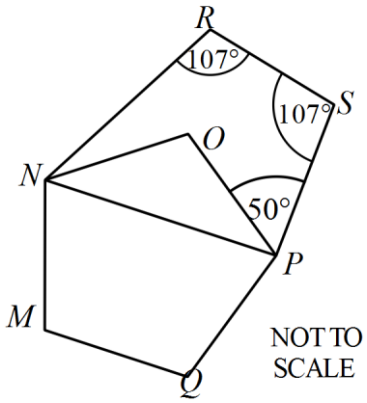
(ii) $3x^2 + 7xy + 2y^2$

$= 3x^2 + 6xy + xy + 2y^2$

$= 3x(x+2y) + y(x+2y)$

$= (x+2y)(3x+y)$ 2 marks
1 mark each bracket

(c)



(i) $\angle NOP = \frac{180(5-2)}{5}$

$= 108°$ 1 mark

NO = PO (equal sides in regular pentagon)

$\therefore \quad \Delta NOP$ is isosceles

let $x = \angle ONP = \angle OPN$
(base angles in isosceles triangle equal)

$180 = 108 + 2x$ 1 mark

$x = 36°$

(ii) $\angle ONR + 2 \times 36 + 50 + 2 \times 107 = 360$
(angle sum quadrilateral)

$\angle ONR = 24°$ 1 mark

(d) Differentiate:

(i) $\frac{d}{dx} = 4x^3 - 6x + 2$ 1 mark

(ii) $x\sqrt{x}$

$= x.x^{\frac{1}{2}}$

$= x^{\frac{3}{2}}$ 1 mark

$\frac{d}{dx} = \frac{3}{2}x^{\frac{1}{2}}$ 1 mark

$= \frac{3\sqrt{x}}{2}$

(iii) $u = x^3$

$u' = 3x^2$ 1 mark

$v = (x^2-5)^7$

$v' = 7(x^2-5)^6 .2x$

$= 14x(x^2-5)^6$ 1 mark

$\frac{d}{dx} = 3x^2(x^2-5)^7 + 14x^4(x^2-5)^6$ 1 mark

$= x^2(x^2-5)^6[3(x^2-5) + 14x^2]$

$= x^2(x^2-5)^6(17x^2-15)$

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) $\sin\theta = \dfrac{2.7}{4.5}$ 1 mark

$\sin\theta = \dfrac{TU}{6.4}$

$\therefore \dfrac{TU}{6.4} = \dfrac{2.7}{4.5}$

$TU = 3.84$ 1 mark

$TV = 3.84 - 2.7$

$= 1.14$ m 1 mark

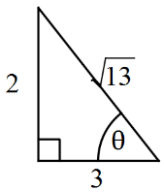
(b) Centre $(2, -3)$ $r = 5$

$-3 \leq x \leq 7$ 1 mark

$-8 \leq y \leq 2$ 1 mark

If incorrect centre and/or radius stated, but domain and range correct from those values, give 1/2.

(c) 2nd quadrant and



(i) $-\dfrac{3}{\sqrt{13}}$ 2 marks

1 for sign, 1 for fraction

(ii) $\dfrac{\sqrt{13}}{2}$ 2 marks

(d) $y = x^2 + 5x - 2$

$\dfrac{dy}{dx} = 2x + 5$ 1 mark

at $x = 1$

$m = 2(1) + 5$

$= 7$ 1 mark

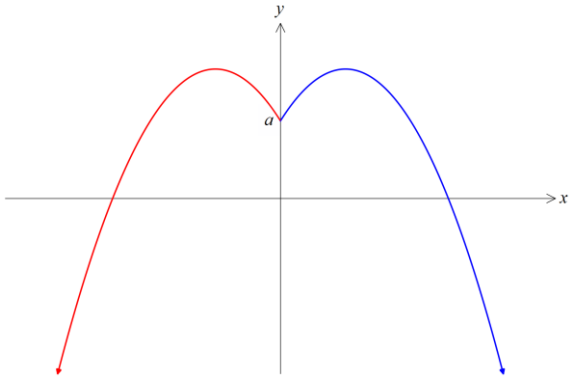
$y - 4 = 7(x - 1)$

$y - 4 = 7x - 7$

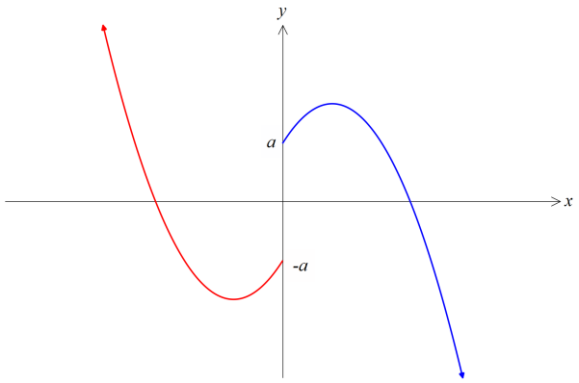
$y = 7x - 3$ 1 mark

(e)

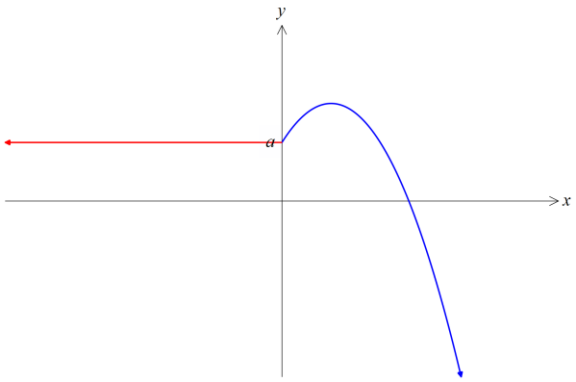
(i) an even function – 1 mark. Must be symmetrical.



(ii) an odd function – 1 mark. Must mark $-a$.



(iii) a function that is neither even nor odd.
1 mark. MUST be a FUNCTION
Multiple answers.

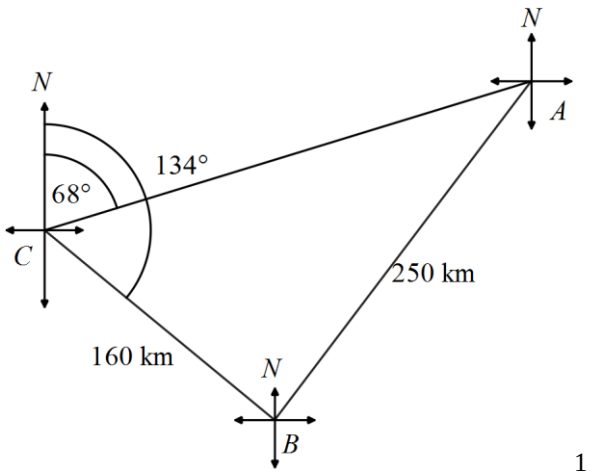


End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet

(a)

(i)



1 mark

(ii) $\angle BCA = 134 - 68$
$= 66°$ 1 mark

$\frac{\sin \angle BAC}{160} = \frac{\sin 66}{250}$

$\angle BAC = 35.77°$
$= 36°$ 1 mark

$\angle CBA = 78°$

$AC^2 = 160^2 + 250^2 - 2 \times 160 \times 250 \times \cos 78$
$= 71\,467.0647...$
$AC = 267.333$ km (to nearest meter) 1 mark

(b)

(i) In triangles DAC and CBD
AD = BC (given)
$\angle ADC = \angle BCD$ (given)
DC is common
$\Delta DAC \equiv \Delta CBD$ (*SAS*) 2 marks

(ii) Let $x = \angle ADC = \angle BCD$
$\angle BDC = \angle ACD = y$ (corresp ∠ in congruent Δ)
1 mark

$\therefore \quad \angle ADB = x - y$ (adjacent ∠)
$\therefore \quad \angle ACB = x - y$ (adjacent ∠) 1 mark

$\therefore \quad \angle ADB = \angle ACB$

(c) $f(x) = 3x - \frac{1}{x}$

$= 3x - x^{-1}$

$f'(x) = 3 + x^{-2}$ 1 mark

$= 3 + \frac{1}{x^2}$

for $m = 7$

$7 = 3 + \frac{1}{x^2}$ 1 mark

$7x^2 = 3x^2 + 1$

$4x^2 = 1$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$

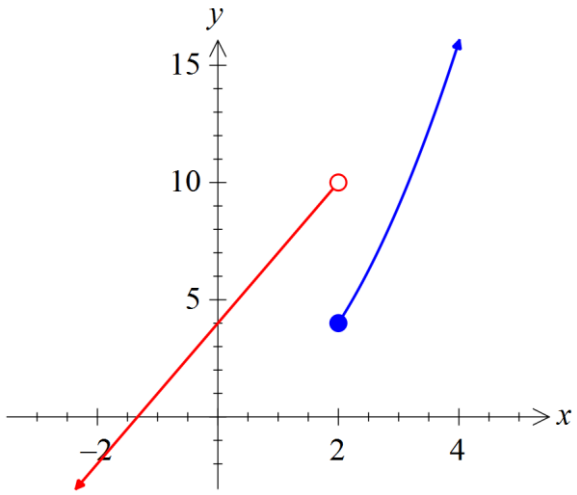
$y = \mp\frac{1}{2}$

$\left(\frac{1}{2}, -\frac{1}{2}\right)$ and $\left(-\frac{1}{2}, \frac{1}{2}\right)$ 1 mark

(d)

(i) $f(2) + f(0)$.
$= (2)^2 + (3 \times 0 + 4)$ 1 mark
$= 4 + 4 = 8$ 1 mark

(ii) .



1 mark general shape
1 mark for open and closed circles

End of Question 15

Question 16 (15 marks)

(a) $f(x) = x^2 - 3x$

$f(x+h) = (x+h)^2 - 3(x+h)$

$= x^2 + 2xh + h^2 - 3x - 3h$ 1 mark

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - (x^2 - 3x)}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 3h}{h}$$ 1 mark

$$= \lim_{x \to 0} \frac{h(2x + h - 3)}{h}$$

$$= \lim_{h \to 0} (2x + h - 3)$$

$= 2x - 3$ 1 mark

(b)

$$\cos^2 x - \sin^2 x = \frac{1}{2}$$

$$(1 - \sin^2 x) - \sin^2 x = \frac{1}{2}$$ 1 mark

$$1 - 2\sin^2 x = \frac{1}{2}$$

$$-2\sin^2 x = -\frac{1}{2}$$

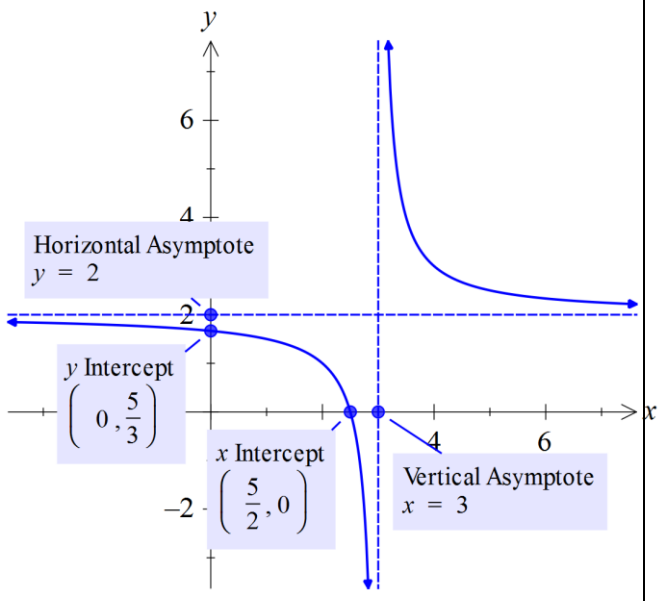
$$\sin^2 x = \frac{1}{4}$$

$$\sin x = \pm\frac{1}{2}$$ 1 mark

$x = 30, 180 - 30, 180 + 30, 360 - 30$

$x = 30°, 150°, 210°, 330°$ 1 mark

(c) 2 marks



(d) $(x - 3y + 5) + k(x + 2y) = 0$

$x - 3y + 5 + kx + 2ky = 0$

$(k+1)x + (2k-3)y + 5 = 0$

(i) $m_1 = -\frac{k+1}{2k-3} = -1$

$k + 1 = 2k - 3$

$k = 4$ 1 mark

(ii) $x^2 + y^2 - 10y + 21 = 0$

$x^2 + (y^2 - 10y + 25) = -21 + 25$

$x^2 + (y-5)^2 = 4$

Centre: $(0, 5)$ 1 mark

$(k+1)(0) + (2k-3)(5) + 5 = 0$

$10k - 10 = 0$

$k = 1$ 1 mark

(iii)

at $k = 4$

$x - 3y + 5 + (4)(x + 2y) = 0$

$x + y + 1 = 0$ ①

at $k = 1$

$x - 3y + 5 + (1)(x + 2y) = 0$

$2x - y + 5 = 0$

$y = 2x + 5$ ② 1 mark subst

subst ② into ①

$x + (2x + 5) + 1 = 0$

$3x = -6$

$x = -2$

$y = 2(-2) + 5$

$= 1$

$P(-2, 1)$ 1 mark

(e) $\frac{\sqrt{20-x} - 5}{5 + x} \times \frac{\sqrt{20-x} + 5}{\sqrt{20-x} + 5}$ 1 mark

$$= \frac{20 - x - 25}{5\sqrt{20-x} + 25 + x\sqrt{20-x} + 5x}$$

$$= \frac{-5 - x}{\sqrt{20-x}(5+x) + 5(5+x)}$$

$$= \frac{-(5+x)}{(\sqrt{20-x} + 5)(5+x)}$$

$$= -\frac{1}{\sqrt{20-x} + 5}$$

$$\lim_{x \to -5} -\frac{1}{\sqrt{20-x} + 5}$$

$$= -\frac{1}{\sqrt{20-(-5)} + 5}$$

$$= -\frac{1}{10}$$ 1 mark

End of Question 16
End of Section II
End of Examination