DETUR GLORIA SOLI DEO

Trinity Grammar School

Mathematics Department

2017

YEARLY EXAMINATION

PRELIMINARY ASSESSMENT
TASK 4

Year 11

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2.5 hours
- Write using black or blue pen
- NESA approved calculators for this course may be used
- A Reference Sheet for this course is supplied
- In Questions **11-16**, show all relevant mathematical reasoning and/or calculations
- Write your NESA Student Number (Year 12 HSC) or Name (Year 10 or 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.
- **Preliminary Assessment Weighting:** 40%

NESA Student Number
(Year 12 only)

Class Teacher:

....

Name:
(Year 10 or 11 only)

....

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Wednesday, 30 August 2017

Total marks – 100

Section I

10 marks

- Attempt all Questions
- Allow about **15** minutes for this section

Section II

90 marks

- Attempt all Questions
- Allow about **2** hours and 15 minutes for this section

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Section I **10 marks**

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Shade the correct response on the multiple-choice answer sheet for Questions 1 – 10
- Each question is worth 1 mark

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1 What is the domain of the function $y = \sqrt{x+8}$?

(A) $x \geq -8$

(B) $x > 8$

(C) $x \geq -8$

(D) $x > -8$

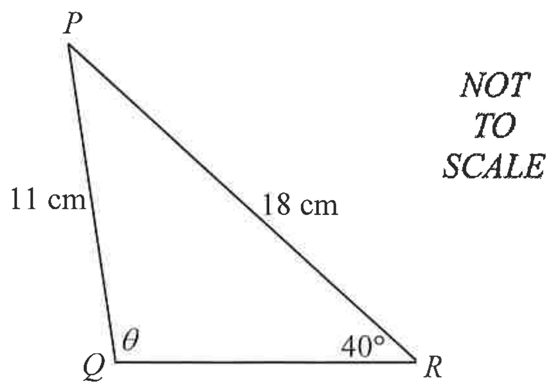
2 The diagram shows the triangle PQR where $PQ = 11\text{cm}$, $PR = 18$ cm and $\angle PRQ = 40°$. Which expression correctly gives the value of $\sin\theta$?

(A) $\dfrac{18}{11\sin 40°}$

(B) $\dfrac{11}{18\sin 40°}$

(C) $\dfrac{11\sin 40°}{18}$

(D) $\dfrac{18\sin 40°}{11}$



3 $A(3, y)$ and $B(7, 2)$ are points on the number plane. The gradient of AB is -3. What is the value of y?

(A) 14

(B) $\dfrac{1}{14}$

(C) -10

(D) $-\dfrac{1}{10}$

4 What are the solutions of the inequation $|3x-5|=10$?

(A) $x = -5$ and $x = 15$

(B) $x = -5$ and $x = 5$

(C) $x = -1\frac{2}{3}$ and $x = 5$

(D) $x = -5$ and $x = 1\frac{2}{3}$

5 How many solutions are there to the equation $x^3 - x^2 - 5x = 0$?

(A) 0

(B) 1

(C) 2

(D) 3

6 Solve $\cos\theta = \frac{-1}{\sqrt{2}}, 0 \le \theta \le 360^\circ$?

(A) $\theta = 45^\circ, 135^\circ$

(B) $\theta = 135^\circ, 225^\circ$

(C) $\theta = 225^\circ, 315^\circ$

(D) $\theta = 45^\circ, 315^\circ$

7 Simplify $\lim_{h\to 0} \frac{4(x+h)^2 - 4x^2}{h}$.

(A) $8x$

(B) $8xh + 8h$

(C) $8x + 8h$

(D) $8h$

8 The line $y = mx + \frac{11}{12}$ is a tangent to the curve $y = 3x^2 - 2x + 1$ at the point $\left(\frac{1}{6}, \frac{3}{4}\right)$.

What is the value of m?

(A) -3

(B) -1

(C) $\frac{1}{2}$

(D) 1

9 The function $y = f(x)$ is defined as:

$$f(x) = \begin{cases} x^2 + 1 & x < 0 \\ 1 - x^2 & x \geq 0 \end{cases}$$

What is the range of $f(x)$?

(A) all real y

(B) $y < -1, y \geq 1$

(C) $y \leq -1, y \geq 1$

(D) $-1 \leq y \leq 1$

10 If $8^{x+3} \times 2^{x-2} = 2^x \times 4^{3x-1}$, what is the value of x?

(A) -3

(B) 0

(C) 2

(D) 3

End of Section I

Section II commences on the next page

Section II **90 marks**

- Attempt Questions 11-16
- Allow about 2 hours and 15 minutes for this section
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
- In Questions 11 – 16, your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 **(15 marks)** Use a SEPARATE writing booklet

(a) Evaluate $\sqrt{\dfrac{2.6^2+5.2^2}{1.3}}$, correct to three significant figures. **1**

(b) Expand and simplify $5(x+3)-x(7-2x)$. **2**

(c) Solve these equations simultaneously: **2**

$$\begin{aligned} 3x + 6y &= -6 \\ 7x - 6y &= 51 \end{aligned}$$

(d) Fully simplify: $\dfrac{x+6}{x^2+11x+30}$ **2**

(e) Find the equation of any asymptotes of the graph of $f(x)=\dfrac{1}{x^2-4}$. **2**

(f) Solve: $\dfrac{5x}{3}+12=\dfrac{3x}{2}$ **2**

(g) Rationalise the denominator of the expression $\dfrac{10}{\sqrt{7}+\sqrt{2}}$. **2**

(h) The point $M(-4,1)$ is the midpoint of $A(2,6)$ and $B(b^2+7b,-4)$.
Find all values of b. **2**

Question 12 **(15 marks)** Use a SEPARATE writing booklet

(a) Given $f(x) = 3 + 2|x|$, find $f(-4)$. **1**

(b) A piecewise function is defined as follows: **2**

$$f(x) = \begin{cases} 3 - x & x < 0 \\ \sqrt{9 - x^2} & 0 \leq x \leq 3 \end{cases}$$

Sketch the curve $y = f(x)$, showing all intercepts.

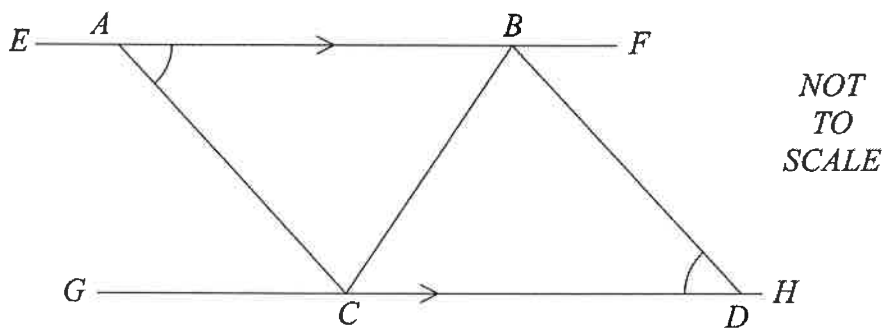
(c) Show by shading, the region where $y \leq 9 - x^2$ and $y \geq x$. **2**

(d) In the diagram below, $EF \parallel GH$.

The points A and B lie on EF and C and D lie on GH.

$\angle BAC = \angle BDC$. **2**

(i) Prove that $\triangle ABC \equiv \triangle BCD$.



(ii) Use the result from (i) to prove that $AC \parallel BD$. **1**

Question 12 continues on next page

(e) Show that $f(x)=\dfrac{x^7-x^3}{x^2+1}$ is an odd function. **3**

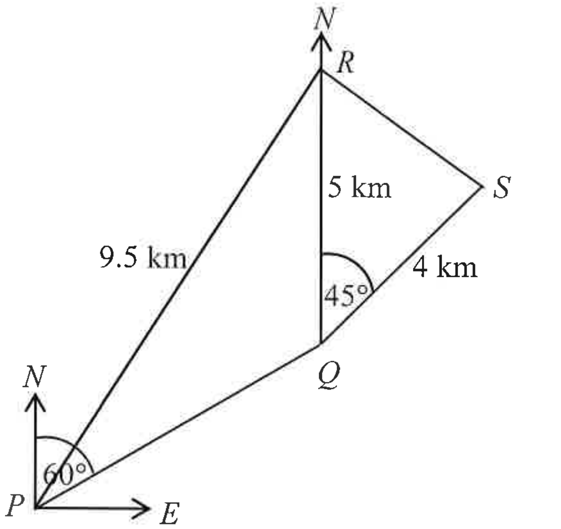
(f) Show that the points $\mathrm{A}(1,4)$, $\mathrm{B}(3,7)$ and $\mathrm{C}(5,10)$ are collinear. **2**

(g) Solve $x^2-6x-10=0$ by completing the square. Write your answer in exact form. **2**

End of Question 12

Question 13 **(15 marks)** Use a SEPARATE writing booklet

(a) Four towns are represented by the points P, Q, R and S. Straight roads connect the towns to each other. Town Q is on a bearing of 060° from town P. Town R is 5 km due north of town Q. Town S is 4 km from town Q, on a bearing of 045°. Town R is 9.5 km from town P.

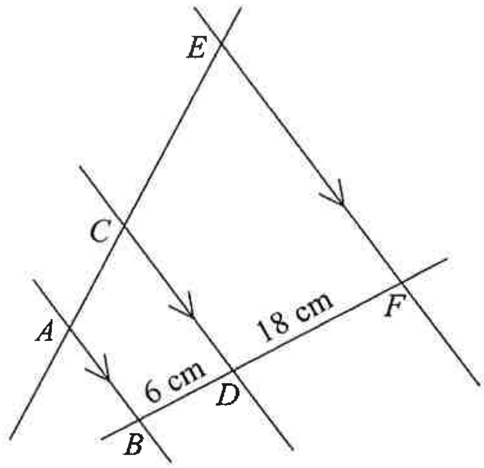


NOT TO SCALE

(i) Find the distance from R to S, correct to one decimal place. **1**

(ii) Find the bearing of R from P, to the nearest degree. **3**

(b) In the diagram below $AB \parallel CD \parallel EF$, $AE = 36$ cm, $BD = 6$ cm and $DF = 18$ cm. Find the length of AC. **2**



NOT TO SCALE

Question 13 continues on next page

(c) Solve $3\tan^2\theta - 1 = 0$ for $0 \leq \theta \leq 360°$. **2**

(d) If $y = \dfrac{1}{\sqrt{2x}}$, find $\dfrac{dy}{dx}$. **2**

(e) If $y = \dfrac{2x}{3+x^2}$;

(i) Show that $\dfrac{dy}{dx} = \dfrac{6-2x^2}{(3+x^2)^2}$. **2**

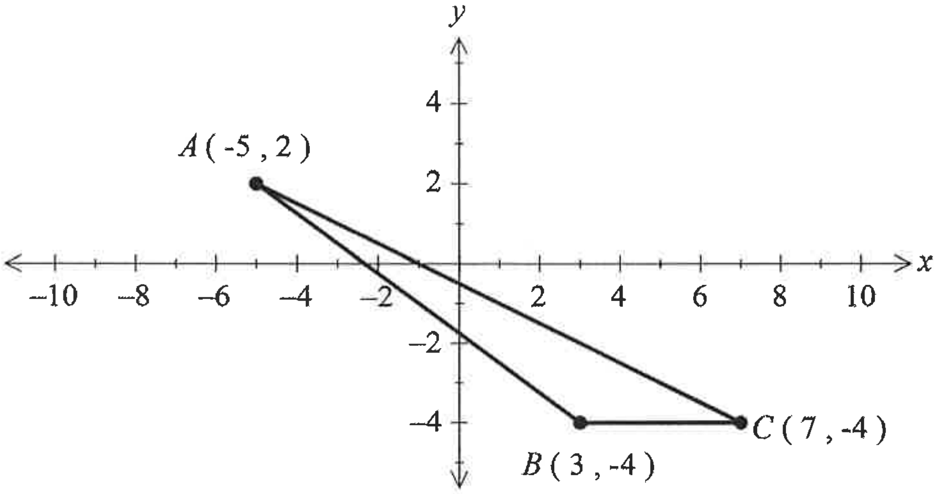
(ii) Find the equation of the tangent to the curve $y = \dfrac{2x}{3+x^2}$ at $x = 1$. **2**

(f) Find the equation of the circle with radius 3 units and centre $(0,-4)$. **1**

End of Question 13

Question 14 **(15 marks)** Use a SEPARATE writing booklet

(a) On a number plane, a triangle is drawn so that its vertices are $A\ (-5\ ,2)$, $B\ (3\ ,-4)$ and $C\ (7\ ,-4)$.



(i) What is the gradient of the line AC ? **1**

(ii) What is the equation of the line AC in general form? **2**

(iii) What is the exact value of the perpendicular distance of the point B from AC? **1**

(iv) Calculate the distance AC. **1**

(v) Hence, or otherwise, calculate the area of $\triangle ABC$. **1**

(vi) A line is drawn parallel to AB, through C. What is the gradient of this line? **1**

(vii) Find the angle that AB makes with the positive direction of the x axis. **1**

(viii) The line AB meets the y axis at D. Find the coordinates of D. **2**

Question 14 continues on next page

(b) Simplify $\dfrac{\cos\left(180^\circ+\theta\right)}{\sin\left(90^\circ-\theta\right)}$. **1**

(c) Find the exact value of $\sin^2 60^\circ + \sin^2 30^\circ$. **1**

(d) If $\cos\theta = \dfrac{a^2-b^2}{a^2+b^2}$, show that the value of $\sin\theta$ is $\dfrac{2ab}{a^2+b^2}$. **3**

End of Question 14

Question 15 **(15 marks)** Use a SEPARATE writing booklet

(a) Simplify $16\sqrt{7} - \sqrt{567}$. **1**

(b) Differentiate $f(x) = 2x - 3$ from first principles. **3**

(c) Evaluate $\lim\limits_{x \to 2} \dfrac{x^3 - 6x^2 + 8x}{x^2 + x - 6}$. **3**

(d) Solve the equation $\sqrt{16^{3x-1}} = \sqrt[3]{64^x}$ for x. **3**

(e) Solve the inequality $|2x - 1| = |1 - x|$ for x. **2**

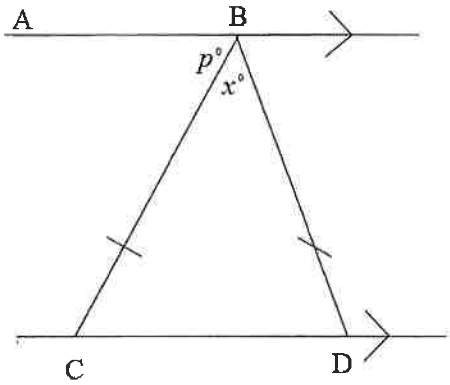
(f) Show that $2\operatorname{cosec}\theta = \dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta}$. **3**

End of Question 15

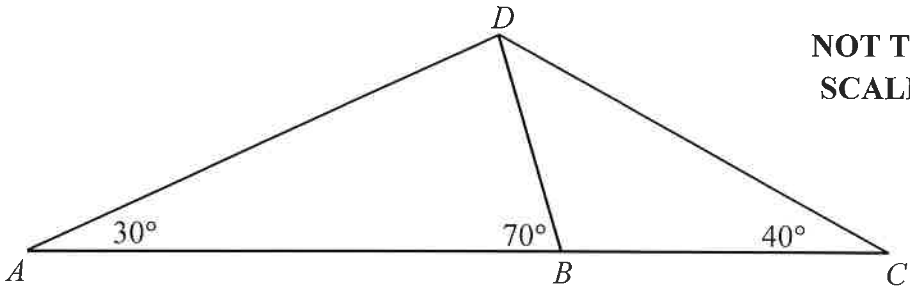
Question 16 **(15 marks)** Use a SEPARATE writing booklet

(a) Given $\angle ABC = p°, \angle CBD = x°,\ BC = BD,\ AB \| CD$

Express p in terms of x giving reasons for your answer. **2**



(b) The diagram shows ΔACD where $\angle DAC = 30°$ and $\angle DCA = 40°$.
The point B lies on AC such that $\angle DBA = 70°$.



(i) Prove that ΔACD and ΔDCB are similar. **2**

(ii) $CD = x$ cm, $AC = (x+3)$ cm and $BC = (x-2)$ cm.
Using (i), or otherwise, find the value of x. **2**

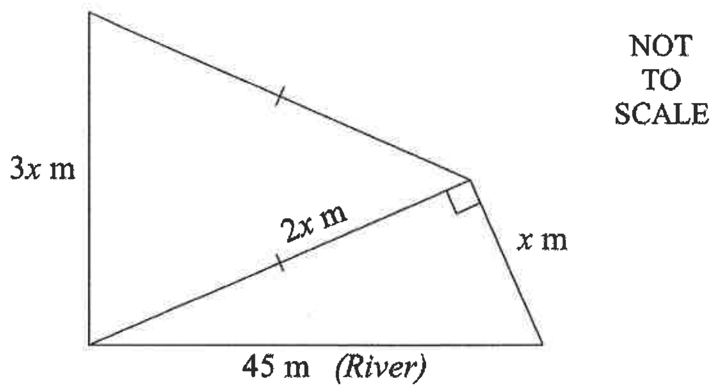
(c) If $y = 5x^2\left(x^2+3\right)^3$, show that $\dfrac{dy}{dx} = 10x\left(4x^2+3\right)\left(x^2+3\right)^2$. **3**

(d) Given that $x = \sqrt{(4-t)^3}$, find the exact value of $\dfrac{dx}{dt}$ when $t = 2$. **2**

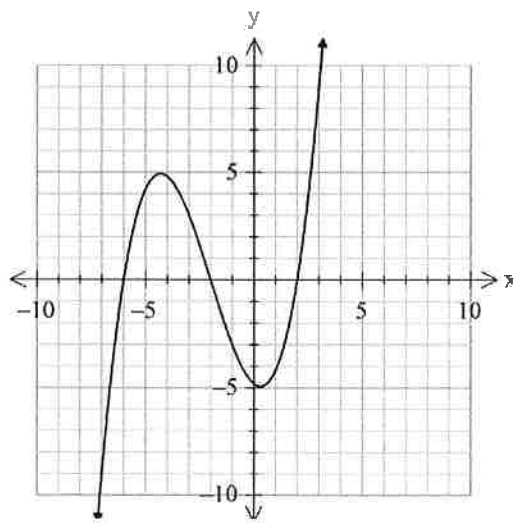
Question 16 continues on next page

(e) Lara needs to fence off two triangular paddocks, using a 45 m stretch of straight river as the side of one triangle, as shown in the diagram below.

What is the exact length of fencing that Lara requires to fence the two paddocks? Assume that the stretch of straight river does not need to be fenced. **2**



(f) Shown below is the graph of $y = f'(x)$.



Given that $f(x)$ is known to be positive definite, meaning it always lies above the x-axis, sketch a possible graph of $f(x)$. **2**

End of Question 16
End of Section II
End of Examination

2017 Year 11 MA Task 4 solutions

Yr 11 2 unit AT4 Solutions 2017 (VINCENT/KESBY)

Section I

Multiple Choice

1. $y = \sqrt{x+8}$

$x+8 \geqslant 0$

$x \geqslant -8$

C

2. $\dfrac{\sin\theta}{18} = \dfrac{\sin 40^\circ}{11}$

$\sin\theta = \dfrac{18\sin 40}{11}$

D

3. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$-3 = \dfrac{2-y}{7-3}$

$-3 = \dfrac{2-y}{4}$

$-12 = 2-y$

$-14 = -y$

$y = 14$

A

4. $|3x-5| = 10$

$3x - 5 = 10$, $\quad 3x = 15$, $\quad x = 5$

$3x-5 = -10$, $\quad 3x = -5$, $\quad x = \dfrac{-5}{3} = -1\frac{2}{3}$

$\therefore x = -1\frac{2}{3}, 5$

C

5. $x(x^2 - x - 5) = 0$

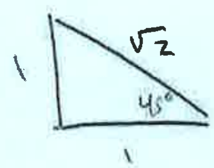
$x = 0$ & $x^2 - x - 5 = 0$

↓

using quadratic formulae, 2 solutions.

$\therefore$ has three solutions.

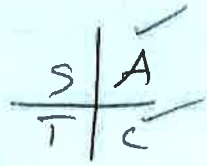
D

6. 

$\cos\theta = -\dfrac{1}{\sqrt{2}}$

$\cos 45 = -\dfrac{1}{\sqrt{2}}$

$\therefore \theta = \cancel{45^\circ}, \cancel{315^\circ}$



~~D~~ ~~C~~ B

$135^\circ, 225^\circ$

7. $\lim_{h \to 0} \dfrac{4(x+h)^2 - 4x^2}{h}$

$= \lim_{h \to 0} \dfrac{4(x^2+2xh+h^2)-4x^2}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{4x^2}+8xh+4h^2 \cancel{-4x^2}}{h}$

$= \lim_{h \to 0} \dfrac{4h(2x+h)}{h}$

$= 4(2x+0)$

$= 8x$

A

8. $y = 3x^2 - 2x + 1$

$y' = 6x - 2$

at $x = \frac{1}{6}$

$y' = 6 \times \frac{1}{6} - 2$

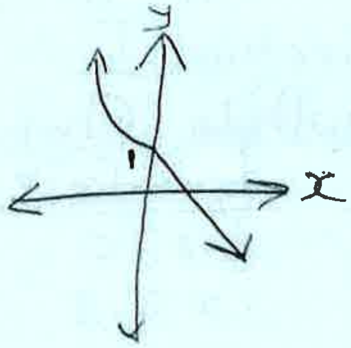
$= 1 - 2$

$y' = -1$

$\therefore m = -1$

B

9.



all real y

A

10. $8^{x+3} \times 2^{x-2} = 2^x \times 4^{3x-1}$

$2^{3(x+3)} \times 2^{(x-2)} = 2^x \times 2^{2(3x-1)}$

$2^{3x+9+x-2} = 2^{x+6x-2}$

$2^{4x+7} = 2^{7x-2}$

$4x+7 = 7x-2$

$+7 = 3x-2$

$9 = 3x$

$x = 3$

D

Section II

Q11 a) 5.099 — must be $\underline{5.10}$

$= 5.10$ (3 sig fig) (1)

b) $5(x+3) - x(7-2x)$

$= 5x + 15 - 7x + 2x^2$ (1)

$= 2x^2 - 2x + 15$ (1)

c) $3x + 6y = -6$ $\boxed{1}$

$7x - 6y = 51$ $\boxed{2}$

$\boxed{1} + \boxed{2}$

$10x = 45$

$x = 4.5$ (1)

sub $x = 4.5$ into $\boxed{1}$

$3(4.5) + 6y = -6$

$13.5 + 6y = -6$

$6y = -19.5$

$y = 3.25$ (1)

d) $\dfrac{(x+6)}{(x+5)(x+6)}$ (1)

$= \dfrac{1}{(x+5)}$ (1)

e) $f(x) = \dfrac{1}{(x-2)(x+2)}$ 1

$\therefore x \neq 2, -2$

$\therefore$ eqn of asymptotes ~~a~~ are $x=2$, $x=-2$ 1
& $y=0$

f) $\dfrac{5x}{3} + 12 = \dfrac{3x}{2}$

$5x + 36 = \dfrac{9x}{2}$ 1

$10x + 72 = 9x$

$x + 72 = 0$

$x = -72$ 1

g) $\dfrac{10}{\sqrt{7}+\sqrt{2}} \times \dfrac{(\sqrt{7}-\sqrt{2})}{(\sqrt{7}-\sqrt{2})}$

$= \dfrac{10\sqrt{7} - 10\sqrt{2}}{7-2}$ 1

$= \dfrac{10\sqrt{7} - 10\sqrt{2}}{5}$

$= 2\sqrt{7} - 2\sqrt{2}$ 1

h) $\dfrac{2 + b^2 + 7b}{2} = -4$ 1

$2 + b^2 + 7b = -8$

$b^2 + 7b + 10 = 0$

$(b+5)(b+2) = 0$

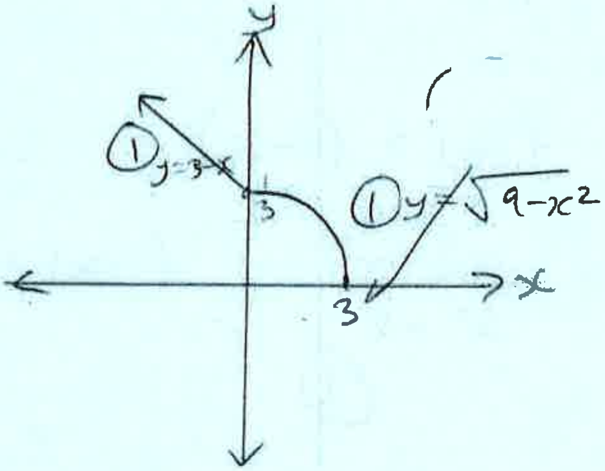
$\therefore b = -5, -2$ 1

Q12 a) $f(-4) = 3 + 2|-4|$

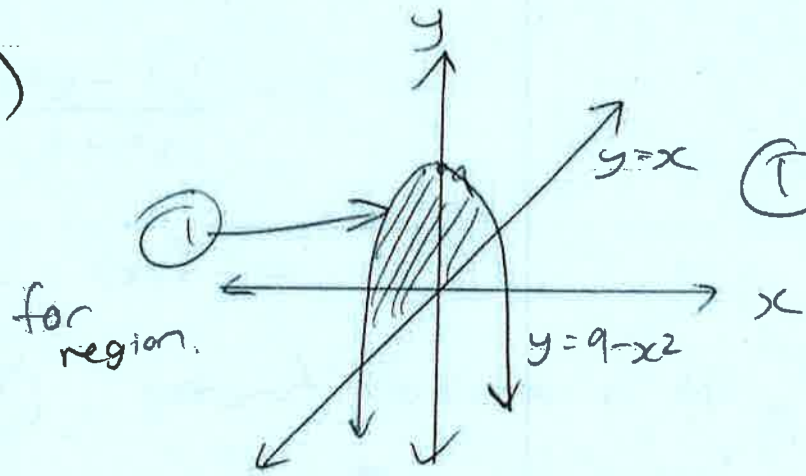
$= 3 + 8$

$= 11$ (1)

b)



c)



(1) for graphs.

d) i) In $\triangle ABC \equiv \triangle BCD$,

$\angle CAB = \angle BDC$ (given) (1)

$\angle ABC = \angle DCB$ (alternate angles, parallel lines).

BC is common.

$\therefore \triangle ABC \equiv \triangle BCD$ (AAS) (1)

ii) $\angle DBC = \angle ACB$ (matching angles, congruent triangles)

$\therefore AC \parallel BD$ (alternate angles are equal on transversal BC) 1

e) odd if $-f(x) = f(-x)$

$$-f(x) = \frac{x^3 - x^7}{x^2+1}$$

1

$$f(-x) = \frac{(-x)^7 - (-x)^3}{(-x)^2+1}$$

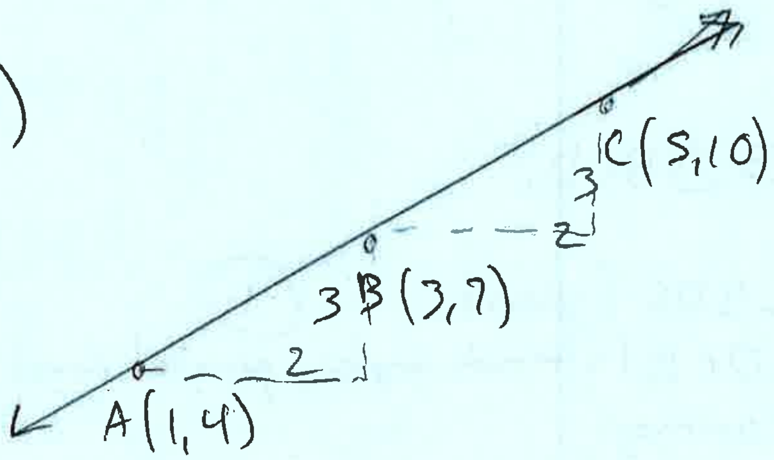
$$= \frac{-x^7 + x^3}{x^2+1}$$

$$= \frac{x^3 - x^7}{x^2+1}$$ 1

$$= -f(x).$$

$\therefore$ it is an odd function. 1

f)



$\cancel{m}$ $mAB = \frac{3}{2}$ 1

$mAC = \frac{6}{4}$

$= \frac{3}{2}$

$mBC = \frac{3}{2}$

$\therefore mAB = mAC = mBC$

$\therefore$ the points A, B & C are collinear 1

12 g) $x^2 - 6x = 10$

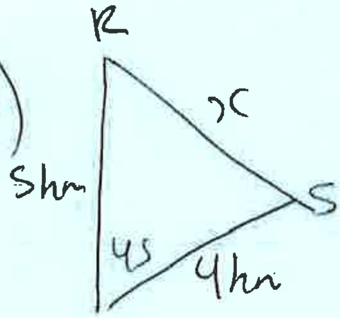
$x^2 - 6x + 9 = 19$ 1

$(x-3)^2 = 19$

$x - 3 = \pm\sqrt{19}$

$x = 3 \pm \sqrt{19}$ 1

Q13 a)

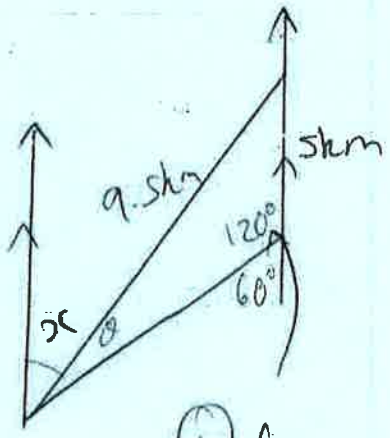


i) $x^2 = 5^2 + 4^2 - 2 \times 5 \times 4 \times \cos 45$

$x^2 = 12.715\ldots$

$x = 3.6$ km (1)

ii)



(1) for finding 120°

$$\frac{\sin\theta}{5} = \frac{\sin 120}{9.5}$$

$$\sin\theta = \frac{5\sin 120}{9.5}$$

$\theta = 27.11660254°$ (1)

$60 - \theta = x$

$60 - 27.11660254 = x$

$x = 32.88339746$

$x = 033°T$ (1)

b) $AC = x$

$CE = 36 - x$

$$\frac{18}{6} = \frac{36-x}{x} \quad (1)$$

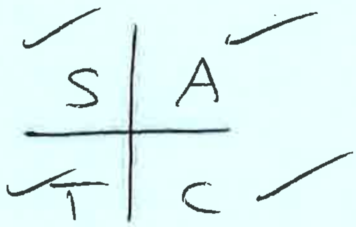
$18x = 216 - 6x$

$24x = 216$

$x = 9$ cm (1)

c) Solve $3\tan^2\theta - 1 = 0$ for $0 \leq \theta \leq 360°$

$$3\tan^2\theta = 1$$
$$\tan^2\theta = \frac{1}{3}$$
$$\tan\theta = \pm\frac{1}{\sqrt{3}} \quad 1 \qquad \theta = 30°$$



$$\theta = 30°, 150°, 210°, 330° \quad 1$$

d) $y = \dfrac{1}{\sqrt{2x}}$

$$y = (2x)^{-\frac{1}{2}} \quad 1$$
$$y' = -\frac{1}{2}(2x)^{-\frac{3}{2}} \times 2$$
$$= \frac{-1}{\sqrt{(2x)^3}} \quad 1$$
$$= \frac{-1}{\sqrt{8x^3}}$$

e) i) $y = \dfrac{2x}{3+x^2}$

$$\begin{aligned} u &= 2x \\ u' &= 2 \\ v &= 3+x^2 \\ v' &= 2x \end{aligned}$$

$$\frac{dy}{dx} = \frac{2(3+x^2) - 4x^2}{(3+x^2)^2} \quad 1$$

$$\frac{dy}{dx} = \frac{6 + 2x^2 - 4x^2}{(3+x^2)^2}$$

$$\therefore \frac{dy}{dx} = \frac{6 - 2x^2}{(3+x^2)^2} \quad 1$$

ii) At $x = 1$

y-coordinate

$$y = \frac{2(1)}{3+(1)^2} = \frac{2}{4} = \frac{1}{2}$$

gradient

$$y' = \frac{6-2(1)^2}{(3+(1))^2} = \frac{4}{16} = \frac{1}{4} \quad 1$$

$$m(x-x_1) = y - y_1$$

$$\frac{1}{4}(x-1) = y - \frac{1}{2} \quad 1$$

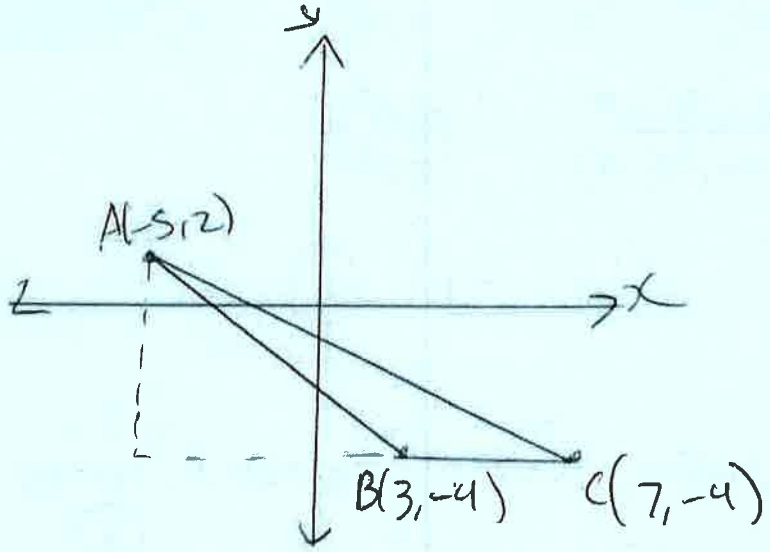
$$x - 4y + 1 = 0$$

f) $x^2 + y^2 = r^2$

$$x^2 + (y+4)^2 = 3^2$$

$$x^2 + (y+4)^2 = 9 \quad 1$$

14 a)



i) $m_{AC} = \frac{-6}{12}$

$= -\frac{1}{2}$ 1

ii) $-\frac{1}{2}(x - -5) = y - 2$

$\frac{-x}{2} - \frac{5}{2} = y - 2$ 1

$-x - 5 = 2y - 4$

$x + 2y + 1 = 0$ 1

iii) $d = \frac{|3 - 8 + 1|}{\sqrt{1^2 + 2^2}}$

$= \frac{4}{\sqrt{5}}$ 1

iv) $6^2 + 12^2 = AC^2$

$AC = \sqrt{180}$ 1

v) $A = \frac{1}{2} \times \sqrt{180} \times \frac{4}{\sqrt{5}}$

$= 12$ units 1

vi) $m_{AB} = m_2$

$\cancel{m_2} = m_{AB} = \frac{-6}{8}$

$= \frac{-3}{4}$

$\therefore m_2 = \frac{-3}{4}$ 1

vii) $\tan\theta = \frac{-3}{4}$

$\theta = 36.869...^\circ$

Angle $= 180^\circ - \theta$

$= 143^\circ$ 1

viii) $-\frac{3}{4}(x - 7) = y + 4$ 1

$-\frac{3x}{4} + \frac{21}{4} = y + 4$

$y = \frac{-3x}{4} + \frac{5}{4}$

1

$\therefore D = (0, \frac{5}{4})$ as y-int $= \frac{5}{4}$

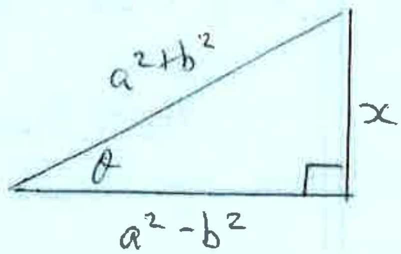
Q14 b) $\dfrac{\cos(180^\circ + \theta)}{\sin(90 - \theta)} = \dfrac{-\cos\theta}{\cos\theta}$

$= -1$ 1

c) $\sin^2 60^\circ + \sin^2 30^\circ = \sin^2 60 + \cos^2 60$

$= 1$ 1

d)



$\cos\theta = \dfrac{a^2-b^2}{a^2+b^2}$

$\sin\theta = \dfrac{x}{a^2+b^2}$ 1

$x^2 = (a^2+b^2)^2 - (a^2-b^2)^2$

$= (a^4 + 2a^2b^2 + b^4) - (a^4 - 2a^2b^2 + b^4)$

$= 4a^2b^2$

$x = 2ab$ 1

$\therefore \sin\theta = \dfrac{2ab}{a^2+b^2}$ 1

15 a) $16\sqrt{7} - \sqrt{567}$

$= 16\sqrt{7} - \sqrt{81} \times \sqrt{7}$

$= 16\sqrt{7} - 9\sqrt{7}$

$= 7\sqrt{7}$

Question updated

Q15 b) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{2(x+h)^2 - 3(x+h) + 7 - (2x^2 - 3x + 7)}{h}$ 1

$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 3x - 3h + 7 - 2x^2 + 3x - 7}{h}$ 1

$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 \cancel{-3x} - 3h \cancel{+7} \cancel{-2x^2} \cancel{+3x} \cancel{-7}}{h}$

$= \lim_{h \to 0} \frac{4x\cancel{h} + 2h^{\cancel{2}} - 3\cancel{h}}{\cancel{h}}$

$= 4x + 2(0) - 3$

$= 4x - 3$ 1

c) $\lim_{x \to 2} \frac{x(x^2 - 6x + 8)}{(x+3)(x-2)}$ 1

$= \lim_{x \to 2} \frac{x(x-4)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$ 1

$= \frac{2(2-4)}{(2+3)}$

$= \frac{-4}{5}$ 1

15 d) $\sqrt{16^{3x-1}} = \sqrt[3]{64^{x}}$

$16^{\frac{3x-1}{2}} = 64^{\frac{x}{3}}$ (1)

$4^{3x-1} = 4^{x}$ (1)

$3x-1 = x$

$2x = 1$

$x = \frac{1}{2}$ (1)

e) $|2x-1| = |1-x|$

Both +ve

$2x-1 = 1-x$

$3x-1 = 1$

$3x = 2$

$x = \frac{2}{3}$

(1)

One +ve, One -ve

$-2x+1 = 1-x$

$1 = 1+x$

$x = 0$

(1)

f) $2\operatorname{cosec}\theta = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

LHS

$2\operatorname{cosec}\theta = 2 \times \frac{1}{\sin\theta}$

$= \frac{2}{\sin\theta}$ (1)

RHS

$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$ (1)

$= \frac{2 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$

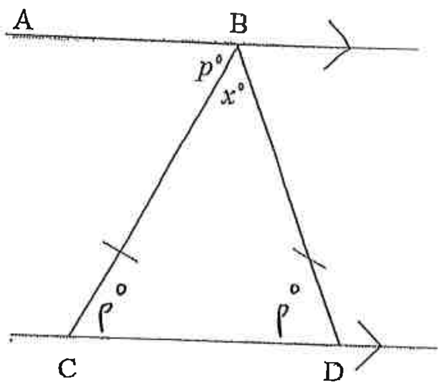
$= \frac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}}$

$= \frac{2}{\sin\theta}$

$= \text{LHS}$ (1)

SOLUTIONS (KESBY)

Q16. (a)



$\angle BCD = p°$ (equal alternate angles, $AB \parallel CD$)

$\angle BDC = p°$ (base angles of isosceles $\triangle BCD$ are equal)

$p° + p° + x° = 180°$ (angle sum of $\triangle BCD$ is 18

$\therefore \quad 2p + x = 180$

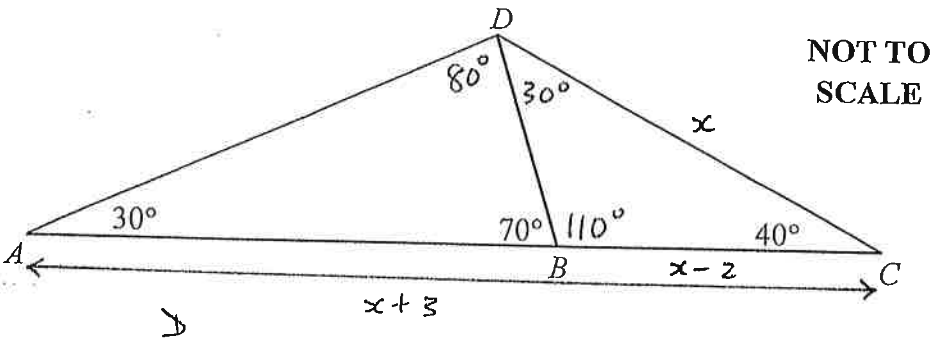
$2p = 180 - x$

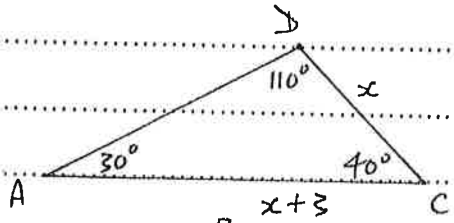
$p = \frac{180 - x}{2}$

Marking scheme

* Correct expression for p in terms of x with 3 thorough reasons – 2 marks
* One reason missing – 1 mark
* Two or more reasons missing – 0 marks

Q16. (b)

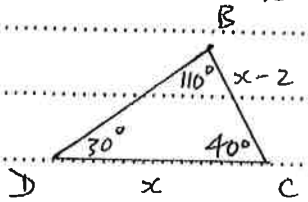




$\angle DBC = 180° - 70° = 110°$ (a straight angle is $180°$)

$\angle BDC = 180° - 110° - 40° = 30°$ ($\angle$ sum of $\triangle DCB$ is 18(

$\angle ADB = 180° - 70° - 30° = 80°$ ($\angle$ sum of $\triangle ADB$ is 18



$$\left.\begin{array}{l} \text{Now, } \angle DAC = \angle BDC = 30° \\ \angle DCA = \angle BCD = 40° \\ \angle ADC = \angle DBC = 110° \end{array}\right\} \therefore \triangle ACD \mathbin{|||} \triangle DCB \text{ (equiangular)}$$

Marking scheme:

* At least 2 reasons with correct test 'equiangular' – 2 marks
* Incorrect test (−1), one reason missing (−1).

Q16. (b) (ii) * Method 1 : (Using (i))

$$\frac{AC}{DC} = \frac{DC}{BC}$$ (matching sides of similar triangles)

$$\frac{x+3}{x} = \frac{x}{x-2}$$ ①

$$(x+3)(x-2) = x^2$$

$$x^2 + x - 6 = x^2$$

$$x - 6 = 0$$

$$\underline{x = 6}$$ ①

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* Method 2 : ('otherwise' method)

$$\frac{x}{\sin 110^\circ} = \frac{(x-2)}{\sin 30^\circ}$$ ①

$$x \sin 30^\circ = (x-2) \sin 110^\circ$$

$$x \sin 30^\circ = x \sin 110^\circ - 2 \sin 110^\circ$$

$$2 \sin 110^\circ = x \sin 110^\circ - x \sin 30^\circ$$

$$2 \sin 110^\circ = x (\sin 110^\circ - \sin 30^\circ)$$

$$\therefore \quad x = \frac{2 \sin 110^\circ}{(\sin 110^\circ - \sin 30^\circ)}$$

$$\underline{x = 4.27} \text{ (2d.p.)}$$ ①

Also, $$\frac{x+3}{\sin 110^\circ} = \frac{x}{\sin 30^\circ}$$ ①

$$(x+3) \sin 30^\circ = x \sin 110^\circ$$

$$x \sin 30^\circ + 3 \sin 30^\circ = x \sin 110^\circ$$

$$x (\sin 30^\circ - \sin 110^\circ) = -3 \sin 30^\circ$$

$$\underline{x = 3.41} \text{ (2d.p.)}$$ ①

Q16. (c) $y = 5x^2(x^2+3)^3$

$u = 5x^2 \qquad v = (x^2+3)^3$

$u' = 10x \qquad v' = 3(x^2+3)^2 \times 2x$

$= 6x(x^2+3)^2$ ✓

$$\frac{dy}{dx} = u'v + uv'$$

$= 10x(x^2+3)^3 + 5x^2 \cdot 6x(x^2+3)^2$ ✓

$= 10x(x^2+3)^3 + 30x^3(x^2+3)^2$

$= 10x(x^2+3)^2\left((x^2+3) + 3x^2\right)$ ✓

$= 10x(x^2+3)^2(4x^2+3)$

$= 10x(4x^2+3)(x^2+3)^2$

Q16. (d) $x = \sqrt{(4-t)^3}$

$x = \left((4-t)^3\right)^{\frac{1}{2}}$

$x = (4-t)^{\frac{3}{2}}$

$\frac{dx}{dt} = \frac{3}{2}(4-t)^{\frac{1}{2}} \times -1$

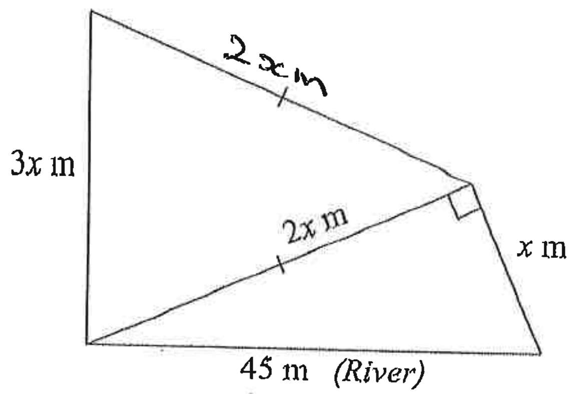
$= -\frac{3}{2}(4-t)^{\frac{1}{2}}$

$= -\frac{3}{2}\sqrt{4-t}$ ✓

When $t = 2$, $\frac{dx}{dt} = -\frac{3}{2} \times \sqrt{2}$

$= \frac{-3\sqrt{2}}{2}$ ✓

(Must be exact!)



Q16. (e)

Using Pythagoras' Theorem,

$$x^2 + (2x)^2 = 45^2$$
$$x^2 + 4x^2 = 2025$$
$$5x^2 = 2025$$
$$x^2 = 405$$
$$x = \pm\sqrt{405}$$

But $x > 0$, $x = \sqrt{405}$ ①

Now, perimeter of fencing $= x + 2x + 2x + 3x$

$= 8x$

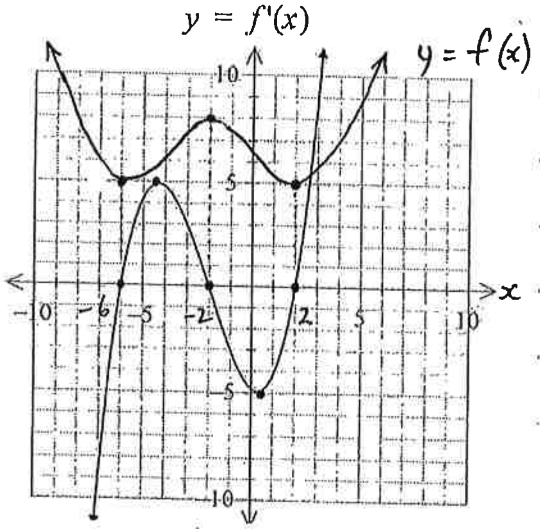
$= 8\sqrt{405}$ m

$= 8 \times \sqrt{81} \times \sqrt{5}$

$= 8 \times 9 \times \sqrt{5}$

$= 72\sqrt{5}$ m ①

Q16. (f)



Marking scheme:

* correct shape – 1 mark
* for turning points – 1 mark at $x = -6, -2, 2$