TRINITY GRAMMAR SCHOOL

Mathematics Department

(NESA Student Number | Year 12 only)

2018

Year 11

Mathematics

YEARLY EXAMINATION

PRELIMINARY ASSESSMENT TASK 3

Date of Assessment Task: Tuesday, 4 September 2018

General Instructions

- Reading time – 5 minutes
- Working time – 2.5 hours
- Write using black pen
- NESA approved calculators may be used
- A formula and data sheet is provided
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations
- Write your NESA Student Number (Year 12 HSC) or Name (Year 11 or 10) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating N/A and your NESA Student Number or Name

Total marks: 100

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6 – 13)

- Attempt Questions 11 – 16
- Allow about 2 hour and 15 minutes for this section

- **Preliminary Assessment Weighting:** 40%

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 What is 38 900 000 in scientific notation?

(A) 0.389×10^{8}

(B) 38.9×10^{6}

(C) 3.89×10^{7}

(D) 3.89×10^{-7}

2 Which of the following is equal to $\dfrac{1}{\sqrt{17}+\sqrt{3}}$?

(A) $\dfrac{\sqrt{17}-\sqrt{3}}{14}$

(B) $\dfrac{14}{\sqrt{17}-\sqrt{3}}$

(C) $\dfrac{\sqrt{17}-\sqrt{3}}{6}$

(D) $\dfrac{\sqrt{17}+\sqrt{3}}{14}$

3 What are the solutions of $\sin x=\dfrac{\sqrt{3}}{2}$, for $0° \le x \le 360°$?

(A) 60° or 240°

(B) 30° or 150°

(C) 30° or 210°

(D) 60° or 120°

4 What is the equation of a circle with centre (2, -1) and radius 3?

(A) $(x-2)^2+(y+1)^2=3$

(B) $(x+2)^2+(y-1)^2=3$

(C) $(x-2)^2+(y+1)^2=9$

(D) $(x+2)^2+(y-1)^2=9$

5 Simplify $2-(3-2x)$

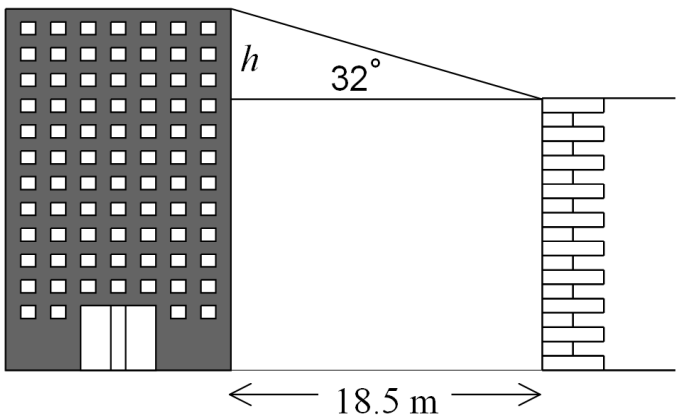
(A) $-1+2x$

(B) $-1-2x$

(C) $1+2x$

(D) $-2+2x$

6 The two buildings below are standing on level ground. The horizontal distance between the buildings is 18.5 metres and the angle of elevation between the buildings is 32º.



NOT TO SCALE

What is the difference in the vertical height (*h*) between the top of the two buildings?

(A) 9.8 m

(B) 11.6 m

(C) 15.7 m

(D) 29.6 m

7 If $y = 2\sqrt{x}$, then which expression for $\frac{dy}{dx}$ is correct?

(A) $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$

(B) $\frac{dy}{dx} = \frac{2}{\sqrt{x}}$

(C) $\frac{dy}{dx} = \frac{\sqrt{x}}{2}$

(D) $\frac{dy}{dx} = 2$

8 What is the solution to the inequality $|2x+1| \leq 3$?

(A) $x \geq 1$ or $x \geq -2$

(B) $x \geq 1$ or $x \geq 2$

(C) $x \leq 1$ or $x \geq -2$

(D) $x \leq 1$ or $x \geq 2$

9 The sum of the interior angles of a regular polygon is $1440°$.

How many sides does the polygon have?

(A) 6

(B) 10

(C) 12

(D) 8

10 What is the point of intersection of the lines, $y = x + 2$ and $y = -x + 4$?

(A) (1, 2)

(B) (2, 1)

(C) (1, 3)

(D) (3, 2)

End of Section I
Section II commences on the next page

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 15 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

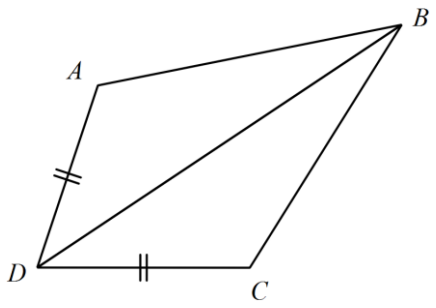
Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Evaluate, to 2 significant figures $\dfrac{(2.4)^2}{\sqrt{2\pi - 5}}$ **2**

(b) Simplify $\dfrac{x^2 - x + xy - y}{x + y}$ **3**

(c) Solve $x^2 + 2x - 5 = 0$, leaving the answer in simplified surd form. **3**

(d) In the diagram shown below $AD = DC$ and BD bisects $\angle ADC$.



(i) Prove $\Delta ADB \equiv \Delta CDB$. **3**

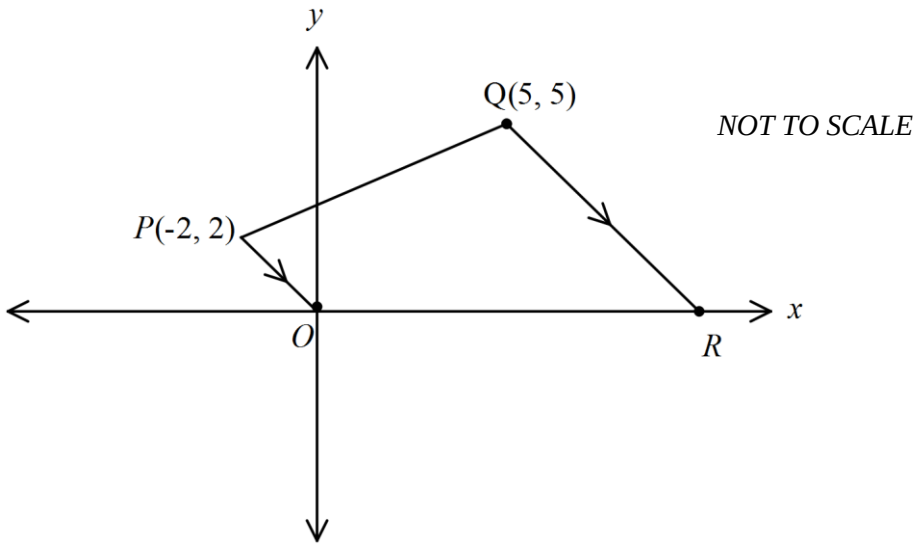
(ii) Prove $AB = CB$. **1**

(e) Find the equation of the tangent to the curve $y = 2x^3 + 5x + 3$ at the point (1, 10). **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet

In the diagram *OPQR* is a trapezium with *OP* parallel to *RQ*. *P* and *Q* are the points (-2, 2) and (5, 5) respectively. Point *O* is the origin and the line *RQ* meets the *x*-axis at *R*.



(a) Write down the midpoint of *PQ*. **1**

(b) Write down the gradient of *OP*. **1**

(c) Show that the equation of the line *RQ* is $x + y - 10 = 0$. **3**

(d) Find the angle the line RQ makes with the positive direction of the *x*-axis. **2**

(e) Find the *x* - coordinate of point *R*. **1**

(f) Find the perpendicular distance from point *O* to the line *RQ*. **2**
Give the answer in exact form.

Question 12 continues on page 8

(g) Find the length of QR, giving the answer in simplified exact form. **2**

(h) Hence or otherwise, find the area of trapezium $ORQP$. **3**

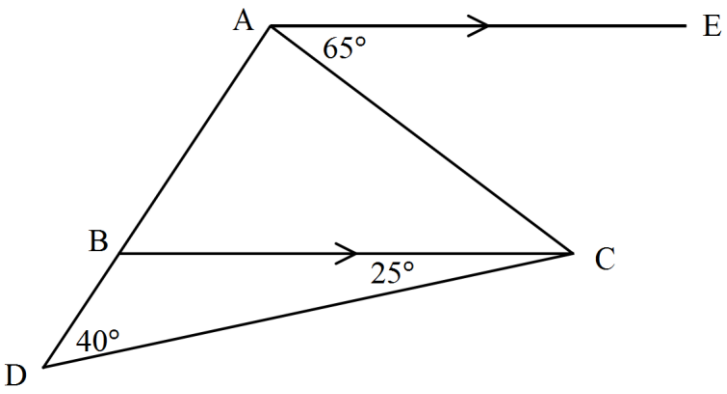
End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) In the diagram below, AE is parallel to BC. **2**

$\angle BDC = 40°$, $\angle BCD = 25°$ and $\angle EAC = 65°$.

Prove that ΔABC is isosceles giving reasons.



NOT TO SCALE

(b) Differentiate:

(i) $y = \dfrac{2}{x}$ **2**

(ii) $f(x) = 3x(x-2)^4$, giving the answer in fully factorised form. **3**

(iii) $g(x) = \dfrac{x}{x-1}$ **2**

(c) (i) Sketch the graph of $f(x)$, showing all the important features.

$$f(x) = \begin{cases} 3^x & \text{for } x \geq 0 \\ 2 - x^2 & \text{for } x < 0 \end{cases}$$ **3**

(ii) For which values of x is $y = f(x)$ not differentiable? **1**

(d) State the domain and range of the curve $y = \sqrt{25 - x^2}$. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) Find the exact value of $\sin(-135^\circ)$ **1**

(b) (i) Sketch the graph of $y = \cos x$ for $-180^\circ \le x \le 180^\circ$. **1**

(ii) How many solutions to the equation $\cos x = \frac{-1}{2}$ are there in the interval $-180^\circ \le x \le 180^\circ$? **1**

(c) Prove that $\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta} = \sec^2\theta \operatorname{cosec}^2\theta$. **3**

(d) Two ships sail from the town of Port Royal (*P*). The ship Black Pearl (*B*) sails on a bearing of 118° and the ship Jolly Roger (*J*) on a bearing of 276°. Both ships sail for 3 hours, the Black Pearl at a speed of 8 knots and the Jolly Roger at a speed of 12 knots.

(1 knot = 1 nautical mile per hour)

(i) Draw a diagram to show the distance and direction of the two ships from *P* after 3 hours. **1**

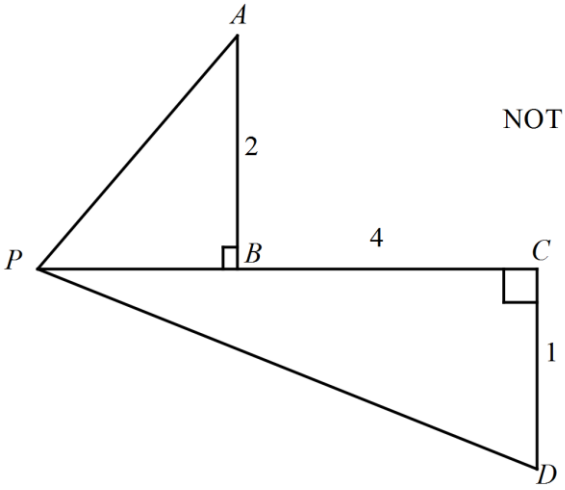
(ii) Find the distance, in nautical miles, between the 2 ships after 3 hours. (Answer correct to 1 decimal place) **2**

(iii) Find the bearing of the Jolly Roger from the Black Pearl after 3 hours. (Answer to the nearest degree) **2**

Question 14 continues on page 11

(e) In the diagram $AB = 2$, $BC = 4$ and $CD = 1$. **4**

If $PD^2 = 2PA^2$, find PB.



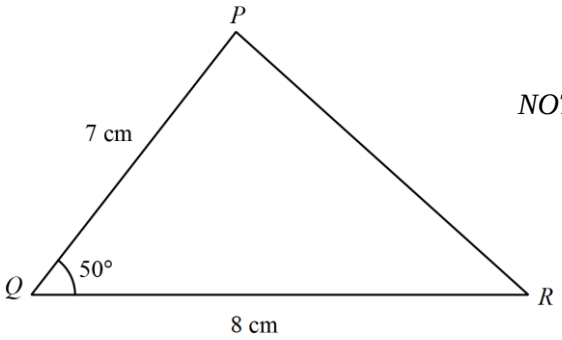
End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet

(a) Solve $5x - x^2 < 0$ **2**

(b) Find the equation of the normal to the curve $y = \dfrac{x^2}{4-x}$ at the point where $x = 2$. **3**

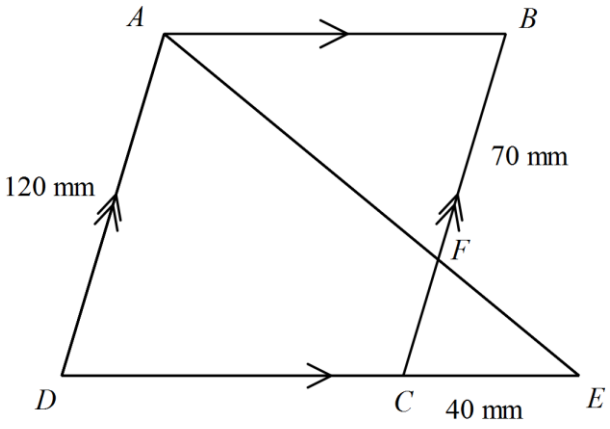
(c) (i) Find the length of *PR*, giving your answer correct to 1 decimal place. **2**



NOT TO SCALE

(ii) Find the area of ΔPQR, correct to the nearest cm^2. **1**

(d) *ABCD* is a parallelogram with *DC* produced to *E*. *BF* = 70 mm, *CE* = 40 mm and *AD* = 120 mm.



NOT TO SCALE

(i) Prove that ΔABF is similar to ΔECF **2**

(ii) Find the length of *AB*. **3**

(e) Find the value of *x* if $2^x \times 4^{x+1} = 0.5$. **2**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet

(a) Using $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, differentiate $f(x) = x^2 + 6x$. **3**

(b) Determine, with working, if the function $f(x) = x^3 + 1$ is odd, even or neither. **2**

(c) Given the function $f(x) = \frac{x^4}{4} - x$, solve the equation $f'(x) = 7$. **2**

(d) (i) Factorise $2x^2 - 7x - 4$. **1**

(ii) Sketch the function $y = 2x^2 - 7x - 4$. **1**

(iii) State the axis of symmetry and the vertex. **2**

(e) Differentiate: $y = \sqrt{4x-1}$. **2**

Give your answer without negative or fractional indices.

(f) Evaluate $\lim_{x \to 3} \frac{x^3 - 27}{(x-3)}$. **2**

End of Question 16

End of Task

2018 Year 11 Mathematics Yearly Solutions

Section 1

1.	C	6.	B
2.	A	7.	A
3.	D	8.	C
4.	C	9.	B
5.	A	10.	C

Section 2

Question 11

a) $5.084845882 \approx 5.1$ ✓ ✓

b) $\dfrac{x^2-x+xy-y}{x+y} = \dfrac{x(x-1)+y(x-1)}{x+y}$ ✓

$= \dfrac{(x+y)(x-1)}{(x+y)}$ ✓

$= x-1$ ✓

c) $x = \dfrac{-2 \pm \sqrt{2^2-4(1)(-5)}}{2(1)}$ ✓

$= \dfrac{-2 \pm \sqrt{24}}{2}$ ✓

$= -1 \pm \sqrt{6}$ ✓

d)(i) In $\triangle ADB$ and $\triangle CDB$

$AD = DC$ (Given)
$\angle ADB = \angle CDB$ (Given) } ✓✓
DB is common

$\therefore \triangle ADB \equiv \triangle CDB$ (SAS test) ✓

(ii) $AB = CB$ (matching sides in congruent triangles are equal) ✓

e) $y' = 6x^2 + 5$ ✓

When $x = 1$, $m_{tangent} = 11$ ✓

$\therefore y - 10 = 11(x - 1)$ ✓

$y - 10 = 11x - 11$

$y = 11x - 1$

Question 12

a) $M = \left(\frac{-2+5}{2}, \frac{2+5}{2}\right)$

$= \left(\frac{3}{2}, \frac{7}{2}\right)$ ✓

b) $m = \frac{2-0}{-2-0}$

$= -1$ ✓

c) $m_{RQ} = 1$ (Parallel to OP) ✓ can use substitution of (5,5) into both equations

$y - 5 = -1(x - 5)$ ✓ ✓

$y - 5 = -x + 5$

$x + y - 10 = 0$

d) $\tan\theta = -1$ ✓ Must see $\tan\theta = -1$ (NOT $\tan\theta = 1$)

$\theta = (180° - 45°)$ ✓ [2 = answer only] [will accept $-45°$ but not $315°$].

$= 135°$

e) Let $y = 0$

$x + 0 - 10 = 0$

$x = 10$ ✓ $\therefore R(10, 0)$

f) $d = \frac{|0 + 0 - 10|}{\sqrt{1+1}}$ ✓

$= \frac{10}{\sqrt{2}}$ or $5\sqrt{2}$ ✓ (did not allow FT because the correct line was given in (c).

i) $d = \sqrt{(5-10)^2 + (5-0)^2}$

$= \sqrt{50}$

$= 5\sqrt{2}$ ✓ (Must be simplified)

ii) $d_{OP} = \sqrt{(-2-0)^2 + (2-0)^2}$

$= \sqrt{8}$

$= 2\sqrt{2}$ ✓

$A = \frac{1}{2} \times \frac{10}{\sqrt{2}} \times (5\sqrt{2} + 2\sqrt{2})$ ✓

$= 35 \text{ units}^2$ ✓

If answer only given max [1].

If $A = 7 \times 5$ without working then max [1]

If OP used (wrong) max [1] given.

[There were several methods used].

Question 13

a) $\angle ACB = 65°$ (Alternate angles, $AE \parallel BC$) ✓

$\angle ABC = 40° + 25°$

$= 65°$ (Exterior angle of a triangle) ✓

$\therefore \Delta ABC$ is isosceles since base angles are equal.

b) (i) $y = 2x^{-1}$ ✓

$y' = -2x^{-2}$ ✓

$= \frac{-2}{x^2}$

(ii) $f'(x) = (x-2)^4 \times 3 + 3x \times 4(x-2)^3$ ✓

$= 3(x-2)^4 + 12x(x-2)^3$ ✓

$= 3(x-2)^3(x-2+4x)$

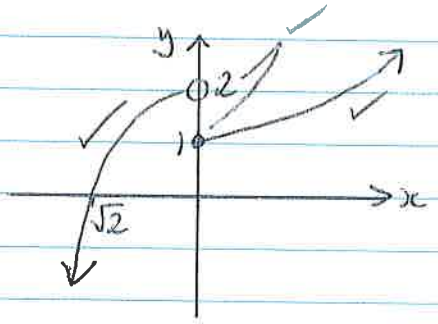
$= 3(x-2)^3(5x-2)$ ✓

(iii) $g'(x) = \dfrac{(x-1)\times 1 - x \times 1}{(x-1)^2}$ ✓

$= \dfrac{x - 1 - x}{(x-1)^2}$

$= \dfrac{-1}{(x-1)^2}$ ✓

c) (i)



✓ ✓ ✓

(ii) Not differentiable at $x = 0$. ✓

d) Domain : $-5 \leq x \leq 5$ ✓

Range : $0 \leq y \leq 5$ ✓

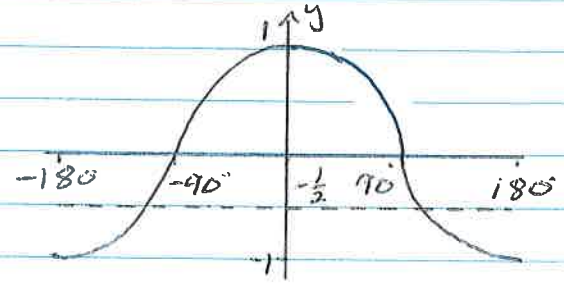
Question 14

a) $\sin(-135°) = -\sin(180° + 45°)$

$= -\sin 45°$

$= -\dfrac{1}{\sqrt{2}}$ ✓

b) (i)



✓

(ii) There are 2 solutions. ✓

c) $LHS = \frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta\cos^2\theta}$ ✓

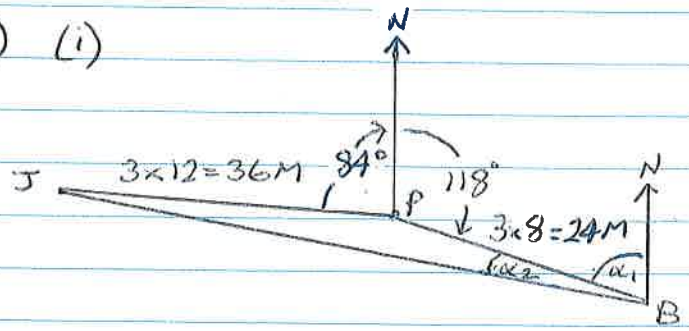
$= \frac{1}{\sin^2\theta\cos^2\theta}$ ✓

$= \frac{1}{\sin^2\theta} \times \frac{1}{\cos^2\theta}$

$= \operatorname{cosec}^2\theta \sec^2\theta$ ✓

$= RHS$

d) (i)



✓

(ii) $\angle JPB = 360° - 118° - 84°$

$= 158°$ ✓

$(JB)^2 = 36^2 + 24^2 - 2\times36\times24\times\cos 158°$

$JB = 58.9\,M$ ✓

(iii) $\frac{\sin\alpha_2}{36} = \frac{\sin 158°}{58.9}$

$\sin\alpha_2 = \frac{\sin 158° \times 36}{58.9}$

$\alpha_2 = 13°$ (nearest degree)

$\alpha_1 = 180° - 118°$ (co-interior angles)

$= 62°$

working ✓

Bearing $= 360° - 13° - 62°$

$= 285°$ ✓

e) $PA^2 = PB^2 + 4$

$PD^2 = (PB+4)^2 + 1^2$ } using Pythagoras

since $PD^2 = 2PA^2$

$2PA^2 = PB^2 + 8PB + 16 + 1$

$2(PB^2 + 4) = PB^2 + 8PB + 17$

$2PB^2 + 8 = PB^2 + 8PB + 17$

$PB^2 - 8PB - 9 = 0$

$(PB-9)(PB+1) = 0$

$PB = 9, \quad PB = -1$

$\therefore$ The solution is $PB = 9$

Question 15

a) $x(5-x) < 0$

Let $x(5-x) = 0$

$x = 0, \quad x = 5$

Now test, $x = 1$

$5(1) - 1^2 \neq 0$

$\therefore$ The solution is $x < 0, \quad x > 5$

b) $\frac{dy}{dx} = \frac{(4-x) \times 2x - x^2 \times -1}{(4-x)^2}$

$= \frac{8x - 2x^2 + x^2}{(4-x)^2}$

$= \frac{8x - x^2}{(4-x)^2}$

At $x = 2$, $\frac{dy}{dx} = \frac{8(2) - 2^2}{(4-2)^2}$

$= 3$

$\therefore$ Gradient of normal is $-\frac{1}{3}$

At $x=2$, $y=\frac{2^2}{4-2}=\frac{4}{2}=2$ and $m=-\frac{1}{3}$

$\therefore \quad y-2=-\frac{1}{3}(x-2)$

$y-2=-\frac{1}{3}x+\frac{2}{3}$

$y=-\frac{1}{3}x+\frac{8}{3}$ or $x+3y-8=0$ ✓

c) (i) $PR^2=7^2+8^2-2\times7\times8\times\cos 50^\circ$ ✓

$PR=6.4\text{ cm}$ ✓

(ii) $A=\frac{1}{2}\times7\times8\times\sin 50^\circ$

$=21\text{ cm}^2$ ✓

d) (i) In $\triangle ABF$ and $\triangle ECF$

$\angle ABC=\angle BCE$ } alternate angles $AB\parallel DE$ ✓
$\angle BAE=\angle AEC$ }

$\therefore \triangle ABF \,|||\, \triangle ECF$ (Equiangular) ✓

(ii) $\frac{BF}{CF}=\frac{AB}{CF}$ (Matching sides in same ratio) ✓

$\frac{70}{50}=\frac{AB}{40}$ ✓

$AB=56$ ✓

e) $2^x\times4^{x+1}=0.5$

$2^x\times(2^2)^{x+1}=2^{-1}$ ✓

$2^{3x+2}=2^{-1}$

$3x+2=-1$ (Equating co-efficients)

$3x=-3$

$x=-1$ ✓

Question 16

must be seen

a) $f'(x) = \lim_{h \to 0} \dfrac{(x+h)^2 + 6(x+h) - (x^2+6x)}{h}$ ✓ ✓

$= \lim_{h \to 0} \dfrac{x^2 + 2xh + h^2 + 6x + 6h - x^2 - 6x}{h}$

$= \lim_{h \to 0} \dfrac{2xh + h^2 + 6h}{h}$

$= \lim_{h \to 0} \dfrac{h(2x + h + 6)}{h}$

$= \lim_{h \to 0} 2x + h + 6$

$= 2x + 6$ ✓

b) $f(x) = x^3 + 1$

$f(-x) = (-x)^3 + 1$ ✓ must see $-x \Rightarrow x$ replacement

$= -x^3 + 1 \neq -(x^3+1)$

✓ $f(-x) = f(x)^3 + 1 = -x^3 + 1 \neq -f(x)$ ↑must see

$f(x)$ is neither odd or even.

[if a value is substituted, that is a verification and only [1] max given].

c) $f'(x) = 4\left(\dfrac{x^3}{4}\right) - 1$

$= x^3 - 1$ ✓

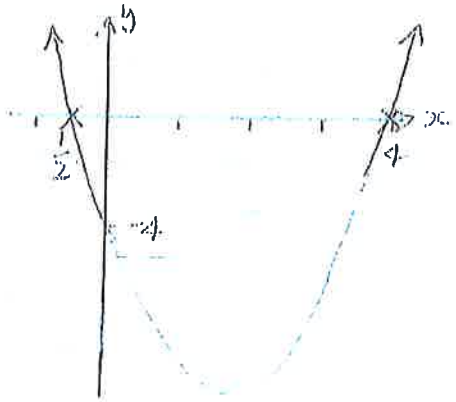
$7 = x^3 - 1$

$8 = x^3$

$x = 2$ ✓

d) (i) $2x^2 - 7x - 4 = (2x+1)(x-4)$ ✓

(ii)



✓ shape, intercepts and vertex to the right of y-axis only.

must have a "subject".

(iii) axis of symmetry : $x = \frac{4 + -\frac{-7}{2}}{2} = 1.75$ ✓

If $x = 1.75$, $y = 2(1.75)^2 - 7(1.75) - 4 = -10.125$

∴ The vertex is $(1.75, -10.125)$ or $\left(\frac{7}{4}, -\frac{81}{8}\right)$ ✓

e) $y = (4x-1)^{\frac{1}{2}}$

$y' = \frac{1}{2}(4x-1)^{-\frac{1}{2}} \times 4$ ✓ [must have all components]

$= 2(4x-1)^{-\frac{1}{2}}$

$= \frac{2}{\sqrt{4x-1}}$ ✓ [if factor of '4' left off then max [1] given]

f) $\lim_{x \to 3} \frac{x^3 - 27}{x-3} = \lim_{x \to 3} \frac{(x-3)(x^2+3x+9)}{x-3}$

$= \lim_{x \to 3} x^2 + 3x + 9$

$= 3^2 + 3(3) + 9$

$= 27$