

Mathematics Advanced

Name: _____	Mark: _____/100
Class: _____	

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11-37, show relevant mathematical reasoning and/or calculations

Weight – 30% of internal assessment

Total marks – 100

Section I — 10 marks (pages 3-6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II — 90 marks (pages 8-27)

- Attempt Questions 11–37
- Allow about 2 hours 45 minutes for this section

Section I**10 marks****Attempt questions 1-10****Allow about 15 minutes for this section**Use the multiple-choice answer sheet for Questions 1-10.

1 What is the value of x for which $4^{3x+1} = 8^x$?

A. 2

B. $\frac{3}{2}$

C. $-\frac{2}{3}$

D. $-\frac{3}{2}$

2 An amount of \$3000 is invested at an interest rate of 3% per annum.

What is the value of the investment after five years if the interest is compounded monthly?

A. \$3485

B. \$3526

C. \$3571

D. \$3654

3 What is the derivative of $\log_2(2x + 4)$?

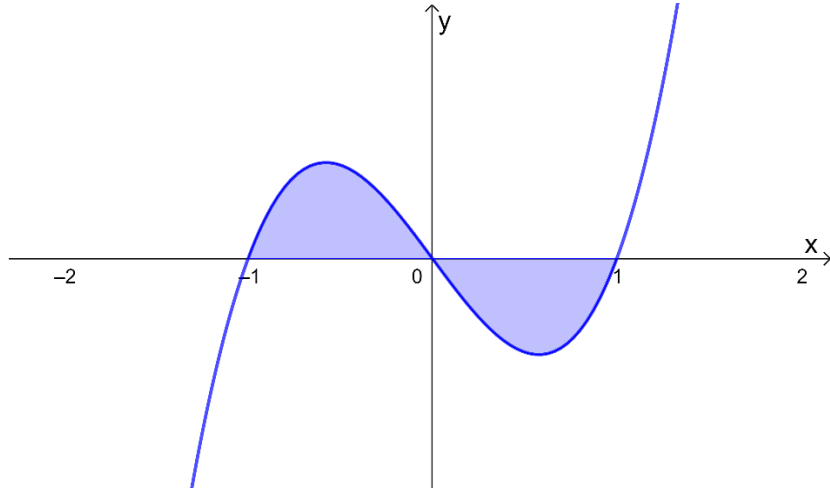
A. $\frac{1}{2x+4}$

B. $\frac{1}{x+2}$

C. $\frac{1}{(2x+4) \ln 2}$

D. $\frac{1}{(x+2) \ln 2}$

- 4 The diagram shows the area bounded by the graph of $y = x^3 - x$ and the x -axis.



Which of the following correctly gives the shaded area?

- A. $2 \int_0^1 (x^3 - x) dx$
- B. $2 \int_{-1}^0 (x^3 - x) dx$
- C. $\int_{-1}^1 (x^3 - x) dx$
- D. $2 \int_{-1}^1 (x^3 - x) dx$
- 5 It is given that $\sin x = \frac{1}{3}$ for $\frac{\pi}{2} < x < \pi$.

What is the value of $\tan x$?

- A. $-\frac{1}{2\sqrt{2}}$
- B. $\frac{2\sqrt{2}}{3}$
- C. $-\frac{3}{2\sqrt{2}}$
- D. $-2\sqrt{2}$

6 What is the range of the function $f(x) = 4 - e^{2x-2}$?

- A. $(-\infty, 4)$
- B. $(1, \infty)$
- C. $(-\infty, 4]$
- D. $(4, \infty)$

7 The function $f(x)$ has derivative $f'(x) = 8 \sin 4x$ and $f\left(\frac{\pi}{4}\right) = 1$.

Which of the following is $f(x)$?

- A. $8 \cos 4x + 7$
- B. $-2 \cos 4x - 1$
- C. $2 \cos 4x + 3$
- D. $2 \sin 4x + 1$

8 Let X be a continuous random variable whose probability density function is

$$f(x) = \frac{3}{4}(2x - x^2) \text{ for } 0 \leq x \leq 2.$$

What is the mode of X ?

- A. 0
- B. 0.5
- C. 0.75
- D. 1

- 9** The heights of the boys in a high school are normally distributed with mean 174 cm and standard deviation S . Daniel's height is $0.25 S$ below the mean, and John's height is $1.5 S$ above the mean.
If John is 14 cm taller than Daniel, what is Daniel's height?

- A. 170 cm
- B. 171 cm
- C. 172 cm
- D. 173 cm

- 10** Given the function, $y = x f(x^2)$, which expression is equal to $\frac{dy}{dx}$?

- A. $2x f(x)$
- B. $f(x^2) + 2x^2 f(x)$
- C. $f(x^2) + 2x^2 f(x) f'(x)$
- D. $f(x^2) + 2x^2 f'(x^2)$

2020

**HSC TRIAL
EXAMINATION**

Mathematics Advanced

Section II Answer Booklet

90 marks

Attempt Questions 11–37

Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
 - Your responses should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
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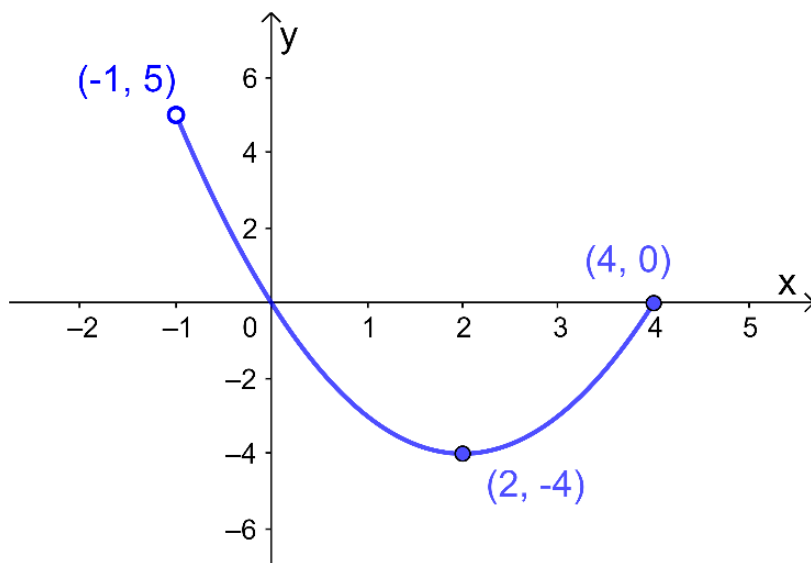
Please turn over

Question 11 (2 marks)Find $\int 24(2x - 7)^5 dx$.

2

Question 12 (2 marks)The graph of a function $f(x)$ is shown.

2

Using interval notation, state the domain and range of $f(x)$.

Question 13 (3 marks)

Describe the three transformations which, when applied in succession, change the graph of $y = \sqrt{x}$ to the graph with equation $y = 2\sqrt{x+1} - 3$.

3

[illegible]

Question 14 (2 marks)

Differentiate $\frac{8}{1-2x}$ with respect to x .

2

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Question 15 (2 marks)

A biased four-sided die is rolled.

The following table gives the probability of each score.

2

Score	1	2	3	4
Probability	0.25	$0.4b$	$2b$	0.15

Calculate the value of b and find the probability of rolling a number at most 2?

Question 16 (2 marks)

The table shows the future values of an annuity of \$1 for different interest rates for 7, 8 and 9 years. The contributions are made at the end of each year.

Years	Future value of an annuity of \$1			
	Interest rate per annum			
	2.0%	2.5%	3.0%	3.5%
7	7.434	7.547	7.662	7.779
8	8.583	8.736	8.892	9.052
9	9.755	9.955	10.159	10.368

- (a) What is the future value of an annuity of \$600 per year for eight years at 3% per annum?

1

- (b) What is the value of an annuity that would provide a future value of \$15 000 after seven years at 2.5% per annum? 1

Question 17 (3 marks)

- Sketch the graph of $y = 2 - \frac{1}{x+4}$, labelling the asymptotes and axis intercepts. 3

Question 18 (3 marks)

Find the equation of tangent to the curve $y = \sin(\cos x)$ at the point $\left(\frac{\pi}{2}, 0\right)$.

3

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Question 20 (7 marks)

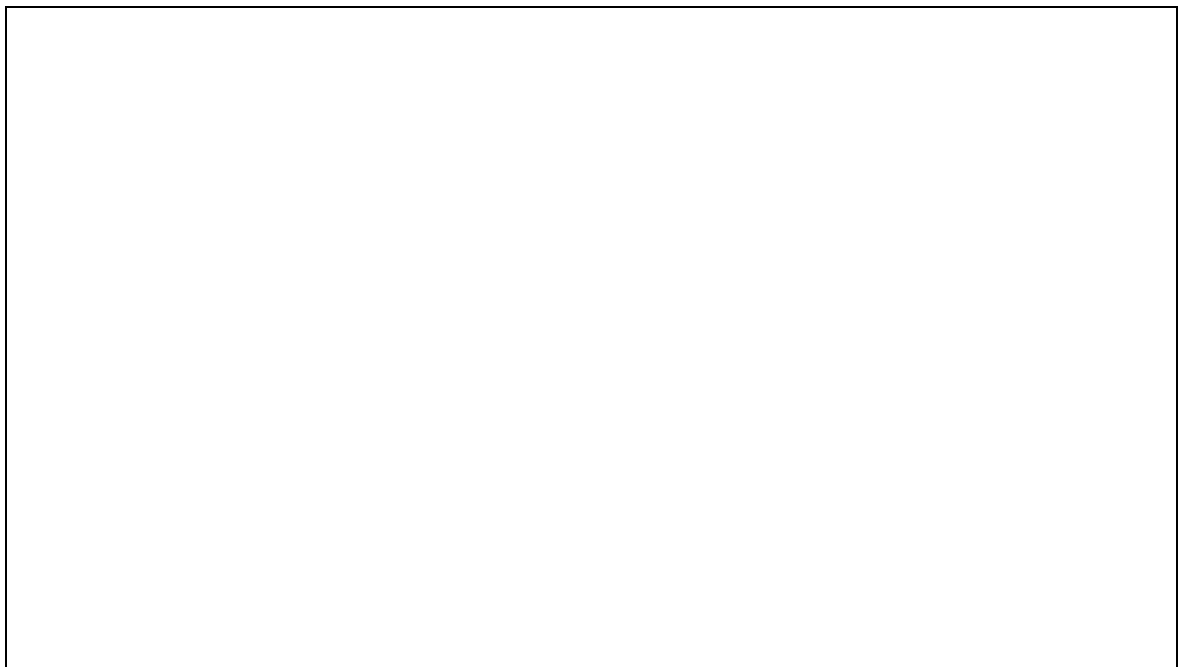
A function is given by $f(x) = 2x^3 - 3x^2$.

- (a) Find the stationary points and determine their nature.

3

- (b) Sketch the curve for $0 \leq x \leq 2$, labelling the stationary points, axis intercepts and endpoints.

3



- (c) How many solutions does the equation $f(x) = \pi$ have?

1

Question 21 (2 marks)

A particle is moving along the x axis. Its displacement, x metres from the origin after time, t seconds, is given by $x = e^t - 3e^{2t}$ for $t \geq 0$.

2

Explain why the particle never comes to rest.

Question 22 (4 marks)

The times taken for a class of Year 12 students to complete a test are normally distributed with a mean of 37.5 minutes and a standard deviation of 1.5 minutes. Use the Empirical rule to answer the questions below.

(a) Find the z score for 34.5 minutes.

1

(b) What percentage of students will complete the test in less than 34.5 minutes?

1

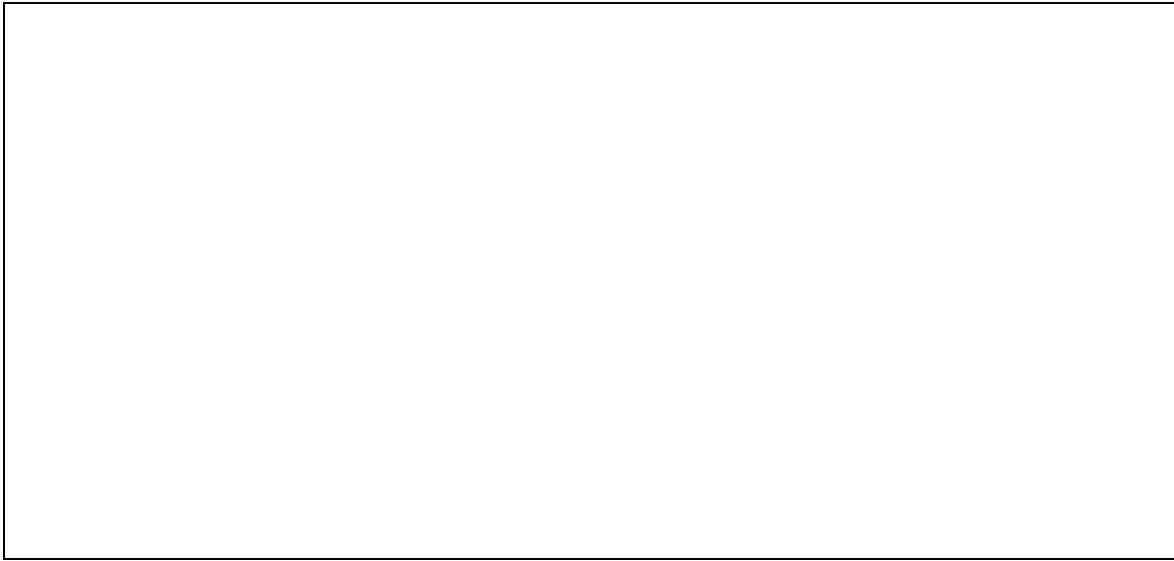
(c) 68% of students complete the test in the interval (a, b) minutes.

2

Find appropriate values for a and b .

Question 23 (4 marks)

- (a) Sketch the graph of $y = |3x - 6|$, labelling the axis intercepts.

2

- (b) Using the graph from part (a), or otherwise, solve $|3x - 6| < x$.

2

Question 24 (2 marks)

Solve $\ln(2x + 3) = 2 \ln x$.

2

Question 25 (3 marks)

Let X be a continuous random variable whose probability density function is

$$f(x) = \frac{3x^2}{8} \text{ for } 0 < x < a.$$

(a) Show that $a = 2$

1

(b) Find the exact value of median of X .

2**Question 26** (2 marks)

A function is defined as follows:

2

$$f(x) = \begin{cases} ax + b & x < 0 \\ e^{2x} & x \geq 0 \end{cases}$$

Find a and b if $f(x)$ is continuous and has a derivative at $x = 0$.

Question 27 (3 marks)

A virus infects 120 people on the first day. On each following day, it infects 10 more new people than it infected the previous day. That is, it infects 130 new people on the second day, 140 new people on the third day and so on.

3

How many days will it take for the virus to infect a total of more than 10 000 people?

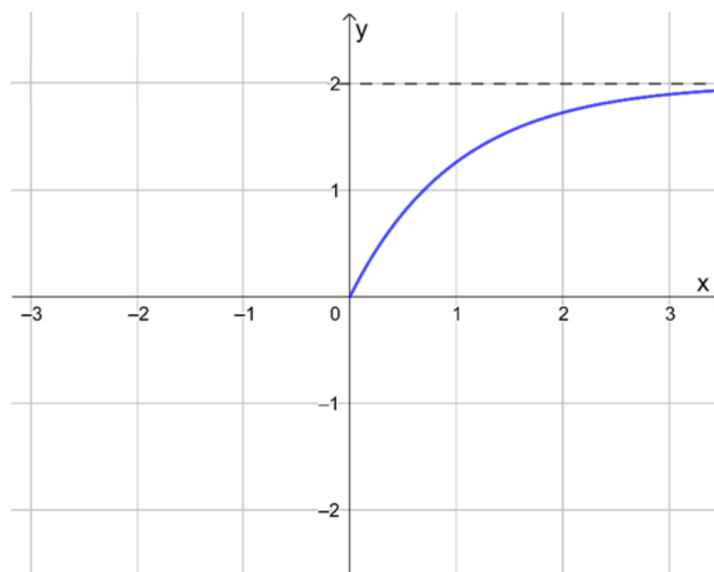
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Question 28 (2 marks)

The diagram shows the part of the graph of an odd function for $x \geq 0$ such that $y = 2$ is a horizontal asymptote.

2

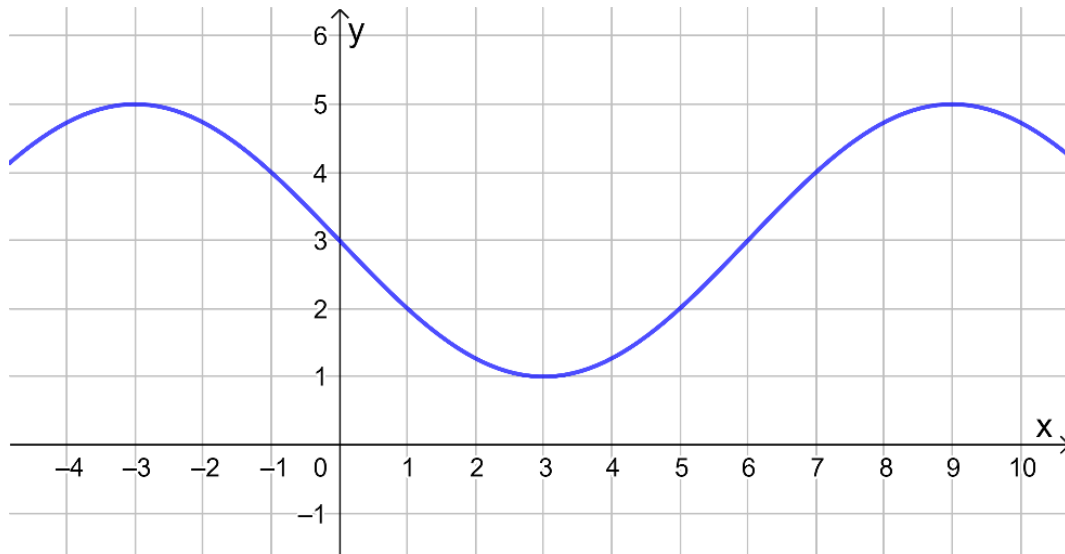
On the same axes complete the rest of the graph for $x < 0$.



Question 29 (3 marks)

The graph of $y = a \sin(bx) + c$ is shown such that $b > 0$.

3



Find the values of a , b , and c .

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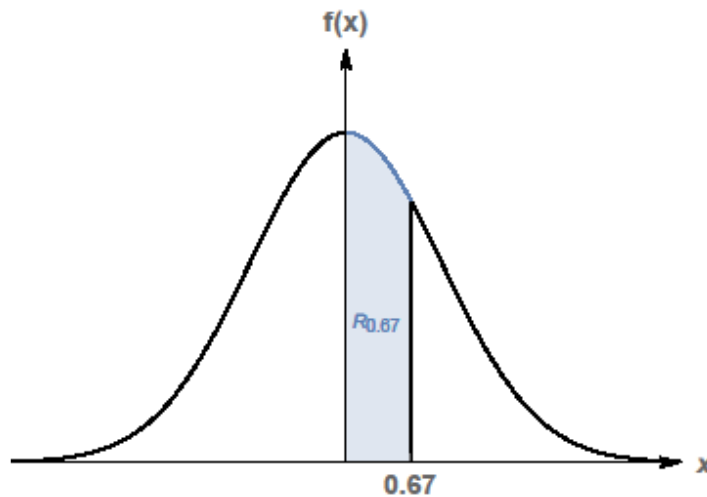
Question 31 (3 marks)

The random variable X has the standard normal distribution, with probability density function

3

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \text{ over the domain } (-\infty, \infty).$$

The graph of $y = f(x)$ is shown, and indicates a region $R_{0.67}$ which indicates the area under $y = f(x)$ over the domain $[0, 0.67]$.



If the area of $R_{0.67}$ is approximately 0.25, determine the range of scores that could be identified as outliers.

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Question 32 (6 marks)

The height, $h(t)$ metres, that the sea water reaches up one particular wooden stump at Coogee Pier during a 24-hour period can be modelled by the rule,

$$h(t) = 2.2 + 1.2 \cos\left(\frac{\pi t}{12}\right)$$

where t is the number of hours after 5 am and $0 \leq t \leq 24$.

- (a) Find the height of the water on the stump at 5 am.

1

- (b) Sketch the graph of $y = h(t)$ for $0 \leq t \leq 24$.

2

Question 32 continues on page 23

Question 32 (continued)

- (c) What will be the time (t) when the height of the water drops to 1.6 metres for the first time? 2

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- (d) For how many hours will the height of the water be above 1.6 metres during a 24-hour period? 1

[illegible]

Question 33 (4 marks)

The body weights and metabolic rate recordings for five women are shown below.

<i>Weight (kg)</i>	52	63	65	47	49
<i>Metabolic rate</i>	1671	1669	1812	1442	1607

- (a) Using your calculator find (r) the correlation coefficient and explain what type and strength of correlation this data gives. Correct your answer to 4 decimal places.

2

- (b) Using your calculator find the **equation** of the least-squares regression line in the form, $y = mx + c$, where x is the weight and y is the metabolic rate. Correct your answer to 3 decimal places.

1

- (c) Use your equation to estimate the weight of a woman with a metabolic rate of 1552. Correct your answer to 1 decimal place.

1

Question 34 (continued)

(b) Let $g(x) = f(ax + b)$.

3

Find a and b if $\int_4^0 g(x)dx + \int_4^6 g(x)dx$ has the same area calculated in part (a).

Question 35 (3 marks)

From the point A, a man observes that the angle of elevation of the summit of a hill is 12° . He then walks towards the hill for 600 m along flat ground. At this point, the summit of the hill is at an angle of elevation of 18° .

3

Find the height of the hill above the level of A.

Question 36 (5 marks)

A car loan of \$48 000 is to be repaid over five years, with equal payments at the end of each month. The interest rate is 8.4% per annum, compounded monthly.

5

Find the amount of monthly payments and the total interest paid over the duration of the loan.

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]

STUDENT ID _____

2020 Mathematics Advanced HSC Trial

Section I Answer Sheet

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Select the alternative A, B, C, or D that best answers the question. Fill in the response circle completely.

- | | | | | |
|-----------|-------------------------|-------------------------|-------------------------|-------------------------|
| 1 | A ☐ | B ☐ | C ☐ | D ☐ |
| 2 | A ☐ | B ☐ | C ☐ | D ☐ |
| 3 | A ☐ | B ☐ | C ☐ | D ☐ |
| 4 | A ☐ | B ☐ | C ☐ | D ☐ |
| 5 | A ☐ | B ☐ | C ☐ | D ☐ |
| 6 | A ☐ | B ☐ | C ☐ | D ☐ |
| 7 | A ☐ | B ☐ | C ☐ | D ☐ |
| 8 | A ☐ | B ☐ | C ☐ | D ☐ |
| 9 | A ☐ | B ☐ | C ☐ | D ☐ |
| 10 | A ☐ | B ☐ | C ☐ | D ☐ |

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$$

$$S = \frac{a}{1-r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

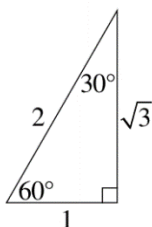
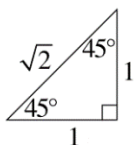
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

**Trigonometric identities**

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1+t^2}$$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A \cos B = \frac{1}{2} [\cos(A-B) + \cos(A+B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$$

$$\sin^2 nx = \frac{1}{2} (1 - \cos 2nx)$$

$$\cos^2 nx = \frac{1}{2} (1 + \cos 2nx)$$

Statistical Analysis

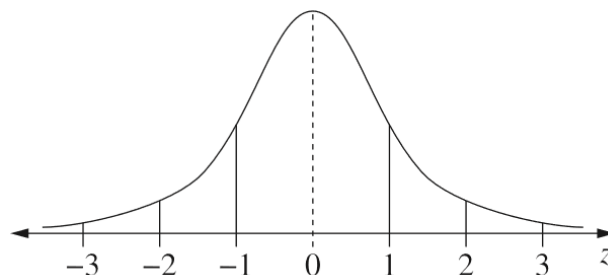
$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score

less than $Q_1 - 1.5 \times IQR$

or

more than $Q_3 + 1.5 \times IQR$

Normal distribution

- approximately 68% of scores have z-scores between -1 and 1
- approximately 95% of scores have z-scores between -2 and 2
- approximately 99.7% of scores have z-scores between -3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x) dx$$

$$P(a < X < b) = \int_a^b f(x) dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function

Derivative

$$y = f(x)^n$$

$$\frac{dy}{dx} = n f'(x) [f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u), u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x) a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int_a^b f(x) dx \approx \frac{b-a}{2n} \{ f(a) + f(b) + 2[f(x_1) + \dots + f(x_{n-1})] \}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \dots + \binom{n}{r}x^{n-r}a^r + \dots + a^n$$

Vectors

$$|\underline{u}| = |x_1\hat{i} + y_1\hat{j}| = \sqrt{x^2 + y^2}$$

$$\underline{u} \cdot \underline{v} = |\underline{u}||\underline{v}|\cos\theta = x_1x_2 + y_1y_2,$$

$$\text{where } \underline{u} = x_1\hat{i} + y_1\hat{j}$$

$$\text{and } \underline{v} = x_2\hat{i} + y_2\hat{j}$$

$$\underline{r} = \underline{a} + \lambda \underline{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta) \\ = re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta) \\ = r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$$

$$x = a \cos(nt + \alpha) + c$$

$$x = a \sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

2020 Mathematics Advanced Year 12

Marking Guidelines

Section 1

Multiple-choice Answer Key

Question	Answer
1	C
2	A
3	D
4	B
5	A
6	A
7	B
8	D
9	C
10	D

Please see multiple-choice explanations over.

Multiple-choice Worked Answers

Question	Solution
1	$2^{6x+2} = 2^{3x} \Rightarrow x = -\frac{2}{3}$
2	$3000 \times \left(1 + \frac{0.03}{12}\right)^{5 \times 12} = 3484.85$
3	$\frac{2}{(2x+4)\ln 2} = \frac{1}{(x+2)\ln 2}$
4	$f(x) = x^3 - x$ is odd, hence Area $= 2 \times \int_{-1}^0 f(x)dx$ since $\int_{-1}^0 f(x)dx > 0$
5	$\cos x = -\frac{\sqrt{8}}{3}$ since $\frac{\pi}{2} < x < \pi$ $\tan x = \frac{\sin x}{\cos x} = -\frac{1}{\sqrt{8}} = -\frac{1}{2\sqrt{2}}$
6	$e^{2x-2} > 0 \Rightarrow f(x) < 4 - 0 \Rightarrow y \in (-\infty, 4)$
7	$f(x) = \int 8 \sin 4x dx = -2 \cos 4x + c$ $f\left(\frac{\pi}{4}\right) = 1 \Rightarrow 2 + c = 1 \Rightarrow c = -1$
8	$f'(x) = 0 \Rightarrow \frac{3}{4}(2 - 2x) = 0 \Rightarrow x = 1$
9	Daniel's height is $174 - 0.25s$ John's height is $174 + 1.5s$ John is 14 cm taller than Daniel, so $174 + 1.5s - (174 - 0.25s) = 14$ $1.75s = 14$ $s = 8$ Hence, Daniel's height is $174 - 0.25s$ $= 174 - 0.25 \times 8 = 172$ cm.
10	$\frac{d}{dx} x f(x^2) = 1 \times f(x^2) + x \times f'(x^2) \times 2x$

Section II

Note: An incorrect answer in a previous part will not necessarily preclude students from achieving full marks in a later part. Answers here are based on correct prior part answers. Marking will need to adapt to pursue correct method with the use of incorrect prior parts.

Question 11

Criteria	Marks
• Provides correct answer	2
• Attempt to use the correct formula for integration	1

Sample answer:

$$\frac{24(2x-7)^6}{6 \times 2} + c = 2(2x-7)^6 + c$$

Question 12

Criteria	Marks
• Provides correct domain and range	2
• Provides correct domain or range, or equivalent merit	1

Sample answer:

Domain is $(-1, 4]$, or $-1 < x \leq 4$

Range is $[-4, 5)$, or $-4 \leq y < 5$

Question 13

Criteria	Marks
• Provides correct answer	3
• Provides correct transformations with wrong order	2
• Provides two correct transformations	1

Sample answer:

Dilate vertically by a factor of 2

Shift left by 1 unit

Shift down by 3 units

Question 14

Criteria	Marks
• Provides correct answer	2
• Attempt to use the correct formula for differentiation	1

Sample answer:

$$\frac{d}{dx} 8(1-2x)^{-1} = -8(1-2x)^{-2} \times (-2) = \frac{16}{(1-2x)^2}$$

Question 15

Criteria	Marks
• Provides correct answer of the probability of at most 2	2
• Provides correct value of b	1

Sample answer:

$$0.25 + 0.4b + 2b + 0.15 = 1$$

$$b = 0.25$$

$$p(\text{at most } 2) = 0.25 + (0.4 \times 0.25) = 0.35$$

Question 16 (a)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\begin{aligned} & \$600 \times 8.8922 \\ & = \$5335.20 \end{aligned}$$

Question 16 (b)

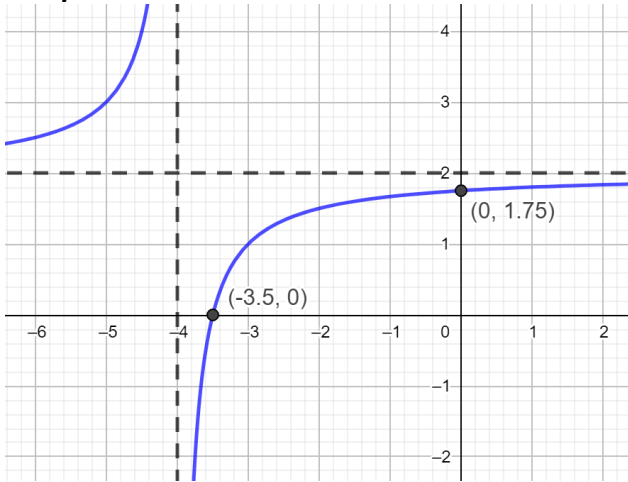
Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\frac{\$15000}{7.547} = \$1987.54$$

Question 17

Criteria	Marks
• Provides correct sketch with the asymptotes and axis intercepts labelled	3
• Provides correct sketch	2
• Provides correct asymptotes or axis intercepts	1

Sample answer:

Question 18

Criteria	Marks
• Provides correct solution	3
• Provides correct gradient	2
• Determines the gradient of the tangent, or equivalent merit	1

Sample answer:

$$\frac{dy}{dx} = \cos(\cos x) \times (-\sin x)$$

$$m_t = y' \left(\frac{\pi}{2} \right) = -1$$

$$y - 0 = -1 \left(x - \frac{\pi}{2} \right) \Rightarrow y = -x + \frac{\pi}{2}$$

Question 19 (a)

Criteria	Marks
• Provides correct solution	2
• Provides correct equation stating that the integral equals one, or equivalent merit	1

Sample answer:

$$\int_1^4 \frac{k}{2x-1} dx = 1 \Rightarrow \left[\frac{k}{2} \ln(2x-1) \right]_1^4 = 1 \Rightarrow \frac{k \ln 7}{2} = 1 \Rightarrow k = \frac{2}{\ln 7}$$

Question 19 (b)

Criteria	Marks
• Provides correct solution	2
• Provides correct integral, or equivalent merit	1

Sample answer:

$$\int_1^2 \frac{k}{2x-1} dx = \left[\frac{k}{2} \ln(2x-1) \right]_1^2 = \frac{k \ln 3}{2} = \frac{\ln 3}{\ln 7} = \log_7 3 \quad (\text{since } k = \frac{2}{\ln 7})$$

Question 20 (a)

Criteria	Marks
• Provides correct solution	3
• Determines the coordinates of the two stationary points	2
• Provides correct derivative	1

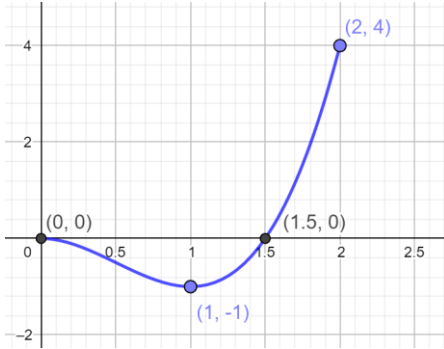
Sample answer:

$$f'(x) = 0 \Rightarrow 6x^2 - 6x = 0 \Rightarrow x = 0 \text{ or } x = 1$$

$$f''(x) = 12x - 6 \Rightarrow \begin{cases} f''(0) < 0 \\ f''(1) > 0 \end{cases} \Rightarrow \begin{matrix} (0, 0) \text{ max} \\ (1, -1) \text{ min} \end{matrix}$$

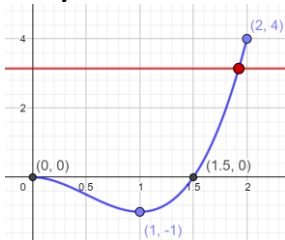
Question 20 (b)

Criteria	Marks
Provides correct sketch with stationary points, axis intercepts and endpoints labelled	3
Provides correct sketch	2
Provides correct axis intercepts or endpoints	1

Sample answer:

Question 20 (c)

Criteria	Marks
Provides correct answer	1

Sample answer:

One solution

Question 21

Criteria	Marks
• Provides correct explanation	2
• Provides correct expression for velocity	1

Sample answer:

$$v = \frac{dx}{dt} = e^t - 6e^{2t}$$

$$v = 0 \Rightarrow e^t(1 - 6e^t) = 0 \Rightarrow e^t = \frac{1}{6} \Rightarrow t = \ln \frac{1}{6} < 0$$

$\therefore$ particle never comes to rest

Question 22 (a)

Criteria	Marks
• Find equivalent z-score	1

Sample answer:

z-score:

$$\left(Z = \frac{34.5 - 37.5}{1.5} = -2 \right)$$

Question 22 (b)

Criteria	Marks
• Provides correct answer	1

Let X be the time taken, and Z be the equivalent z-score

$$P(X < 34.5) = P\left(Z < \frac{34.5 - 37.5}{1.5} = -2\right) = \frac{1 - 0.95}{2} = 0.025 = 2.5\%$$

Question 22 (c)

Criteria	Marks
• Provides correct answer	2
• Find equivalent z-score for a and b	1

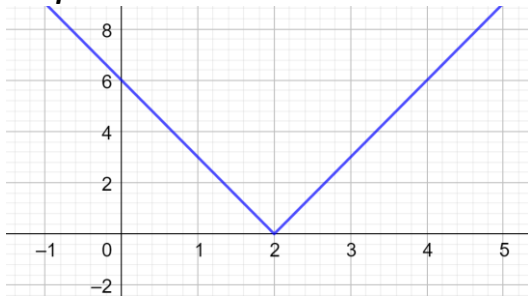
Sample answer:

Let X be the time taken, and Z be the equivalent z-score

$$P(a < X < b) = 68\% = P(-1 < Z < 1) \Rightarrow \begin{cases} a = 37.5 - 1.5 \\ b = 37.5 + 1.5 \end{cases} \Rightarrow \begin{cases} a = 36 \\ b = 39 \end{cases}$$

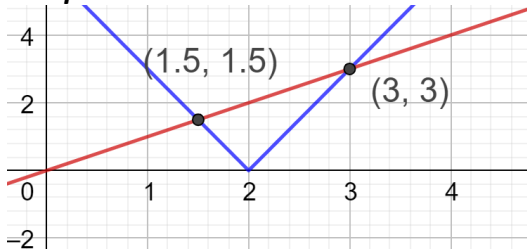
Question 23 (a)

Criteria	Marks
• Provides correct sketch with intercepts	2
• Provides correct intercepts or sketch	1

Sample answer:

Question 23 (b)

Criteria	Marks
• Provides correct solution	2
• Provides correct x values of intersection, or equivalent merit	1

Sample answer:

Using graphs, $x \in (1.5, 3)$

Question 24

Criteria	Marks
• Provides correct answer	2
• Obtains the quadratic equation, or equivalent merit	1

Sample answer:

$$\ln(2x+3) = \ln(x^2) \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow \begin{cases} x = -1 \text{ (reject since } x > 0) \\ x = 3 \end{cases}$$

$$\therefore x = 3$$

Question 25 (a)

Criteria	Marks
• Provides correct solution of $a=2$	2
• Provides correct integral expression	1

Sample answer:

$$\int_0^a \frac{3x^2}{8} dx = 1$$

$$\frac{3}{8} \left[\frac{x^3}{3} \right]_0^a = 1$$

$$\frac{1}{8} (a^3 - 0^3) = 1$$

$$a^3 = 8$$

$$a = \sqrt[3]{8} = 2$$

Question 25 (b)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\int_0^x \frac{3x^2}{8} dx = \frac{1}{2}$$

$$\frac{3}{8} \left[\frac{x^3}{3} \right]_0^x = \frac{1}{2}$$

$$\frac{1}{8} (x^3 - 0^3) = \frac{1}{2}$$

$$a^3 = 4$$

$$a = \sqrt[3]{4}$$

Question 26

Criteria	Marks
• Provides correct solution	2
• Provides correct equations involving $f(0)$ or $f'(0)$	1

Sample answer:

$$\begin{cases} f(0) = a \times 0 + b = e^{2 \times 0} \\ f'(0) = a = 2e^{2 \times 0} \end{cases} \Rightarrow a = 2, b = 1$$

Question 27

Criteria	Marks
• Provides correct solution	3
• Provides correct roots to the quadratic equation	2
• Provides correct equation involving sum of arithmetic series, or equivalent merit	1

Sample answer:

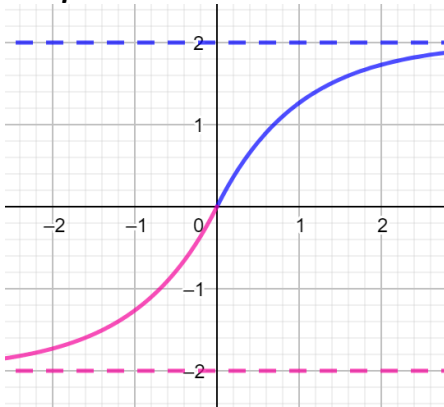
$$a_1 = 120, d = 10, S_n = 10000, n = ?$$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d) \Rightarrow 10000 = \frac{n}{2}(240 + 10(n-1)) \Rightarrow 5n^2 + 115n - 10000 = 0 \Rightarrow \begin{cases} n = -57.68 \\ n = 34.68 \end{cases}$$

35 days

Question 28

Criteria	Marks
• Provides correct sketch including the asymptote	2
• Provides correct sketch	1

Sample answer:

Question 29

Criteria	Marks
• Provides correct solution	3
• Provides correct roots to the quadratic equation	2
• Provides correct equation involving sum of arithmetic series, or equivalent merit	1

Sample answer:

$$\text{amplitude} = \frac{5-1}{2} = 2 = |a| \Rightarrow a = -2 \text{ since graph is reflected vertically}$$

$$\text{period} = 9 - (-3) = 12 = \frac{2\pi}{b} \Rightarrow b = \frac{\pi}{6}$$

$$c = 3 \text{ since graph is translated up by 3 units}$$

Question 30 (a)

Criteria	Marks
• Provides correct solution	3
• Progress towards solution	2
• Provides correct integral equation, or equivalent merit	1

Sample answer:

$$\int_0^{t_1} v(t) dt = 0 \Rightarrow \int_0^5 (4t - t^2) dt - \int_5^{t_1} 5 dt = 0$$

$$\left[2t^2 - \frac{t^3}{3} \right]_0^5 - (t_1 - 5) \times 5 = 0 \Rightarrow t_1 = \frac{20}{3}$$

Question 30 (b)

Criteria	Marks
• Provides correct solution, or equivalent merit	1

Sample answer:

$$t = 2 \text{ or } t \geq 5 \text{ (accept } t > 5)$$

Question 31

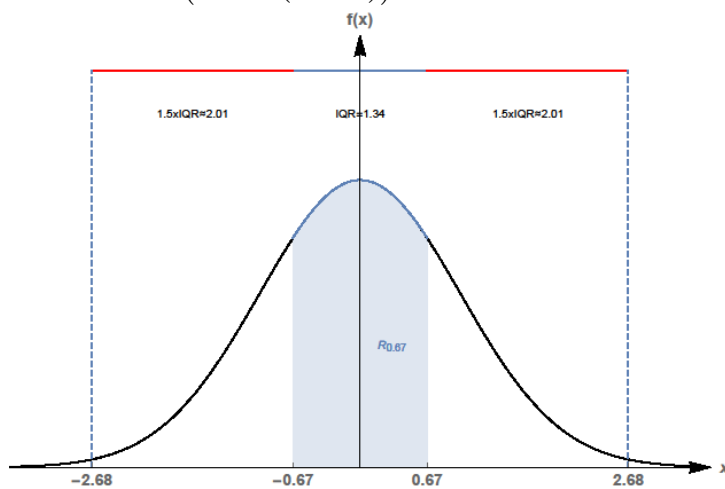
Criteria	Marks
• Provides correct solution	3
• Correctly determines outlier region above the mean, or equivalent merit	2
• Attempts to calculate <i>IQR</i>	1

Sample Answer:

Since $R_{0.67}$ has area of 0.25, $Q_3 = 0.67$ and $Q_1 = -0.67$ by symmetry.

Hence, the outliers will be below $-2.68 = -0.67 - 1.5 \times (0.67 - (-0.67))$ or above 2.68

$$= 0.67 + 1.5 \times (0.67 - (-0.67))$$



Note to markers: $R_{0.67}$ is not exactly 0.25 so these calculations are approximate

Question 32 (a)

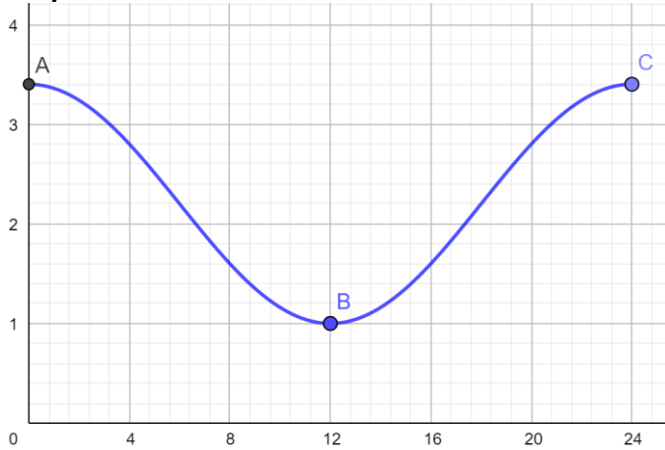
Criteria	Marks
• Provides correct answer	1

Sample answer:

$$h(0) = 3.4 \text{ m}$$

Question 32 (b)

Criteria	Marks
• Provides correct sketch with accurate points A, B and C	2
• Provides correct sketch	1

Sample answer:

Question 32 (c)

Criteria	Marks
• Provides correct solution	2
• Provides one solution involving $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$ correct sketch	1

Sample answer:

$$2.2 + 1.2 \cos\left(\frac{\pi t}{12}\right) = 1.6 \Rightarrow \cos\left(\frac{\pi t}{12}\right) = -\frac{1}{2} \Rightarrow \begin{cases} \frac{\pi t}{12} = \frac{2\pi}{3} \\ \frac{\pi t}{12} = \frac{4\pi}{3} \end{cases} \Rightarrow \begin{cases} t = 8 \\ t = 16 \end{cases}$$

$$\therefore t = 8$$

Question 32 (d)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$24 - (16 - 8) = 16 \text{ hr}$$

Question 33 (a)

Criteria	Marks
Provides correct answer	1

Sample answer:

0.8233

Question 33 (b)

Criteria	Marks
Provides correct answer	1

Sample answer: $m = 13.346$ $c = 903.499$

Question 33 (c)

Criteria	Marks
Provides correct calculations resulting in m	2
Provides correct values for s_x and s_y	1

Sample answer:

$$\begin{cases} s_y = 119.732 \\ s_x = 7.386 \end{cases} \Rightarrow r \times \frac{s_y}{s_x} = 0.8233 \times 16.2096 = 13.346 = m$$

Question 34 (a)

Criteria	Marks
Provides correct solution	3
Provides correct area for one part	2
Provides correct integral expression for the total area	1

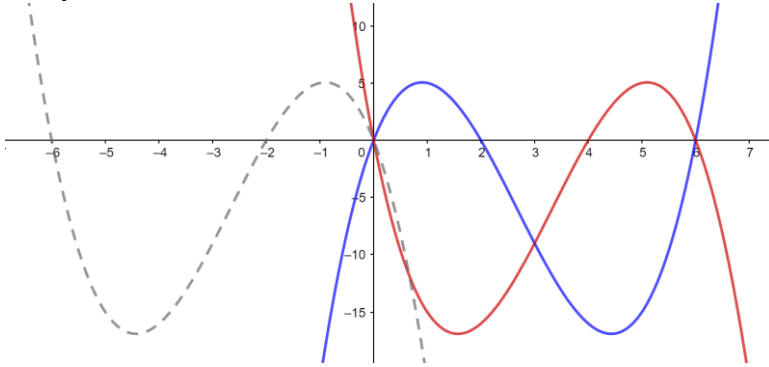
Sample answer:

$$\int_0^2 f(x)dx - \int_2^6 f(x)dx = \left[\frac{x^4}{4} - \frac{8x^3}{3} + 6x^2 \right]_0^2 - \left[\frac{x^4}{4} - \frac{8x^3}{3} + 6x^2 \right]_2^6 = \frac{20}{3} + \frac{128}{3} = \frac{148}{3} \text{ units}^2$$

Question 34 (b)

Criteria	Marks
• Provides correct solution	3
• Describes the correct transformations	2
• Recognises the graph of $g(x)$, or equivalent merit	1

Sample answer:



$$\int_4^0 g(x)dx + \int_4^6 g(x)dx = -\int_0^4 g(x)dx + \int_4^6 g(x)dx$$

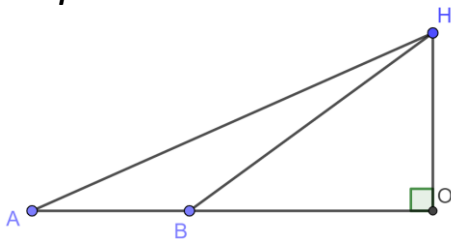
$g(x)$ is obtained by reflecting $f(x)$ horizontally and then shifting it 6 units to right

$$\therefore g(x) = f(-x+6) \Rightarrow a = -1, b = 6$$

Question 35

Criteria	Marks
• Provides correct solution	3
• Provides an equation in terms of the height of the hill, or equivalent merit	2
• Provides a trigonometric ratio involving one of the given angles	1

Sample answer:



$$\angle HAO = \alpha = 12^\circ, \angle HBO = \beta = 18^\circ, AB = 600, OH = ?$$

$$\tan \alpha = \frac{OH}{600 + OB} \Rightarrow OB = \frac{OH}{\tan \alpha} - 600$$

$$\tan \beta = \frac{OH}{OB} \Rightarrow OB = \frac{OH}{\tan \beta}$$

$$\frac{OH}{\tan \alpha} - 600 = \frac{OH}{\tan \beta} \Rightarrow OH = \frac{600}{\frac{1}{\tan \alpha} - \frac{1}{\tan \beta}} = 368.79$$

$$= 368.79\text{m}$$

Question 36

Criteria	Marks
• Provides correct solution including the monthly payment and interest paid	5
• Provides correct solution including the monthly payment	4
• Provides correct expression for the loan remaining after n th payment involving the formula for sum of geometric series	3
• Provides correct expression for the loan remaining after n th payment	2
• Provides correct expression for the loan remaining after first payment, or equivalent merit	1

Sample answer:

Let A_n be the remaining loan after n th payment and M be the monthly payment

$$P = 48000, r = 1 + \frac{0.084}{12}$$

$$A_1 = P \times r - M$$

$$A_2 = A_1 r - M = Pr^2 - Mr - M$$

$$A_3 = A_2 r - M = Pr^3 - Mr^2 - Mr - M$$

$$A_n = Pr^n - M(1 + r + r^2 + \dots + r^{n-1}) = Pr^n - \frac{M(r^n - 1)}{r - 1}$$

$$A_{60} = 0 \Rightarrow M = \frac{Pr^n(r - 1)}{r^n - 1} = 982.48$$

$$\text{Interest paid} = 60M - 48000 = 10948.9$$

$$= \$10948.90$$

Question 37

Criteria	Marks
• Provides correct solution	4
• Provides correct root for the derivative of the area	3
• Provides correct expression for the derivative of the area, or equivalent merit	2
• Provides correct expression for the area, or equivalent merit	1

Sample answer:

$$P = (x, \sqrt{1-x^2})$$

$$\text{Area} = A(x) = 2x\sqrt{1-x^2}$$

$$\frac{dA}{dx} = 2\sqrt{1-x^2} + 2x \times \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \times (-2x)$$

$$\frac{dA}{dx} = 0 \Rightarrow \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}} = 0 \Rightarrow 1-2x^2 = 0 \Rightarrow x = \frac{1}{\sqrt{2}}$$

$$A\left(\frac{1}{\sqrt{2}}\right) = 1$$

$$A = 1 \text{ units}^2$$

2020 Mathematics Advanced Year 12 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes	Targeted performance bands
1	1	MA-E1 Logarithms and Exponentials	MA11-6	2-3
2#	1	MA-M1 Modelling Financial Situations	MA12-2	2-3
3	1	MA-C2 Differential Calculus	MA12-6	2-3
4	1	MA-C4 Integral Calculus	MA12-7	2-4
5	1	MA-T1 Trigonometry and Measure of Angles	MA11-3	2-4
6	1	MA-E1 Logarithms and Exponentials	MA11-8	2-3
7	1	MA-C4 Integral Calculus	MA12-7	3-4
8	1	MA-S3 Random Variables	MA12-9	3-5
9	1	MA-C3 Applications of Differentiation	MA12-9	5-6
10	1	MA-C2 Differential Calculus	MA12-6	5-6

Section II

Question	Marks	Content	Syllabus outcomes	Targeted performance bands
11	2	MA-C4 Integral Calculus	MA12-7	2-3
12	2	MA-F1 Working with Functions	MA11-2	2-3
13	3	MA-F2 Graphing Techniques	MA12-1	2-3
14	2	MA-C1 Introduction to Differentiation	MA11-5	2-3
15#	2	MA-S1 Probability and Discrete Probability Distributions	MA11-7	3-4
16 (a)#	1	MA-M1 Modelling Financial Situations	MA12-2	2-3
16 (b)#	1	MA-M1 Modelling Financial Situations	MA12-2	2-3
17	3	MA-F1 Working with Functions	MA11-8	2-3
18	3	MA-C1 Introduction to Differentiation	MA11-5	3-4
19 (a)	2	MA-S3 Random Variables	MA12-10	3-4
19 (b)	2	MA-S3 Random Variables	MA12-8	3-4
20 (a)	3	MA-C3 Applications of Differentiation	MA12-6	2-3
20 (b)	3	MA-C3 Applications of Differentiation	MA12-6	2-3
20 (c)	1	MA-F2 Graphing Techniques	MA12-1	2-3
21 (a)#	2	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8	3-4
21 (b)#	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-10	2-4
22	2	MA-C3 Applications of Differentiation	MA12-10	3-4
23 (a)#	2	MA-S3 Random Variables	MA12-9	3-4
23 (b)#	2	MA-S3 Random Variables	MA12-9	2-3

24 (a)	2	MA-F1 Working with Functions	MA11-8	2-3
24 (b)	2	MA-F2 Graphing Techniques	MA12-9	3-4
25	2	MA-E1 Logarithms and Exponentials	MA11-6	2-4
26	3	MA-S3 Random Variables	MA12-8	3-4
27	2	MA-C1 Introduction to Differentiation	MA11-5	4-5
28	3	MA-M1 Modelling Financial Situations	MA12-4	3-4
29	2	MA-F1 Working with Functions	MA11-2	2-3
30	3	MA-T3 Trigonometric Functions and Graphs MA-F2 Graphing Techniques	MA12-5	4-5
31 (a)	3	MA-C4 Integral Calculus	MA12-7	3-5
31 (b)	1	MA-C3 Applications of Differentiation	MA12-3	3-4
32 (a)	1	MA-T3 Trigonometric Functions and Graphs	MA12-5	2-3
32 (b)	2	MA-T3 Trigonometric Functions and Graphs	MA12-1	3-4
32 (c)	2	MA-T3 Trigonometric Functions and Graphs	MA12-5	3-4
32 (d)	1	MA-T3 Trigonometric Functions and Graphs	MA12-5	3-4
33 (a)#	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-9	3-4
33 (b)#	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-9	3-4
33 (c)	2	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-10	3-5
34 (a)	3	MA-C4 Integral Calculus	MA12-7	3-4
34 (b)	3	MA-C4 Integral Calculus MA-F2 Graphing Techniques	MA12-9	5-6
35	3	MA-T1 Trigonometry and Measure of Angles	MA11-1	4-6
36	5	MA-M1 Modelling Financial Situations	MA12-2	5-6
37	4	MA-C3 Applications of Differentiation	MA12-3	5-6

- denotes a question that could be common to Mathematics Standard 2 HSC examination.