amity college

2021

HIGHER
SCHOOL
CERTIFICATE
TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- No reading time
- Working time – 2 hours
- Write using a black pen

Total marks: 60

Section I –10 marks
- Multiple choice questions
- Attempt Questions 1-10
- You have 20 minutes for this section
- You will not be able to move onto the next section until the time is up for this section

Section II –25 marks
- You have 50 minutes for this section
- You will not be able to move onto the next section until the time is up for this section

Section III –25 marks
- You have 50 minutes for this question

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TRIAL EXAMINATION

Mathematics Advanced

Section I

10 marks
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You have 20 minutes for this section

Instructions

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Section I

10 marks
Attempt questions 1-10

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1 Given that $\log_a b = 3.75$ and $\log_a c = 0.25$, find $\log_a (bc)^2$.

A. 0.879

B. 0.9375

C. 3.5

D. 8

2 Which interval gives the range of the function $y = 6\sin x - 10$?

A. $[6,-10]$

B. $[4,16]$

C. $[-16,-4]$

D. $[-4,16]$

3 A random variable, T, has the flowing probability distribution:

t	$w-3$	$w-2$	$w-1$	w	$w+1$
P(T=t)	0.2	0.5	0.1	0.05	0.15

Given that E(T)=8.45, what is the value of w?

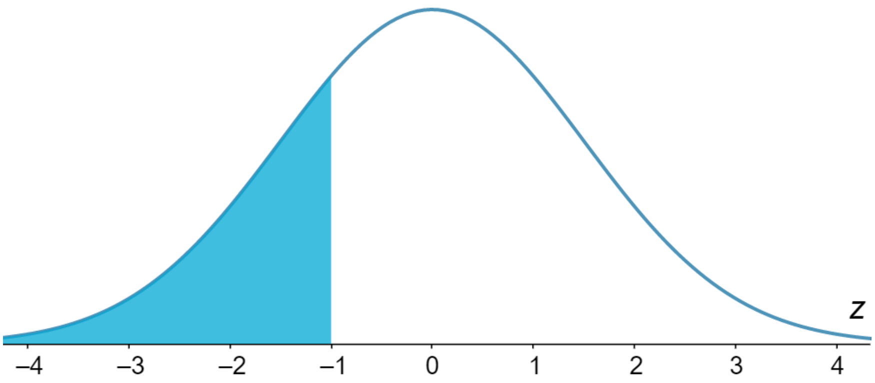
A. 2.69

B. 3

C. 6.9

D. 10

4 The graph shows the area under a normal distribution curve.



Which of the following is equivalent to the shaded area?

A. $P(z \geq 1)$

B. $P(z \geq -1)$

C. $1 - P(-1 \leq z \leq 1)$

D. $P(z \leq 1)$

5 If $f(x) = \sqrt{x}$ and $g(x) = 4 - x^2$, what is the domain of the function $f\left(g(x)\right)$?

A. $\left[-2, 2\right]$

B. $\left(0, \infty\right)$

C. $\left(-\infty, 2\right] \cup \left[2, \infty\right)$

D. $\left(-\infty, \infty\right)$

6 Which expression is a term of the geometric series $2x - 6x^3 + 18x^5 - \ldots$?

A. $-1458x^{13}$

B. $1458x^{13}$

C. $-1458x^{15}$

D. $1458x^{15}$

7 How many solutions does $2\sin^2 x - 1 = 0$ have for $-\frac{\pi}{2} \le x \le \pi$?

A. 1

B. 2

C. 3

D. 4

8 Which of the following is NOT a probability density function?

A. $f(x) = \begin{cases} 3x^2 & \text{for } 0 \le x \le 1 \\ 0 & \text{for all other values of } x \end{cases}$

B. $f(x) = \begin{cases} 2x^4 & \text{for } 0 \le x \le 1 \\ 0 & \text{for all other values of } x \end{cases}$

C. $f(x) = \begin{cases} 4x^3 & \text{for } 0 \le x \le 1 \\ 0 & \text{for all other values of } x \end{cases}$

D. $f(x) = \begin{cases} 2x & \text{for } 0 \le x \le 1 \\ 0 & \text{for all other values of } x \end{cases}$

9 The equation of the tangent to the curve $y = f(x)$ at the point $(2,7)$ is $y = 5x - 3$.

What is the equation of the tangent to the curve $f(x+2)+1$ at the point $(0,8)$?

A. $y = 3x - 4$

B. $y = 5x - 13$

C. $y = 5x - 12$

D. $y = 5x + 8$

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Tina drank tea on a Monday morning.
What is the probability that she drinks coffee on the Wednesday morning?

A. 0.12

B. 0.24

C. 0.36

D. 0.40

End of Section I

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Mathematics Advanced

Section II

25 marks
You have 50 minutes for this section
Attempt Questions 11-18 (25 marks)

Instructions

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Question 11 (2 marks)

A function $f(x)$ has the derivative $f'(x) = 4x^3 - 6x^2 + 5$ and $f(1) = 10$. **2**

Find the function.

Question 12 (2 marks)

2

Find $\int \frac{x-3}{x^2-6x+5}\,dx$.

Question 13 (2 marks)

Explain whether or not the lowest score 3 is an outlier for the data set with 5-number summary. **2**

Justify your answer with mathematical calculations.

L	Q_1	Q_2	Q_3	H
3	11	14	17	19

Question 14 (3 marks)

The graph of $y = \sqrt{x}$ undergoes the following sequence of transformations: **3**

- Translation 3 units right
- Dilation horizontally by a factor of $\frac{1}{2}$
- Translation 1 unit down
- Dilation vertically by a factor of 4

Find the rule of the resulting graph.

Question 15 (4 marks)

(a) Show that $1-\frac{4}{x-1}=\frac{x-5}{x-1}$. **1**

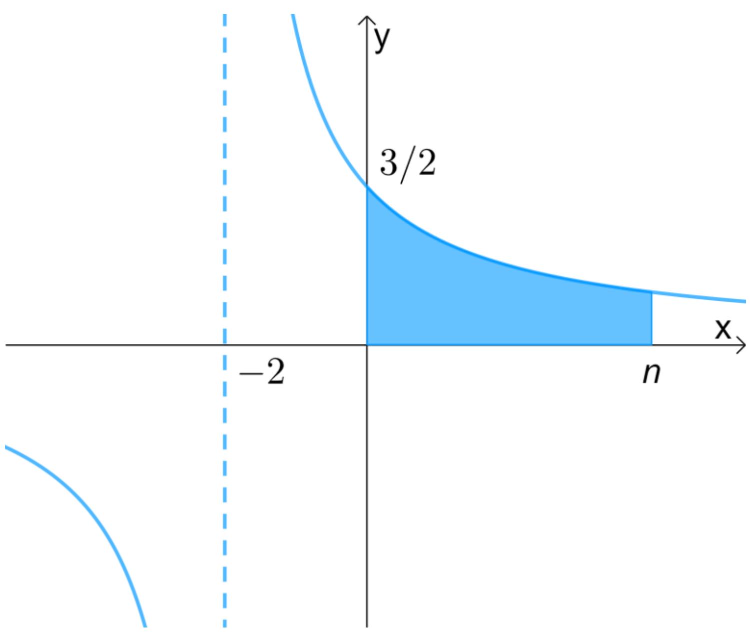
(b) Hence, or otherwise, find $\int_2^e \frac{2x-10dx}{x-1}$, leave your answer in exact form. **3**

Question 16 (4 marks)

Sketch the graph of the curve $y=x(x^2+2x)^2$, labelling the stationary points and intercepts. **4**

Question 17 (4 marks)

Part of the graph of the function $f(x)=\frac{a}{x+b}$ is given: **4**



If the shaded area is $\ln 27$, find the values of *a*, *b* and *n*.

Question 18 (4 marks)

Boris makes yearly contributions of \$2000 into an investment fund. His investment earns 6% interest compounded annually. **4**

After how many years will the total investment exceed \$45 000?

End of Section II

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Section III

25 marks
You have 50 minutes for this section
Attempt Questions 19-23 (25 marks)

Instructions

- Answer the questions in the writing space provided
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Question 19 (2 marks)

Evaluate $\int_{-1}^{4}\left(x-3f(x)\right)dx$ given that $\int_{-1}^{4} f(x)dx=8$. **2**

Question 20 (4 marks)

An object is moving in a straight line with acceleration function:

$$\ddot{x}(t)=-4te^{-t^2}\ m/s^2$$

It is initially at the origin with a velocity of 1*m/s*.

(a) Show that as *t* increases the object approaches a limiting velocity and state what the limiting velocity is. **2**

(b) Determine the time when the particle is stationary. **2**

Question 21 (8 marks)

A continuous random variable, *X*, has the following probability density function.

$$f(x)=\begin{cases} k\cos\left(\dfrac{\pi x}{2}\right) & \text{for } 0\le x\le 1 \\ 0 & \text{for all other values of } x \end{cases}$$

(a) Show that $k=\dfrac{\pi}{2}$. **2**

(b) Find $\mathrm{P}\left(X\le\dfrac{1}{3}\,|\,X\le\dfrac{1}{2}\right)$. **3**

(c) Find the median of *X*. **3**

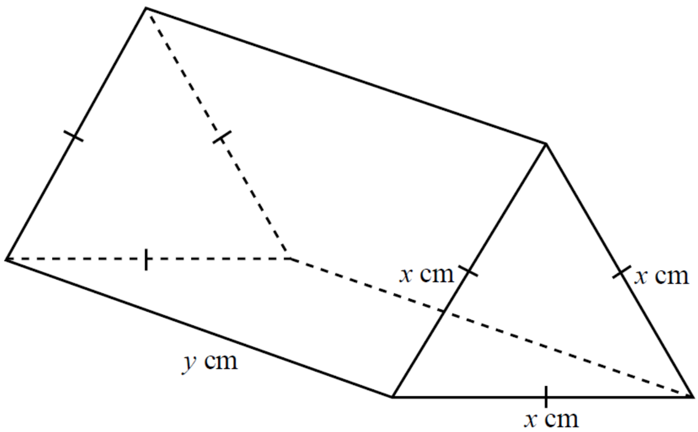
Question 22 (3 marks)

Solve $1+\left(\frac{x+1}{x-1}\right)+\left(\frac{x+1}{x-1}\right)^2+\left(\frac{x+1}{x-1}\right)^3+\ldots=\frac{x^2}{2}$. Leave your answer in exact form. **3**

Question 23 (8 marks)

A gift box is made in the shape of a triangular prism.

The triangular end is an equilateral triangle with length x cm and the length of the box is y cm. The volume of the box is 300 cm^3.



(a) Find an expression for y in terms of x. **2**

(b) Show that the total surface area A cm^2, of the box, is given by **2**

$$A=\frac{1200\sqrt{3}}{x}+\frac{\sqrt{3}x^2}{2}.$$

(c) Find the value of x that gives the minimum total surface area. Justify your answer. **4**

End of Paper

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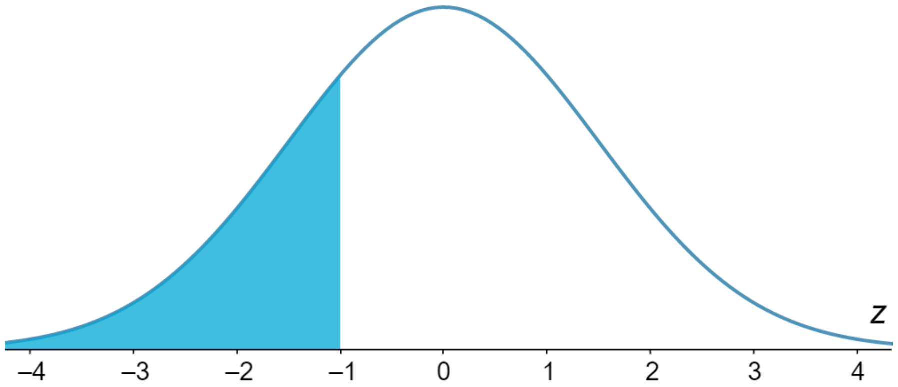

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Question 11 (2 marks)

A function $f(x)$ has the derivative $f'(x) = 4x^3 - 6x^2 + 5$ and $f(1) = 10$. **2**

Find the function.

$f(x) = \int 4x^3 - 6x^2 + 5 \, dx$

$f(x) = \frac{4x^4}{4} - \frac{6x^3}{3} + 5x + C$

$f(x) = x^4 - 2x^3 + 5x + C$ (1)

$f(1) = 10$

$10 = (1)^4 - 2(1)^3 + 5(1) + C$

$C = 6$

$\therefore f(x) = x^4 - 2x^3 + 5x + 6$ (1)

Question 12 (2 marks)

Find $\int \frac{x-3}{x^2 - 6x + 5} dx$. **2**

$f(x) = x^2 - 6x + 5$

$f'(x) = 2x - 6$

$= 2(x-3)$

$= \frac{1}{2} \int \frac{2(x-3)}{x^2 - 6x + 5} dx$ (1)

$= \frac{1}{2} \ln(x^2 - 6x + 5) + c$ (1)

Question 13 (2 marks)

Explain whether or not the lowest score 3 is an outlier for the data set with 5-number summary. **2**

Justify your answer with mathematical calculations.

L	Q_1	Q_2	Q_3	H
3	11	14	17	19

$Q_1 - 1.5 \times IQR$ (1)

$= 11 - 1.5 \times 6$

$= 2$

$IQR = Q_3 - Q_1$

$= 17 - 11$

$= 6$

Lowest score

$\therefore 2 < 3$ Hence 3 is not an outlier (1)

Question 14 (3 marks)

The graph of $y=\sqrt{x}$ undergoes the following sequence of transformations: **3**

- Translation 3 units right
- Dilation horizontally by a factor of $\frac{1}{2}$
- Translation 1 unit down
- Dilation vertically by a factor of 4

Find the rule of the resulting graph.

$y=\sqrt{x-3}$

$y=\sqrt{2x-3}$ (1)

$y=\sqrt{2x-3}-1$ (1)

$y=4\sqrt{2x-3}-4$ (1)

OR

$y=\sqrt{32x-48}-4$

Question 15 (4 marks)

(a) Show that $1-\dfrac{4}{x-1}=\dfrac{x-5}{x-1}$. **1**

(b) Hence, or otherwise, find $\displaystyle\int_2^e \frac{2x-10dx}{x-1}$, leave your answer in exact form. **3**

(a) $\text{LHS} = 1-\dfrac{4}{x-1}$

$=\dfrac{x-1}{x-1}-\dfrac{4}{x-1}$

$=\dfrac{x-1-4}{x-1}$ (1)

$=x-5$

$=\text{RHS}$

(b) $\displaystyle\int_2^e \frac{2x-10}{x-1}\,dx$

$\displaystyle= 2\int_2^e \frac{x-5}{x-1}\,dx$

$\displaystyle= 2\int_2^e \left(1-\frac{4}{x-1}\right)dx$

$\displaystyle= 2\int_2^e 1\,dx - 8\int_2^e \frac{1}{x-1}\,dx$ (1)

$= 2\left[x\right]_2^e - 8\left[\ln(x-1)\right]_2^e$

$= 2(e-2)-8\left(\ln(e-1)-\ln(2-1)\right)$ (1)

$= 2e-4-8\ln(e-1)$

(1)

Question 16 (4 marks)

Sketch the graph of the curve $y = x(x^2+2x)^2$, labelling the stationary points and intercepts. **4**

$u = x \quad v = (x^2+2x)^2$

$u' = 1 \quad v' = 2(x^2+2x)\cdot(2x+2)$

$y' = 2x(2x+2)(x^2+2x) + (x^2+2x)^2$

$y' = (x^2+2x)\left(2x(2x+2) + (x^2+2x)\right)$

$y' = x(x+2)(4x^2+4x+x^2+2x)$

$y' = x^2(x+2)(5x+6)$

$y' = 0 \rightarrow$ Stat points

$0 = x^2(x+2)(5x+6)$

$\therefore \begin{pmatrix} x=0 \\ y=0 \end{pmatrix} \quad \begin{pmatrix} x=-2 \\ y=0 \end{pmatrix} \quad \begin{pmatrix} x=-\frac{6}{5} \\ y=-\frac{3456}{3125} \end{pmatrix}$

hor inf (1) max (1) min (1)

test $x=-2$

x	-2.1	-2	-1.9
y'	+	0	−

max

$x = -\frac{6}{5}$

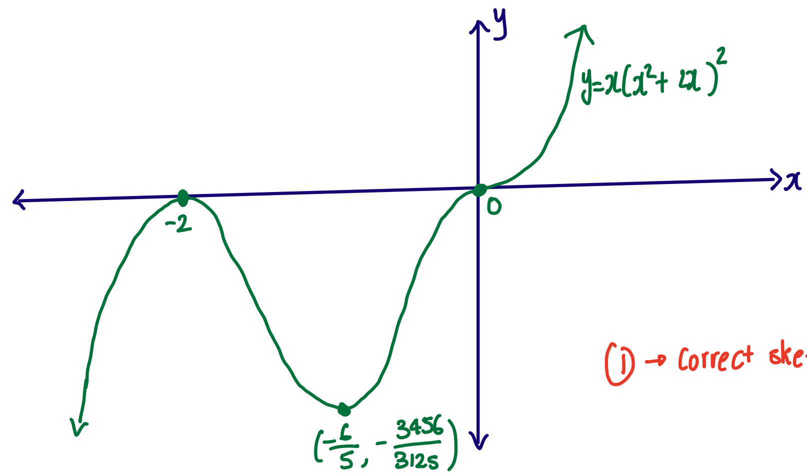
x	-1.3	$-\frac{6}{5}$	-1.1
y'	−	0	+

min

$x = 0$

x	-0.1	0	0.1
y'	+	0	+

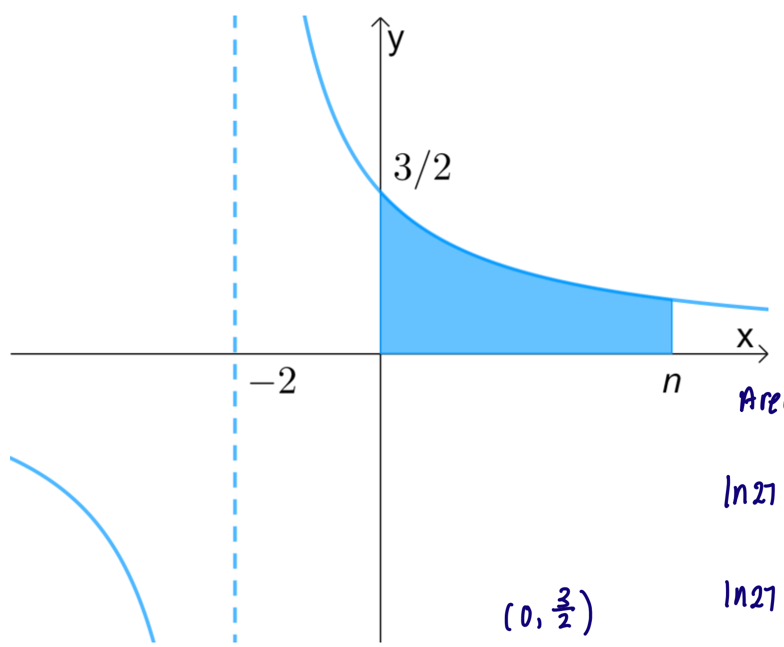
hor. pt. inf



(1) → Correct sketch

Question 17 (4 marks)

Part of the graph of the function $f(x) = \dfrac{a}{x+b}$ is given: **4**



$\left(0, \frac{3}{2}\right)$

$f(0) = \frac{3}{2}$

$\frac{3}{2} = \frac{a}{0+2}$

$\frac{3}{2} = \frac{a}{2}$

$\therefore a = 3$ (1)

If the shaded area is $\ln 27$, find the values of a, b and n.

asym: $x + b = 0$

$x = -2$

$b = 2$ (1)

Area $= \ln 27$

$\ln 27 = \int_0^n \frac{3}{x+2}\,dx$

$\ln 27 = 3\int_0^n \frac{1}{x+2}\,dx$

$\ln 27 = 3\left[\ln(x+2)\right]_0^n$ (1)

$\ln 27 = 3\left(\ln(n+2) - \ln(0+2)\right)$

$\ln 27 = 3\ln\frac{(n+2)}{2}$

$3\ln 3 = 3\ln\left(\frac{n+2}{2}\right)$

$\therefore 3 = \frac{n+2}{2}$

$6 = n+2$

$\therefore n = 4$ (1)

Question 18 (4 marks)

Boris makes yearly contributions of \$2000 into an investment fund. His investment earns 6% interest compounded annually. **4**

After how many years will the total investment exceed \$45 000?

A_n = value of the investment after n years

$A_1 = 2000 \times 1.06$

$A_2 = (A_1 + 2000) \cdot 1.06$

$= 2000 \cdot (1.06)^2 + 2000(1.06)$

$A_3 = (A_2 + 2000) \cdot 1.06$

$A_3 = 2000(1.06)^3 + 2000(1.06)^2 + 2000(1.06)^1$

$\vdots$

$A_n = 2000(1.06)^n + 2000(1.06)^{n-1} + \cdots + 2000(1.06)^1$

$A_n = 2000(1.06^1 + \cdots + 1.06^{n-1} + 1.06^n)$

$A_n = 2000 \cdot 1.06(1 + \cdots + 1.06^{n-2} + 1.06^{n-1})$

$A_n = 2000 \times 1.06 \times \dfrac{1 \times (1.06^n - 1)}{1.06 - 1}$

1

$45000 = 2120 \times \dfrac{1.06^n - 1}{0.06}$ 1

$1.273584906 = 1.06^n - 1$

$2.273584906 = 1.06^n$

$\dfrac{\ln 2.273584906}{\ln 1.06} = n$

$\therefore \; n = 14.09$ 1

$\therefore$ It will take 15 years. 1

End of Section II

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Section III

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Attempt Questions 19-23 (25 marks)

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Question 19 (2 marks)

Evaluate $\int_{-1}^{4}\left(x-3f(x)\right)dx$ given that $\int_{-1}^{4}f(x)dx=8$. **2**

$= \int_{-1}^{4} x\,dx - 3\int_{-1}^{4} f(x)\,dx$

$= \left[\frac{x^2}{2}\right]_{-1}^{4} - 3\times 8$ (1)

$= \left(\frac{4^2}{2} - \frac{(-1)^2}{2}\right) - 24$

$= -\frac{33}{2}$ (1)

Question 20 (4 marks)

An object is moving in a straight line with acceleration function:

$$\ddot{x}(t) = -4te^{-t^2}\ m/s^2$$

It is initially at the origin with a velocity of $1m/s$.

(a) Show that as t increases the object approaches a limiting velocity and state what the limiting velocity is. **2**

(b) Determine the time when the particle is stationary. **2**

(a) $\dot{x} = \int -4te^{-t^2}\,dt$

$f(x) = -t^2 \quad f'(x) = -2t$

$\dot{x} = 2\int\left(-2t \cdot e^{-t^2}\right)dt$

$\dot{x} = 2e^{-t^2} + c$

$\dot{x} = 1,\ t = 0$

$1 = 2e^{-0^2} + c$

$c = -1$

$\therefore\ \dot{x} = 2e^{-t^2} - 1$ (1)

$t \to \infty$

$\dot{x} = \frac{1}{2e^{t^2}} - 1$

$\dot{x} = \frac{1 \to 0}{\infty} - 1$

$\dot{x} = -1$

$\therefore$ as $t \to \infty$

$\dot{x} \to -1\ m/s$ (1)

(b) particle stationary $\to \dot{x} = 0$

$t = ?$

$0 = 2e^{-t^2} - 1$

$1 = 2e^{-t^2}$ (1)

$1 = \frac{2}{e^{t^2}}$

$e^{t^2} = 2$

$t^2 = \ln 2$

$t = \pm\sqrt{\ln 2}$

$t > 0$

$\therefore\ t = \sqrt{\ln 2}$ (1)

Question 21 (8 marks)

A continuous random variable, X, has the following probability density function.

$$f(x)=\begin{cases} k\cos\left(\dfrac{\pi x}{2}\right) & \text{for } 0\le x\le 1 \\ 0 & \text{for all other values of } x \end{cases}$$

(a) Show that $k=\dfrac{\pi}{2}$. **2**

$$1=\int_0^1 k\cos\left(\frac{\pi x}{2}\right)dx$$

$$1=\frac{2k}{\pi}\left[\sin\left(\frac{\pi x}{2}\right)\right]_0^1 \quad (1)$$

$$1=\frac{2k}{\pi}\left(\sin\frac{\pi}{2}-\sin 0\right)$$

$$1=\frac{2k}{\pi}(1-0)$$

$$\pi=2k$$

$$\therefore k=\frac{\pi}{2} \quad (1)$$

(b) Find $P\left(X\le\dfrac{1}{3}\,|\,X\le\dfrac{1}{2}\right)$. **3**

$$=\frac{P(x\le\frac{1}{3}\cap x\le\frac{1}{2})}{P(x\le\frac{1}{2})}$$

$$=\frac{\int_0^{\frac{1}{3}}\frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)dx}{\int_0^{\frac{1}{2}}\frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)dx} \quad (1)$$

$$=\frac{\left[\sin\left(\frac{\pi x}{2}\right)\right]_0^{\frac{1}{3}}}{\left[\sin\left(\frac{\pi x}{2}\right)\right]_0^{\frac{1}{2}}}$$

$$=\frac{\left(\sin\left(\frac{\pi}{2}\cdot\frac{1}{3}\right)-\sin 0\right)}{\left(\sin\left(\frac{\pi}{2}\cdot\frac{1}{2}\right)-\sin 0\right)} \quad (1)$$

$$=\frac{\sin\frac{\pi}{6}}{\sin\frac{\pi}{4}}$$

$$=\frac{\frac{1}{2}}{\frac{\sqrt{2}}{2}}$$

$$=\frac{1}{\sqrt{2}} \quad (1)$$

(c) Find the median of X. **3**

median $\Rightarrow$

$$\frac{1}{2}=\int_0^m\frac{\pi}{2}\cos\left(\frac{\pi}{2}x\right)dx \quad (1)$$

$$\frac{1}{2}=\left[\sin\left(\frac{\pi}{2}\cdot x\right)\right]_0^m$$

$$\frac{1}{2}=\left[\sin\left(\frac{\pi}{2}\cdot m\right)-\sin\left(\frac{\pi}{2}\cdot 0\right)\right] \quad (1)$$

$$\frac{1}{2}=\sin\frac{\pi m}{2}$$

$$\therefore\ \frac{\pi m}{2}=\frac{\pi}{6}$$

$$\pi m=\frac{\pi}{3}$$

$$m=\frac{1}{3} \quad (1)$$

Question 22 (3 marks)

Solve $1+\left(\dfrac{x+1}{x-1}\right)+\left(\dfrac{x+1}{x-1}\right)^2+\left(\dfrac{x+1}{x-1}\right)^3+\ldots=\dfrac{x^2}{2}$. Leave your answer in exact form. **3**

$a = 1$

$r = \dfrac{x+1}{x-1}$

$S_\infty = \dfrac{a}{1-r}$

$\dfrac{x^2}{2} = \dfrac{1}{1-\dfrac{x+1}{x-1}}$

$\dfrac{x^2}{2} = \dfrac{1}{\dfrac{\cancel{x}-1-\cancel{x}-1}{x-1}}$

$\dfrac{x^2}{2} = \dfrac{x-1}{-2}$ (1)

$-2x^2 = 2x - 2$

$0 = 2x^2 + 2x - 2$

$0 = x^2 + x - 1$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2}$

$x = \dfrac{-1 \pm \sqrt{5}}{2}$ (1)

Sum will only exist if $|r| < 1$

$\left|\dfrac{x+1}{x-1}\right| < 1$

when $x = \dfrac{-1+\sqrt{5}}{2}$ $\left|\dfrac{x+1}{x-1}\right| = 4.2\ldots \not< 1$

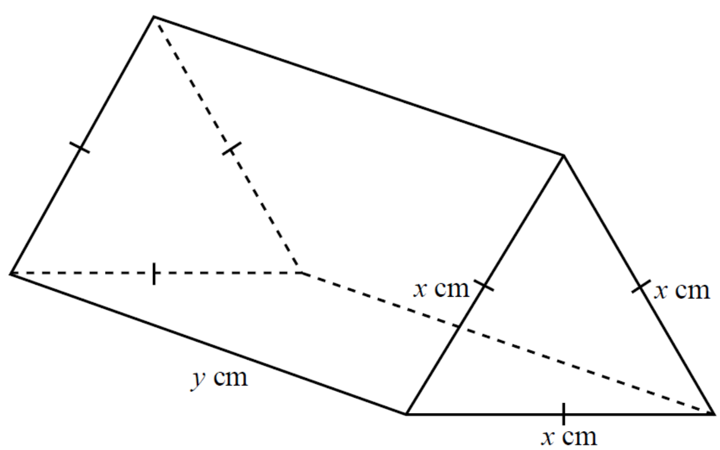
$x = \dfrac{-1-\sqrt{5}}{2}$ $\left|\dfrac{x+1}{x-1}\right| = 0.23\ldots < 1$

$\therefore x = \dfrac{-1-\sqrt{5}}{2}$ (1)

Question 23 (8 marks)

A gift box is made in the shape of a triangular prism.

The triangular end is an equilateral triangle with length x cm and the length of the box is y cm. The volume of the box is 300 cm^3.



(a) Find an expression for y in terms of x. **2**

Area of $\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2}x \cdot x \cdot \sin 60^\circ$

$= \frac{1}{2} \cdot x^2 \cdot \frac{\sqrt{3}}{2}$

$= \dfrac{\sqrt{3}x^2}{4}$ (1)

$V = A \times h$

$300 = \dfrac{\sqrt{3}}{4}x^2 \cdot y$

$1200 = \sqrt{3}x^2 \cdot y$

$y = \dfrac{1200}{\sqrt{3}x^2}$ or $\dfrac{400\sqrt{3}}{x^2}$ (1)

(b) Show that the total surface area A cm^2, of the box, is given by **2**

$$A = \frac{1200\sqrt{3}}{x} + \frac{\sqrt{3}x^2}{2}.$$

$SA = 3 \times \square + 2 \times \triangle$

$= 3xy + 2 \times \frac{\sqrt{3}}{4} x^2$ (1)

$= 3\cancel{x} \cdot \frac{400\sqrt{3}}{x^{\cancel{2}}} + \frac{\cancel{2}\sqrt{3}x^2}{\cancel{4}_{2}}$

$= \frac{1200\sqrt{3}}{x} + \frac{\sqrt{3}x^2}{2}$ (1)

(c) Find the value of x that gives the minimum total surface area. Justify your answer. **4**

$S = 1200\sqrt{3}x^{-1} + \frac{\sqrt{3}}{2}x^2$

$\frac{dS}{dx} = \frac{-1200\sqrt{3}}{x^2} + \sqrt{3}x$ (1)

$\frac{dS}{dx} = 0 \rightarrow$ Stat points

$0 = \frac{-1200\sqrt{3}}{\cancel{x^2}} + \frac{\sqrt{3}x^3}{\cancel{x^2}}$

$0 = -1200\sqrt{3} + \sqrt{3}x^3$

$1200\cancel{\sqrt{3}} = \cancel{\sqrt{3}}x^3$ (1)

$x = \sqrt[3]{1200}$

$\frac{dS}{dx} = -1200\sqrt{3}x^{-2} + \sqrt{3}x$

$\frac{d^2S}{dx^2} = \frac{2400\sqrt{3}}{x^3} + \sqrt{3}$ (1)

$\frac{d^2S}{dx^2}\left(\sqrt[3]{1200}\right) = \frac{2400\sqrt{3}}{1200} + \sqrt{3}$

$= 3\sqrt{3} > 0$ ✓ min

(1)

$\therefore x = \sqrt[3]{1200}$ gives minimum SA.

End of Paper