amity college

HSC
TRIAL EXAMINATION
2022

Mathematics Advanced

Name:____________________________

Class:____________________________

Mark:____________/100

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11-32, show relevant mathematical reasoning and/or calculations

Weight – 30% of internal assessment

Total marks – 100

Section I — 10 marks (pages 3-6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II — 90 marks (pages 8-27)

- Attempt Questions 11–32
- Allow about 2 hours 45 minutes for this section

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1 The first term of an arithmetic sequence is 12. The fifteenth term of the same sequence is 67.

What is the common difference of the sequence?

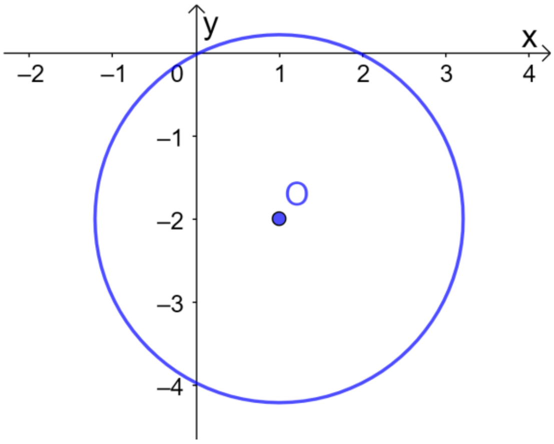
A. $\frac{11}{3}$

B. $\frac{55}{14}$

C. 3

D. $\frac{45}{14}$

2 In the diagram, the graph of the circle with centre $O(1,-2)$ is shown.



Which of the following gives the equation of the circle?

A. $(x+1)^2+(y-2)^2=25$

B. $(x-1)^2+(y+2)^2=25$

C. $(x-1)^2+(y+2)^2=5$

D. $(x+1)^2+(y-2)^2=5$

3 How many stationary points does the function $f(x) = 4x^3 - 6x^2 + 3x$ have?

A. 0

B. 1

C. 2

D. 3

4 Which of the following is equivalent to $\sec^2 x - \sin^2 x - \cos^2 x$?

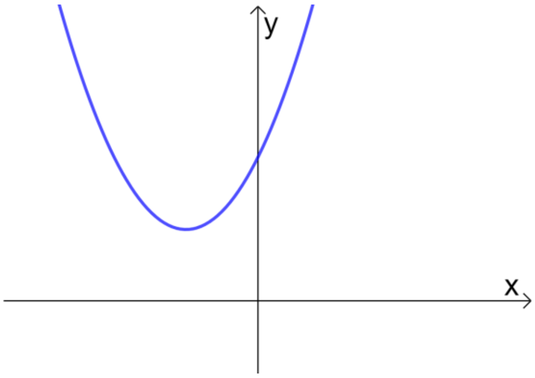
A. $\tan^2 x + 1$

B. $1 - \tan^2 x$

C. $\tan^2 x - 1$

D. $\tan^2 x$

5 The diagram shows the graph of $y = x^2 + bx + 2$.



Which of the following could be the value of *b*?

A. 2

B. −2

C. 3

D. −3

6 Given the function $y = \frac{2}{1-e^x}$, which expression is equal to $\frac{dy}{dx}$?

A. $\frac{2e^x}{(1-e^x)^2}$

B. $\frac{-2e^x}{1-e^x}$

C. $\frac{-2}{(1-e^x)^2}$

D. $\frac{e^x}{(1-e^x)^2}$

7 What is $\int 4\sqrt{2x-3}\,dx$?

A. $\frac{1}{3}\left(\sqrt{2x-3}\right)^3 + c$

B. $\frac{3}{4}\left(\sqrt{2x-3}\right)^3 + c$

C. $\frac{1}{2\sqrt{2x-3}} + c$

D. $\frac{4}{3}\left(\sqrt{2x-3}\right)^3 + c$

8 What transformation is required to produce the graph of $y = 3^{x+1}$ from the graph of $y = 3^x$?

A. Shift left by 3 units

B. Shift right by 1 unit

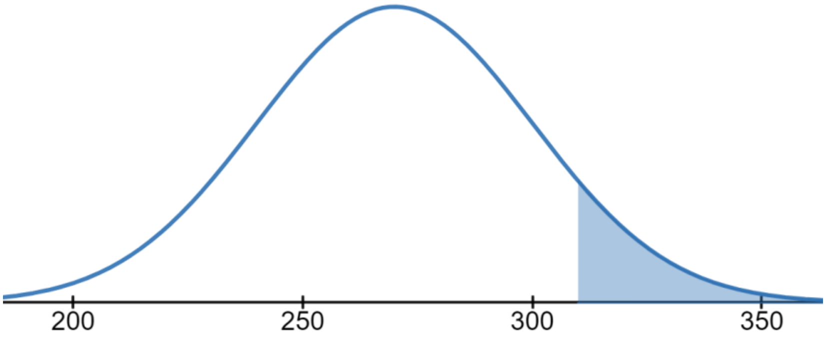
C. Dilate vertically by a factor of 3

D. Dilate horizontally by a factor of 3

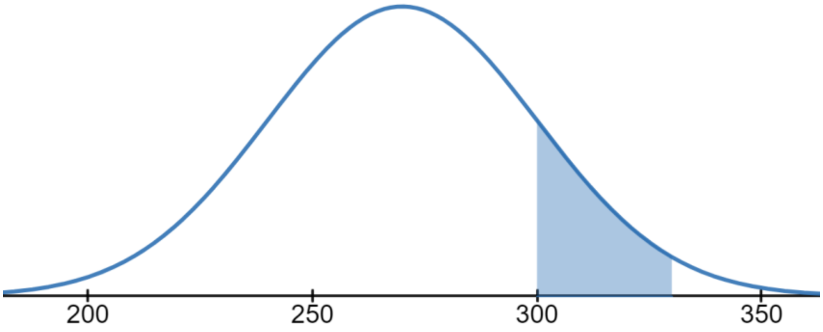
9 The heights of trees in a street are normally distributed with a mean of 270 cm and standard deviation of 30 cm. A particular tree is taller than 92% of the trees in the same street.

Which shaded region most accurately represents where the height of this tree lies?

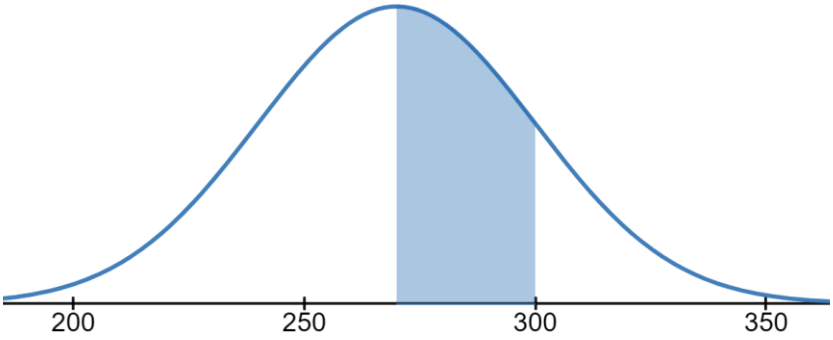
A.



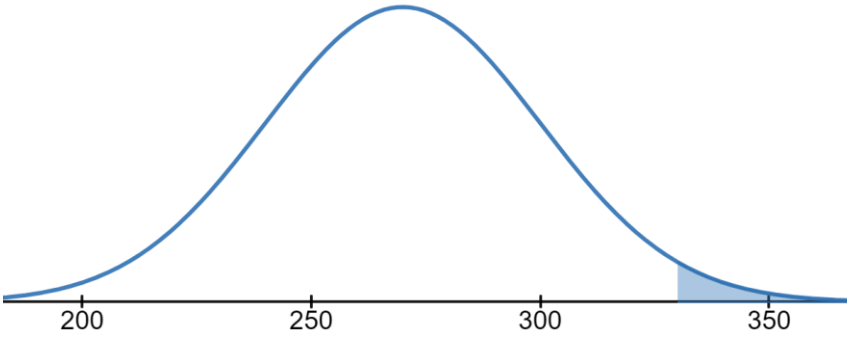
B.



C.



D.



10 Which of the following represents the domain of the function $f(x)=\dfrac{1}{\ln(x-2)}$?

A. $\left[2,\infty\right)$

B. $\left(2,\infty\right)$

C. $\left(2,3\right)\cup\left(3,\infty\right)$

D. $\left(3,\infty\right)$

Student NAME: ______________________________

2022 | **YEAR 12 TRIAL EXAMINATION**

Mathematics Advanced

Section II
Answer Booklet 1

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section

Booklet 1 - Attempt Questions 11–28 (64 Marks)
Booklet 2 - Attempt Questions 29–32 (26 marks)

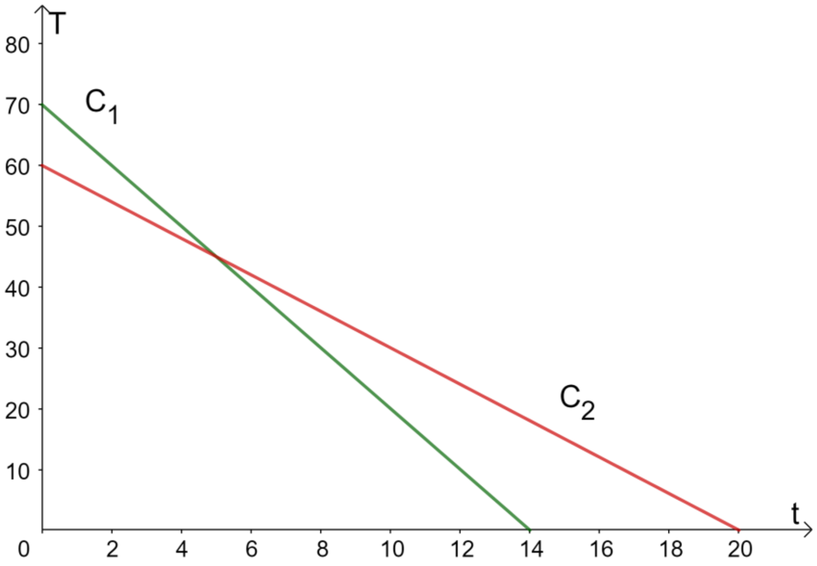
Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NAME above

Please turn over

Question 11 (4 marks)

The diagram shows the temperatures of two containers, C_1 and C_2, after t hours of the initial measurement.



(a) State the linear model for the temperature of each container. **2**

(b) For how many hours will C_2 be cooler than C_1? **2**

Question 12 (2 marks)

Solve $\left|3x-5\right|=\frac{1}{2}$. **2**

Question 13 (3 marks)

(a) Differentiate e^{x^2}. **1**

(b) Hence, evaluate $\int_0^2 xe^{x^2}\,dx$. **2**

Question 14 (4 marks)

For a sample of twelve students studying a certain course, the number of absent days and their final exam scores are recorded.

Absent Days	*Exam Score*
0	92
0	76
1	87
2	90
2	83
3	83
4	77
4	68
4	69
5	72
6	66
7	58

(a) Determine the equation of the least-squares regression line, $y = mx + b$, for this data. State the coefficients correct to three decimal places. **1**

(b) State the gradient of the regression line and interpret the value of this gradient in the given context. **2**

(c) Another student studying the same course was absent for 10 days. **1**

Calculate the predicted exam score for this student using your answer to part (a). Give your answer correct to the nearest whole number.

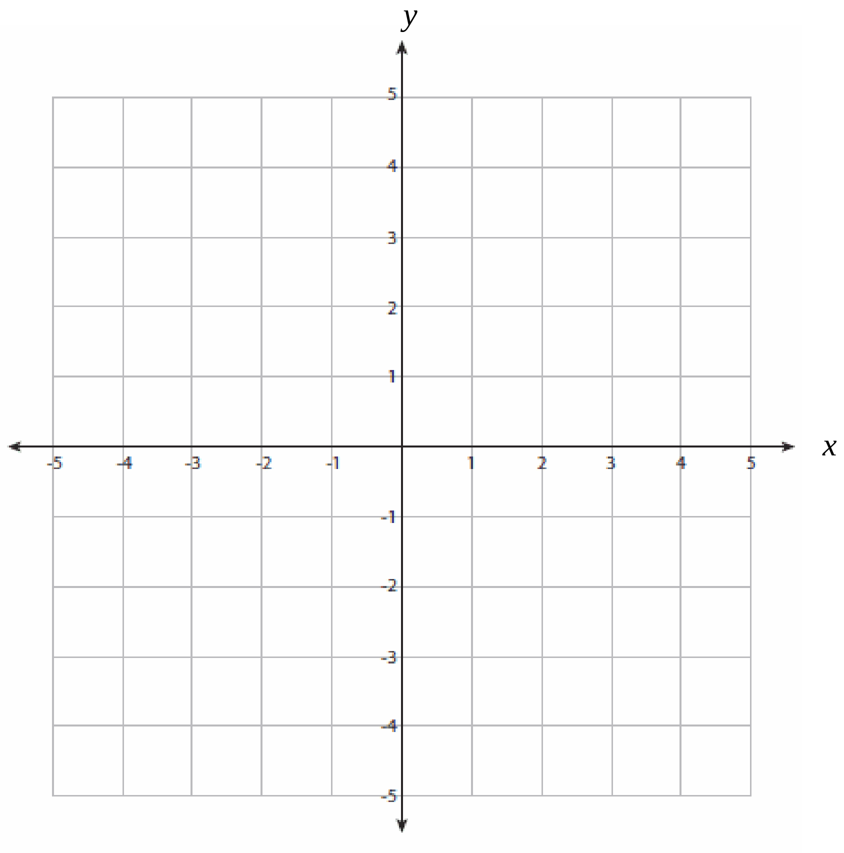
Question 15 (2 marks)

The curve $y = f(x)$ passes through the point $(0,1)$ such that $f'(x) = \dfrac{8x}{2x^2+1}$. **2**

Find its equation.

Question 16 (3 marks)

Sketch the graph of $y = \dfrac{4}{x+2} - 1$, labelling the asymptotes and axes intercepts. **3**



Question 17 (3 marks)

3

Let $0 < x < \frac{\pi}{2}$.

Explain why the limiting sum $1 + \sin^2 x + \sin^4 x + \sin^6 x + \ldots$ exists.

Hence, find the limiting sum in terms of *x*, leave your answer in the simpliest form.

Question 18 (3 marks)

A table for the present value for an annuity of $1 is shown.

The present value of an annuity of $1

Period	1%	2%	4%	6%	8%	10%
1	0.9901	0.9804	0.9615	0.9434	0.9259	0.9091
2	1.9704	1.9416	1.8861	1.8334	1.7833	1.7355
3	2.941	2.8839	2.7751	2.673	2.5771	2.4869
4	3.902	3.8077	3.6299	3.4651	3.3121	3.1699
5	4.8534	4.7135	4.4518	4.2124	3.9927	3.7908
6	5.7955	5.6014	5.2421	4.9173	4.6229	4.3553

(a) What would be the present value of a $5000 per year annuity at 6% per annum for six years, with interest compounding annually? Give your answer correct to the nearest dollar. **1**

(b) An annuity of $2000 per quarter is invested at 8% per annum, compounded quarterly for one year. **1**

What is the present value, correct to the nearest dollar, of the annuity?

(c) An annuity has a present value of $46 000. **1**

What is the amount of each yearly payment, compounded at 8% per annum, that is required to accumulate this annuity after five years? Give your answer correct to the nearest dollar.

Question 19 (3 marks)

Describe the three transformations which, when applied in succession, change the graph of $y = 2\sin x$ to the graph with equation $y = \sin\left(\frac{x+\pi}{3}\right)$. **3**

Question 20 (2 marks)

If $\int_0^8 f(x)\,dx = -7$ and $\int_0^4 f(x) = 6$, find the value of $\int_8^4 f(x)\,dx$. **2**

Question 21 (4 marks)

In a marathon, numbered flags are placed at regular intervals along the course. Flag number 1 is 800 metres from the start and all the rest are 500 metres apart.

(a) Show that the formula for the distance, F_n, that a competitor must run to reach flag number n is $F_n = 500n + 300$. **1**

(b) How many flags are needed if the length of the course is 10 000 m? **1**

(c) A runner retires from the race after completing 6000 m. **2**

Find the nearest flag number and the runner's distance from that flag when he retired.

Question 22 (6 marks)

On any given day, the number X of times Aleeza waits at a red light while driving to work is a random variable with probability distribution given by

x	0	1	2	3
$\mathrm{P}(X = x)$	a^2	$\frac{3a^2}{2}$	$\frac{7-32a}{6}$	$4a$

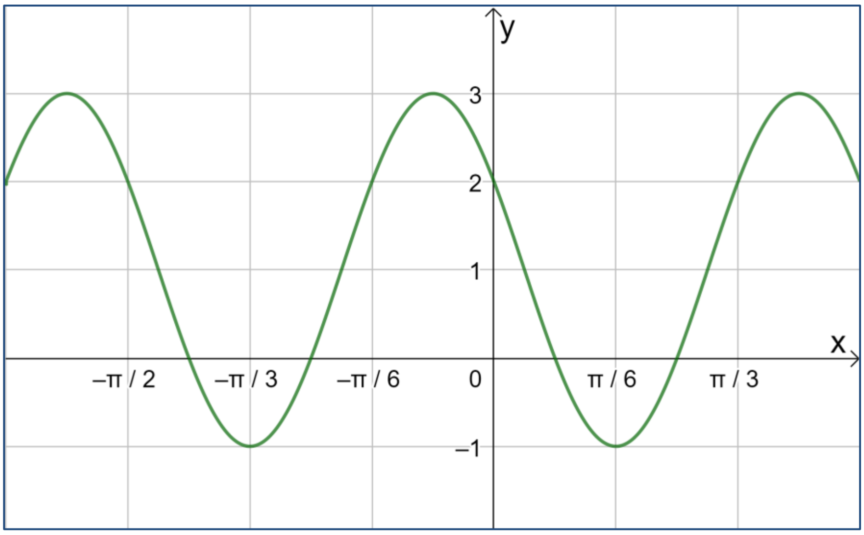
(a) Show that the only answer for a is $\frac{1}{5}$. **3**

(b) Let μ be the expected value of X. **3**

Find $\mathrm{P}(X < \mu)$.

Question 23 (5 marks)

The diagram shows the graph of $y = a\cos(bx + c) + d$.



(a) State the range. **1**

(b) State the period. **1**

(c) Find the values of a, b, c and d. **3**

Question 24 (4 marks)

The lifespan in weeks, X, of a certain butterfly species is given by a probability density function.

$$f(x) = \begin{cases} \frac{\pi}{8}\sin\left(\frac{\pi x}{4}\right) & \text{for } 0 \le x \le 4 \\ 0 & \text{for all other values of } x \end{cases}$$

(a) In a sample of 100 butterflies, how many are expected to live longer than three weeks? Give your answer correct to the nearest integer. **2**

(b) It is given that a butterfly has lived for more than two weeks. **2**

What is the probability that the same butterfly lives for more than three weeks? Give your answer to two decimal places.

Question 25 (3 marks)

A random variable Z is normally distributed with mean 0 and standard deviation 1. **3**

The table gives the probability of this random variable being greater than different values of z.

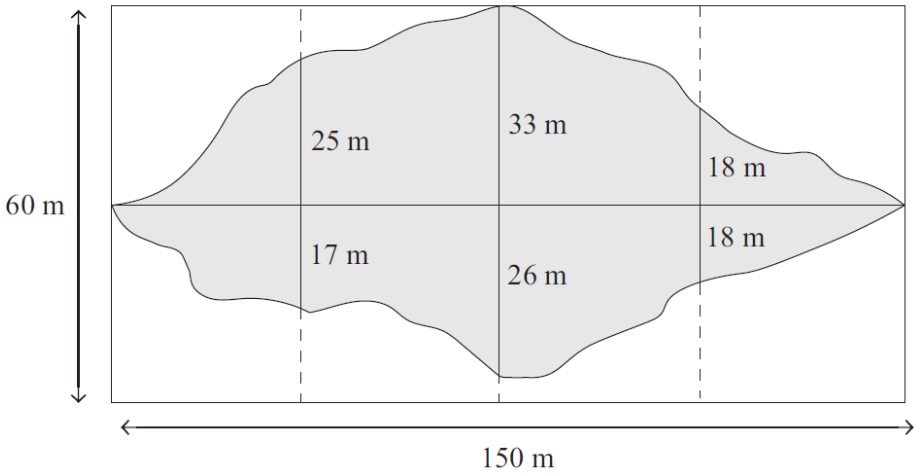
z	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
$P(Z > z)$	0.5	0.4013	0.3085	0.2266	0.1587	0.1057	0.668	0.0401	0.0228

In a hospital of 3125 patients, it is found that 708 patients have a body temperature higher than $36.8^\circ C$. The body temperatures are distributed normally with a mean μ and standard deviation $0.3^\circ C$.

Using the given table, find the value of μ, correct to two decimal places.

Question 26 (3 marks)

Archaeologists are excavating a rectangular site with dimensions of 150 metres by 60 metres. The shaded region indicates the portion of the site that has been excavated. **3**



NOT TO SCALE

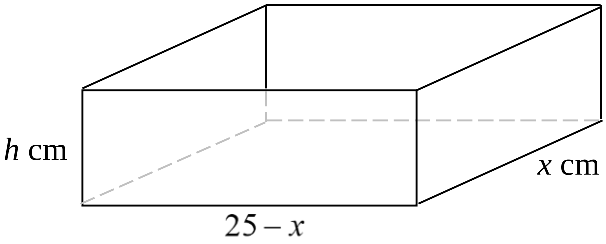
Use the trapezoidal rule to best approximate the total area that has been excavated. Give your answer 2 correct to the nearest whole number.

Question 27 (3 marks)

Simplify $a^{\frac{\log_b c}{1+\log_b\left(\frac{a}{b}\right)}}$. **3**

Question 28 (6 marks)

The base of an **open** topped toy box has length $25-x$ cm and width x cm. It has a total external surface area of 300 cm^2.



(a) Show that the height h of the toy box is given by $h=\dfrac{x^2-25x+300}{50}$. **2**

(b) Find the dimensions (length and width) of the base which give the maximum volume. **5**

Question 28 (continued)

End of Questions 28

Proceed to Booklet 2 for Questions 29−32

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student NAME: ______________________________

2022 | **YEAR 12 TRIAL EXAMINATION**

Mathematics Advanced

Section II
Answer Booklet 2

Booklet 2 - Attempt Questions 29–32 (26 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NAME above

Please turn over

Question 29 (10 marks)

Consider the function $f(x)=e^{2x}-4e^{x}-5$.

(a) Solve $f(x)=0$. **2**

(b) Find and classify the coordinates of the stationary point. **3**

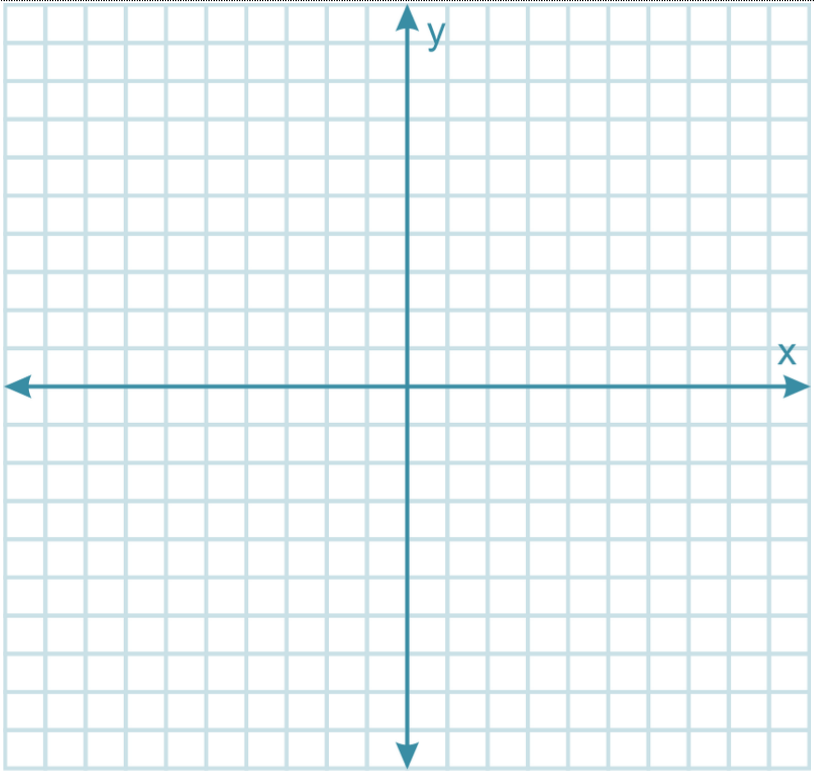
(c) Find the range of the function. **1**

Question 29 continues on page 23

Question 29 (continued)

(d) Find the interval for which the graph of the function is concave down. **2**

(e) Sketch the graph $y = e^{2x} - 4e^{x} - 5$, labelling all important features **2**



End of Question 29

Question 30 (5 marks)

On his date of retirement Nabil has \$650 000 in his superannuation fund. **5**

His fund is earning interest at a rate of 0.5% per month. Each month, immediately after the interest has been paid, Nabil withdraws \$5000.

How many years, correct to one decimal place, will the fund last?

Question 31 (6 marks)

The concentration of a chemical in a container changes according to the formula $C(t) = te^{-\frac{t}{10}}$, where t is time in minutes.

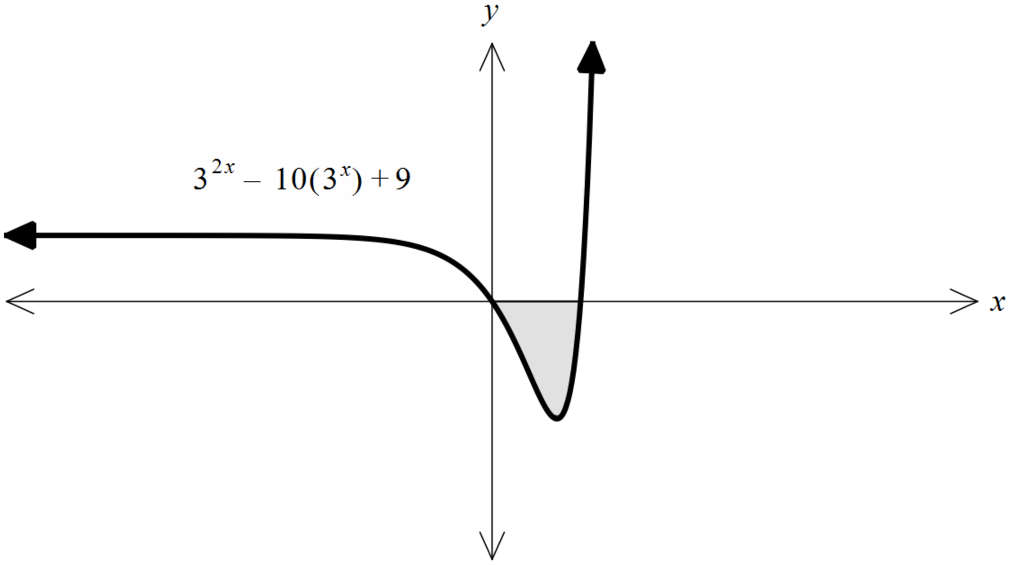
(a) After how many minutes will the concentration reach its maximum? **2**

(b) Find the time when the concentration of the chemical decreases at the fastest rate. **3**

(c) Find the maximum rate of increase. **1**

Question 32 (5 marks)

The graph of $f(x) = 3^{2x} - 10\left(3^x\right) + 9$ is shown below. Find the exact area of the shaded region. **5**



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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

STUDENT ID______________________________

2022 Mathematics Advanced Yr 12

Section I Answer Sheet

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response circle completely.

1	A ○	B ○	C ○	D ○
2	A ○	B ○	C ○	D ○
3	A ○	B ○	C ○	D ○
4	A ○	B ○	C ○	D ○
5	A ○	B ○	C ○	D ○
6	A ○	B ○	C ○	D ○
7	A ○	B ○	C ○	D ○
8	A ○	B ○	C ○	D ○
9	A ○	B ○	C ○	D ○
10	A ○	B ○	C ○	D ○

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r-1} = \frac{a(1-r^n)}{1-r}, r \neq 1$$

$$S = \frac{a}{1-r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

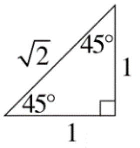
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

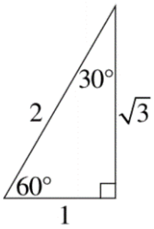
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$





Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

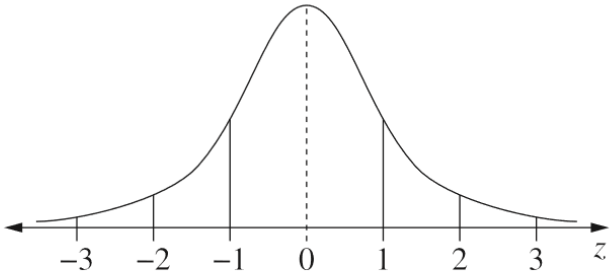
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have *z*-scores between -1 and 1
- approximately 95% of scores have *z*-scores between -2 and 2
- approximately 99.7% of scores have *z*-scores between -3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X-\mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)dx$$

$$P(a < X < b) = \int_a^b f(x)dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$\boldsymbol{X} \sim Bin(n,p)$$

$$\Rightarrow \boldsymbol{P(X = x)}$$

$$= \boldsymbol{\binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, \ldots, n}$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function	**Derivative**
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u),\ u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	
$\frac{dy}{dx} = (\ln a)\, f'(x)a^{f(x)}$	
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)\, f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \ldots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r}=\frac{n!}{(n-r)!}$$

$$\binom{n}{r}={}^{n}C_{r}=\frac{n!}{r!(n-r)!}$$

$$(x+a)^{n}=x^{n}+\binom{n}{1}x^{n-1}a+\ldots+\binom{n}{r}x^{n-r}a^{r}+\ldots+a^{n}$$

Vectors

$$|\underset{\sim}{u}|=\left|x\underset{\sim}{i}+y\underset{\sim}{j}\right|=\sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v}=|\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta=x_{1}x_{2}+y_{1}y_{2},$$

where $\underset{\sim}{u}=x_{1}\underset{\sim}{i}+y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v}=x_{2}\underset{\sim}{i}+y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r}=\underset{\sim}{a}+\lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned}z=a+ib&=r(\cos\theta+i\sin\theta)\\&=re^{i\theta}\end{aligned}$$

$$\begin{aligned}\left[r(\cos\theta+i\sin\theta)\right]^{n}&=r^{n}(\cos n\theta+i\sin n\theta)\\&=r^{n}e^{in\theta}\end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x=a\cos(nt+\alpha)+c$$

$$x=a\sin(nt+\alpha)+c$$

$$\ddot{x}=-n^{2}(x-c)$$

amity college

HSC
TRIAL EXAMINATION
2022

Mathematics Advanced

Name:______________________________	
Class:______________________________	Mark:____________/100

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11-32, show relevant mathematical reasoning and/or calculations

Weight – 30% of internal assessment

Total marks – 100

Section I — 10 marks (pages 3-6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II — 90 marks (pages 8-27)

- Attempt Questions 11–32
- Allow about 2 hours 45 minutes for this section

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

---

1 The first term of an arithmetic sequence is 12. The fifteenth term of the same sequence is 67.

What is the common difference of the sequence?

A. $\frac{11}{3}$

[✓ B.] $\frac{55}{14}$

C. 3

D. $\frac{45}{14}$

$a_1 = 12$

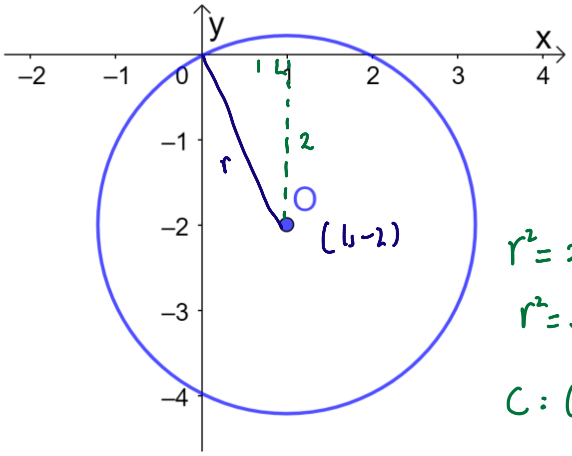
$a_{15} = 167$

$a_{15} = a_1 + (15-1)\,d$

$167 = 12 + 14d$

$d = \frac{55}{14}$

2 In the diagram, the graph of the circle with centre $O(1,-2)$ is shown.



$r^2 = 2^2 + 1^2$

$r^2 = 5$

C : (1, -2)

Which of the following gives the equation of the circle?

A. $(x+1)^2 + (y-2)^2 = 25$

B. $(x-1)^2 + (y+2)^2 = 25$

[✓ C.] $(x-1)^2 + (y+2)^2 = 5$

D. $(x+1)^2 + (y-2)^2 = 5$

Eqn : $(x-1)^2 + (y+2)^2 = 5$

3 How many stationary points does the function $f(x) = 4x^3 - 6x^2 + 3x$ have?

A. 0

[✓ B.] 1

C. 2

D. 3

$f'(x) = 12x^2 - 12x + 3$

$= 3(4x^2 - 4x + 1)$

$0 = 3(4x^2 - 4x + 1)$

$0 = 4x^2 - 4x + 1$

$0 = (2x-1)^2 \rightarrow x = \frac{1}{2}$ only one sol$^{\underline{n}}$

4 Which of the following is equivalent to $\sec^2 x - \sin^2 x - \cos^2 x$?

A. $\tan^2 x + 1$

B. $1 - \tan^2 x$

C. $\tan^2 x - 1$

[✓ D.] $\tan^2 x$

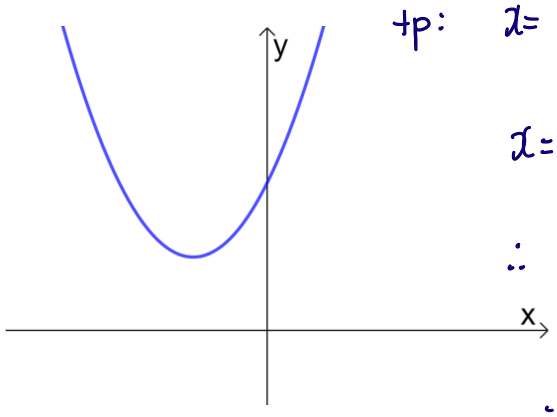
$\sec^2 x = 1 + \tan^2 x$

$1 + \tan^2 x - (\sin^2 x + \cos^2 x)$

$= 1 + \tan^2 x - 1$

$= \tan^2 x$

5 The diagram shows the graph of $y = x^2 + bx + 2$.



$\Delta = b^2 - 4ac$

$= b^2 - 4 \times 1 \times 2$

$= b^2 - 8$

$b^2 - 8 < 0$

$b < \pm 2\sqrt{2}$

tp: $x = \frac{-b}{2a} < 0$ (negative)

$x = \frac{-b}{2 \cdot 1} < 0$

$\therefore x = \frac{-b}{2} < 0$

$\therefore b < 2\sqrt{2}$

So only choice is $b = 2$

Which of the following could be the value of *b*?

[✓ A.] 2

B. −2

C. 3

D. −3

6 Given the function $y = \frac{2}{1-e^x}$, which expression is equal to $\frac{dy}{dx}$?

[✓ A.] $\frac{2e^x}{(1-e^x)^2}$

B. $\frac{-2e^x}{1-e^x}$

C. $\frac{-2}{(1-e^x)^2}$

D. $\frac{e^x}{(1-e^x)^2}$

$y = 2(1-e^x)^{-1}$

$y' = -2(1-e^x)^{-2} \cdot (-e^x)$

$y' = \frac{2e^x}{(1-e^x)^2}$

7 What is $\int 4\sqrt{2x-3}\,dx$?

A. $\frac{1}{3}\left(\sqrt{2x-3}\right)^3 + c$

B. $\frac{3}{4}\left(\sqrt{2x-3}\right)^3 + c$

C. $\frac{1}{2\sqrt{2x-3}} + c$

[✓ D.] $\frac{4}{3}\left(\sqrt{2x-3}\right)^3 + c$

$\int 4(2x-3)^{\frac{1}{2}}\,dx$

$= \frac{4\,(2x-3)^{3/2}}{2 \cdot 3/2} + C$

$= \frac{4}{3}\sqrt{(2x-3)^3} + C$

8 What transformation is required to produce the graph of $y = 3^{x+1}$ from the graph of $y = 3^x$?

A. Shift left by 3 units

B. Shift right by 1 unit

[✓ C.] Dilate vertically by a factor of 3

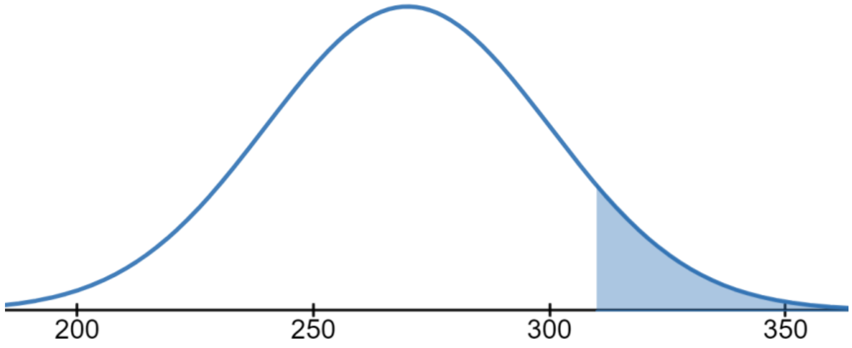
D. Dilate horizontally by a factor of 3

$y = \boxed{3} \cdot 3^x$

9 The heights of trees in a street are normally distributed with a mean of 270 cm and standard deviation of 30 cm. A particular tree is taller than 92% of the trees in the same street.

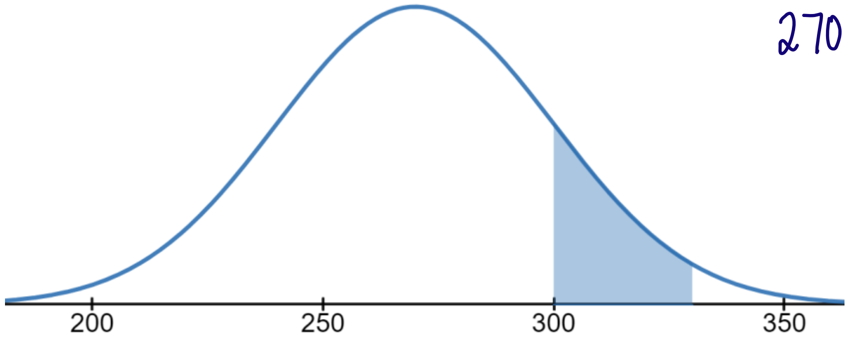
Which shaded region most accurately represents where the height of this tree lies?

A.



Height is between 1 to 2 SD above the mean

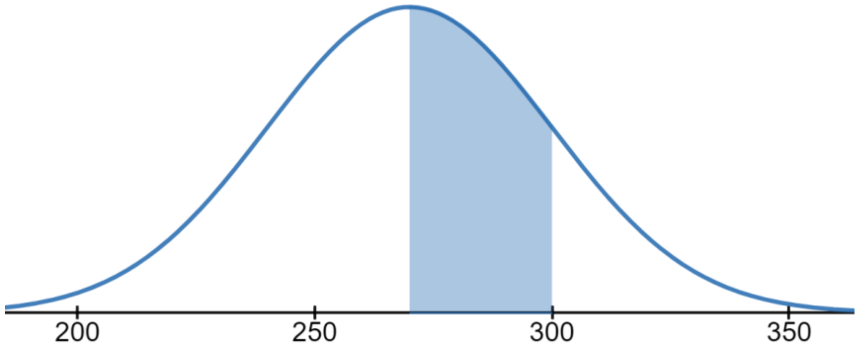
[✓ B.]



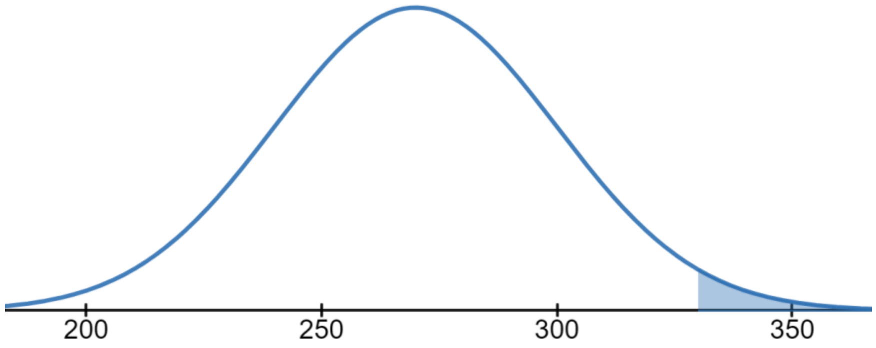
$270+30 < h < 270 + 60$

$300 < h < 330$

C.



D.



10 Which of the following represents the domain of the function $f(x)=\dfrac{1}{\ln(x-2)}$?

A. $[2,\infty)$

B. $(2,\infty)$

[✓ C.] $(2,3)\cup(3,\infty)$

D. $(3,\infty)$

$$\begin{cases} x-2>0 \\ \ln(x-2)\neq 0 \end{cases} \Rightarrow \begin{cases} x>2 \\ x\neq 3 \end{cases}$$

Student NAME: ______________________________

2022 | **YEAR 12 TRIAL EXAMINATION**

Mathematics Advanced

Section II
Answer Booklet 1

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section

Booklet 1 - Attempt Questions 11–28 (64 Marks)
Booklet 2 - Attempt Questions 29–32 (26 marks)

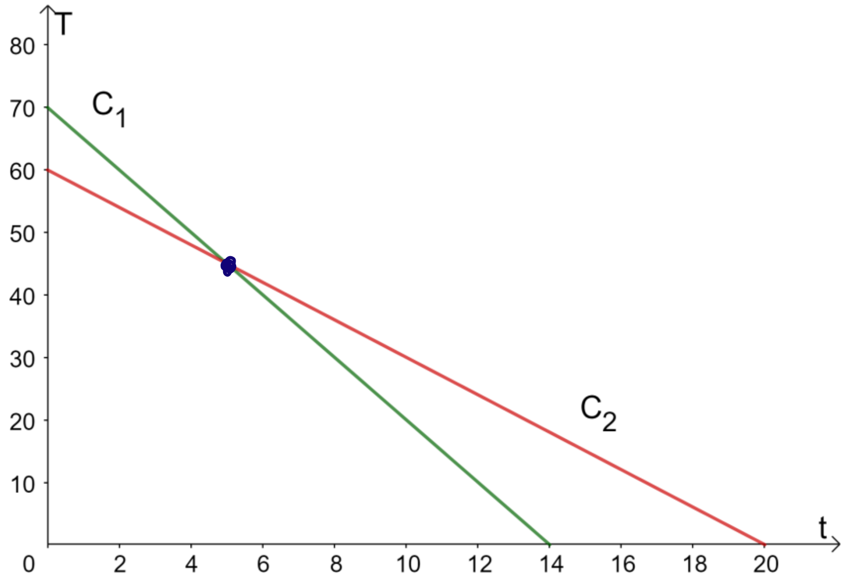
Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NAME above

Please turn over

Question 11 (4 marks)

The diagram shows the temperatures of two containers, C_1 and C_2, after t hours of the initial measurement.



(a) State the linear model for the temperature of each container. **2**

$m_1 = \frac{-70}{14} = -5$, $b = 70$ $C_1 = -5t + 70$ ①

$m_2 = -\frac{60}{20} = -3$, $b = 60$ $C_2 = -3t + 60$ ①

(b) For how many hours will C_2 be cooler than C_1? **2**

$C_2 > C_1$

$-3t + 60 = -5t + 70$ ①

$2t = 10$

$t = 5$

$\therefore$ 5 hours ①

Question 12 (2 marks)

Solve $|3x-5| = \frac{1}{2}$. **2**

+ve

$3x - 5 = \frac{1}{2}$

$x = \frac{11}{6}$ ①

-ve

$-(3x-5) = \frac{1}{2}$

$x = \frac{3}{2}$ ①

Question 13 (3 marks)

(a) Differentiate e^{x^2}. **1**

$\frac{d}{dx}\left(e^{x^2}\right) = 2x\,e^{x^2}$ ①

(b) Hence, evaluate $\int_0^2 x e^{x^2}\,dx$. **2**

$= \frac{1}{2}\int_0^2 2x\,e^{x^2}\,dx$

$= \frac{1}{2}\left[e^{x^2}\right]_0^2$ ①

$= \frac{1}{2}\left[e^{2^2} - e^{0^2}\right]$

$= \frac{e^4 - 1}{2}$ ① or $\frac{1}{2}(e^4 - 1)$

Question 14 (4 marks)

For a sample of twelve students studying a certain course, the number of absent days and their final exam scores are recorded.

Absent Days	*Exam Score*
0	92
0	76
1	87
2	90
2	83
3	83
4	77
4	68
4	69
5	72
6	66
7	58

(a) Determine the equation of the least-squares regression line, $y = mx + b$, for this data. State the coefficients correct to three decimal places. **1**

$y = -3.889x + 89.066$ (1)

(b) State the gradient of the regression line and interpret the value of this gradient in the given context. **2**

gradient $= -3.889$ (1)

(1)

More absenteeism will result in less exam score.

(c) Another student studying the same course was absent for 10 days. **1**

Calculate the predicted exam score for this student using your answer to part (a). Give your answer correct to the nearest whole number.

$-3.889 \times 10 + 89.066 = 50$ (1)

Question 15 (2 marks)

The curve $y = f(x)$ passes through the point $(0,1)$ such that $f'(x) = \dfrac{8x}{2x^2+1}$. **2**

Find its equation.

$y = \int \frac{8x}{2x^2+1}\,dx$

$f(x) = 2x^2+1 \qquad f'(x) = 4x$

$y = 2\cdot\int \frac{4x}{2x^2+1}\,dx$

$y = 2\ln(2x^2+1) + C$ ①

pt $(\overset{x}{0}, \overset{y}{1})$

$1 = 2\ln(2\cdot 0^2+1) + C$

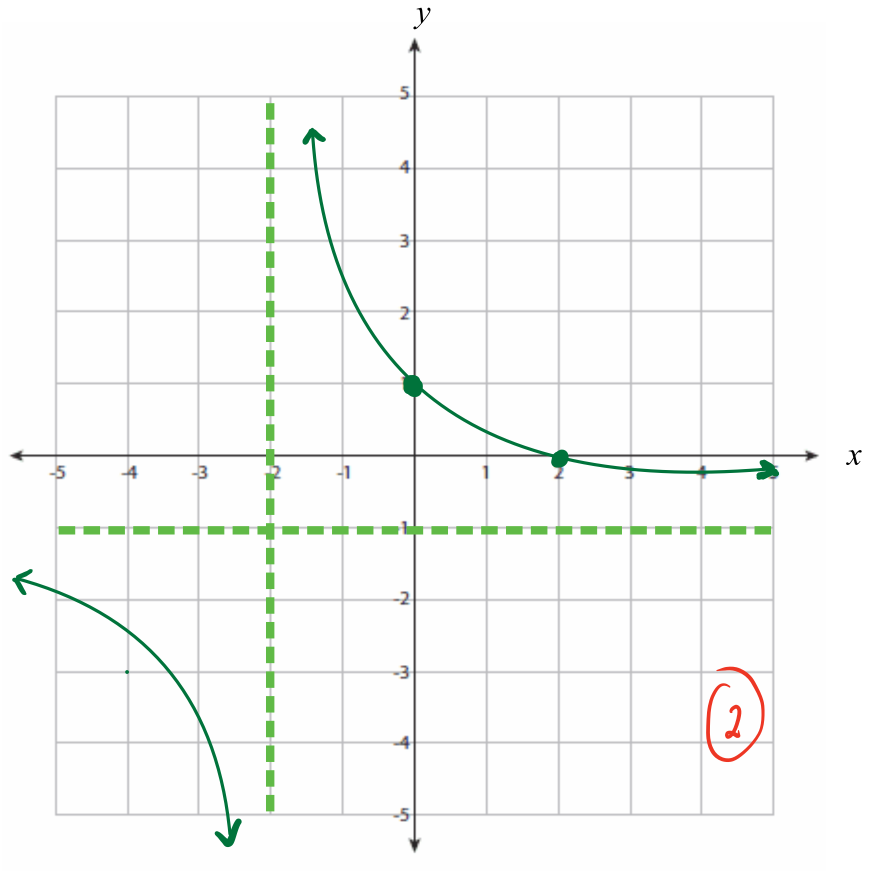
$1 = 2\ln 1 + C$

$C = 1$

$\therefore\ y = 2\ln(2x^2+1) + 1$ ①

Question 16 (3 marks)

Sketch the graph of $y = \dfrac{4}{x+2} - 1$, labelling the asymptotes and axes intercepts. **3**



②

Asymptotes: $x = -2$

$y = -1$ ①

y-int.

$y = \frac{4}{0+2} - 1$

$y = 1$

$(0,1)$

x-int

$0 = \frac{4}{x+2} - 1$

$1 = \frac{4}{x+2}$

$x + 2 = 4$

$x = 2$ $(2,0)$

Question 17 (3 marks)

3

Let $0 < x < \frac{\pi}{2}$.

Explain why the limiting sum $1 + \sin^2 x + \sin^4 x + \sin^6 x + \ldots$ exists.

Hence, find the limiting sum in terms of *x*, leave your answer in the simpliest form.

$r = \sin^2 x$

$0 < x < \frac{\pi}{2}$

$\therefore \quad 0 < \sin^2 x < 1$

$0 < r < 1$ ①

$\therefore$ limiting sum exists.

$S_\infty = \frac{a}{1-r}$ ①

$= \frac{1}{1-\sin^2 x}$

$= \frac{1}{\cos^2 x}$ or

$= \sec^2 x$ ①

Question 18 (3 marks)

A table for the present value for an annuity of $1 is shown.

The present value of an annuity of $1

Period	1%	2%	4%	6%	8%	10%
1	0.9901	0.9804	0.9615	0.9434	0.9259	0.9091
2	1.9704	1.9416	1.8861	1.8334	1.7833	1.7355
3	2.941	2.8839	2.7751	2.673	2.5771	2.4869
4	3.902	3.8077	3.6299	3.4651	3.3121	3.1699
5	4.8534	4.7135	4.4518	4.2124	3.9927	3.7908
6	5.7955	5.6014	5.2421	4.9173	4.6229	4.3553

(a) What would be the present value of a $5000 per year annuity at 6% per annum for six years, with interest compounding annually? Give your answer correct to the nearest dollar. **1**

$r = 0.06$ $A = 5000 \times 4.9173$

$n = 6$ $= \$24587$ ①

(b) An annuity of $2000 per quarter is invested at 8% per annum, compounded quarterly for one year. **1**

What is the present value, correct to the nearest dollar, of the annuity?

$r = \frac{8}{4}\% = 2\%$ $A = 2000 \times 3.8077$

$n = 4$ $A = \$7615$ ①

(c) An annuity has a present value of $46 000. **1**

What is the amount of each yearly payment, compounded at 8% per annum, that is required to accumulate this annuity after five years? Give your answer correct to the nearest dollar.

$r = 8\%$ $x \times 3.9927 = 46000$

$n = 5$

$x = \$11521$ ①

Question 19 (3 marks)

Describe the three transformations which, when applied in succession, change the graph of $y = 2\sin x$ to the graph with equation $y = \sin\left(\frac{x+\pi}{3}\right)$. **3**

$2\sin x \dashrightarrow \sin x \dashrightarrow \sin\left(\frac{x}{3}\right) \dashrightarrow \sin\left(\frac{1}{3}(x+\pi)\right)$

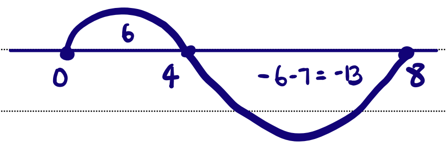
① Vertical dilation by factor $\frac{1}{2}$ ①

② horizontal dilation by factor of 3 ①

③ translation π units left. ①

Question 20 (2 marks)

If $\int_0^8 f(x)\,dx = -7$ and $\int_0^4 f(x) = 6$, find the value of $\int_8^4 f(x)\,dx$. **2**



$$\int_0^8 f(x)\,dx = \int_0^4 f(x)\,dx + \int_4^8 f(x)\,dx$$

$$-7 = 6 + x \quad ①$$

$$-13 = x$$

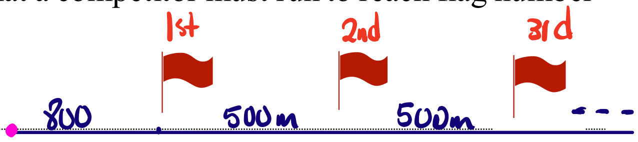
$$\therefore \int_8^4 f(x)\,dx = -\int_4^8 f(x)\,dx$$

$$= -(-13) = 13 \quad ①$$

Question 21 (4 marks)

In a marathon, numbered flags are placed at regular intervals along the course. Flag number 1 is 800 metres from the start and all the rest are 500 metres apart.

(a) Show that the formula for the distance, F_n, that a competitor must run to reach flag number n is $F_n = 500n + 300$. **1**



$F_n = F_1 + (n-1)d$

$= 800 + (n-1) \times 500$

$= 800 + 500n - 500$

$F_n = 500n + 300$ ①

(b) How many flags are needed if the length of the course is 10 000 m? **1**

$F_n = 10\,000$

$10000 = 500n + 300$

$n = 19.4$

$\therefore$ 19 flags ①

(c) A runner retires from the race after completing 6000 m. **2**

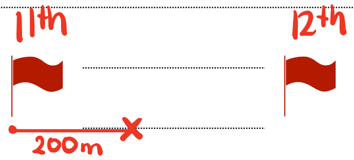
Find the nearest flag number and the runner's distance from that flag when he retired.

$F_n = 6000$

$6000 = 500n + 300$ ①

$n = 11.4$

$0.4 \times 500 = 200$



$\therefore$ 200 m from the 11th flag ①

Question 22 (6 marks)

On any given day, the number X of times Aleeza waits at a red light while driving to work is a random variable with probability distribution given by

x	0	1	2	3
$\mathrm{P}(X = x)$	a^2	$\frac{3a^2}{2}$	$\frac{7-32a}{6}$	$4a$

(a) Show that the only answer for a is $\frac{1}{5}$. **3**

Sum = 1

$6\times 1 = a^{2\,\times 6} + \frac{3a^2}{2}^{\times 6} + \frac{7-32a}{6}^{\times 6} + 4a^{\times 6}$ ①

$6 = 6a^2 + 9a^2 + 7 - 32a + 24a$

$0 = 15a^2 - 8a + 1$

$0 = (3a-1)(5a-1)$

$\therefore \quad a = \frac{1}{3} \qquad a = \frac{1}{5}$ ①

$a \neq \frac{1}{3}$ as

$P(X=3) = 4\times\frac{1}{3}$

$= \frac{4}{3} > 1$ ①

not possible

$\therefore \quad a = \frac{1}{5}$ only.

(b) Let μ be the expected value of X. **3**

Find $\mathrm{P}(X < \mu)$.

$\mu = 0\times\left(\frac{1}{5}\right)^2 + 1\times 3\cdot\frac{\left(\frac{1}{5}\right)^2}{2} + 2\times\frac{7-32\left(\frac{1}{5}\right)}{6} + 3\times 4\left(\frac{1}{5}\right)$

$\mu = \frac{131}{50} \approx 2.62$ ①

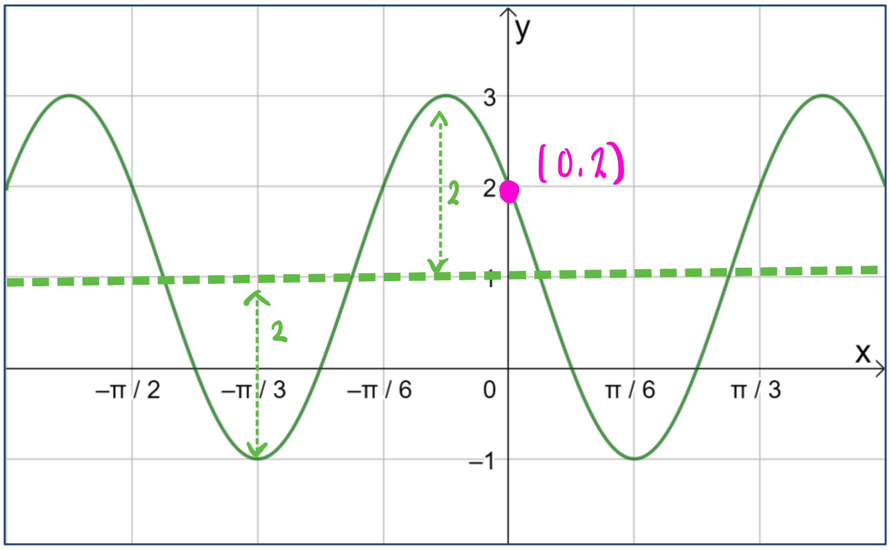
$P\left(X < \underset{(2.6)}{\frac{131}{50}}\right) = 1 - P(X=3)$ ①

$= 1 - \frac{4}{5}$

$= \frac{1}{5}$ ①

Question 23 (5 marks)

The diagram shows the graph of $y = a\cos(bx + c) + d$.



(a) State the range. **1**

$-1 \leq y \leq 3$ or $[-1, 3]$ ①

(b) State the period. **1**

$\frac{\pi}{6} - (-\frac{\pi}{3}) = \frac{\pi}{2}$ ①

(c) Find the values of *a*, *b*, *c* and *d*. **3**

Pt $(0,2)$

① $\{ d = 1$
$a = 2$

$y = 2\cos(4x + c) + 1$

$2 = 2\cos(4\times0 + c) + 1$

① $\frac{2\pi}{b} = \frac{\pi}{2}$
$b = 4$

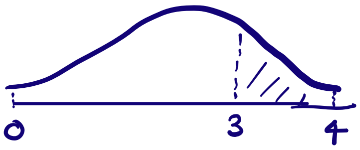
$\frac{1}{2} = \cos(c)$

$c = \frac{\pi}{3}$ ①

Question 24 (4 marks)

The lifespan in weeks, X, of a certain butterfly species is given by a probability density function.

$$f(x)=\begin{cases}\frac{\pi}{8}\sin\left(\frac{\pi x}{4}\right) & \text{for } 0\le x\le 4\\ 0 & \text{for all other values of } x\end{cases}$$

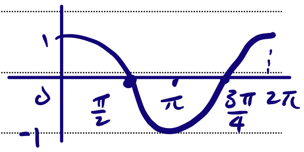


(a) In a sample of 100 butterflies, how many are expected to live longer than three weeks? Give your answer correct to the nearest integer. **2**

$$P(x>3) = \int_3^4 \frac{\pi}{8}\sin\left(\frac{\pi}{4}x\right)dx$$

$$\frac{\pi}{8}\times\frac{4}{\pi}$$

$$= \cancel{\frac{\pi}{8}}_{2}\cdot\frac{\cancel{4}}{\cancel{\pi}}\left[-\cos\left(\frac{\pi}{4}x\right)\right]_3^4 \quad ①$$



$$= \frac{1}{2}\left[\underset{(-1)}{-\cos(\pi)} + \underset{\frac{\sqrt{2}}{2}}{\cos\left(\frac{3\pi}{4}\right)}\right]$$

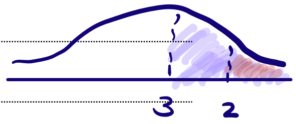
$$= \frac{1}{2}+\frac{\sqrt{2}}{4} \approx 0.146 \quad ①$$

100x 0.146 ≈ 15 butterflies.

(b) It is given that a butterfly has lived for more than two weeks. **2**

What is the probability that the same butterfly lives for more than three weeks? Give your answer to two decimal places.

$$P(x>3\,|\,x>2) = \frac{P(x>3)\cap P(x>2)}{P(x>2)}$$



$$= \frac{P(x>3)}{P(x>2)} \quad ①$$

$$P(x>2) = \frac{\pi}{8}\int_2^4 \sin\left(\frac{\pi}{4}x\right)dx$$

$$= \frac{1}{2}\left[\underset{-1}{-\cos(\pi)} + \underset{0}{\cos\left(\frac{\pi}{2}\right)}\right]$$

$$= \frac{1}{2}$$

$$= \frac{\frac{1}{2}+\frac{\sqrt{2}}{4}}{\frac{1}{2}}$$

$$= \frac{2-\sqrt{2}}{2}$$

$$\doteqdot 0.29 \quad (2\text{ dp}) \quad ①$$

Question 25 (3 marks)

A random variable Z is normally distributed with mean 0 and standard deviation 1. **3**

The table gives the probability of this random variable being greater than different values of z.

z	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
$P(Z > z)$	0.5	0.4013	0.3085	0.2266	0.1587	0.1057	0.668	0.0401	0.0228

In a hospital of 3125 patients, it is found that 708 patients have a body temperature higher than $36.8^\circ C$. The body temperatures are distributed normally with a mean μ and standard deviation $0.3^\circ C$.

Using the given table, find the value of μ, correct to two decimal places.

$$P(T > 36.8) = \frac{708}{3125}$$

$$= 0.2266 \quad ①$$

So $P(Z > 0.75) = 0.2266$

$$\frac{x - \mu}{\sigma} = z$$

$$\frac{36.8 - \mu}{0.3} \overset{①}{=} 0.75$$

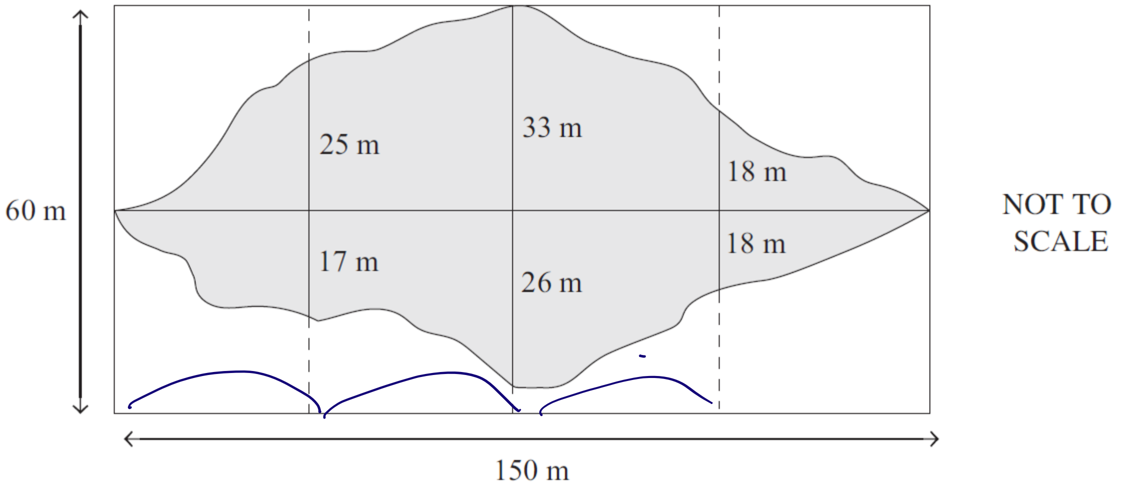
$$36.8 - \mu = 0.225$$

$$\mu = 36.58 \quad ①$$

Question 26 (3 marks)

Archaeologists are excavating a rectangular site with dimensions of 150 metres by 60 metres. **3**

The shaded region indicates the portion of the site that has been excavated.



Use the trapezoidal rule to best approximate the total area that has been excavated. Give your answer 2 correct to the nearest whole number.

h=37.5

x	0	37.5	75	112.5	150
y	0	42	59	36	0
	y_0	y_1	y_2	y_3	y_4
		25+17	33+26	18+18	

①

$$A = \frac{h}{2}\left[y_0+y_4+2\times(y_1+y_2+y_3)\right]$$ ①

$$= \frac{37.5}{2}\left[0+0+2\times(42+59+36)\right] = 5137.5\ m^2$$

①

Question 27 (3 marks)

Simplify $a^{\frac{\log_b c}{1+\log_b\left(\frac{a}{b}\right)}}$. **3**

$$= a^{\frac{\log_b c}{\log_b b+\log_b\left(\frac{a}{b}\right)}}$$

$$= a^{\frac{\log_b c}{\log_b\left(\cancel{b}\times\frac{a}{\cancel{b}}\right)}}$$ ①

$$= a^{\frac{\log_b c}{\log_b a}}$$

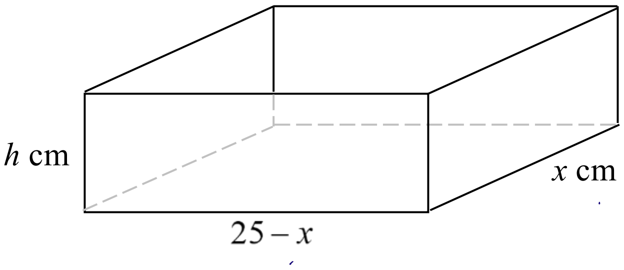
①

$$= \cancel{a}^{\cancel{\log_a c}}$$

$$= c$$ ①

Question 28 (6 marks)

The base of an **open** topped toy box has length $25-x$ cm and width x cm. It has a total external surface area of 300 cm^2.



(a) Show that the height h of the toy box is given by $h=\dfrac{x^2-25x+300}{50}$. **2**

$SA=300$

$SA = 2\times h(25-x) + 2\times hx + x(25-x)$

$300 = 50h - \cancel{2hx} + \cancel{2xh} + 25x - x^2$ ①

$300 = -x^2 + 25x + 50h$

$x^2 - 25x + 300 = 50h$

$\therefore\ h = \dfrac{x^2-25x+300}{50}$ ①

(b) Find the dimensions (length and width) of the base which give the maximum volume. ~~5~~ 4

$V = lbh$

$= x(25-x)\cdot\dfrac{x^2-25x+300}{50}$

$= \frac{1}{50}(25x-x^2)(x^2-25x+300)$

$u = 25x - x^2 \qquad v = x^2 - 25x + 300$

$u' = 25 - 2x \qquad v' = 2x - 25$

$\dfrac{dv}{dx} = \frac{1}{50}\left((2x-25)(25x-x^2) + \overset{-(2x-25)}{(25-2x)}(x^2-25x+300)\right)$ ①

$= \frac{1}{50}(2x-25)\left(25x - x^2 - x^2 + 25x - 300\right)$

$v' = 0 \qquad = \frac{1}{50}(2x-25)(-2x^2+50x-300)$

$0 = (2x-25)(2x^2-50x+300)$

$0 = 2(2x-25)(x-15)(x-10)$

$\therefore\ x = \frac{25}{2},\quad x = 15,\quad x = 10$ ①

min max max

$V = 449.219 \qquad V = 450 \qquad V = 450$

OR test using V' (table) 1

$\therefore$ base : x. $25-x$

15, 10

15 cm by 10 cm. ①

Question 28 (continued)

End of Questions 28

Proceed to Booklet 2 for Questions 29−32

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student NAME: ________________________

2022 | **YEAR 12 TRIAL**
EXAMINATION

Mathematics Advanced

Section II
Answer Booklet 2

Booklet 2 - Attempt Questions 29–32 (26 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NAME above

Please turn over

Question 29 (10 marks)

Consider the function $f(x) = e^{2x} - 4e^{x} - 5$.

(a) Solve $f(x) = 0$. **2**

let $u = e^{x}$

$f(x) = u^2 - 4u - 5$

$= (u-5)(u+1)$

$\therefore u = 5$ or $u = -1$

$e^{x} = 5$ or $e^{x} = -1$ ①

$x = \ln 5$ no sol^{n}.

①

(b) Find and classify the coordinates of the stationary point. **3**

$f(x) = e^{2x} - 4e^{x} - 5$

$f'(x) = 2e^{2x} - 4e^{x}$

$0 = 2e^{x}(e^{x} - 2)$

$e^{x} = 0$ $e^{x} - 2 = 0$

no sol^{n} $e^{x} = 2$ ①

$x = \ln 2$

$y = -9$

$f''(x) = 4e^{2x} - 4e^{x}$

$f''(\ln 2) = 4(e^{\ln 2})^2 - 4e^{\ln 2}$

$= 16 - 8$

$= 8 > 0$ ∪ min ①

$\therefore (\ln 2, -9)$ is a min tp.

①

(c) Find the range of the function. **1**

Range : $[-9, \infty)$

OR $y \geq -9$ ①

Question 29 continues on page 23

Question 29 (continued)

(d) Find the interval for which the graph of the function is concave down. **2**

$f''(x) < 0$

$f''(x) = 4e^{2x} - 4e^{x}$

$4(e^{x})^2 - 4e^{x} < 0$

$4e^{x}(e^{x} - 1) < 0$

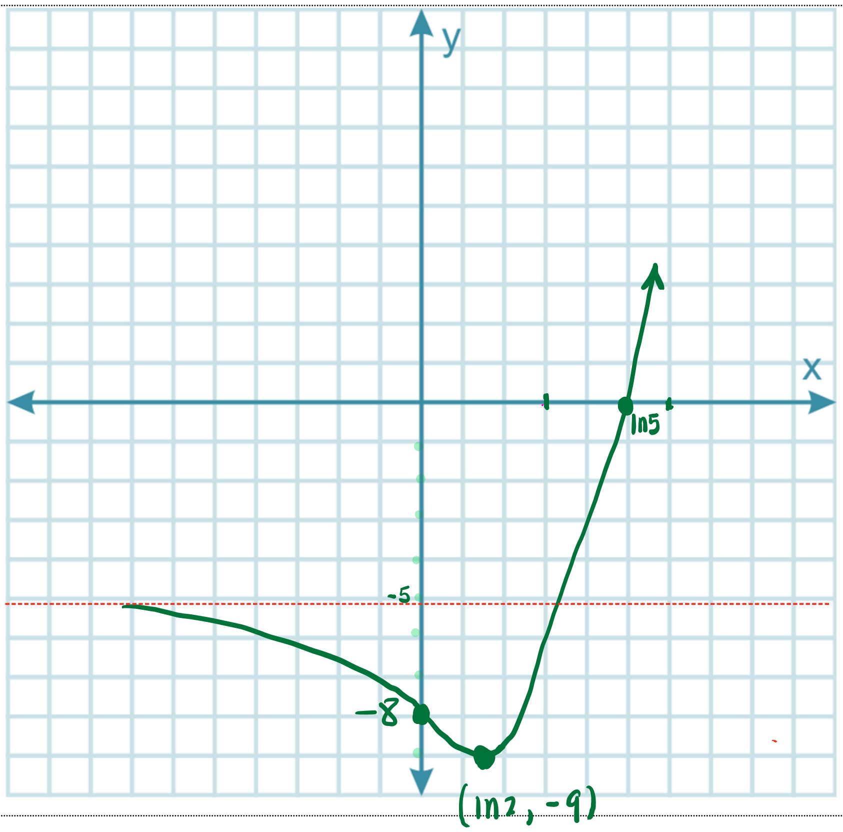
✓

$4e^{x} = 0$ $\quad e^{x} = 1$ $\quad$ ✓ − + 0

no sol$^{\underline{n}}$ $\quad x = \ln 1$ ①

$x = 0$ $\quad \therefore \quad x < 0$ concave down. ①

(e) Sketch the graph $y = e^{2x} - 4e^{x} - 5$, labelling all important features **2**



inf pt (0, -8)

tp V (ln2, -9) (b)

0.69

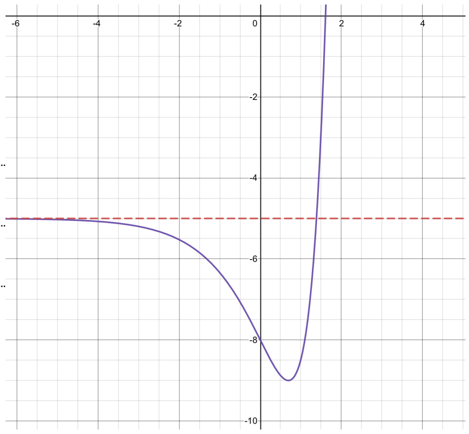
x-int (ln5, 0) (a)

1.6

y-int (0, -8)

as $x \to -\infty$

$y \to -5$



① - Correct shape & tp

① - Correct asym.

End of Question 29

Question 30 (5 marks)

On his date of retirement Nabil has \$650 000 in his superannuation fund. **5**

His fund is earning interest at a rate of 0.5% per month. Each month, immediately after the interest has been paid, Nabil withdraws \$5000.

$r = 1 + 0.5\% = 1.005$

How many years, correct to one decimal place, will the fund last?

$A_1 = 650000r - 5000$

$A_2 = A_1 r - 5000$

$= 650000r^2 - 5000r - 5000$

$A_3 = A_2 r - 5000$

$= 650000r^3 - 5000r^2 - 5000r - 5000$

$= 650000r^3 - 5000(r^2 + r + 1)$

$\vdots$

$A_n = 650000r^n - 5000(1 + r + \dots + r^{n-1})$ ①

$A_n = 0$

$0 = 650000r^n - 5000\left(\frac{a\cdot(r^n - 1)}{r-1}\right)$ ①

$\times 0.005$ $\times 0.005$

$0 = 650000(1.005)^n - 5000\left(\frac{1.005^n - 1}{0.\cancel{00}5}\right)$ $\times 0.\cancel{00}5$

$0 = 3250(1.005)^n - 5000(1.005^n) + 5000$

$1750(1.005)^n = 5000$

$1.005^n = \frac{20}{7}$ ①

$n = \ln\left(\frac{20}{7}\right) \div \ln(1.005)$

$n = 210.4888996$ ①

$\therefore n \doteqdot 210$ months

$n = 17.5$ years. ①

Question 31 (6 marks)

The concentration of a chemical in a container changes according to the formula $C(t) = te^{-\frac{t}{10}}$, where t is time in minutes.

(a) After how many minutes will the concentration reach its maximum? **2**

$C = te^{-\frac{1}{10}t}$

$u = t \quad v = e^{-\frac{1}{10}t}$

$u' = 1 \quad v' = -\frac{1}{10}e^{-\frac{1}{10}t}$

$\frac{dC}{dt} = -\frac{1}{10}t \cdot e^{-\frac{1}{10}t} + e^{-\frac{1}{10}t}$

$= e^{-\frac{1}{10}t}\left(-\frac{1}{10}t + 1\right)$ ①

$\frac{dC}{dt} = 0$

$0 = e^{-\frac{t}{10}}\left(-\frac{t}{10} + 1\right)$

$e^{-\frac{t}{10}} = 0$ or $-\frac{t}{10} + 1 = 0$

no sol$^{\underline{n}}$ $\quad t = 10$ ①

t	9	10	11
C'	+	0	−

∴ max at t=10 min.

(b) Find the time when the concentration of the chemical decreases at the fastest rate. **3**

$C' = e^{-\frac{1}{10}t}\left(-\frac{1}{10}t + 1\right)$

$u = e^{-\frac{1}{10}t} \quad v = -\frac{1}{10}t + 1$

$u' = -\frac{1}{10}e^{-\frac{1}{10}t} \quad v' = -\frac{1}{10}$

$C'' = -\frac{1}{10}e^{-\frac{1}{10}t} + \frac{1}{10}e^{-\frac{1}{10}t}\left(\frac{1}{10}t - 1\right)$ ①

$= \frac{1}{10}e^{-\frac{t}{10}}\left(-\cancel{1} + \frac{1}{10}t - \cancel{1}\right)$

$= \frac{1}{10}e^{-\frac{t}{10}}\left(\frac{1}{10}t + 2\right)$

$C'' = 0$

$0 = \frac{1}{10}e^{-\frac{t}{10}}\left(\frac{1}{10}t + 2\right)$

$\frac{1}{10}e^{-\frac{t}{10}} = 0$ no sol$^{\underline{n}}$ $\quad \frac{1}{10}t + 2$ $\quad t = 20$ ①

hence : t=20 is a point of inf.

t	19	20	21
C''	−	0	+

∴ decrease at fastest rate at t= 20 min ①

(c) Find the maximum rate of increase. **1**

t=0

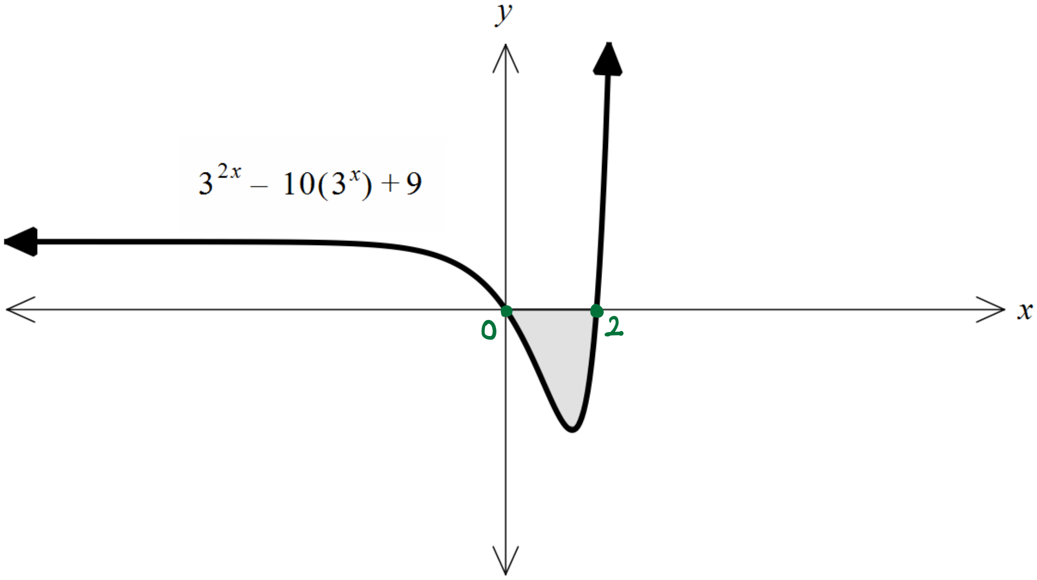
C''(t)=0 only have one sol$^{\underline{n}}$ at t=20

However at t=20 rate of decarease is at maximum (fastest)

∴ max rate of increase can only happen initially. ①

Question 32 (5 marks)

The graph of $f(x)=3^{2x}-10\left(3^{x}\right)+9$ is shown below. Find the exact area of the shaded region. **5**



let $u=3^x$

x-Int

$0=u^2-10u+9$

$0=(u-9)(u-1)$

$u=9$ or $u=1$

$3^x=9$ $\quad 3^x=1$

$x=2$ $\quad x=0$ ①

Area is under the x-axis.

$\therefore A=\int_2^0 3^{2x}-10(3^x)+9\,dx$ ①

$=\left[\dfrac{3^{2x}}{2\ln3}-\dfrac{10(3^x)}{\ln3}+9x\right]_2^0$

$=\left(\dfrac{3^0}{2\ln3}-\dfrac{10(3^0)}{\ln3}+0\right)-\left(\dfrac{3^4}{2\ln3}-\dfrac{10(3^2)}{\ln3}+9\times2\right)$ ①

$A=\dfrac{1}{2\ln3}-\dfrac{10}{\ln3}-\dfrac{81}{2\ln3}+\dfrac{90}{\ln3}-18$

$A=\dfrac{1-20-81+180-36\ln3}{2\ln3}$ ①

$A=\dfrac{40-18\ln3}{\ln3}$ ①

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

STUDENT ID______________________________

2022 Mathematics Advanced Yr 12

Section I Answer Sheet

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response circle completely.

1	A ○	B ○	C ○	D ○
2	A ○	B ○	C ○	D ○
3	A ○	B ○	C ○	D ○
4	A ○	B ○	C ○	D ○
5	A ○	B ○	C ○	D ○
6	A ○	B ○	C ○	D ○
7	A ○	B ○	C ○	D ○
8	A ○	B ○	C ○	D ○
9	A ○	B ○	C ○	D ○
10	A ○	B ○	C ○	D ○

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r-1} = \frac{a(1-r^n)}{1-r}, r \neq 1$$

$$S = \frac{a}{1-r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

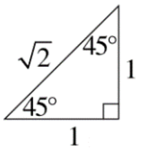
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

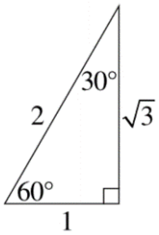
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$





Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \dfrac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

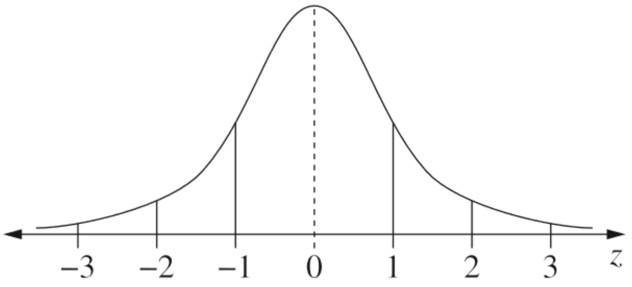
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have *z*-scores between -1 and 1
- approximately 95% of scores have *z*-scores between -2 and 2
- approximately 99.7% of scores have *z*-scores between -3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X-\mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \; P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$\boldsymbol{X} \sim Bin(n, p)$$

$$\Rightarrow \boldsymbol{P(X = x)}$$

$$= \boldsymbol{\binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, \ldots, n}$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function	**Derivative**
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u),\ u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	
$\dfrac{dy}{dx} = \left(\ln a\right) f'(x)a^{f(x)}$	
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\left(\ln a\right) f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \ldots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r}=\frac{n!}{(n-r)!}$$

$$\binom{n}{r}={}^{n}C_{r}=\frac{n!}{r!(n-r)!}$$

$$(x+a)^{n}=x^{n}+\binom{n}{1}x^{n-1}a+\ldots+\binom{n}{r}x^{n-r}a^{r}+\ldots+a^{n}$$

Vectors

$$|\underset{\sim}{u}|=\left|x\underset{\sim}{i}+y\underset{\sim}{j}\right|=\sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v}=|\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta=x_{1}x_{2}+y_{1}y_{2},$$

where $\underset{\sim}{u}=x_{1}\underset{\sim}{i}+y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v}=x_{2}\underset{\sim}{i}+y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r}=\underset{\sim}{a}+\lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned}z=a+ib&=r(\cos\theta+i\sin\theta)\\&=re^{i\theta}\end{aligned}$$

$$\begin{aligned}\left[r(\cos\theta+i\sin\theta)\right]^{n}&=r^{n}(\cos n\theta+i\sin n\theta)\\&=r^{n}e^{in\theta}\end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x=a\cos(nt+\alpha)+c$$

$$x=a\sin(nt+\alpha)+c$$

$$\ddot{x}=-n^{2}(x-c)$$