HSC
TRIAL EXAMINATION
2023

Mathematics Advanced

Name:____________________________

Maths Class:____________________________

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11–31 show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – **10 marks** (pages 1–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – **90 marks** (pages 7–30)

- Attempt Questions 11–31
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Which inequality gives the range of $y = x^2 - 6x$?

A. $y \geq 9$

B. $y \geq -9$

C. $y \leq 9$

D. $y \leq -9$

2 Given $x + \frac{1}{x} = 4$, what is $x^2 + \frac{1}{x^2}$?

A. 14

B. 16

C. 18

D. 20

3 Which of the following graphs have **<u>no</u>** asymptotes?

A. $y = ln(x - 2)$

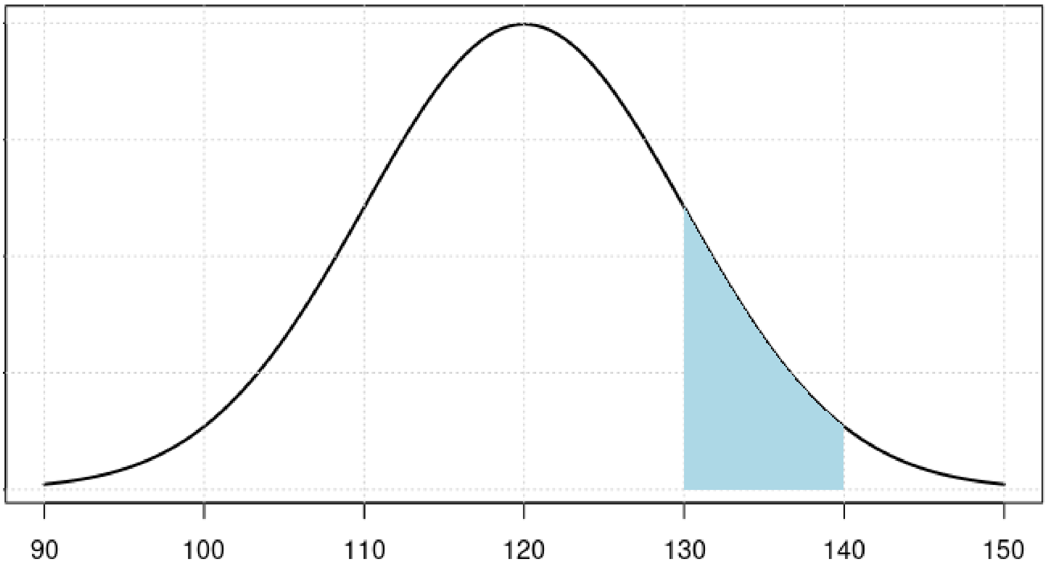
B. $y = e^{2x}$

C. $y = \sqrt{4 - x^2}$

D. $y = \dfrac{3}{x - 4}$

4 The number of cars in a car park is normally distributed with a mean of 120 and standard deviation of 10.

The shaded region represents the probability of the number of cars lying in a certain range at some time of the day.



A random variable Z is normally distributed with mean 0 and standard deviation 1.

Which of the following is equivalent to the shaded area?

A. $P(0 < Z < 1)$

B. $P(0 < Z < 2)$

C. $P(-1 < Z < 0)$

D. $P(-2 < Z < -1)$

5 What is the y-intercept of the tangent line to the curve $y = x^3 - 5x$ at $x = 3$?

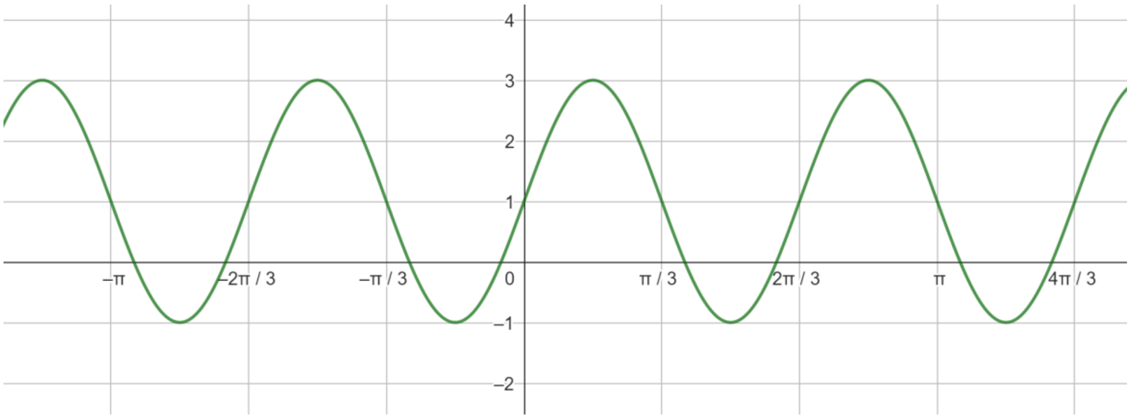
A. -54

B. -50

C. 38

D. 42

6 In the diagram, a graph of a trigonometric function is given.



Which of the following could be the equation of the given graph?

A. $y = 2\sin\left(3x - \dfrac{\pi}{2}\right) + 1$

B. $y = 2\sin(2x - \pi) + 1$

C. $y = 2\cos\left(3x - \dfrac{\pi}{2}\right) + 1$

D. $y = 2\cos\left(2x + \dfrac{\pi}{2}\right) + 1$

7 What is $\int \frac{4}{(2x-1)^2}\, dx$?

A. $2\ln(2x - 1) + c$

B. $\dfrac{-2}{2x - 1} + c$

C. $\dfrac{-2}{(2x - 1)^3} + c$

D. $\dfrac{8}{2x - 1} + c$

8 What is $\frac{d}{dx}(x^n e^{2x})$?

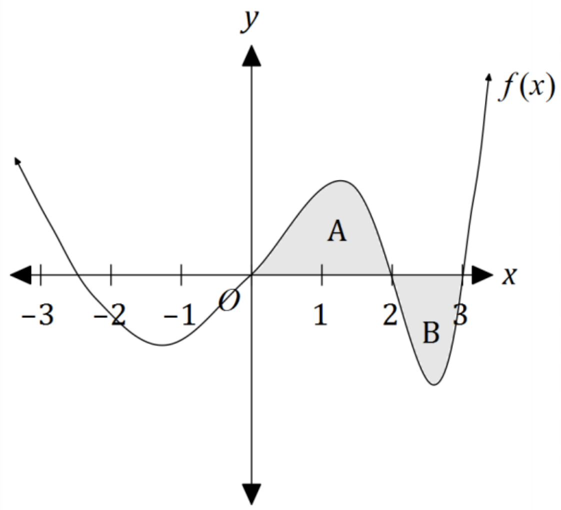
A. $x^{(n-1)}(2x+n+1)e^{2x}$

B. $x^n(2x+n-1)e^{2x}$

C. $x^n(2x+n)e^{2x}$

D. $x^{(n-1)}(2x+n)e^{2x}$

9 The graph of a function $f(x)$ is shown below, where the shaded area labelled A is $\frac{3}{2}$ of the shaded area labelled B.



If $\int_0^3 f(x)dx = 2$, what is the area of A?

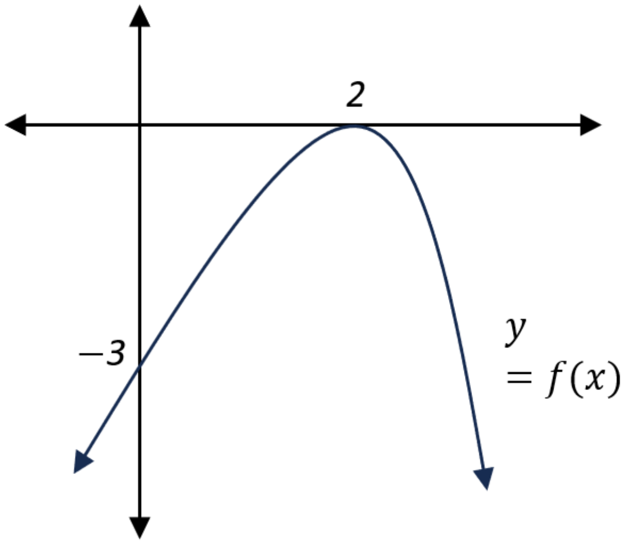
A. 6

B. 4

C. $\frac{6}{5}$

D. $\frac{4}{5}$

10 A graph of the function $f(x)$ is shown below. It has a single stationary point at $(2,0)$ and is defined for all real x:



Define the function $g(x) = f\left(f(x)\right)$. Which of the following is false?

A. $g(2) = -3$

B. $g(x)$ is negative for the entirety of its domain

C. $g'(2) = 0$

D. $g'(x)$ is negative for $x < 2$

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Student Name: ____________________

2023	**HSC TRIAL EXAMINATION**

Mathematics Advanced

Section II

Answer Booklet - I

30 marks
Attempt Questions 11–19
Allow about 45 minutes for this booklet

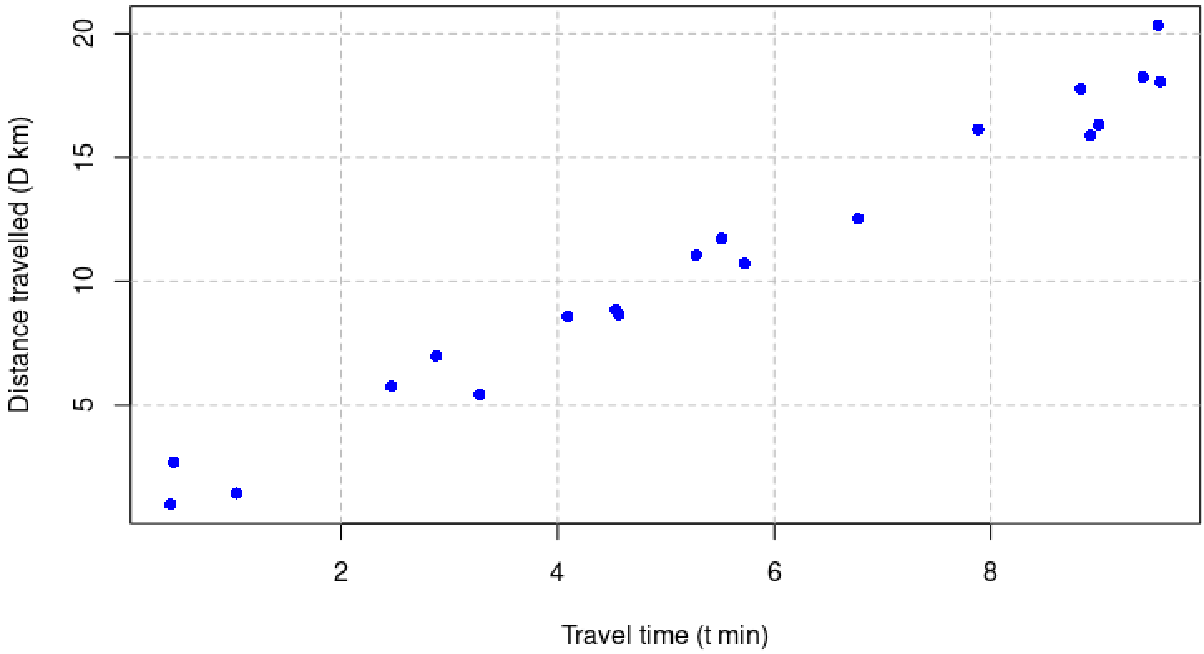
Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your name above

Please turn over

Question 11 (4 marks)

The relationship between the time (t minutes) taken for a taxi to travel a short distance (D kilometres) in a city centre is displayed in the following graph for 20 instances.



The equation of the least square regression line is $D = 1.87t + 0.62$.

(a) State the gradient and interpret its value in the given context. **2**

(b) On the graph above, sketch the line of best fit. **1**

(c) Use the equation of the least square regression line to find the expected travel time for a distance of 10 km? Give your answer in minutes, correct to two decimal places. **1**

Question 12 (2 marks)

A function $f(x)$ has derivative $f'(x) = 4\sin(2x)$ and $f\left(\dfrac{\pi}{6}\right) = 0.$ **2**

Find $f(x)$.

Question 13 (4 marks)

(a) Differentiate $\dfrac{x^2}{x^2+1}$. **2**

(b) Hence, evaluate $\displaystyle\int_1^2 \frac{x}{(x^2+1)^2}\,dx$. **2**

Question 14 (3 marks)

Describe three transformations which, when applied in succession, change the graph of $y = x^2 + 5$ to the graph with equation $y = \left(\dfrac{x+1}{2}\right)^2$. **3**

Question 15 (4 marks)

Two bags contain a number of red and green marbles:

- bag A contains 3 red marbles and 4 green marbles
- bag B contains 5 red marbles and 2 green marbles.

A six-sided die is rolled. If the face of the die shows a number that is 1 or 2, then a marble from bag A will be selected at random. Otherwise, a marble from bag B will be selected.

(a) What is the probability that a green marble will be selected? **2**

(b) It is known that a green marble is selected. **2**

What is the probability that the marble comes from bag B?

Question 16 (2 marks)

Use two applications of the trapezoidal rule to approximate $\int_1^5 \frac{12}{x^2+1}\,dx$. **2**

Give your answer correct to two decimal places.

Question 17 (4 marks)

Sketch the graph of the curve $y = -x^3 - 6x^2 - 9x$, labelling the stationary points and point of inflection. **4**

Question 18 (4 marks)

The table below gives the future value of an annuity of $1 per period for various periods and interest rates.

Table of Future Value Interest Factors								
	Interest rate per period							
Number of Periods	0.25%	0.30%	0.35%	0.40%	0.45%	0.50%	0.55%	0.60%
53	56.5961	57.3530	58.1230	58.9063	59.7033	60.5141	61.3391	62.1785
54	57.7376	58.5250	59.3264	60.1419	60.9719	61.8167	62.6765	63.5516
55	58.8819	59.7006	60.5340	61.3825	62.2463	63.1258	64.0212	64.9329
56	60.0291	60.8797	61.7459	62.6280	63.5264	64.4414	65.3733	66.3225
57	61.1792	62.0624	62.9620	63.8786	64.8123	65.7636	66.7329	67.7204
58	62.3322	63.2485	64.1824	65.1341	66.1040	67.0924	68.0999	69.1267
59	63.4880	64.4383	65.4070	66.3946	67.4014	68.4279	69.4744	70.5415
60	64.6467	65.6316	66.6359	67.6602	68.7047	69.7700	70.8565	71.9647
61	65.8083	66.8285	67.8692	68.9308	70.0139	71.1189	72.2463	73.3965
62	66.9729	68.0290	69.1067	70.2065	71.3290	72.4745	73.6436	74.8369
63	68.1403	69.2331	70.3486	71.4874	72.6499	73.8368	75.0487	76.2859
64	69.3106	70.4408	71.5948	72.7733	73.9769	75.2060	76.4614	77.7436
65	70.4839	71.6521	72.8454	74.0644	75.3098	76.5821	77.8820	79.2101
66	71.6601	72.8670	74.1004	75.3607	76.6487	77.9650	79.3103	80.6854

(a) Berra invests $250 per month in an annuity which pays 5.4% p.a. compounding monthly. What will be the value of the annuity after 5 years? **2**

(b) Yousif finds that he can only get an interest rate of 3.6% p.a. compounding mothly. **2**

If he wants to achieve the same total amount as Berra after the same period, what amount should he invest each month?

Question 19 (3 marks)

Solve $\log_2 x + \log_2 (x-3) = 2$. **3**

End of Section 1I – Booklet 1

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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2023 | **HSC TRIAL EXAMINATION**

Mathematics Advanced

Section II

Answer Booklet - II

30 marks
Attempt Questions 20–25
Allow about 60 minutes for this booklet

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
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Please turn over

Question 20 (5 marks)

During the mushroom season, every day, Asna would go into the forest to collect mushrooms.

On the first day she collects 34 mushrooms and each consecutive day she collects 6 more mushrooms than she did on the previous day. That is, 40 mushrooms on the second day and 46 mushrooms on the third day.

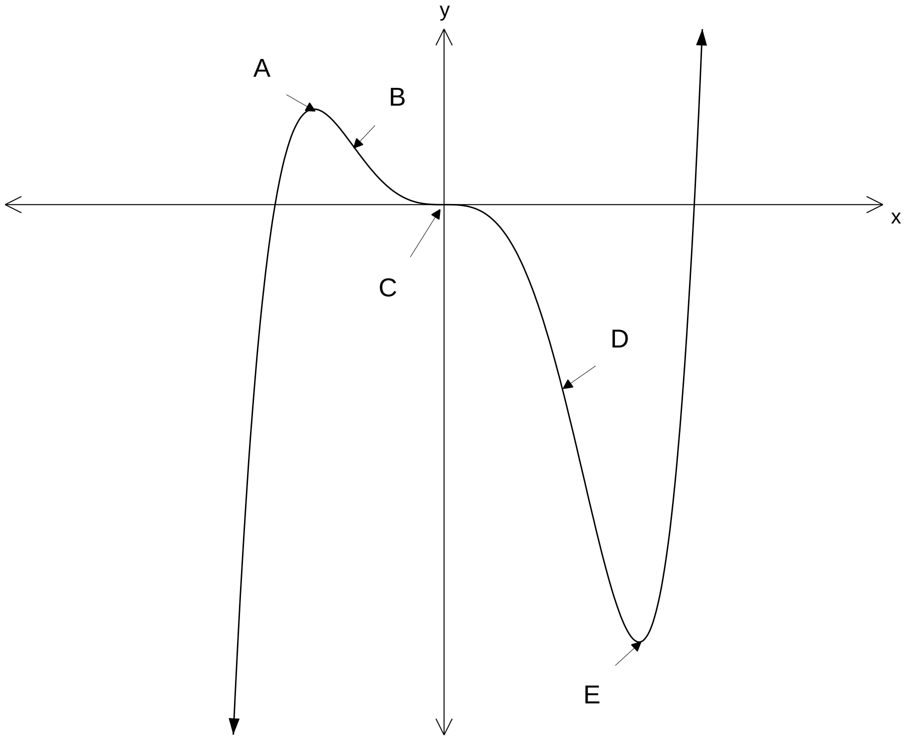
(a) On which day will Asna collect more than 120 mushrooms for the first time? **2**

(b) During the entire mushroom season, Asna collected 3220 mushrooms. **3**

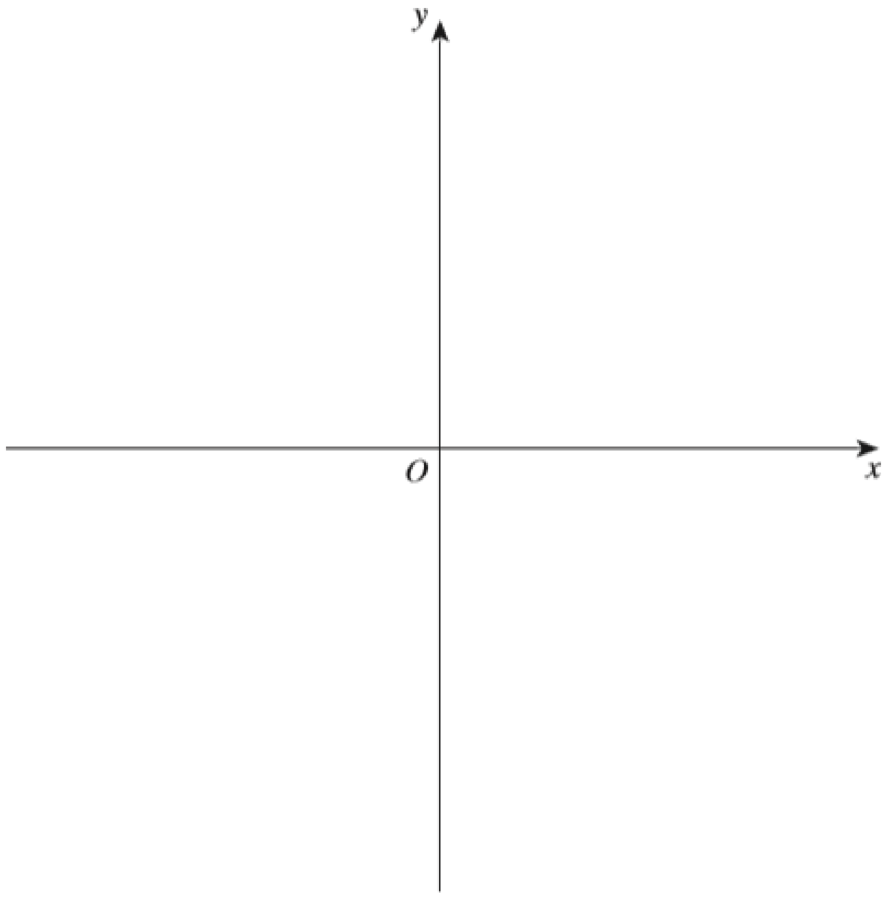
For how many days did she collect mushrooms?

Question 21 (2 marks)

The graph of the curve $y = f(x)$ is drawn below: **2**



On the axes below, sketch the graph of $y = f'(x)$, Clearly showing the behaviour of the points: A,B,C,D and E.



Question 22 (7 marks)

The temperature, T degrees, of a liquid is given according to the model $T(t) = 20 + Ae^{kt}$, where t is time in hours. The initial temperature of the liquid is 35 degrees.

(a) Show that $A = 15$. **1**

(b) After four hours, the temperature of the liquid is 80 degrees. **3**

Show that $k = ln(\sqrt{2})$.

Question 22 continues on page 20

Question 22 (continued)

(c) Show that $T(t) = 15 \cdot 2^{\frac{t}{2}} + 20$ and hence sketch its graph. **3**

End of Question 22

Question 23 (9 marks)

The water at a certain beach rises and falls in a periodic pattern. The height of the water at any time can be modelled by the equation $h(t) = 8 + 5\,sin\left(\frac{\pi t}{12}\right)$, where t is time in hours after midnight.

(a) Sketch the graph of $y = h(t)$ for $0 \leq t \leq 24$. **2**

(b) At what rate does the height of water change at 8:00 am? Leave your answer in exact form. **2**

Question 23 continues on page 22

Question 23 (continued)

(c) A boat with a depth of 5.5 metres, is scheduled to arrive at the beach at 10:00 am one day. **2**

Explain if the boat will be able to dock.

(d) On each day, between what times will the boat in part (c) be able to dock? **3**

End of Question 23

Question 24 (3 marks)

Find the value of k such that $\int_0^k e^x(e^x - 2)\,dx = \frac{3}{2}$. **3**

Question 25 (4 marks)

A continuous random variable *X* has probability density function given by:

$$f(x)=\begin{cases}\dfrac{x+2}{k} & 1\le x\le 3\\ 0 & \text{elsewhere}\end{cases}$$

(a) Show that $k=8$. **1**

(b) Find $P(X<2\,|\,X>1.5)$. **3**

End of Section 1I – Booklet II

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student Name: ______________________________

2023 | **HSC TRIAL EXAMINATION**

Mathematics Advanced

Section II

Answer Booklet - III

30 marks
Attempt Questions 26–31
Allow about 60 minutes for this booklet

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your name above

Please turn over

Question 26 (5 marks)

A random variable is normally distributed with mean 0 and standard deviation 1.

The table gives the probability that this random variable is less than z for different values of z.

z	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
$P(Z < z)$	0.0228	0.0668	0.1587	0.3085	0.5000	0.6915	0.8413	0.9332	0.9772

(a) Using the table, find the probability that a value from a random variable that is normally distributed with mean 0 and standard deviation 1, lies between -0.5 and 1. **1**

(b) The blood pressure readings of patients with a certain condition are normally distributed, with a mean of μ and a standard deviation of σ. It is known that 2.28% of these patients have a blood reading of less than 105 and 6.68% of these patients have a blood reading of more than 133. **4**

Using the table, find μ and σ.

Question 27 (4 marks)

Find the global maximum and minimum values of $f(x) = sin^2 x + 4\, cos\, x$, where $\frac{\pi}{3} \leq x \leq \frac{3\pi}{2}$. **4**

Question 28 (5 marks)

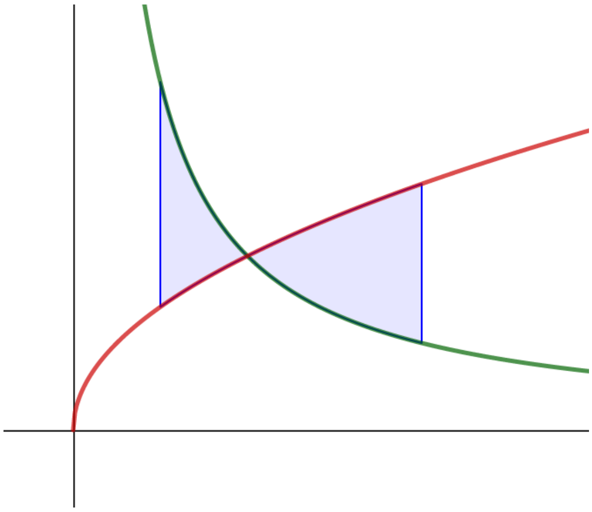
A container in the shape of a right circular cylinder with no top, has outside surface area 12π square metres.

(a) Show that the height (h) is $\dfrac{12-r^2}{2r}$, where r is the radius. **1**

(b) Find and verify the height (h) and radius (r) that give the maximum volume. **4**

Question 29 (4 marks)

In the diagram, the region between the graphs of $y = \frac{4}{x}$ and $y = \sqrt{2x}$ for $1 \leq x \leq 4$ is shaded. **4**



Find the exact area of the shaded region.

Question 30 (8 marks)

Eman invests \$9000 in an account earning interest at a rate of 0.6% per month. Each month, immediately after the interest has been paid, Eman contributes \$200.

The amount in the account after the *n*th contribution can be determined using the recurrence relation $A_n = 1.006A_{n-1} + 200$, where $A_0 = 9000$.

(a) Find the amount in the account after the first contribution. **1**

(b) Find the total value of the investment after the 12^{th} contribution. **3**

Question 30 continues on page 32

Question 30 (continued)

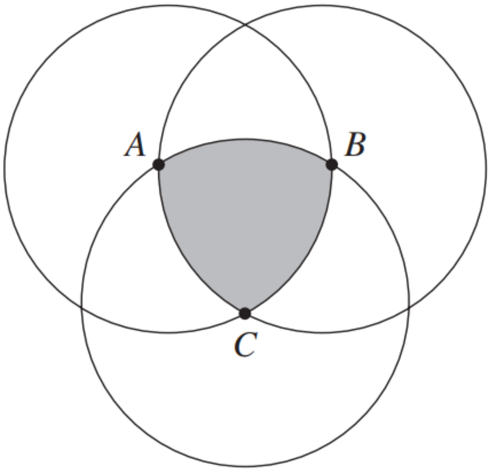
(c) After the 24th contribution the total value of the investment is \$15 536. At that time, Eman stops contributing and starts to withdraw \$450 every following month. The balance in the account will still earn interest at a rate of 0.6% per month. **4**

How many complete months will Eman be able to withdraw \$450 from this account ?

End of Question 30

Question 31 (4 marks)

Three circles with radius r are drawn such that their centres lie on the circumference of the other two circles, as shown in the diagram. **4**



Points A, B and C are the centres of the three circles. Prove that the area of the shaded region is $\frac{1}{2}r^2\left(\pi-\sqrt{3}\right)$.

End of EXAM

Section II extra writing space

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student Name: ______________________________

2023 HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Advanced Yr 12

Section I Answer Sheet

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response circle completely.

1	A ○	B ○	C ○	D ○
2	A ○	B ○	C ○	D ○
3	A ○	B ○	C ○	D ○
4	A ○	B ○	C ○	D ○
5	A ○	B ○	C ○	D ○
6	A ○	B ○	C ○	D ○
7	A ○	B ○	C ○	D ○
8	A ○	B ○	C ○	D ○
9	A ○	B ○	C ○	D ○
10	A ○	B ○	C ○	D ○

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r-1} = \frac{a(1-r^n)}{1-r}, r \neq 1$$

$$S = \frac{a}{1-r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$

$A = \frac{1}{2}ab\sin C$

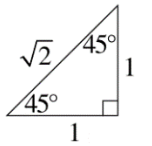
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

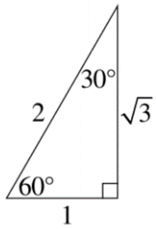
$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$

$A = \frac{1}{2}r^2\theta$





Trigonometric identities

$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$\cos A = \frac{1-t^2}{1+t^2}$

$\tan A = \frac{2t}{1-t^2}$

$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$

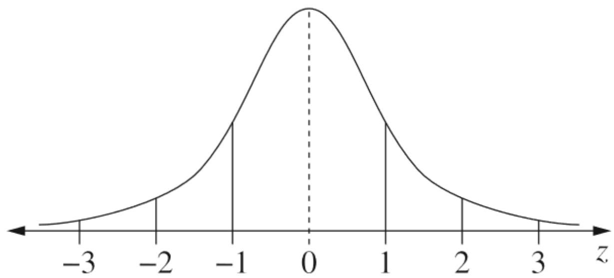
$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \frac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have *z*-scores between -1 and 1
- approximately 95% of scores have *z*-scores between -2 and 2
- approximately 99.7% of scores have *z*-scores between -3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X-\mu)^2\right] = E(X^2) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \le x) = \int_a^x f(x)\,dx$

$P(a < X < b) = \int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r\, p^r (1-p)^{n-r}$

$\boldsymbol{X} \sim Bin(n, p)$

$\Rightarrow \boldsymbol{P(X = x)}$

$= \boldsymbol{\binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, \ldots, n}$

$E(X) = np$

$\operatorname{Var}(X) = np(1-p)$

Differential Calculus

Function	**Derivative**
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u),\ u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	

$$\frac{dy}{dx} = (\ln a)\, f'(x)a^{f(x)}$$

$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)\, f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1 + \left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \ldots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \ldots + \binom{n}{r}x^{n-r}a^{r} + \ldots + a^{n}$$

Vectors

$$|\underset{\sim}{u}| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

HSC
TRIAL EXAMINATION
2023

Mathematics Advanced

Name: Solutions

Maths Class:

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11–31 show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 1–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7–30)

- Attempt Questions 11–31
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

---

1 Which inequality gives the range of $y = x^2 - 6x$?

A. $y \geq 9$

B. $y \geq -9$

C. $y \leq 9$

D. $y \leq -9$

$y = x^2 - 6x$, concave up

tp $(3, -9)$

$(3, -9)$

$\therefore \quad y \geq -9$

2 Given $x + \frac{1}{x} = 4$, what is $x^2 + \frac{1}{x^2}$?

A. 14

B. 16

C. 18

D. 20

$(x + \frac{1}{x})^2 = 4^2$

$x^2 + 2 \cdot \cancel{x} \cdot \frac{1}{\cancel{x}} + \frac{1}{x^2} = 16$

$x^2 + \frac{1}{x^2} = 14$

3 Which of the following graphs have **no** asymptotes?

A. $y = ln(x - 2)$ $\rightarrow$ $x = 2$

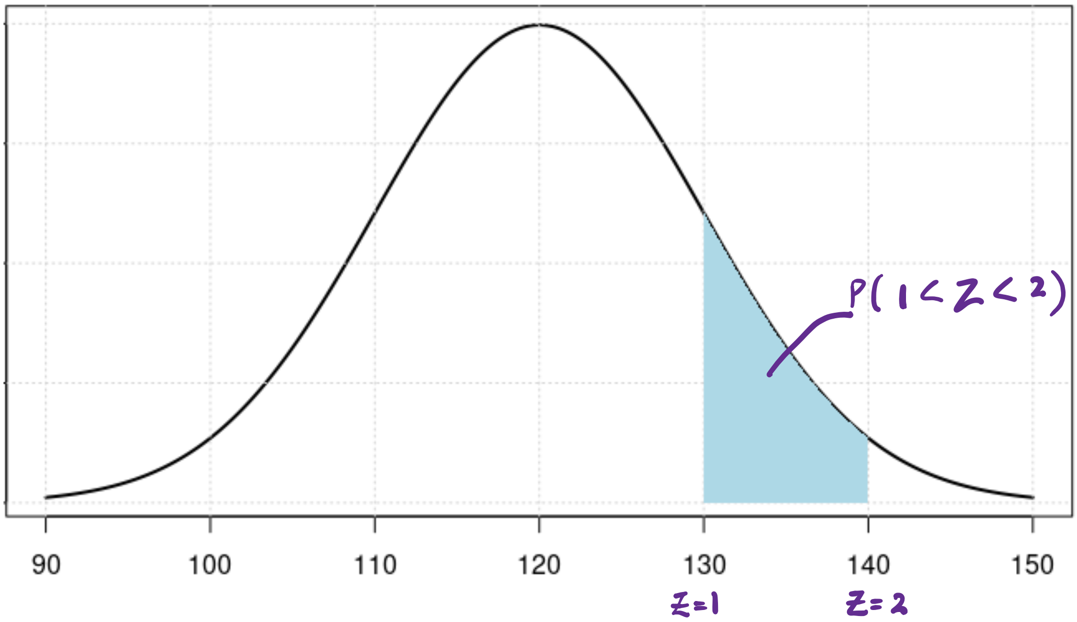
B. $y = e^{2x}$ $\rightarrow$ $y = 0$

C. $y = \sqrt{4 - x^2}$ $\rightarrow$ semicircle no asymptote.

D. $y = \dfrac{3}{x - 4}$ $\rightarrow$ $x = 4, \ y = 0$

4 The number of cars in a car park is normally distributed with a mean of 120 and standard deviation of 10.

The shaded region represents the probability of the number of cars lying in a certain range at some time of the day.



A random variable Z is normally distributed with mean 0 and standard deviation 1.

Which of the following is equivalent to the shaded area?

A. $P(0 < Z < 1)$

B. $P(0 < Z < 2)$

C. $P(-1 < Z < 0)$

D. $P(-2 < Z < -1)$

5 What is the y-intercept of the tangent line to the curve $y = x^3 - 5x$ at $x = 3$?

A. -54

B. -50

C. 38

D. 42

$y(3) = 12$

$y' = 3x^2 - 5$

at $x = 3$

$y'(3) = 3 \cdot 3^2 - 5 = 22$

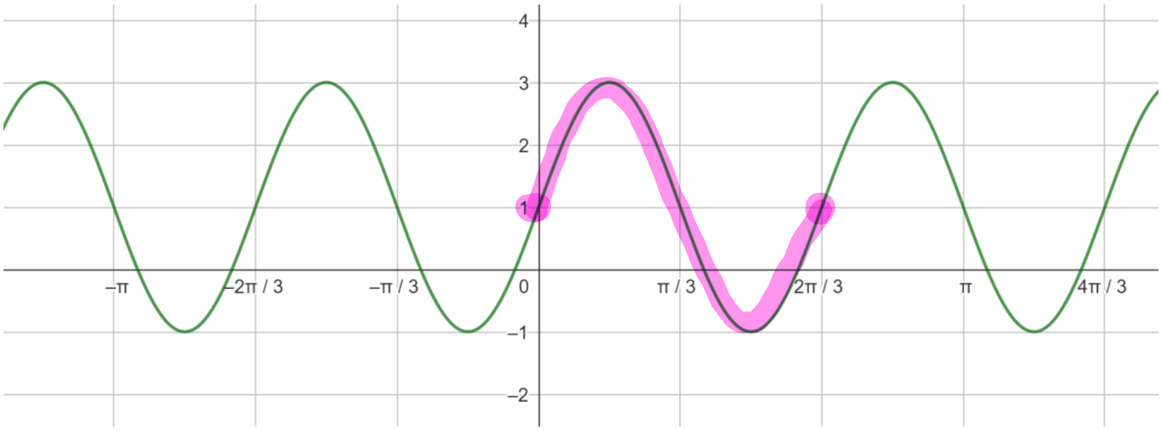
$m_{tangent} = 22$

$y - 12 = 22(x - 3)$

$y = 22x - 66 + 12$

$y = 22x - 54$

6 In the diagram, a graph of a trigonometric function is given.



Which of the following could be the equation of the given graph?

A. $y=2\sin\left(3x-\frac{\pi}{2}\right)+1$

B. $y=2\sin(2x-\pi)+1$

C. $y=2\cos\left(3x-\frac{\pi}{2}\right)+1$

D. $y=2\cos\left(2x+\frac{\pi}{2}\right)+1$

period $= \frac{2\pi}{3}$

range $= [-1, 3]$

check y-int when $x=0$

$y=2\cos(3\cdot 0-\frac{\pi}{2})+1$

$y=2(0)+1$

$y=1$

7 What is $\int \frac{4}{(2x-1)^2}\,dx$?

A. $2\ln(2x-1)+c$

B. $\frac{-2}{2x-1}+c$

C. $\frac{-2}{(2x-1)^3}+c$

D. $\frac{8}{2x-1}+c$

$\int 4(2x-1)^{-2}dx$

$= \frac{4(2x-1)^{-1}}{-1\times 2}+C$

$= \frac{-2}{(2x-1)}+C$

8 What is $\frac{d}{dx}(x^n e^{2x})$?

A. $x^{(n-1)}(2x+n+1)e^{2x}$

B. $x^n(2x+n-1)e^{2x}$

C. $x^n(2x+n)e^{2x}$

D. $x^{(n-1)}(2x+n)e^{2x}$

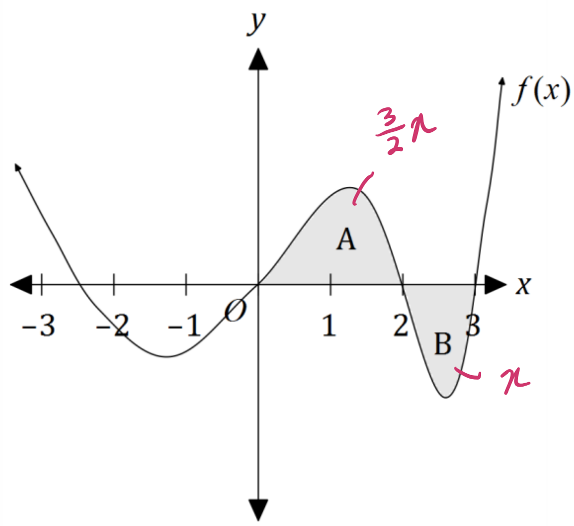
$u = x^n \quad v = e^{2x}$
$u' = nx^{n-1} \quad v' = 2e^{2x}$

$y' = 2 \cdot x^n e^{2x} + n \cdot x^{n-1} e^{2x}$

$y' = x^{n-1} e^{2x}(2 \cdot x + n)$

$y' = x^{n-1}(2x+n) \cdot e^{2x}$

9 The graph of a function $f(x)$ is shown below, where the shaded area labelled A is $\frac{3}{2}$ of the shaded area labelled B.



If $\int_0^3 f(x)dx = 2$, what is the area of A?

A. 6

B. 4

C. $\frac{6}{5}$

D. $\frac{4}{5}$

$B = x$
$A = \frac{3}{2}x$

$\int_0^3 f(x)\,dx = 2$

$\frac{3}{2}x - x = 2$

$\frac{1}{2}x = 2$

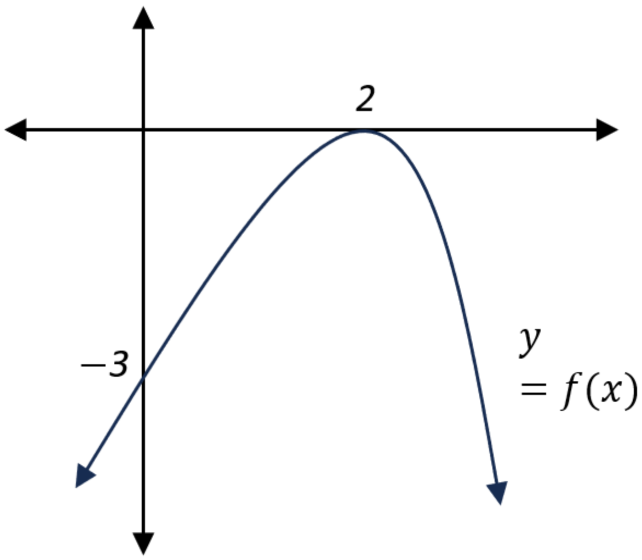
$x = 4$

$A = \frac{3}{2}x$

$= \frac{3}{2} \times 4$

$= 6$

10 A graph of the function $f(x)$ is shown below. It has a single stationary point at $(2,0)$ and is defined for all real x:



Define the function $g(x) = f\big(f(x)\big)$. Which of the following is false?

A. $g(2) = -3$

B. $g(x)$ is negative for the entirety of its domain

C. $g'(2) = 0$

D. $g'(x)$ is negative for $x < 2$

ws

Student Name: ______________________

2023 | **HSC TRIAL EXAMINATION**

Mathematics Advanced

Section II

Answer Booklet - I

30 marks
Attempt Questions 11–19
Allow about 45 minutes for this booklet

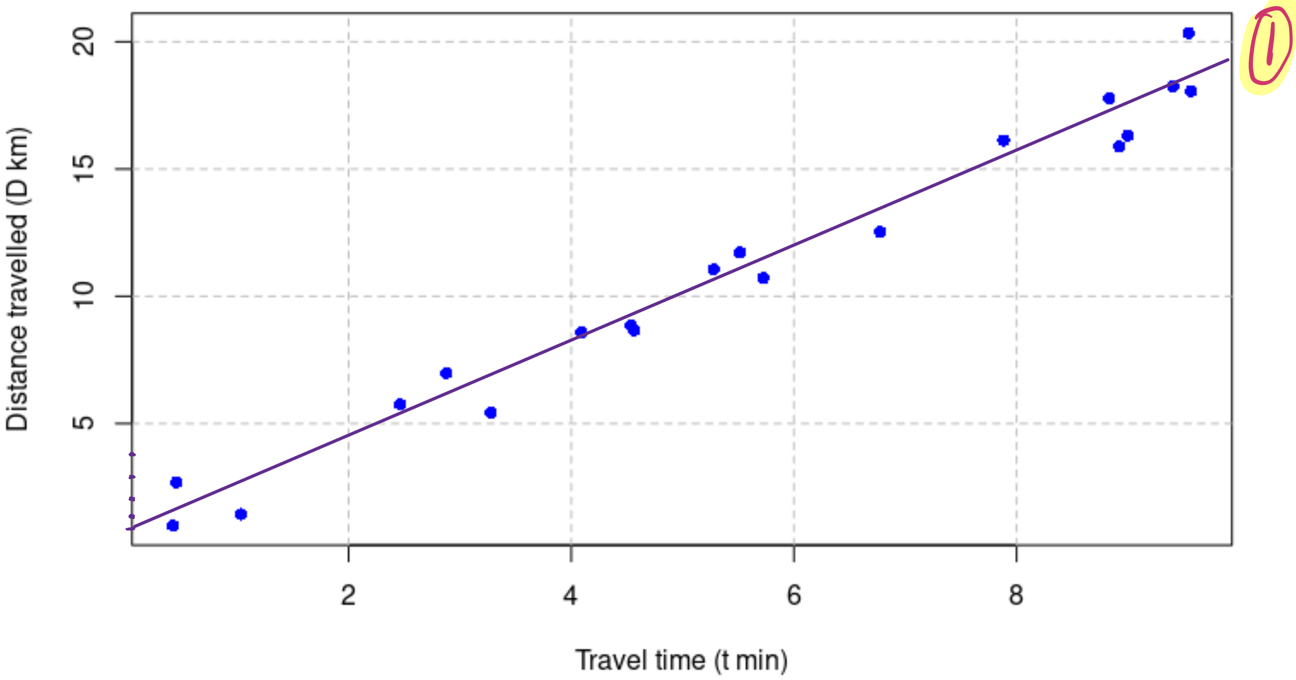
Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your name above

Please turn over

Question 11 (4 marks)

The relationship between the time (t minutes) taken for a taxi to travel a short distance (D kilometres) in a city centre is displayed in the following graph for 20 instances.



The equation of the least square regression line is $D = 1.87t + 0.62$.

(a) State the gradient and interpret its value in the given context. **2**

$M = 1.87$ 1

- for every 1 min increase in time distance increase by 1.87 Km 1

OR

- Distance travelled is almost double the travel time.

(b) On the graph above, sketch the line of best fit. **1**

(c) Use the equation of the least square regression line to find the expected travel time for a distance of 10 km? Give your answer in minutes, correct to two decimal places. **1**

$D = 1.87t + 0.62$

$10 = 1.87t + 0.62$

$t \doteqdot 5.016042781$

$t \approx 5.02$ mins. 1

Question 12 (2 marks)

A function $f(x)$ has derivative $f'(x) = 4\sin(2x)$ and $f\left(\frac{\pi}{6}\right) = 0$. **2**

Find $f(x)$.

$f(x) = \int 4\sin(2x)\,dx$

$= -\frac{\cancel{4}^{2}\cos(2x)}{\cancel{2}} + C$

$= -2\cos(2x) + C$

$f\left(\frac{\pi}{6}\right) = 0$

$0 = -2\cos\left(\cancel{2}\cdot\frac{\pi}{\cancel{6}_3}\right) + C$

$0 = -2\cos\left(\frac{\pi}{3}\right) + C$ (1)

$0 = -2\left(\frac{1}{2}\right) + C$

$C = 1$

(1)

$\therefore f(x) = -2\cos(2x) + 1$

Question 13 (4 marks)

(a) Differentiate $\dfrac{x^2}{x^2+1}$. **2**

$u = x^2 \quad v = x^2+1$

$u' = 2x \quad v' = 2x$

$\frac{2x(x^2+1) - 2x\cdot x^2}{(x^2+1)^2}$ (1)

$= \frac{\cancel{2x^3} + 2x - \cancel{2x^3}}{(x^2+1)^2}$

$= \frac{2x}{(x^2+1)^2}$ (1)

(b) Hence, evaluate $\displaystyle\int_1^2 \frac{x}{(x^2+1)^2}\,dx$. **2**

$= \frac{1}{2}\int_1^2 \frac{2x}{(x^2+1)^2}\,dx$

$= \frac{1}{2}\left[\frac{x^2}{x^2+1}\right]_1^2$ (1)

$= \frac{1}{2}\left[\frac{2^2}{2^2+1} - \frac{1^2}{1^2+1}\right]$

$= \frac{1}{2}\left(\frac{4}{5} - \frac{1}{2}\right)$

$= \frac{3}{20}$ (1)

Question 14 (3 marks)

Describe three transformations which, when applied in succession, change the graph of $y = x^2 + 5$ to the graph with equation $y = \left(\frac{x+1}{2}\right)^2$. **3**

$y = x^2 + 5 \rightarrow x^2 \rightarrow \left(\frac{x}{2}\right)^2 \rightarrow \left(\frac{x+1}{2}\right)^2$

- 5 units down (1)
- H. Dilation by factor of 2 (1)
- 1 unit left (1)

OR:

- $\downarrow 5$, $\leftarrow 1$, $D \times \frac{1}{4}$ vertically

OR: • $\downarrow 5$, $D \times \frac{1}{4}$, $\leftarrow 1$

OR: $y = \left(\frac{x}{2} + \frac{1}{2}\right)^2$

- $5\downarrow$, $\leftarrow \frac{1}{2}$, $2 \times$ H.D

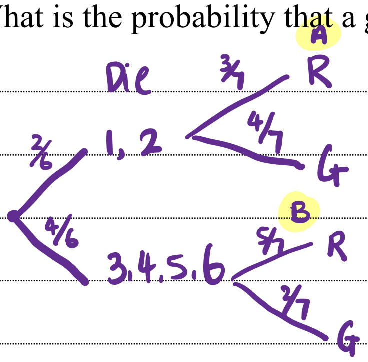
Question 15 (4 marks)

Two bags contain a number of red and green marbles:

- bag A contains 3 red marbles and 4 green marbles
- bag B contains 5 red marbles and 2 green marbles.

A six-sided die is rolled. If the face of the die shows a number that is 1 or 2, then a marble from bag A will be selected at random. Otherwise, a marble from bag B will be selected.

(a) What is the probability that a green marble will be selected? **2**



$P(\text{Green}) = P(D_{1,2}, A_G) + P(D_{3,4,5,6}, B_G)$ (1)

$= \left(\frac{2}{6} \times \frac{4}{7}\right) + \left(\frac{4}{6} \times \frac{2}{7}\right)$

$= \frac{8}{21}$ (1)

(b) It is known that a green marble is selected. **2**

What is the probability that the marble comes from bag B?

$$P(B/G) = \frac{P(B \cap G)}{P(G)}$$

$$= \frac{\frac{4}{6} \times \frac{2}{7}}{\frac{8}{21}} \quad (1)$$

$$= \frac{\frac{8}{42}}{\frac{8}{21}}$$

$$= \frac{1}{2} \quad (1)$$

Question 16 (2 marks)

Use two applications of the trapezoidal rule to approximate $\int_1^5 \frac{12}{x^2+1}\,dx$. **2**

Give your answer correct to two decimal places.

$h=2 \quad h=2$

x	1	3	5
$f(x)$	6	$\frac{12}{10}$	$\frac{12}{26}$

$\frac{12}{1^2+1} = 6 \qquad \frac{12}{3^2+1} = 1.2 \qquad \frac{12}{5^2+1}$

$$\int_1^5 \frac{12}{x^2+1}\,dx$$

$$= \frac{h}{2}\left[y_0 + 2 \times y_1 + y_2\right]$$

$$= \frac{2}{2}\left[6 + 2 \times \frac{12}{10} + \frac{12}{26}\right] \quad (1)$$

$$\doteqdot 8.86 \quad (1)$$

Question 17 (4 marks)

Sketch the graph of the curve $y = -x^3 - 6x^2 - 9x$, labelling the stationary points and point of inflection. **4**

$y = -x^3 - 6x^2 - 9x$

$y' = -3x^2 - 12x - 9$

$y' = 0 \rightarrow$ stat pt

$0 = -3x^2 - 12x - 9$

$0 = x^2 + 4x + 3$

$0 = (x+3)(x+1)$

(1) $\begin{pmatrix} x = -1 \\ y = 4 \end{pmatrix}$ or $\begin{pmatrix} x = -3 \\ y = 0 \end{pmatrix}$ (1)

$\cap$ max tp $\qquad \cup$ min tp

$y'' = -6x - 12$

$y'' = 0 \rightarrow$ inf pts

$0 = -6x - 12$

$0 = x + 2$

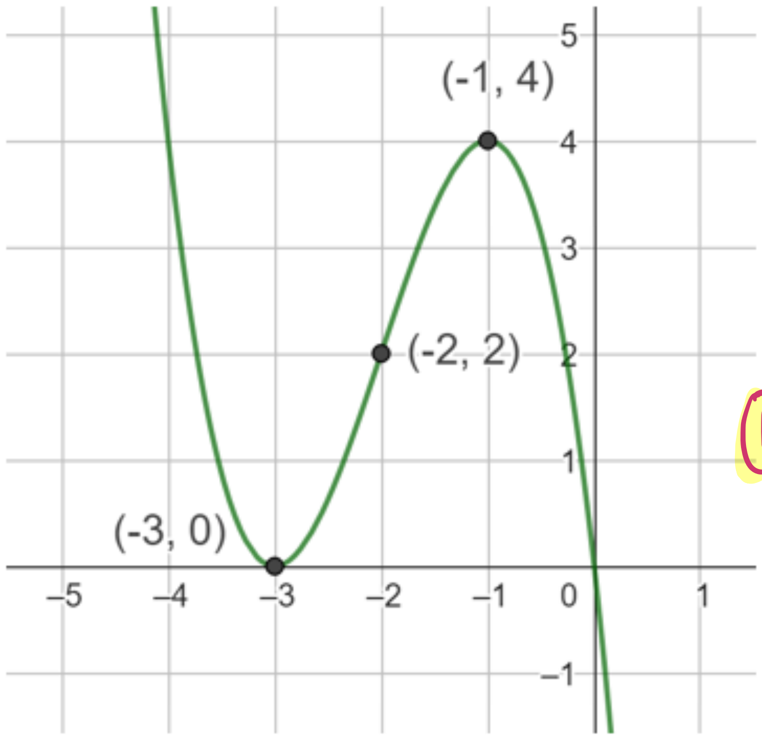
$\begin{pmatrix} x = -2 \\ y = 2 \end{pmatrix}$ inf pt (1)

$y''(-1) < 0$ $\cap$ max

$y''(-3) > 0$ $\cup$ min

test $x = -2$

x	-2.1	-2	-1.9
y''	$+$	0	$-$



(1)

Question 18 (4 marks)

The table below gives the future value of an annuity of $1 per period for various periods and interest rates.

Table of Future Value Interest Factors								
	Interest rate per period							
Number of Periods	0.25%	0.30%	0.35%	0.40%	0.45%	0.50%	0.55%	0.60%
53	56.5961	57.3530	58.1230	58.9063	59.7033	60.5141	61.3391	62.1785
54	57.7376	58.5250	59.3264	60.1419	60.9719	61.8167	62.6765	63.5516
55	58.8819	59.7006	60.5340	61.3825	62.2463	63.1258	64.0212	64.9329
56	60.0291	60.8797	61.7459	62.6280	63.5264	64.4414	65.3733	66.3225
57	61.1792	62.0624	62.9620	63.8786	64.8123	65.7636	66.7329	67.7204
58	62.3322	63.2485	64.1824	65.1341	66.1040	67.0924	68.0999	69.1267
59	63.4880	64.4383	65.4070	66.3946	67.4014	68.4279	69.4744	70.5415
60	64.6467	65.6316	66.6359	67.6602	68.7047	69.7700	70.8565	71.9647
61	65.8083	66.8285	67.8692	68.9308	70.0139	71.1189	72.2463	73.3965
62	66.9729	68.0290	69.1067	70.2065	71.3290	72.4745	73.6436	74.8369
63	68.1403	69.2331	70.3486	71.4874	72.6499	73.8368	75.0487	76.2859
64	69.3106	70.4408	71.5948	72.7733	73.9769	75.2060	76.4614	77.7436
65	70.4839	71.6521	72.8454	74.0644	75.3098	76.5821	77.8820	79.2101
66	71.6601	72.8670	74.1004	75.3607	76.6487	77.9650	79.3103	80.6854

(a) Berra invests $250 per month in an annuity which pays 5.4% p.a. compounding monthly. What will be the value of the annuity after 5 years? **2**

$\frac{5.4\%}{12} = 0.45\%$ per month

5yr $= 5 \times 12 = 60$ month

Factor $= 68.7047$ 1

$FV = 68.7047 \times 250$

$= \$17176.18$ 1

(b) Yousif finds that he can only get an interest rate of 3.6% p.a. compounding mothly. **2**

If he wants to achieve the same total amount as Berra after the same period, what amount should he invest each month?

$\frac{3.6\%}{12} = 0.30\%$ per month

5yr $= 5 \times 12 = 60$ mon

Factor $= 65.6316$ 1

$17176.18 = x \times 65.6316$

$\$261.71 = x$ 1

Question 19 (3 marks)

Solve $\log_2 x + \log_2 (x-3) = 2$. **3**

$x > 0$ & $x-3 > 0$

$\boxed{x > 3}$

$\log_2 x + \log_2 (x-3) = 2$

$\log_2 x \cdot (x-3) = 2$

$2^2 = x^2 - 3x$ 1

$0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$x = 4$ or $x = -1$ 1

1 not valid as $x > 3$

$\therefore \quad x = 4$

End of Section 1I – Booklet 1

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student Name: ______________________________

2023 | **HSC TRIAL EXAMINATION**

Mathematics Advanced

Section II

Answer Booklet - II

30 marks
Attempt Questions 20–25
Allow about 60 minutes for this booklet

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your name above

Please turn over

Question 20 (5 marks)

During the mushroom season, every day, Asna would go into the forest to collect mushrooms.

On the first day she collects 34 mushrooms and each consecutive day she collects 6 more mushrooms than she did on the previous day. That is, 40 mushrooms on the second day and 46 mushrooms on the third day.

(a) On which day will Asna collect more than 120 mushrooms for the first time? **2**

34, 40, 46 ....

$a_1 = 34$

$d = 6$

$a_n > 120$

$a_n = a_1 + (n-1)d$

$34 + (n-1)\cdot 6 > 120$ (1)

$34 + 6n - 6 > 120$

$6n > 92$

$n > 92/6$

$n > 15.3\ldots$

$n = 16^{th}$ day (1)

(b) During the entire mushroom season, Asna collected 3220 mushrooms. **3**

For how many days did she collect mushrooms?

$S_n = 3220$

$\frac{n}{2}[2a_1 + (n-1)d] = 3220$

$\frac{n}{2}[2(34) + (n-1)\cdot 6] = 3220$ (1)

$34n + 3n^2 - 3n = 3220$

$3n^2 + 31n - 3220 = 0$

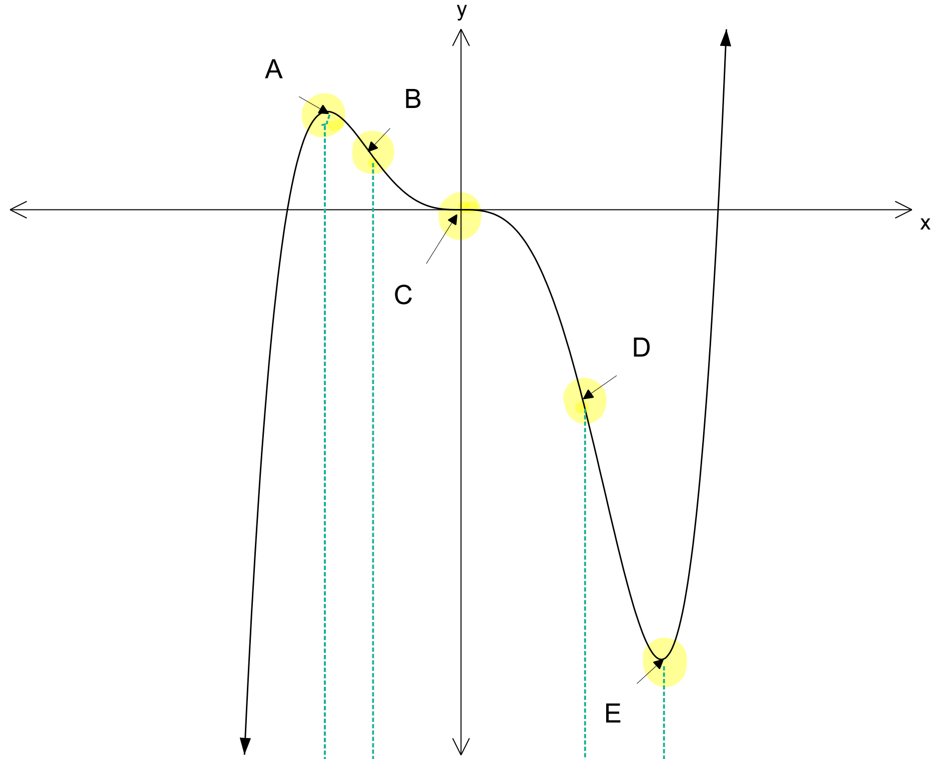
$n = 28$ or $n = -\frac{115}{3}$ (1)

not valid as $n > 0$

$\therefore n = 28$ days (1)

Question 21 (2 marks)

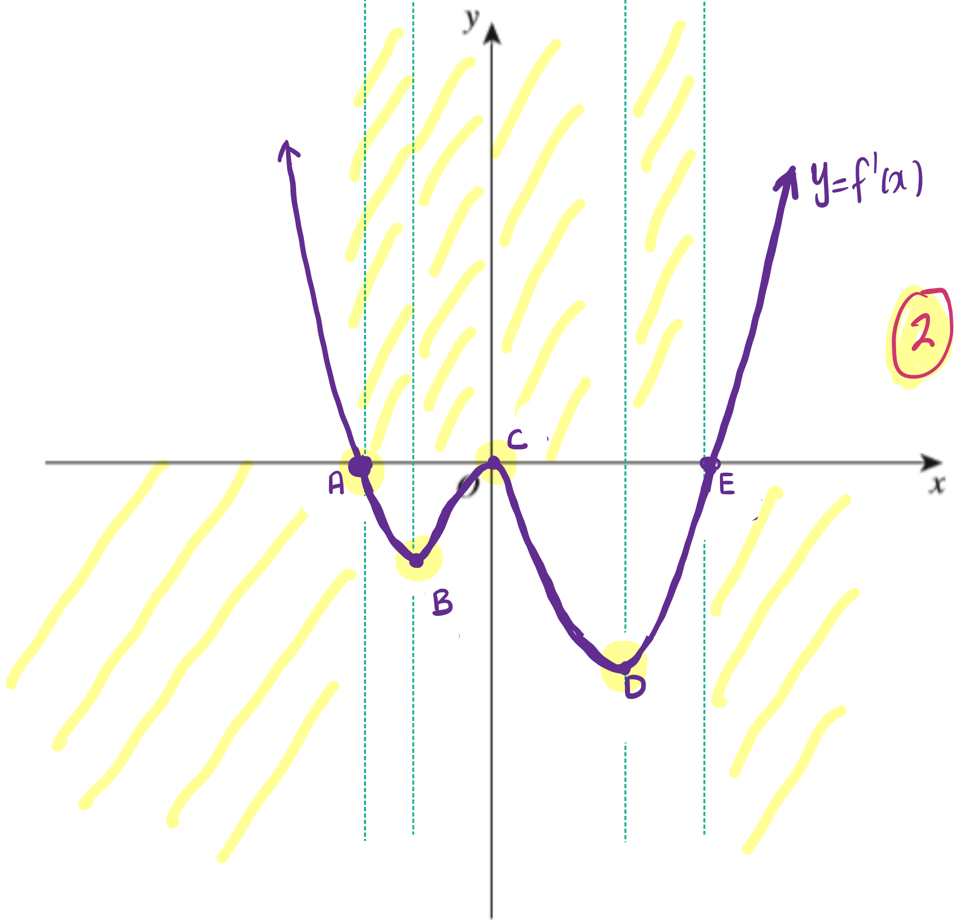
The graph of the curve $y = f(x)$ is drawn below: **2**



Inf pt → tp → x-int

hor inf → tp & x-int

On the axes below, sketch the graph of $y = f'(x)$, Clearly showing the behaviour of the points: A,B,C,D and E.



2

Question 22 (7 marks)

The temperature, T degrees, of a liquid is given according to the model $T(t) = 20 + Ae^{kt}$, where t is time in hours. The initial temperature of the liquid is 35 degrees.

(a) Show that $A = 15$. **1**

$t=0$, $T(0)=35$

$35 = 20 + A \cdot e^{k\times 0}$

$35 = 20 + A$

$15 = A$ 1

$\therefore A = 15$

(b) After four hours, the temperature of the liquid is 80 degrees. **3**

Show that $k = ln(\sqrt{2})$.

$t=4$, $T(4)=80$, $k=?$

$80 = 20 + 15 \cdot e^{k\cdot 4}$

$60 = 15\, e^{k\cdot 4}$ 1

$4 = e^{4k}$

$\ln 4 = 4k$

$k = \frac{\ln 4}{4}$ 1

$k = \frac{1}{4} \ln 2^2$

$k = \frac{1}{2} \ln 2$

$k = \ln \sqrt{2}$. 1

Question 22 continues on page 20

Question 22 (continued)

(c) Show that $T(t) = 15 \cdot 2^{\frac{t}{2}} + 20$ and hence sketch its graph. **3**

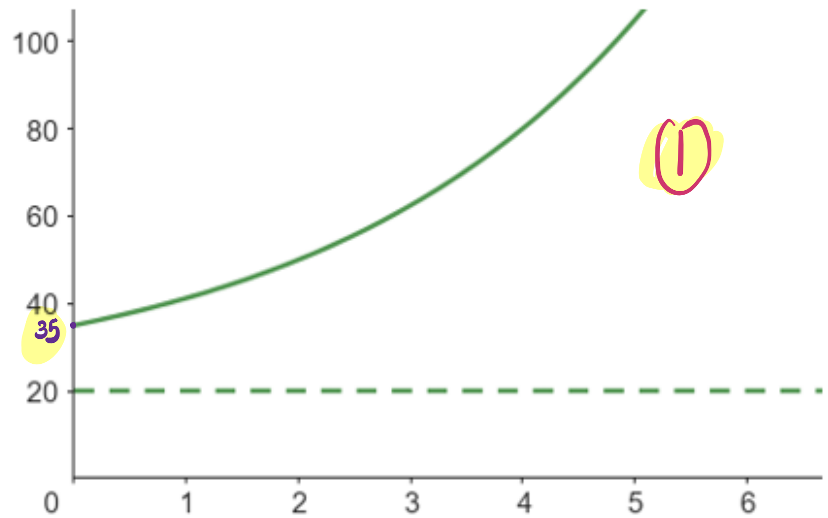
$T(t) = 20 + 15e^{\ln\sqrt{2}\, t}$

$T(t) = 20 + 15\left(\cancel{e}^{\cancel{\ln}\sqrt{2}}\right)^t$ 1

$= 20 + 15 \cdot (\sqrt{2})^t$

$= 20 + 15 \cdot 2^{\frac{1}{2} \cdot t}$

$\therefore\ T(t) = 15 \cdot 2^{\frac{t}{2}} + 20$ 1

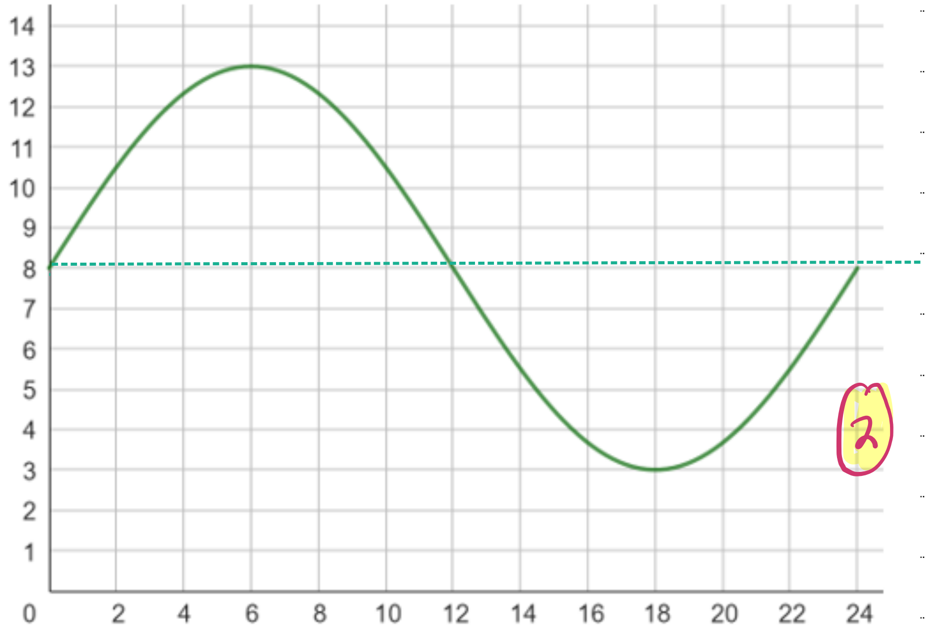


End of Question 22

Question 23 (9 marks)

The water at a certain beach rises and falls in a periodic pattern. The height of the water at any time can be modelled by the equation $h(t) = 8 + 5\,sin\left(\frac{\pi t}{12}\right)$, where t is time in hours after midnight.

(a) Sketch the graph of $y = h(t)$ for $0 \leq t \leq 24$. **2**



Period $= \frac{2\pi}{\frac{\pi}{12}}$

$= 24$

$8 \uparrow$

max $= 13$

min $= 3$

(b) At what rate does the height of water change at 8:00 am? Leave your answer in exact form. **2**

$h = 8 + 5\sin\left(\frac{\pi t}{12}\right)$

$h' = 5 \cdot \frac{\pi}{12} \cdot \cos\left(\frac{\pi}{12} t\right)$

$h' = \frac{5\pi}{12} \cos\left(\frac{\pi}{12} t\right)$ 1

at 8:00 am $\rightarrow$ $t = 8$

$h'(8) = \frac{5\pi}{12} \cdot \cos\left(\frac{\pi}{\cancel{12}_3} \times \cancel{8}^2\right)$

$= \frac{5\pi}{12} \cdot \cos\left(\frac{2\pi}{3}\right)$

$h'(8) = \frac{5\pi}{12} \cdot \left(-\frac{1}{2}\right)$

$h'(8) = -\frac{5\pi}{24}$ 1

Question 23 continues on page 22

Question 23 (continued)

(c) A boat with a depth of 5.5 metres, is scheduled to arrive at the beach at 10:00 am one day. **2**

Explain if the boat will be able to dock.

$h(10) = 8 + 5\sin\left(\frac{5\pi}{6}\right)$

$= 10.5$ 1

$10.5 > 5.5$ So Yes able to dock. 1

(d) On each day, between what times will the boat in part (c) be able to dock? **3**

$8 + 5\sin\left(\frac{\pi t}{12}\right) = 5.5$

$5\sin\left(\frac{\pi}{12}t\right) = -2.5$

$\sin\left(\frac{\pi}{12}t\right) = -\frac{1}{2}$ 1

$\left(\sin\frac{\pi}{6} = \frac{1}{2} \quad \theta = \frac{\pi}{6}\right)$

S | A
T ✓ | C ✓
$(\pi+\theta)$ $(2\pi-\theta)$

$\frac{\pi}{12}t = \left(\pi + \frac{\pi}{6}\right), \left(2\pi - \frac{\pi}{6}\right)$

$\frac{\pi}{12}t = \frac{7\pi}{6}, \frac{11\pi}{6}$

$t = 14, 22$ 1

$\therefore$ $t =$ 2pm or 10pm

$\therefore$ $t <$ 2pm $t >$ 10pm

before 2pm or after 10pm. 1

End of Question 23

Question 24 (3 marks)

Find the value of k such that $\int_0^k e^x(e^x - 2)dx = \frac{3}{2}$. **3**

$$\int_0^k (e^{2x} - 2e^x)\,dx = \frac{3}{2}$$

$$\left[\frac{1}{2}e^{2x} - 2e^x\right]_0^k = \frac{3}{2}$$

$$\left[\frac{1}{2}e^{2k} - 2e^k\right] - \left[\frac{1}{2}e^{\cancel{2\cdot 0}} - 2e^{0}\right] = \frac{3}{2}$$

$$\left[\frac{1}{2}e^{2k} - 2e^k\right] - \left[-\frac{3}{2}\right] = \frac{3}{2}$$ (1)

$$\frac{1}{2}e^{2k} - 2e^k = 0$$

$$e^{2k} - 4e^k = 0$$

$$e^k(e^k - 4) = 0$$

$\therefore$ $e^k = 0$ (1) or $e^k = 4$

no soln $k = \ln 4$ (1)

OR $k = \ln 2^2$

$k = 2\ln 2.$

Question 25 (4 marks)

A continuous random variable X has probability density function given by:

$$f(x)=\begin{cases}\dfrac{x+2}{k} & 1\le x\le 3\\ 0 & \text{elsewhere}\end{cases}$$

(a) Show that $k=8$. **1**

$$\int_1^3 \frac{x+2}{k}\,dx = 1$$

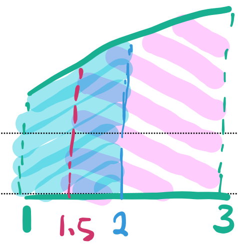
$$\frac{1}{k}\int_1^3 (x+2)\,dx = 1$$

$$\left[\frac{x^2}{2}+2x\right]_1^3 = k \quad 1$$

$$\left[\frac{9}{2}-6\right]-\left[\frac{1}{2}+2\right]=k$$

$$8=k$$

(b) Find $P(X<2\mid X>1.5)$. **3**



$$P(x<2 / x>1.5) = \frac{P(x<2)\cap P(x>1.5)}{P(x>1.5)}$$

$$= \frac{P(1.5<x<2)}{P(1.5<x<3)}$$

$$= \frac{\cancel{\tfrac{1}{8}}\int_{1.5}^{2}(x+2)\,dx}{\cancel{\tfrac{1}{8}}\int_{1.5}^{3}(x+2)\,dx}$$

$$\frac{\left[\frac{1}{2}x^2+2x\right]_{1.5}^{2}}{\left[\frac{1}{2}x^2+2x\right]_{1.5}^{3}} \quad 1 \quad 1$$

$$= \frac{\left(\frac{1}{2}\cdot 2^2+2\cdot 2\right)-\left(\frac{1}{2}\cdot 1.5^2+2\cdot 1.5\right)}{\left(\frac{1}{2}\cdot 3^2+2\cdot 3\right)-\left(\frac{1}{2}\cdot 1.5^2+2\cdot 1.5\right)}$$

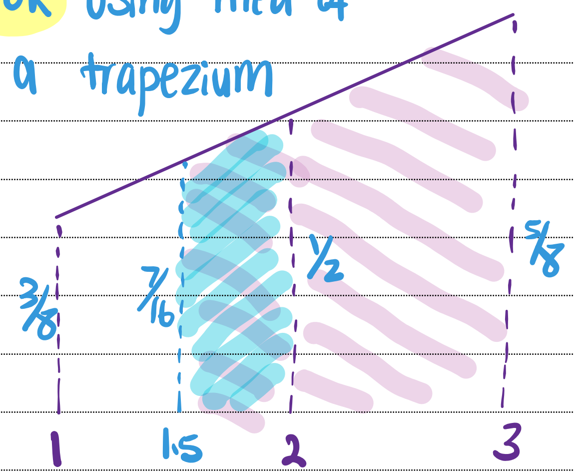
$$= \frac{5}{17} \quad 1$$

End of Section 1I – Booklet II

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

OR using Area of a trapezium



$\frac{1}{8}(x+2)$

$x=1, \ \frac{1}{8}(1+2) = 3/8$

$x=1.5, \ \frac{1}{8}(1.5+2) = 7/16$

$x=2 \ \ \frac{1}{8}(2+2) = 1/2$

$x=3 \ \ \frac{1}{8}(3+2) = 5/8$

using Area $= \frac{h}{2}(a+b)$

$$\frac{P(1.5 < x < 2)}{P(1.5 < x < 3)} = \frac{\frac{0.5}{2}\left(\frac{1}{2}+\frac{7}{16}\right)}{\frac{1.5}{2}\left(\frac{7}{16}+\frac{5}{8}\right)} \ \ 1 \ \ 1$$

$$= \frac{15/64}{51/64} = 5/17 \ \ 1$$

Student Name: ______________________________

2023 | **HSC TRIAL EXAMINATION**

Mathematics Advanced

Section II

Answer Booklet - III

30 marks
Attempt Questions 26–31
Allow about 60 minutes for this booklet

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your name above

Please turn over

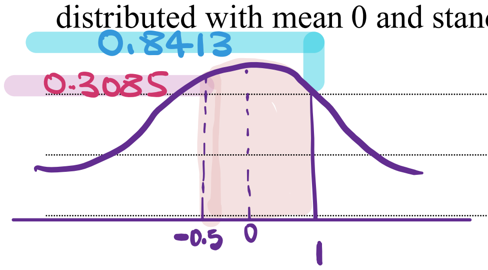
Question 26 (5 marks)

A random variable is normally distributed with mean 0 and standard deviation 1.

The table gives the probability that this random variable is less than z for different values of z.

z	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
$P(Z < z)$	0.0228	0.0668	0.1587	0.3085	0.5000	0.6915	0.8413	0.9332	0.9772

(a) Using the table, find the probability that a value from a random variable that is normally distributed with mean 0 and standard deviation 1, lies between −0.5 and 1. **1**



$0.8413 - 0.3085$
$= 0.5328$ (1)

(b) The blood pressure readings of patients with a certain condition are normally distributed, with a mean of μ and a standard deviation of σ. It is known that 2.28% of these patients have a blood reading of less than 105 and 6.68% of these patients have a blood reading of more than 133. **4**

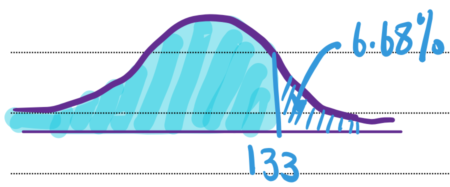
Using the table, find μ and σ.

$2.28\% = 0.0228 \rightarrow P(Z < -2) = 0.0228$

$\frac{105 - \mu}{\sigma} = -2$ (1)

6.68% more than 133

$105 - \mu = -2\sigma$ --- (1)



$\frac{133 - \mu}{\sigma} = 1.5$ (1)

$100\% - 6.68\% = 93.32\%$
$= 0.9332$
which gives $Z = 1.5$

$133 - \mu = 1.5\sigma$ --- (2)

(2) − (1)

$28 = 3.5\sigma$
$8 = \sigma$ (1)

subst $\sigma = 8$ into (1)

$105 - \mu = -2(8)$

$\mu = 121$ (1)

Question 27 (4 marks)

Find the global maximum and minimum values of $f(x) = sin^2 x + 4\,cos\,x$, where $\frac{\pi}{3} \leq x \leq \frac{3\pi}{2}$. **4**

$f(x) = \sin^2 x + 4\cos x$

$f'(x) = 2\sin x \cdot \cos x - 4\sin x$ (1)

$f'(x) = 0 \rightarrow$ stat points

$0 = 2\sin x \cos x - 4\sin x$

$0 = 2\sin x(\cos x - 2)$

$2\sin x = 0$ $\quad\quad$ $\cos x - 2 = 0$

$\sin x = 0$ $\quad\quad$ $\cos x = 2$

$x = \cancel{0}, \pi, \cancel{2\pi}$ $\quad\quad$ no sol$^{\underline{n}}$

(1) D: $\frac{\pi}{3} < x < \frac{3\pi}{2}$

$\therefore x = \pi$

$f\left(\frac{\pi}{3}\right) = \frac{11}{4}$ $\quad\quad$ $f(x)\ max = \frac{11}{4}$ (1)

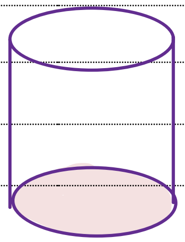
$f(\pi) = -4$ $\quad\quad$ $f(x)\ min = -4$ (1)

$f\left(\frac{3\pi}{2}\right) = 1$

Question 28 (5 marks)

A container in the shape of a right circular cylinder with no top, has outside surface area 12π square metres.

(a) Show that the height (h) is $\dfrac{12 - r^2}{2r}$, where r is the radius. **1**



$SA = \pi r^2 + 2\pi r h$

$12\pi = \pi r^2 + 2\pi r h$

$12\pi - \pi r^2 = 2\pi r h$

$h = \dfrac{12\pi - \pi r^2}{2\pi r}$

$h = \dfrac{\cancel{\pi}(12 - r^2)}{2\cancel{\pi} r} = \dfrac{12 - r^2}{2r}$ 1

(b) Find and verify the height (h) and radius (r) that give the maximum volume. **4**

$V = \pi r^2 h$

$V = \pi \cdot r^2 \cdot \dfrac{12 - r^2}{2r}$

1 $V = \dfrac{\pi}{2}(12r - r^3)$

$\dfrac{dV}{dr} = \dfrac{\pi}{2}(12 - 3r^2)$

$0 = \dfrac{\pi}{2}(12 - 3r^2)$ 1

$3r^2 = 12$

$r^2 = 4$

$r = \pm 2$

$r > 0$

$\therefore\ r = 2$

$V'' = \dfrac{\pi}{2}(-6r)$

$V''(2) = \dfrac{\pi}{2}(-6 \cdot 2)$

$= -6\pi < 0$ $\frown$ max

$\therefore\ r = 2$ 1

$h = \dfrac{12 - 2^2}{2 \cdot 2}$

$h = \dfrac{12 - 4}{4}$

$h = 2$ 1

$\therefore$ max volume is given by

$r = 2, \quad h = 2$

Question 29 (4 marks)

In the diagram, the region between the graphs of $y = \frac{4}{x}$ and $y = \sqrt{2x}$ for $1 \le x \le 4$ is shaded. **4**

P.O.I :

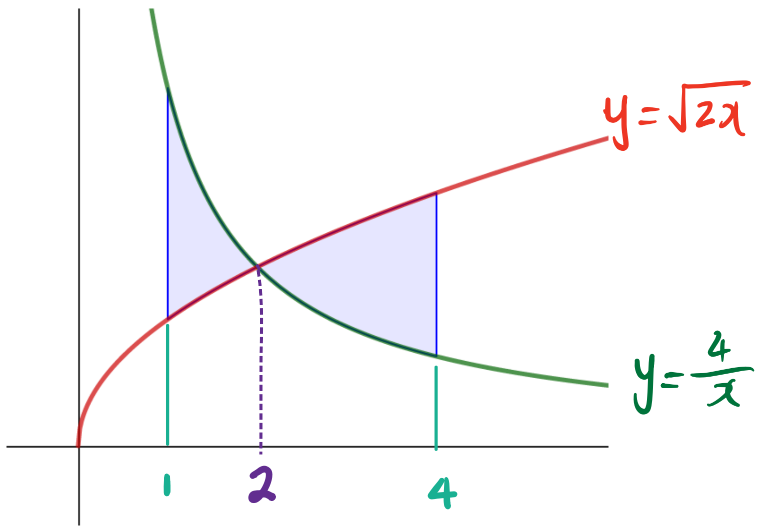
$\sqrt{2x} = \frac{4}{x}$

$2x = \frac{16}{x^2}$

$2x^3 = 16$

$x^3 = 8$

$x = 2$ (1)



Find the exact area of the shaded region.

$A_1 = \int_1^2 \frac{4}{x} - \sqrt{2}x^{\frac{1}{2}}\, dx$

$= \left[4\ln|x| - \frac{\sqrt{2}x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^2$

$= \left[4\ln 2 - \frac{2\sqrt{2}}{3}(2)^{3/2}\right] - \left[4\ln 1 - \frac{2\sqrt{2}}{3}(1)^{3/2}\right]$

$= \left(4\ln 2 - \frac{8}{3}\right) + \frac{2\sqrt{2}}{3}$ (1)

$= 4\ln 2 - \frac{8}{3} + \frac{2\sqrt{2}}{3}$

$A_2 = \int_2^4 \left(\sqrt{2}\, x^{\frac{1}{2}} - \frac{4}{x}\right) dx$

$= \left[\frac{2\sqrt{2}}{3} x^{3/2} - 4\ln|x|\right]_2^4$ (1)

$= \left(\frac{2\sqrt{2}}{3}(4)^{3/2} - 4\ln 4\right) - \left(\frac{2\sqrt{2}}{3}\cdot 2^{3/2} - 4\ln 2\right)$ $\quad 2^2$

$= \frac{16\sqrt{2}}{3} - 8\ln 2 - \frac{8}{3} + 4\ln 2$

$A_1 + A_2 = 4\cancel{\ln 2} - \frac{8}{3} + \frac{2\sqrt{2}}{3} + \frac{16\sqrt{2}}{3} - \cancel{8\ln 2} - \frac{8}{3} + 4\cancel{\ln 2}$

$= \underbrace{\frac{18\sqrt{2}}{3} - \frac{16}{3} = 6\sqrt{2} - \frac{16}{3}}_{(1)}$.

Question 30 (8 marks)

Eman invests \$9000 in an account earning interest at a rate of 0.6% per month. Each month, immediately after the interest has been paid, Eman contributes \$200.

The amount in the account after the nth contribution can be determined using the recurrence relation $A_n = 1.006A_{n-1} + 200$, where $A_0 = 9000$.

(a) Find the amount in the account after the first contribution. **1**

$A_1 = A_0 \cdot r + 200$

$A_1 = 9000(1.006) + 200$

$A_1 = 9254$ 1

(b) Find the total value of the investment after the 12th contribution. **3**

$A_2 = A_1(1.006) + 200$

$= 9000(1.006)^2 + 200(1.006) + 200$

$A_3 = 9000(1.006)^3 + 200(1.006)^2 + 200(1.006) + 200$

$\vdots$

$A_{12} = 900(1.006)^{12} + 200(1.006)^{11} + \dots + 200$ 1

$A_{12} = 900(1.006)^{12} + 2000\left[(1.006)^{11} + \dots + 1.006 + 1\right]$

$= 900(1.006)^{12} + 200\left(\frac{1.006^{12} - 1}{1.006 - 1}\right)$ 1

$= 12150.60$ 1

Question 30 continues on page 32

Question 30 (continued)

(c) After the 24th contribution the total value of the investment is \$15 536. At that time, Eman stops contributing and starts to withdraw \$450 every following month. The balance in the account will still earn interest at a rate of 0.6% per month. **4**

How many complete months will Eman be able to withdraw \$450 from this account ?

$A_{24} = 15536$

Let: $B_0 = 15536$, $M = \$450$, $r = 1.006$, $B_n = 0$, $n = ?$

$B_1 = B_0 r - M$

$B_2 = B_0 r^2 - Mr - M$

$\vdots$

$B_n = B_0 r^n - M r^{n-1} - \dots - M$ 1

$0 = 15536(1.006)^n - 450(1 + 1.006 + \dots + 1.006^{n-1})$

$450 \times \dfrac{(1.006^n - 1)}{1.006 - 1} = 15536(1.006)^n$ 1

$75000(1.006)^n - 75000 = 15536(1.006)^n$

$59464(1.006)^n = 75000$

$(1.006)^n = \dfrac{75000}{59464}$

$n = \ln\left(\dfrac{75000}{59464}\right) \div \ln(1.006)$

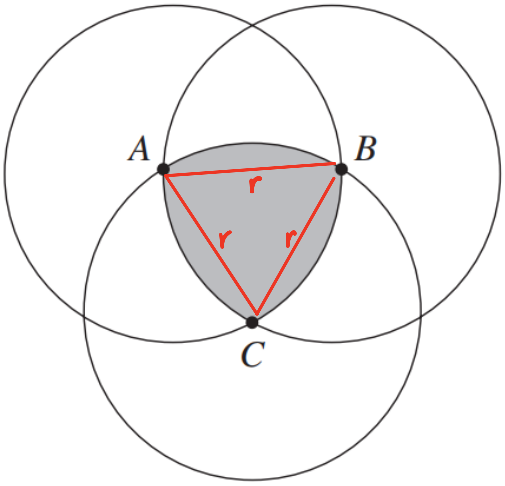
$n \doteqdot 38.8$ 1

$n = 38$ months. 1

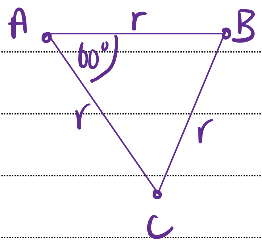
End of Question 30

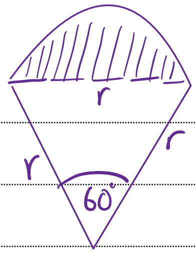
Question 31 (4 marks)

Three circles with radius r are drawn such that their centres lie on the circumference of the other two circles, as shown in the diagram. **4**



Points A, B and C are the centres of the three circles. Prove that the area of the shaded region is $\frac{1}{2}r^2\left(\pi - \sqrt{3}\right)$.





$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \cdot r \cdot r \sin 60°$

$= \frac{1}{2}r^2 \cdot \frac{\sqrt{3}}{2}$

$= \frac{r^2\sqrt{3}}{4}$ (1)

$A = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}r^2\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$

$= \frac{1}{2}r^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= \frac{\pi r^2}{6} - \frac{\sqrt{3}r^2}{4}$ (1)

Total Area $= \frac{r^2\sqrt{3}}{4} + 3\times\left(\frac{\pi r^2}{6} - \frac{\sqrt{3}r^2}{4}\right)$ (1)

$= \frac{r^2\sqrt{3}}{4} + \frac{\pi r^2}{2} - \frac{3\sqrt{3}r^2}{4}$

$= \frac{\pi r^2}{2} - \frac{\cancel{2}^{1}\sqrt{3}r^2}{\cancel{4}_{2}}$

$= \frac{r^2}{2}\left(\pi - \sqrt{3}\right)$ (1)

End of EXAM

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student Name: ______________________________

2023 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Advanced Yr 12

Section I Answer Sheet

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response circle completely.

1	A ○	B ○	C ○	D ○
2	A ○	B ○	C ○	D ○
3	A ○	B ○	C ○	D ○
4	A ○	B ○	C ○	D ○
5	A ○	B ○	C ○	D ○
6	A ○	B ○	C ○	D ○
7	A ○	B ○	C ○	D ○
8	A ○	B ○	C ○	D ○
9	A ○	B ○	C ○	D ○
10	A ○	B ○	C ○	D ○

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a\left(r^n - 1\right)}{r-1} = \frac{a\left(1-r^n\right)}{1-r}, r \neq 1$$

$$S = \frac{a}{1-r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

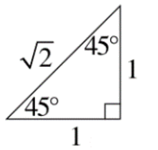
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$

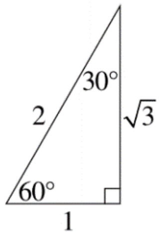
$A = \frac{1}{2}ab\sin C$



$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$



$l = r\theta$

$A = \frac{1}{2}r^2\theta$

Trigonometric identities

$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$\cos A = \frac{1-t^2}{1+t^2}$

$\tan A = \frac{2t}{1-t^2}$

$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$

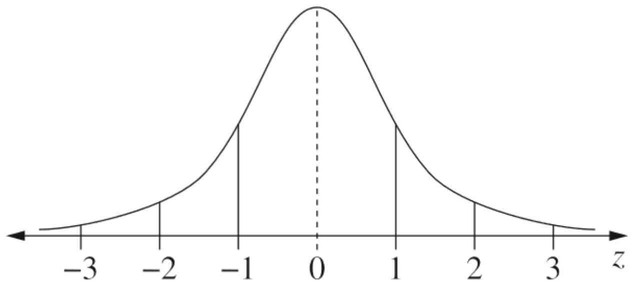
$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \frac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between -1 and 1
- approximately 95% of scores have z-scores between -2 and 2
- approximately 99.7% of scores have z-scores between -3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X-\mu)^2\right] = E(X^2) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq x) = \int_a^x f(x)\,dx$

$P(a < X < b) = \int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$

$\boldsymbol{X \sim Bin(n, p)}$

$\boldsymbol{\Rightarrow P(X = x)}$

$\boldsymbol{= \binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, \ldots, n}$

$E(X) = np$

$\operatorname{Var}(X) = np(1-p)$

Differential Calculus

Function	**Derivative**
$y=f(x)^n$	$\frac{dy}{dx}=nf'(x)\left[f(x)\right]^{n-1}$
$y=uv$	$\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$
$y=g(u),\ u=f(x)$	$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$
$y=\frac{u}{v}$	$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
$y=\sin f(x)$	$\frac{dy}{dx}=f'(x)\cos f(x)$
$y=\cos f(x)$	$\frac{dy}{dx}=-f'(x)\sin f(x)$
$y=\tan f(x)$	$\frac{dy}{dx}=f'(x)\sec^2 f(x)$
$y=e^{f(x)}$	$\frac{dy}{dx}=f'(x)e^{f(x)}$
$y=\ln f(x)$	$\frac{dy}{dx}=\frac{f'(x)}{f(x)}$
$y=a^{f(x)}$	$\frac{dy}{dx}=(\ln a)f'(x)a^{f(x)}$
$y=\log_a f(x)$	$\frac{dy}{dx}=\frac{f'(x)}{(\ln a)f(x)}$
$y=\sin^{-1} f(x)$	$\frac{dy}{dx}=\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y=\cos^{-1} f(x)$	$\frac{dy}{dx}=-\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y=\tan^{-1} f(x)$	$\frac{dy}{dx}=\frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx=\frac{1}{n+1}\left[f(x)\right]^{n+1}+c$$
where $n\neq -1$

$$\int f'(x)\sin f(x)dx=-\cos f(x)+c$$

$$\int f'(x)\cos f(x)dx=\sin f(x)+c$$

$$\int f'(x)\sec^2 f(x)dx=\tan f(x)+c$$

$$\int f'(x)e^{f(x)}dx=e^{f(x)}+c$$

$$\int \frac{f'(x)}{f(x)}dx=\ln\left|f(x)\right|+c$$

$$\int f'(x)a^{f(x)}dx=\frac{a^{f(x)}}{\ln a}+c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}dx=\sin^{-1}\frac{f(x)}{a}+c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}dx=\frac{1}{a}\tan^{-1}\frac{f(x)}{a}+c$$

$$\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx\approx\frac{b-a}{2n}\left\{f(a)+f(b)+2\left[f(x_1)+\ldots+f(x_{n-1})\right]\right\}$$
where $a=x_0$ and $b=x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \ldots + \binom{n}{r}x^{n-r}a^{r} + \ldots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2} + y^{2}}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^{2}(x - c)$$