HSC
TRIAL EXAMINATION
2024

Mathematics Advanced

Name:____________________________

Maths Class:____________________________

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11–32 show relevant mathematical reasoning and/or calculations

Total marks:
100

Section I – **10 marks** (pages 1–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – **90 marks** (pages 8–36)

- Attempt Questions 11–32
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 What is the domain of $y = \ln(x-1) + \sqrt{4-x}$?

A. $(1,4)$

B. $[4,\infty)$

C. $(-\infty,4)$

D. $(1,4]$

2 What is $\int \frac{10}{(2x+3)^2}\,dx$?

A. $-5\ln(2x+3)+c$

B. $10\ln(2x+3)+c$

C. $\frac{-5}{2x+3}+c$

D. $\frac{-20}{(2x+3)^3}+c$

3 There are ten pieces fruit in a basket. Four are apples and six are oranges. Mia randomly picks a piece from the basket and eats it. Liam then randomly picks and eats one of the remaining fruits.

What is the probability that Liam ate an orange?

A. $\frac{3}{5}$

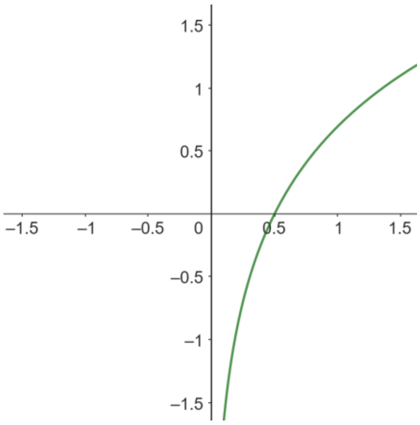
B. $\frac{4}{5}$

C. $\frac{27}{50}$

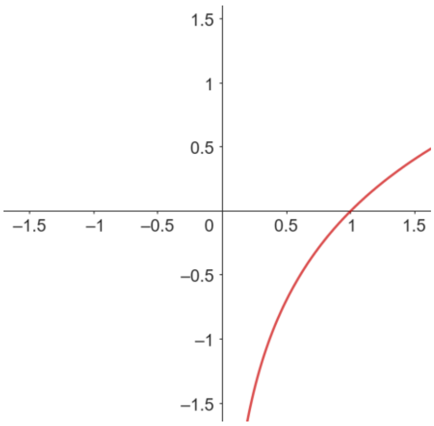
D. $\frac{22}{45}$

4 Which of the following best represents the graph of $y = \ln(-2x)$?

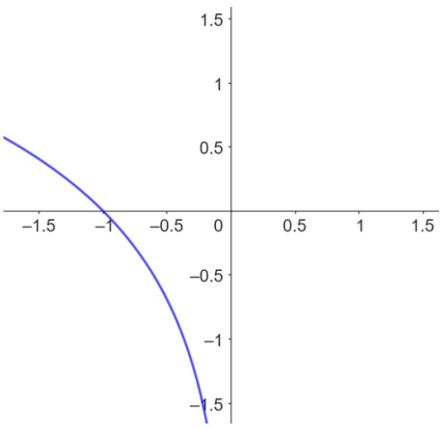
A.



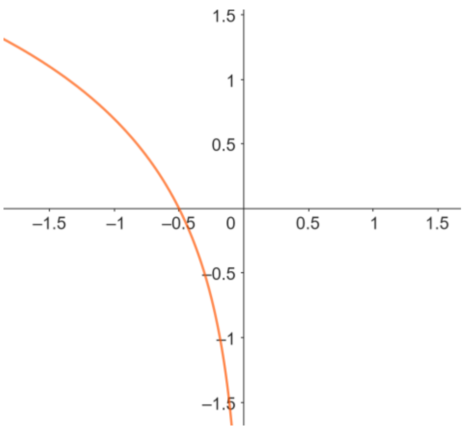
B.



C.



D.



5 It is given that $y = f(2x+3)$, where $f(7)=5$, $f(2)=3$, $f'(7)=-1$ and $f'(2)=2$.

What is the value of $y'(2)$?

A. -2

B. 4

C. 6

D. 10

6 A data set consists of 20 numbers. Its mean is X and its median is Y.

Each number in the data set is increased by 5.

What is the mean and median of the new data set?

A. mean = $X + 5$, median = Y

B. mean = $X + 5$, median = $Y + 5$

C. mean = $X + 0.4$, median = Y

D. mean = $X + 0.4$, median = $Y + 5$

7 What is the range of $y = x^3 - 3x^2$ for $-1 < x \le 1$?

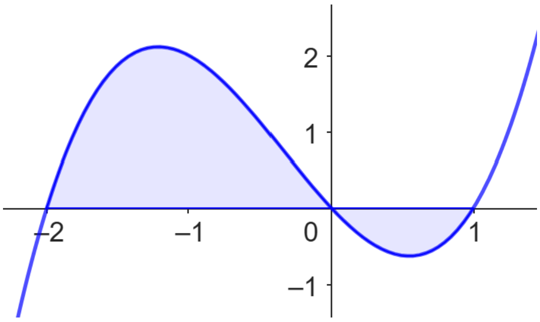
A. $[-4, 0]$

B. $(-4, 0]$

C. $(-4, 1]$

D. $(-4, -2]$

8 The graph of $y = f(x)$ is shown.



Which of the following gives the area of the shaded region?

A. $-\int_{-2}^{0} f(x)dx + \int_{0}^{1} f(x)dx$

B. $\int_{-2}^{1} f(x)dx$

C. $\int_{-2}^{0} f(x)dx + \int_{1}^{0} f(x)dx$

D. $\int_{-2}^{-1} f(x)dx - \int_{-1}^{1} f(x)dx$

9 A cylindrical tank with a radius of 3 metres is being filled with water. The height of the water in the tank, $h(t)$ in metres, is given by the function $h(t) = 5\sin(t) + 6$, where t is the time in hours.

What is the rate of change of the volume of water in the tank at $t = \frac{\pi}{2}$ hours?

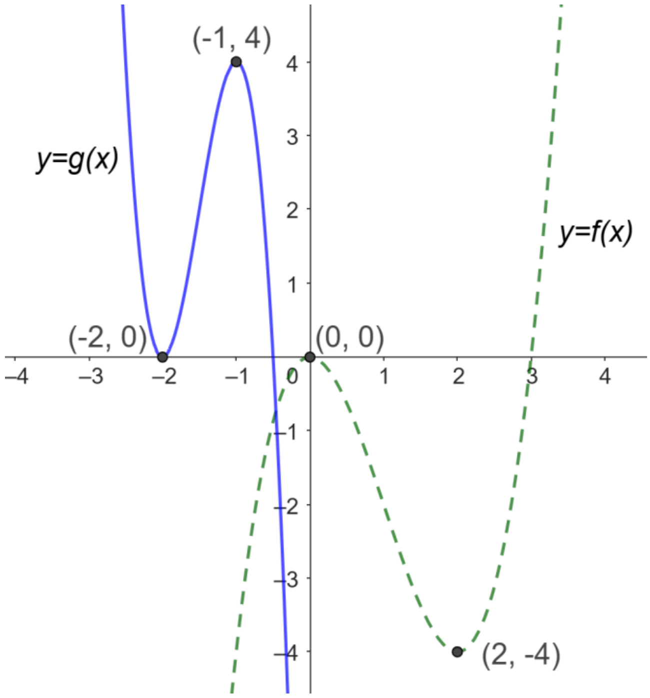
A. $0 \text{ m}^3/\text{h}$

B. $15\pi \text{ m}^3/\text{h}$

C. $15\sqrt{2}\pi \text{ m}^3/\text{h}$

D. $15\sqrt{3}\pi \text{ m}^3/\text{h}$

10 The graphs of $y = f(x)$ and $y = g(x)$ are shown. The function $g(x)$ can be obtained by applying a transformation to $f(x)$.



Which of the following is correct?

A. $g(x) = -f(x+4)$

B. $g(x) = -f(2x+4)$

C. $g(x) = f(-2x)+4$

D. $g(x) = f(-x+2)+4$

2024 | **YEAR 12 TRIAL EXAMINATION**

Mathematics Advanced

Section II
Answer Booklet - I

30 marks
Attempt Questions 11–20
Allow about 45 minutes for this booklet

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
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- Write your name above

Please turn over

Question 11 (2 marks)

A certain chemical is used to control the population growth of an invasive plant species in the city. The amount of the chemical in the soil decays exponentially according to the function $C = C_0 e^{kt}$, where C_0 is the initial amount of the chemical in kilograms, and t is the time in years. Initially, 50 kg of chemical is applied. After one year, 10 kg of chemical remains in the soil.

(a) Show that $k = \ln\left(\frac{1}{5}\right)$. **1**

(b) Find the amount of chemical in the soil after five years. **1**

Question 12 (2 marks)

Find the sum of the arithmetic series $-17 - 9 - 1 + 7 + 15 + ... + 495$. **2**

Question 13 (2 marks)

Find $\int \frac{x+2}{x^2+4x}\,dx$. **2**

Question 14 (2 marks)

Sam is measuring the width of a field at regular intervals along its length to estimate the total area of the field. The measurements are taken every 2 metres, and the table gives the width $w(x)$ in metres at each distance x metres from the starting point. **2**

x	0	2	4	6	8	10
$w(x)$	5	6	5.5	6.5	7	6.5

The area of the field is given by $\int_0^{10} w(x)\,dx$.

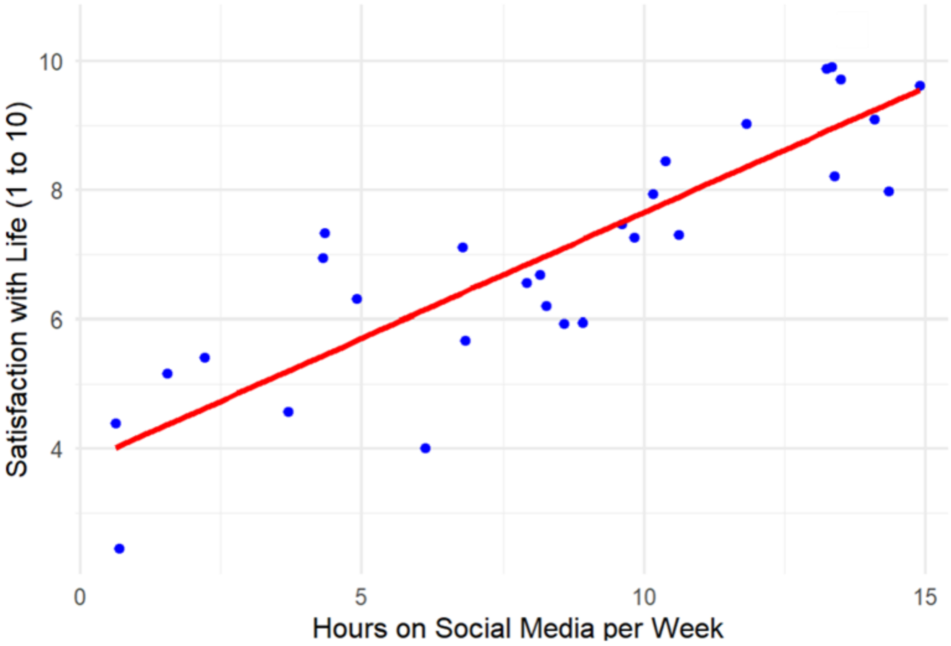
Use the trapezoidal rule and the width at each of the six distance values to find the approximate area of the field in square metres.

Question 15 (3 marks)

Taylor is researching the relationship between the number of hours teenagers spend on social media per week and their overall satisfaction with life, measured on a scale from 1 to 10. **3**

After collecting the data, Taylor finds that the correlation coefficient is 0.868.

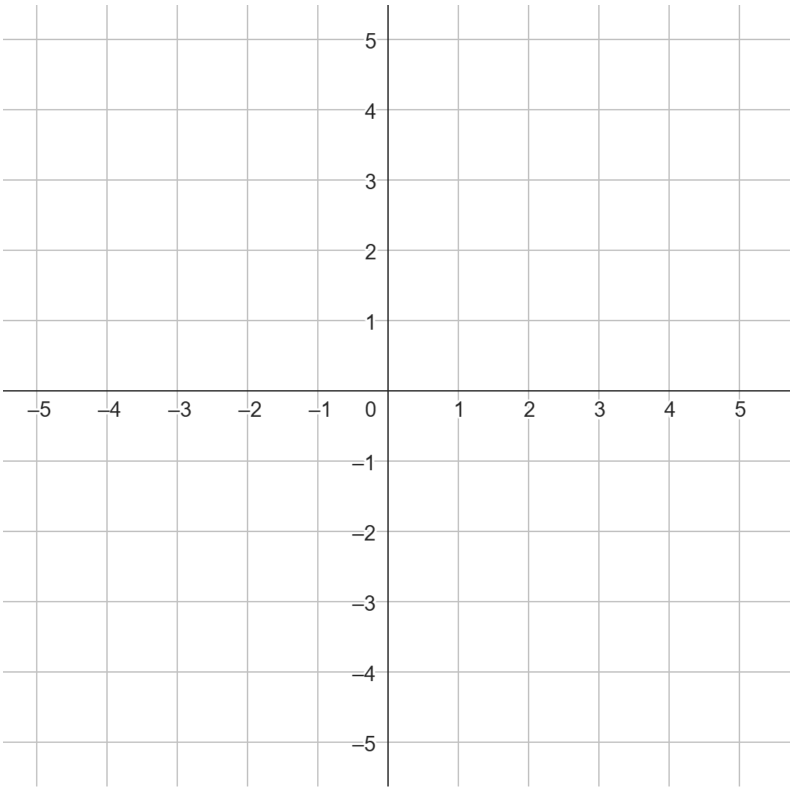
A scatterplot showing the data is drawn. The line of best fit with the equation, $y = 3.77 + 0.39x$ is also drawn.



Describe and interpret the data and other information provided, with reference to the context given.

Question 16 (5 marks)

(a) Sketch the graph of the functions $y = \dfrac{4}{x}$ and $y = |x+1| + 2$, showing coordinates of their intersection point and intercept(s). **4**



(b) Hence, or otherwise, solve $\dfrac{4}{x} \geq |x+1| + 2$. **1**

Question 17 (3 marks)

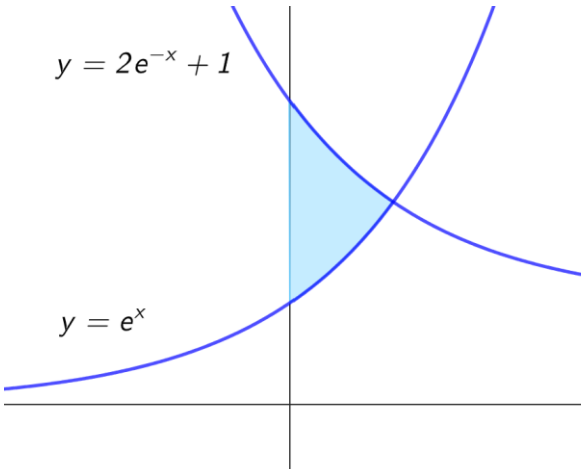
Find the values of x for which the series $1+(3x-5)+(3x-5)^2+(3x-5)^3+...$ has a limiting value. Hence, state all possible values of the limiting value if it exists. **3**

Question 18 (3 marks)

Show that $\log_e(1+\sin x)-\log_e(1-\sin x)=2\log_e(\sec x+\tan x)$. **3**

Question 19 (6 marks)

The curves, $y = e^x$ and $y = 2e^{-x} + 1$, intersect at one point as shown in the diagram.



(a) By solving an appropriate equation, show that the two curves intersect at $x = \ln 2$. **2**

(b) Find the exact area as shaded in the diagram. **3**

(c) State all values of *k* such that the equation $e^x + k = 2e^{-x} + 1$ has a positive solution. **1**

Question 20 (3 marks)

(a) Show that: $\dfrac{\sec^2 x}{\tan^2 x} = \operatorname{cosec}^2 x.$ **1**

(b) Hence, or otherwise evaluate $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\operatorname{cosec}^2 x)\,dx.$ Leave your answer in exact form. **2**

End of booklet 1

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student Name: ______________________________

2024 | **HSC TRIAL EXAMINATION**

Mathematics Advanced

Section II

Answer Booklet - II

30 marks
Attempt Questions 21–27
Allow about 60 minutes for this booklet

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
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Please turn over

Question 21 (2 marks)

Let $f(x)$ be an even function. If $\int_0^8 f(x)dx = 10$ and $\int_4^8 f(x)dx = -2$ evaluate $\int_{-1}^1 f(4x)dx$. **2**

Question 22 (5 marks)

(a) On the day that Alex was born, his grandmother deposited \$4000 into an account earning 4% per annum compounded annually. **2**

On each of Alex's birthdays after this, his grandmother deposited \$1200 into the same account, making her final deposit on Alex's 15th birthday. That is, a total of 16 deposits were made.

Let B_n be the amount in the account on Alex's nth birthday, after the deposit is made.

Show that $B_4 = \$9775.19$

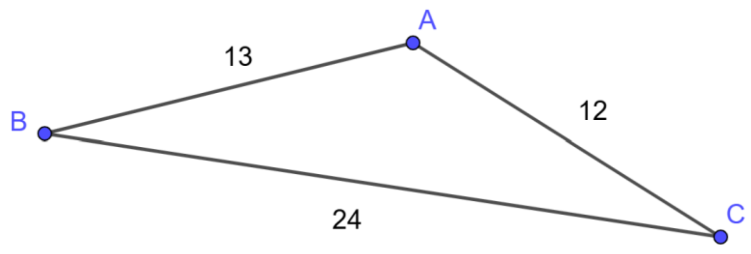
(b) On his 15th birthday, just after the final deposit is made, Alex has \$31 232.08 in his account. Alex then decides to leave all the money in the same account, continuing to earn interest at 4% per annum compounded annually. **3**

On his 16th birthday, and on each birthday after this, Alex withdraws \$2500 from the account.

How many withdrawals of \$2500 will Alex be able to make?

Question 23 (2 marks)

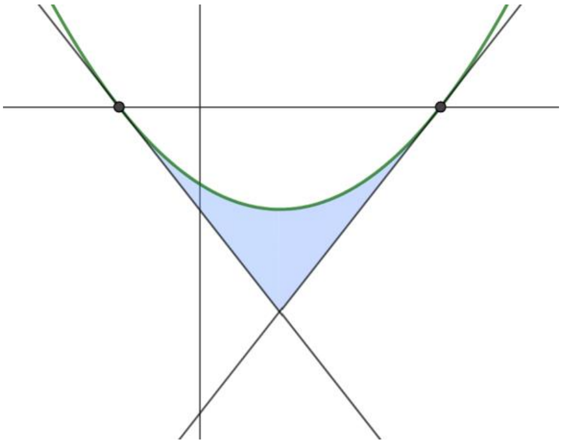
The diagram shows a triangle ABC where $AB = 13$, $AC = 12$ and $BC = 24$. **2**



Find the area of the triangle ABC. Give your answer correct to two decimal places.

Question 24 (7 marks)

The graph of $y = x^2 - 2x - 3$ and its tangents through x-intercepts are shown.



(a) Show the equations of the tangents are $y = 4x - 12$ and $y = -4x - 4$. **3**

(b) Find the shaded area between the parabola and its tangents. **4**

Question 25 (4 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table gives the probability that this random variable is less than *z* for different values of *z*.

z	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2
$P(Z < z)$	0.023	0.067	0.159	0.309	0.500	0.692	0.841	0.933	0.977

In a particular city, the monthly rent prices for apartments are normally distributed with a mean of \$1200 and a standard deviation of \$200.

(a) An apartment is chosen at random. Using the table, find the probability that its rent is between \$1000 and \$1500 per month. **2**

(b) Two apartments are chosen at random. Using the table, find the probability that at least one of them has rent less than \$1300 per month. Give your answer correct to three decimal places. **2**

Question 26 (7 marks)

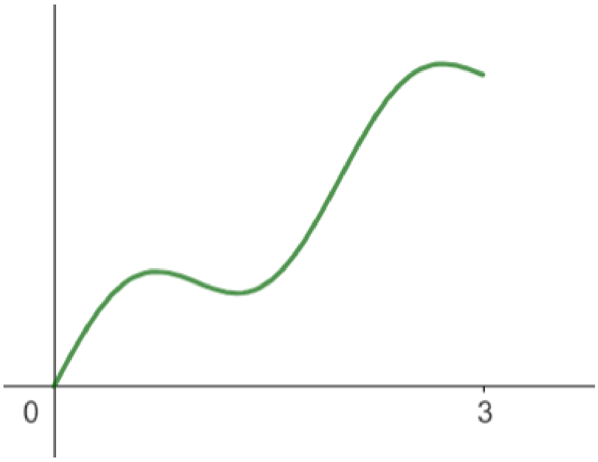
A continuous random variable X has the probability density function $f(x)$ given by

$$f(x)=\begin{cases} k\left(\sin(\pi x)+2x\right), & \text{for } 0 \le x \le 3 \\ 0, & \text{for all other values of } x \end{cases}$$

(a) Show that $k=\dfrac{\pi}{2+9\pi}$. **3**

Question 26 (continued)

(b) The graph of the density function is given. **4**



Find the mode of X. Give your answer correct to two decimal places.

Question 27 (3 marks)

Solve $2\log_2(x-6) = \log_2 x$. **3**

End of booklet 2

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student Name: ____________________

2024 | **HSC TRIAL EXAMINATION**

Mathematics Advanced

Section II

Answer Booklet - III

30 marks
Attempt Questions 28–32
Allow about 60 minutes for this booklet

Instructions

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Please turn over

Question 28 (5 marks)

(a) Show the equation of the tangent to the graph of $y = \ln(ax + a)$ at $x = 1$ is $y = \dfrac{x}{2} + \dfrac{2\ln(2a) - 1}{2}$. **3**

(b) Find all values of *a* such that the tangent from part (a) has a negative *y*-intercept. **2**

Question 29 (8 marks)

The water depth, y in metres, in a lake can be modelled by the function $y = 5 + 3\cos\left(\frac{\pi t}{6}\right)$, where t is the time in hours after 6:00 am.

(a) Determine the time between successive high tides as well as the maximum and minimum depth. **2**

(b) Sketch the graph of $y(t)$ for $0 \le t \le 24$. **3**

(c) A boat is to sail from a dock to a canal and then out to the main lake. An overhead bridge 7 metres above the bottom of the lake obstructs the boat's exit from the dock. The boat itself has a height of three metres from the waterline to its highest point. **3**

What is the earliest time that the boat can leave the dock? Give your answer correct to the nearest minute.

Question 30 (4 marks)

(a) Show that the function $f(x)=\dfrac{\sin x}{1+\cos x}$ is increasing for $0<x<\pi$. **2**

(b) Find in simplest exact form $\displaystyle\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{\sin x}{1+\cos x}\,dx$. **2**

Question 31 (8 marks)

Consider the function $f(x) = x^3\sqrt{2x+4}$.

(a) Find the domain of $f(x)$. **1**

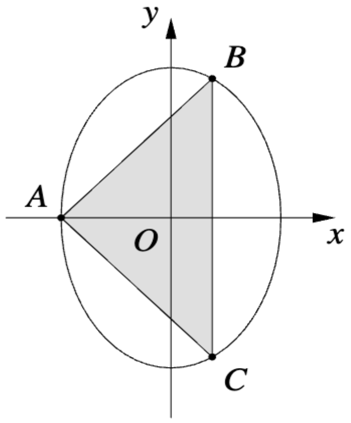
(b) Show that $f'(x) = \dfrac{x^2(7x+12)}{\sqrt{2x+4}}$. **3**

(c) State the coordinates and nature of the stationary points of f. **2**

(d) Sketch the graph of $y = f(x)$. Indicate coordinates of intercepts and stationary points. **2**

Question 32 (5 marks)

The diagram below shows the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$. The point A has coordinates $(-2,0)$. Points B (in the first quadrant) and C both lie on the ellipse such that the interval BC is vertical. Triangle ABC is shaded.



(a) Show that for points in the top half of the ellipse, $y=\sqrt{9-\frac{9}{4}x^2}$. **1**

(b) Hence, find the x-coordinate of B that maximises the area of triangle ABC. Justify your answer. **4**

End of EXAM

Section II extra writing space

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HSC
TRIAL EXAMINATION
2024

Mathematics Advanced

Name: Solutions

Maths Class:

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
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Section I – 10 marks (pages 1–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8–36)

- Attempt Questions 11–32
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
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Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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A. $(1,4)$

B. $[4,\infty)$

C. $(-\infty,4)$

D. $(1,4]$

2 What is $\int \frac{10}{(2x+3)^2}\,dx$?

A. $-5\ln(2x+3)+c$

B. $10\ln(2x+3)+c$

C. $\frac{-5}{2x+3}+c$

D. $\frac{-20}{(2x+3)^3}+c$

3 There are ten pieces fruit in a basket. Four are apples and six are oranges. Mia randomly picks a piece from the basket and eats it. Liam then randomly picks and eats one of the remaining fruits.

What is the probability that Liam ate an orange?

A. $\frac{3}{5}$

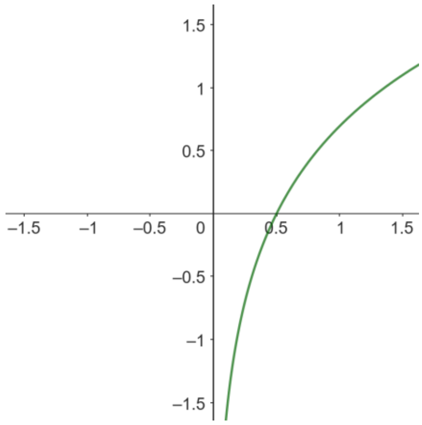
B. $\frac{4}{5}$

C. $\frac{27}{50}$

D. $\frac{22}{45}$

4 Which of the following best represents the graph of $y = \ln(-2x)$?

A.



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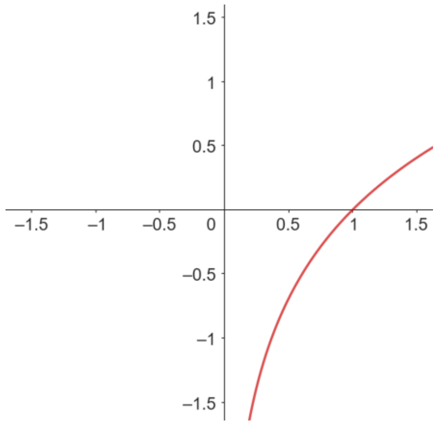
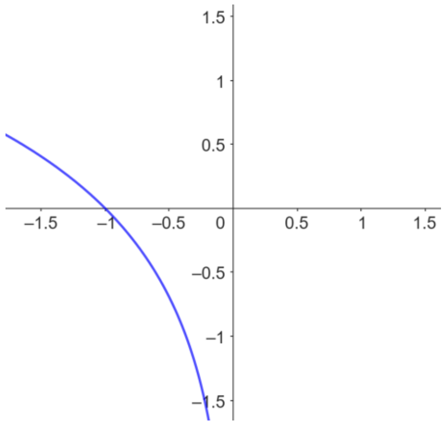
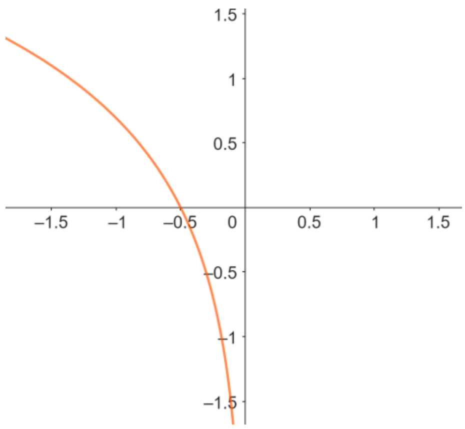


C.



D.



5 It is given that $y = f(2x+3)$, where $f(7) = 5$, $f(2) = 3$, $f'(7) = -1$ and $f'(2) = 2$.

What is the value of $y'(2)$?

A. -2

B. 4

C. 6

D. 10

6 A data set consists of 20 numbers. Its mean is X and its median is Y.

Each number in the data set is increased by 5.

What is the mean and median of the new data set?

A. mean $= X + 5$, median $= Y$

B. mean $= X + 5$, median $= Y + 5$

C. mean $= X + 0.4$, median $= Y$

D. mean $= X + 0.4$, median $= Y + 5$

7 What is the range of $y = x^3 - 3x^2$ for $-1 < x \leq 1$?

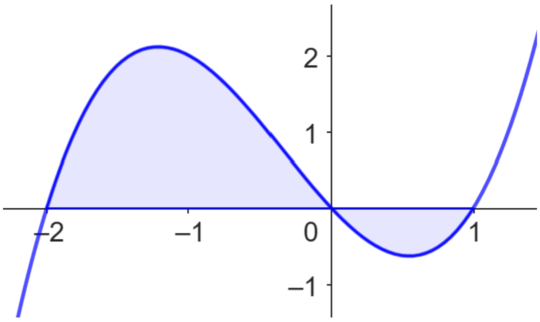
A. $[-4, 0]$

B. $(-4, 0]$

C. $(-4, 1]$

D. $(-4, -2]$

8 The graph of $y = f(x)$ is shown.



Which of the following gives the area of the shaded region?

A. $-\int_{-2}^{0} f(x)dx + \int_{0}^{1} f(x)dx$

B. $\int_{-2}^{1} f(x)dx$

C. $\int_{-2}^{0} f(x)dx + \int_{1}^{0} f(x)dx$

D. $\int_{-2}^{-1} f(x)dx - \int_{-1}^{1} f(x)dx$

9 A cylindrical tank with a radius of 3 metres is being filled with water. The height of the water in the tank, $h(t)$ in metres, is given by the function $h(t) = 5\sin(t) + 6$, where t is the time in hours.

What is the rate of change of the volume of water in the tank at $t = \frac{\pi}{2}$ hours?

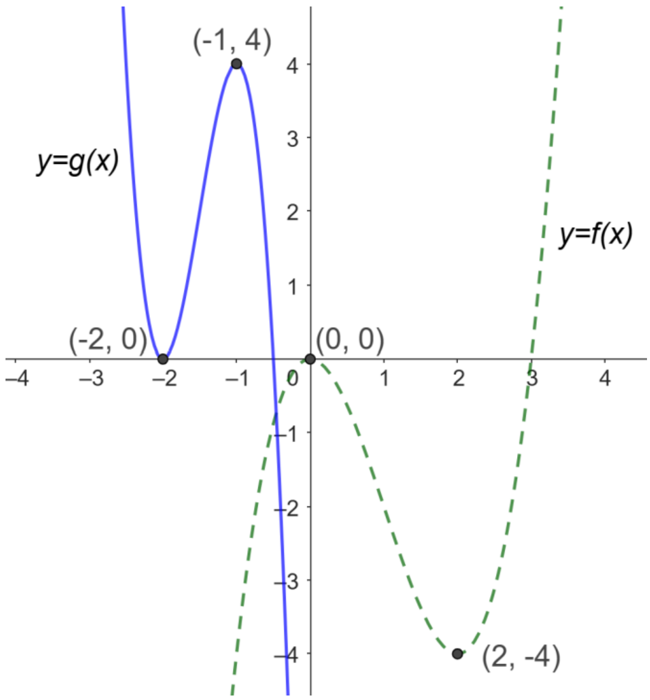
A. 0 m^3/h

B. 15π m^3/h

C. $15\sqrt{2}\pi$ m^3/h

D. $15\sqrt{3}\pi$ m^3/h

10 The graphs of $y = f(x)$ and $y = g(x)$ are shown. The function $g(x)$ can be obtained by applying a transformation to $f(x)$.



Which of the following is correct?

A. $g(x) = -f(x+4)$

B. $g(x) = -f(2x+4)$

C. $g(x) = f(-2x)+4$

D. $g(x) = f(-x+2)+4$

2024 | **YEAR 12 TRIAL EXAMINATION**

Mathematics Advanced

Section II
Answer Booklet - I

30 marks
Attempt Questions 11–20
Allow about 45 minutes for this booklet

Instructions

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- Write your name above

Please turn over

Question 11 (2 marks)

A certain chemical is used to control the population growth of an invasive plant species in the city. The amount of the chemical in the soil decays exponentially according to the function $C = C_0 e^{kt}$, where C_0 is the initial amount of the chemical in kilograms, and t is the time in years. Initially, 50 kg of chemical is applied. After one year, 10 kg of chemical remains in the soil.

(a) Show that $k = \ln\left(\frac{1}{5}\right)$. **1**

$C_0 = 50$ $\quad 10 = 50e^{k \cdot 1}$

$t = 1$ $\quad \frac{1}{5} = e^k$

$C = 10$

$\therefore \ln\left(\frac{1}{5}\right) = k$ (1)

(b) Find the amount of chemical in the soil after five years. **1**

$t = 5$ $\quad C = 50e^{k \cdot 5}$

$C = ?$

$C = 50e^{\ln\left(\frac{1}{5}\right)5}$

$C = 0.016\text{kg}$

or $C = 16\text{g}$ (1)

Question 12 (2 marks)

Find the sum of the arithmetic series $-17 - 9 - 1 + 7 + 15 + \ldots + 495$. **2**

$a_1 = -17$

$a_n = 495$ $\quad S_n = \frac{n}{2}(a + l)$

$d = 8$

$= \frac{65}{2}(-17 + 495)$

$a_n = a_1 + (n-1)d$ $\quad = 15535$ (1)

$495 = -17 + (n-1) \cdot 8$

(1)

$n = 65$

Question 13 (2 marks)

Find $\int \frac{x+2}{x^2+4x}\,dx$. **2**

$\frac{1}{2}\int \frac{2(x+2)}{x^2+4x}\,dx$ (1)

$f(x) = x^2+4x$

$f'(x) = 2x+4$

$= 2(x+2)$

$= \frac{1}{2}\ln|x^2+4x| + C$ (1)

Question 14 (2 marks)

Sam is measuring the width of a field at regular intervals along its length to estimate the total area of the field. The measurements are taken every 2 metres, and the table gives the width $w(x)$ in metres at each distance x metres from the starting point. **2**

x	0	2	4	6	8	10
$w(x)$	5	6	5.5	6.5	7	6.5
	y_0	y_1	y_2	y_3	y_4	y_5

$h=2$ $h=2$ $h=2$ $h=2$ $h=2$

The area of the field is given by $\int_0^{10} w(x)\,dx$.

Use the trapezoidal rule and the width at each of the six distance values to find the approximate area of the field in square metres.

$A = \frac{h}{2}\left[y_0 + 2\times(y_1+y_2+y_3+y_4) + y_5\right]$ (1)

$= \frac{2}{2}\left[5 + 2(6+5.5+6.5+7) + 6.5\right]$

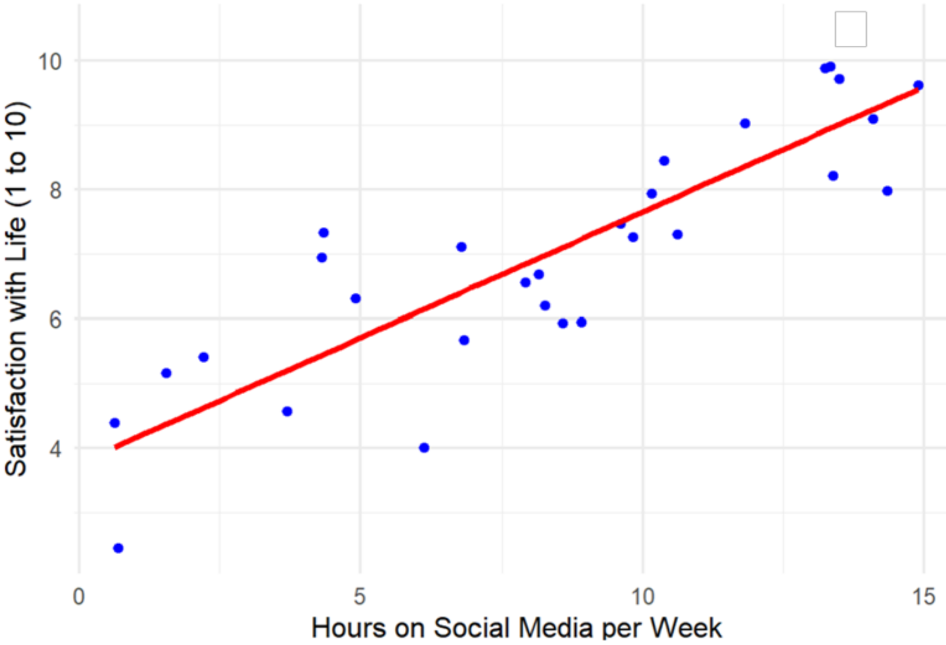
$= 61.5\ m^2$ (1)

Question 15 (3 marks)

Taylor is researching the relationship between the number of hours teenagers spend on social media per week and their overall satisfaction with life, measured on a scale from 1 to 10. **3**

After collecting the data, Taylor finds that the correlation coefficient is 0.868.

A scatterplot showing the data is drawn. The line of best fit with the equation, $y = 3.77 + 0.39x$ is also drawn.



Describe and interpret the data and other information provided, with reference to the context given.

1 • r= 0.868 Strong relationship between # of hours spend on social Median and life Satisfaction Index

1 • Lineal positive , Increase in one quantity means that the other quantity also increases.

1 • M=0.39 means for every one unit increas in hours on social media , Satisfaction Index Increases by 0.39

Question 16 (5 marks)

(a) Sketch the graph of the functions $y = \frac{4}{x}$ and $y = |x+1| + 2$, showing coordinates of their intersection point and intercept(s). **4**

$y = |x+1| + 2$

$x = 0$

$y = |0+1| + 2$

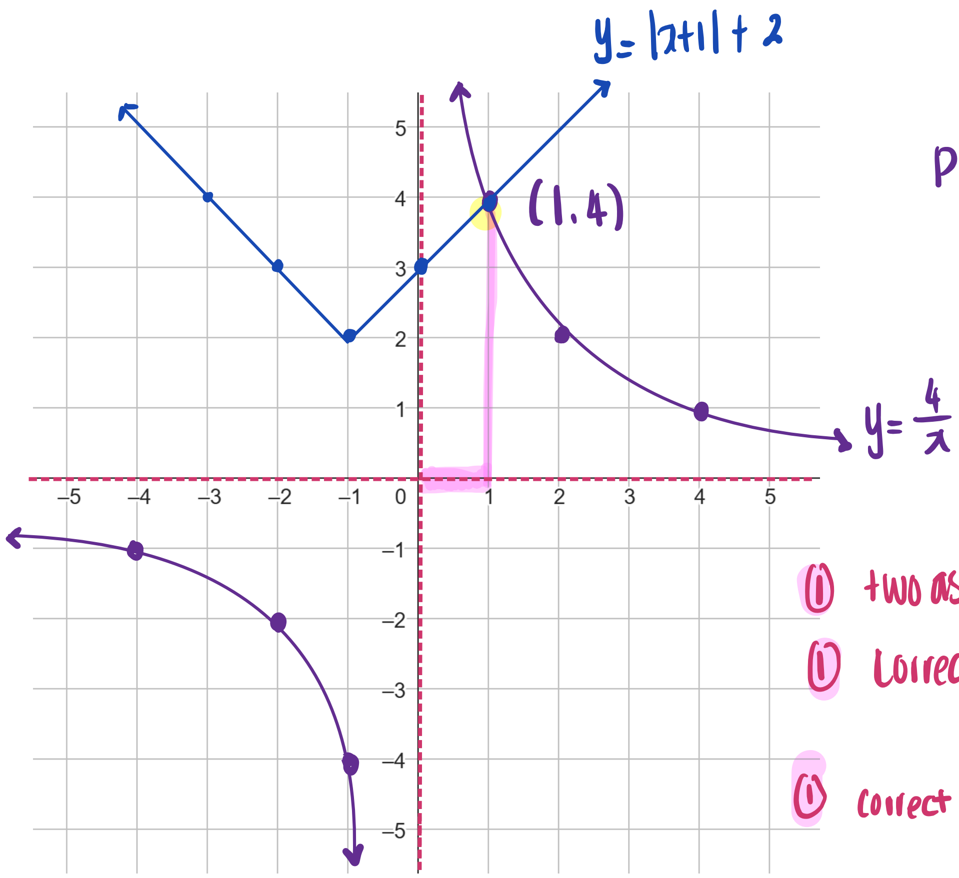
$y = 3$

$y = \frac{4}{x}$

$x \to +\infty \quad y \to 0^+$

$x \to -\infty \quad y \to 0^-$

asym at $y = 0$



POI (1, 4)

(1) two asymptotes

(1) Correct shape of $y = \frac{4}{x}$

(1) Correct shape of $y = |x+1| + 2$

(1) Correct y-int and key points

(b) Hence, or otherwise, solve $\frac{4}{x} \geq |x+1| + 2$. **1**

$0 < x \leq 1$ (1)

Question 17 (3 marks)

Find the values of x for which the series $1+(3x-5)+(3x-5)^2+(3x-5)^3+\dots$ has a limiting value. Hence, state all possible values of the limiting value if it exists. **3**

$r = 3x-5$

$|r|<1$

$|3x-5|<1$

$-1<3x-5<1$

$4<3x<6$

$\frac{4}{3}<x<2$ (1)

limiting value exist when $\frac{4}{3}<x<2$

$S_\infty = \frac{a}{1-r}$

$\therefore S_\infty = \frac{1}{1-(3x-5)}$

$S_\infty = \frac{1}{6-3x}$

$\frac{4}{3}<x<2$

$x \to \frac{4}{3}$ $S_\infty = \frac{1}{6-3(\frac{4}{3})} \to \frac{1}{2}$

$x \to 2$ $S_\infty = \frac{1}{0.0000\dots 1} \to \infty$

(1) any

$\therefore S_\infty > \frac{1}{2}$ (1)

Question 18 (3 marks)

Show that $\log_e(1+\sin x)-\log_e(1-\sin x) = 2\log_e(\sec x+\tan x)$. **3**

$LHS = \log_e \frac{1+\sin x}{1-\sin x}$

$= \log_e \frac{(1+\sin x)(1+\sin x)}{(1-\sin x)(1+\sin x)}$

$= \log_e \frac{(1+\sin x)^2}{1-\sin^2 x}$ (1)

$= \log_e \frac{(1+\sin x)^2}{\cos^2 x}$

$= \log_e \left(\frac{1+\sin x}{\cos x}\right)^2$ (1)

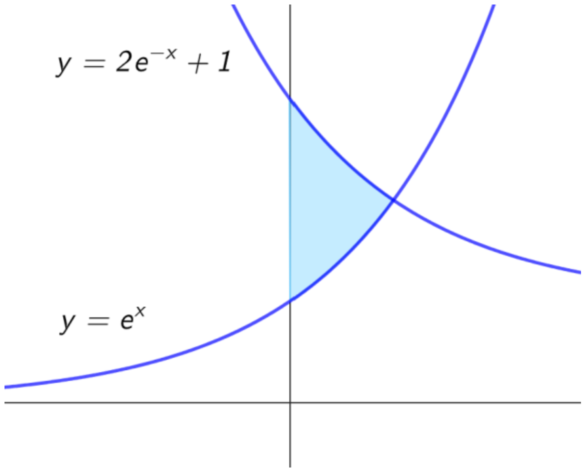
$= 2\log_e\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)$ (1)

$= 2\log_e(\sec x + \tan x)$

$= RHS$

Question 19 (6 marks)

The curves, $y = e^x$ and $y = 2e^{-x} + 1$, intersect at one point as shown in the diagram.



(a) By solving an appropriate equation, show that the two curves intersect at $x = \ln 2$. **2**

$y = e^x \quad \cdots ①$

$y = 2e^{-x} + 1 \quad \cdots ②$

$① = ②$

$e^x = 2e^{-x} + 1$

$\times e^x \quad \times e^x \quad \times e^x$

$(e^x)^2 = 2 + e^x$ 1

let $u = e^x$

$u^2 - u - 2 = 0$

$(u-2)(u+1) = 0$

$u = 2 \quad u = -1$

$2 = e^x$ or $-1 = e^x$

$\ln 2 = x$ 1 no sol^n

$\therefore$ two curves intersect at $x = \ln 2$

(b) Find the exact area as shaded in the diagram. **3**

$A = \int_0^{\ln 2} \left[(2e^{-x}+1) - (e^x) \right] dx$ 1

$= \int_0^{\ln 2} (2e^{-x}+1-e^x)\, dx$

$= \left[-2e^{-x} + x - e^x \right]_0^{\ln 2}$ 1

$= (-2e^{-\ln 2} + \ln 2 - \cancel{e}^{\cancel{\ln} 2}) - (-2e^0 + 0 - e^0)$

$= (-2\cancel{e}^{\cancel{\ln} 2^{-1}} + \ln 2 - 2) - (-2 - 1)$

$= (-\cancel{2} \cdot \frac{1}{\cancel{2}} + \ln 2 - 2) + 3$

$= -3 + \ln 2 + 3$ 1

$= \ln 2$

(c) State all values of k such that the equation $e^x + k = 2e^{-x} + 1$ has a positive solution. **1**

$y = e^x + k$ $y = 2e^{-x} + 1$

when $x = 0$ when $x = 0$

$y = 1 + k$ $y = 3$

when y-int of $y = 2e^{-x}+1$ > y-int of $y = e^x + k$ positive sol^n

$3 > 1 + k$

$2 > k$

$\therefore k < 2$ 1

Question 20 (3 marks)

(a) Show that : $\dfrac{\sec^2 x}{\tan^2 x} = \cos ec^2 x.$ **1**

$$LHS = \frac{\left(\frac{1}{\cos x}\right)^2}{\left(\frac{\sin x}{\cos x}\right)^2}$$

$$= \frac{1}{\cancel{\cos^2 x}} \times \frac{\cancel{\cos^2 x}}{\sin^2 x}$$

$$= \frac{1}{\sin^2 x}$$ 1

$$= \operatorname{cosec}^2 x$$

(b) Hence, or otherwise evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\cos ec^2 x)dx$. Leave your answer in exact form. **2**

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{cosec}^2 x \, dx$$

$f(x) = \tan x$

$f'(x) = \sec^2 x$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan^2 x} dx$$

f'(x) f(x) n

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x \cdot (\tan x)^{-2} \, dx$$

$$= \left[\frac{1}{-2+1} (\tan x)^{-2+1}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$ 1

$$= \left[-(\tan x)^{-1}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= -\frac{1}{(\tan\frac{\pi}{3})} - \left(-\frac{1}{(\tan\frac{\pi}{6})}\right)$$

$$= -\frac{1}{\sqrt{3}} + \frac{1}{\frac{1}{\sqrt{3}}}$$

$$= -\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{1}\frac{\times\sqrt{3}}{\times\sqrt{3}}$$

OR

$$= \frac{2}{\sqrt{3}}$$

1 any exact answer

End of booklet 1

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student Name: ______________________________

2024 | **HSC TRIAL EXAMINATION**

Mathematics Advanced

Section II

Answer Booklet - II

30 marks
Attempt Questions 21–27
Allow about 60 minutes for this booklet

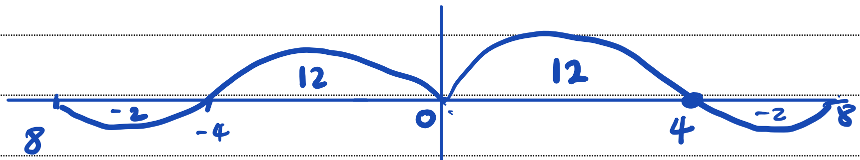
Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your name above

Please turn over

Question 21 (2 marks)

Let $f(x)$ be an even function. If $\int_0^8 f(x)dx = 10$ and $\int_4^8 f(x)dx = -2$ evaluate $\int_{-1}^1 f(4x)dx$. **2**



$\int_0^4 f(x) = \int_0^8 f(x)\,dx - \int_4^8 f(x)\,dx = 10-(-2) = 12$

even $\int_{-4}^4 f(x) = 2 \times 12 = 24$ } 1

$\int_{-1}^1 f(4x) = \frac{1}{4}\int_{-4}^4 f(x) = \frac{1}{4} \times 24 = 6$ 1

Question 22 (5 marks)

(a) On the day that Alex was born, his grandmother deposited \$4000 into an account earning 4% per annum compounded annually. **2**

On each of Alex's birthdays after this, his grandmother deposited \$1200 into the same account, making her final deposit on Alex's 15th birthday. That is, a total of 16 deposits were made.

Let B_n be the amount in the account on Alex's nth birthday, after the deposit is made.

Show that $B_4 = \$9775.19$

$B_0 = 4000$ $\quad r = 1.04$

$B_1 = 4000(1.04) + 1200$

$B_2 = B_1(1.04) + 1200$
$\quad = 4000(1.04)^2 + 1200(1.04) + 1200$

$B_3 = B_2(1.04) + 1200$
$\quad = 4000(1.04)^3 + 1200(1.04)^2 + 1200(1.04) + 1200$ } 1

$B_4 = B_3(1.04) + 1200$
$B_4 = 4000(1.04)^4 + 1200(1.04)^3 + 1200(1.04)^2 + 1200(1.04) + 1200$ 1
$\quad = 9775.19$

(b) On his 15th birthday, just after the final deposit is made, Alex has \$31 232.08 in his account. Alex then decides to leave all the money in the same account, continuing to earn interest at 4% per annum compounded annually. **3**

On his 16th birthday, and on each birthday after this, Alex withdraws \$2500 from the account.

How many withdrawals of \$2500 will Alex be able to make?

15th B'day $B_{16} = \$31\,232.08$

$\cancel{B_{17}}$ $A_1 = 31232.08(1.04) - 2500$

$A_2 = A_1(1.04) - 2500$

$= 31232.08(1.04)^2 - 2500(1.04) - 2500$

$A_3 = A_2(1.04) - 2500$

$= 31232.08(1.04)^3 - 2500(1.04)^2 - 2500(1.04)^1 - 2500$

$\vdots$

$A_n = 31232.08(1.04)^n - 2500(1 + 1.04 + \cdots + 1.04^{n-1})$ 1

$A_n = 0$

$$0 = 31232.08(1.04)^n - 2500\,\frac{1\cdot(1.04^n - 1)}{1.04 - 1}$$

$$0 = 31232.08(1.04)^n - 62500(1.04)^n + 62500$$ 1

$$\frac{31267.92(1.04)^n}{31267.92} = \frac{62500}{31267.92}$$

$$1.04^n = 1.99885377$$

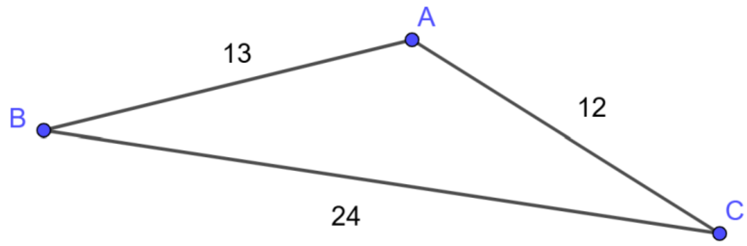
$$n = \frac{\ln(1.998\cdots)}{\ln(1.04)}$$

$n = 17.65837103$ 1

$\therefore n = 17$ $\therefore$ 17 withdrawals

Question 23 (2 marks)

The diagram shows a triangle ABC where $AB = 13, AC = 12$ and $BC = 24$. **2**



Find the area of the triangle ABC. Give your answer correct to two decimal places.

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \dfrac{13^2 + 12^2 - 24^2}{2 \times 13 \times 12}$

$A = \cos^{-1}\left(-\dfrac{263}{312}\right)$

$A = 147^\circ 27' 10.15$ 1

$A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 13 \times 12 \times \sin 147^\circ 27' 10.15$

$= 41.96352583$ 1

$= 41.96 \, u^2$

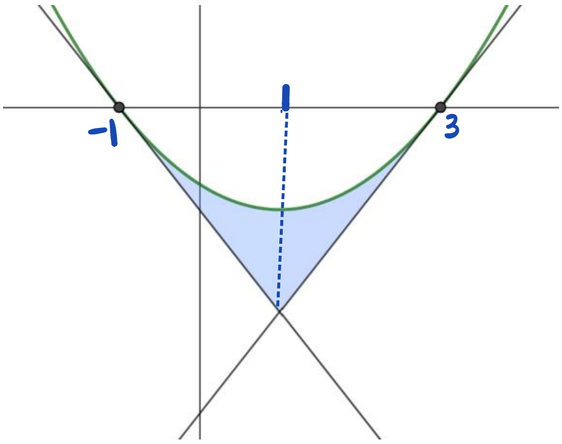
Question 24 (7 marks)

The graph of $y = x^2 - 2x - 3$ and its tangents through x-intercepts are shown.

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = 3$ or $x = -1$



(a) Show the equations of the tangents are $y = 4x - 12$ and $y = -4x - 4$. **3**

$y = x^2 - 2x - 3$

$y' = 2x - 2$

at $(-1, 0)$

$m = 2(-1) - 2$

$= -2 - 2$

$= -4$

(1)

$y - y_1 = m(x - x_1)$

$y - 0 = -4(x - -1)$

$y = -4x - 4$ (1)

at $(3, 0)$

$m = 2(3) - 2$

$m = 4$

$y - y_1 = m(x - x_1)$

$y - 0 = 4(x - 3)$

$y = 4x - 12$ (1)

(b) Find the shaded area between the parabola and its tangents. **4**

$y = 4x - 12 \quad \cdots \text{①}$

$y = -4x - 4 \quad \cdots \text{②}$

$\text{①} = \text{②}$

$4x - 12 = -4x - 4$

$8x = 8$

$x = 1$ ①

$A_1 = \int_{-1}^{1} (x^2 - 2x - 3 + 4x + 4)\, dx$

$= \int_{-1}^{1} (x^2 + 2x + 1)\, dx$

$= \left[\frac{x^3}{3} + \frac{2x^2}{2} + x \right]_{-1}^{1}$

$= \left[\frac{1}{3} + 1^2 + 1 \right] - \left[\frac{-1}{3} + (-1)^2 + (-1) \right]$

$= \frac{1}{3} + 1 + 1 + \frac{1}{3}$

$= \frac{8}{3}$ ①

$A_2 = \int_{1}^{3} (x^2 - 2x - 3 - 4x + 12)\, dx$

$= \int_{1}^{3} (x^2 - 6x + 9)\, dx$

$= \left[\frac{x^3}{3} - 3x^2 + 9x \right]_{1}^{3}$

$= (9 - 27 + 27) - \left(\frac{1}{3} - 3 + 9 \right)$

$= \frac{8}{3}$ ①

$\therefore$ Total Area $= \frac{8}{3} + \frac{8}{3}$ ①

$= \frac{16}{3}$

Question 25 (4 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table gives the probability that this random variable is less than z for different values of z.

z	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
$P(Z < z)$	0.023	0.067	0.159	0.309	0.500	0.692	0.841	0.933	0.977

In a particular city, the monthly rent prices for apartments are normally distributed with a mean of \$1200 and a standard deviation of \$200.

(a) An apartment is chosen at random. Using the table, find the probability that its rent is between \$1000 and \$1500 per month. **2**

$\mu = 1200$

$\sigma = 200$

$z = \frac{1000-1200}{200}$

$z = -1$

$z = \frac{1500-1200}{200}$

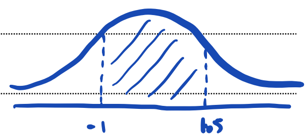
$z = 1.5$

(1)

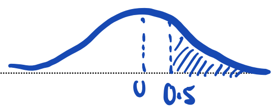
$P(-1 < z < 1.5)$

$= 0.933 - 0.159$

$= 0.774$ (1)



(b) Two apartments are chosen at random. Using the table, find the probability that at least one of them has rent less than \$1300 per month. Give your answer correct to three decimal places. **2**



$z = \frac{1300-1200}{200}$

$= 0.5$

$P(\text{at least one} < 1300)$

$= 1 - P(\text{rent} \geq 1300) \cdot P(\text{rent} \geq 1300)$

$= 1 - P(z > 0.5) \cdot P(z > 0.5)$ (1)

$= 1 - (1-0.692) \cdot (1-0.692)$

$= 0.905$ (1)

Question 26 (7 marks)

A continuous random variable X has the probability density function $f(x)$ given by

$$f(x)=\begin{cases} k\left(\sin(\pi x)+2x\right), & \text{for } 0\le x\le 3 \\ 0, & \text{for all other values of } x \end{cases}$$

(a) Show that $k=\dfrac{\pi}{2+9\pi}$. **3**

$$k\int_0^3 \left(\sin(\pi x)+2x\right)dx = 1$$

$$k\left[-\frac{1}{\pi}\cos(\pi x)+x^2\right]_0^3 = 1 \quad 1$$

$$\left[-\frac{1}{\pi}\cos(\pi\cdot 3)+3^2\right]-\left[-\frac{1}{\pi}\cos(0)+0^2\right]=\frac{1}{k}$$

$$\left(\frac{1}{\pi}+9\right)-\left(-\frac{1}{\pi}\right)=\frac{1}{k}$$

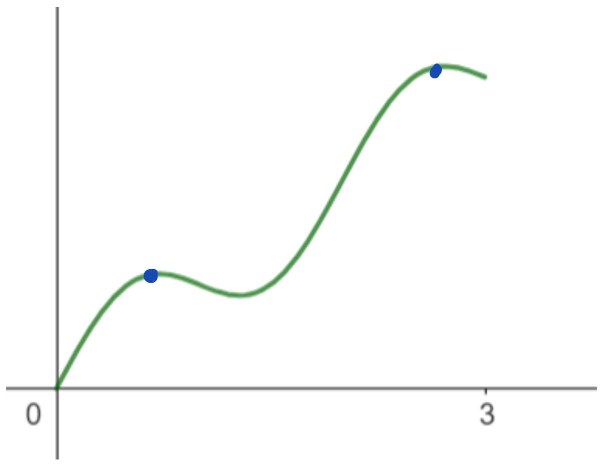
$$\frac{2}{\pi}+9=\frac{1}{k} \quad 1$$

$$\frac{2+9\pi}{\pi}=\frac{1}{k}$$

$$\therefore \quad k=\frac{\pi}{2+9\pi} \quad 1$$

Question 26 (continued)

(b) The graph of the density function is given. **4**



Find the mode of *X*. Give your answer correct to two decimal places.

$f(x) = k(\sin(\pi x) + 2x)$

$f'(x) = k(\pi\cos(\pi x) + 2)$ (1)

$f'(x) = 0 \rightarrow$ Stat pts

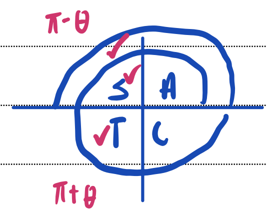
$0 = k(\pi\cos(\pi x) + 2)$

$-2 = \pi\cos(\pi x)$ (1)

$\frac{-2}{\pi} = \cos(\pi x)$

$0 \leq x \leq 3$

$0 \leq \pi x \leq 3\pi$



$\pi x = \cos^{-1}\left(\frac{2}{\pi}\right)$

$\pi x = 0.880689$

$\pi x = \pi - 0.880689,\ \pi + 0.880689,\ 2\pi + \pi - 0.880689$

$x = 0.71966,\ 1.28033,\ 2.719667\ldots$ (1)

From the graph it can be seen that the largest x gives the max $\therefore$ mode $= 2.72$. (1)

Question 27 (3 marks)

Solve $2\log_2(x-6) = \log_2 x$. **3**

$\log_2 (x-6)^2 = \log_2 x$

$(x-6)^2 = x$

$x^2 - 12x + 36 - x = 0$ 1

$x^2 - 13x + 36 = 0$

$(x-9)(x-4) = 0$

$x = 9$ or $x = 4$ 1

not valid as $(x-6) > 0$

$x > 6$

$\therefore x = 9$ only. 1

End of booklet 2

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student Name: ______________________________

2024 | **HSC TRIAL EXAMINATION**

Mathematics Advanced

Section II

Answer Booklet - III

30 marks
Attempt Questions 28–32
Allow about 60 minutes for this booklet

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your name above

Please turn over

Question 28 (5 marks)

(a) Show the equation of the tangent to the graph of $y = \ln(ax + a)$ at $x = 1$ is $y = \frac{x}{2} + \frac{2\ln(2a) - 1}{2}$. **3**

$y = \ln(ax + a)$

$f(x) = ax + a$

$f'(x) = a$

$y' = \frac{a}{ax + a}$

at $x = 1$

$m_{\text{tangent}} = \frac{a}{2a}$

$= \frac{1}{2}$ (1)

$m = \frac{1}{2}$ $(1, \ln(2a))$ (1)

$y - y_1 = m(x - x_1)$

$y - \ln 2a = \frac{1}{2}(x - 1)$

$y = \frac{x}{2} - \frac{1}{2} + \ln 2a$

$y = \frac{x}{2} + \frac{\ln 2a}{1} - \frac{1}{2}$ (1)

$y = \frac{x}{2} + \frac{2\ln(2a) - 1}{2}$

(b) Find all values of a such that the tangent from part (a) has a negative y-intercept. **2**

y-int let $x = 0$

$y = \frac{0}{2} + \frac{2\ln(2a) - 1}{2}$

$y = \frac{2\ln(2a) - 1}{2} < 0$

$2\ln(2a) - 1 < 0$

$\ln 2a < \frac{1}{2}$

$2a < e^{\frac{1}{2}}$ (1)

$a < \frac{1}{2}e^{\frac{1}{2}}$

Since $2a > 0$

$\therefore$

$0 < a < \frac{1}{2}e^{\frac{1}{2}}$ (1)

Question 29 (8 marks)

The water depth, y in metres, in a lake can be modelled by the function $y = 5 + 3\cos\left(\frac{\pi t}{6}\right)$, where t is the time in hours after 6:00 am.

(a) Determine the time between successive high tides as well as the maximum and minimum depth. ~~2~~ 3

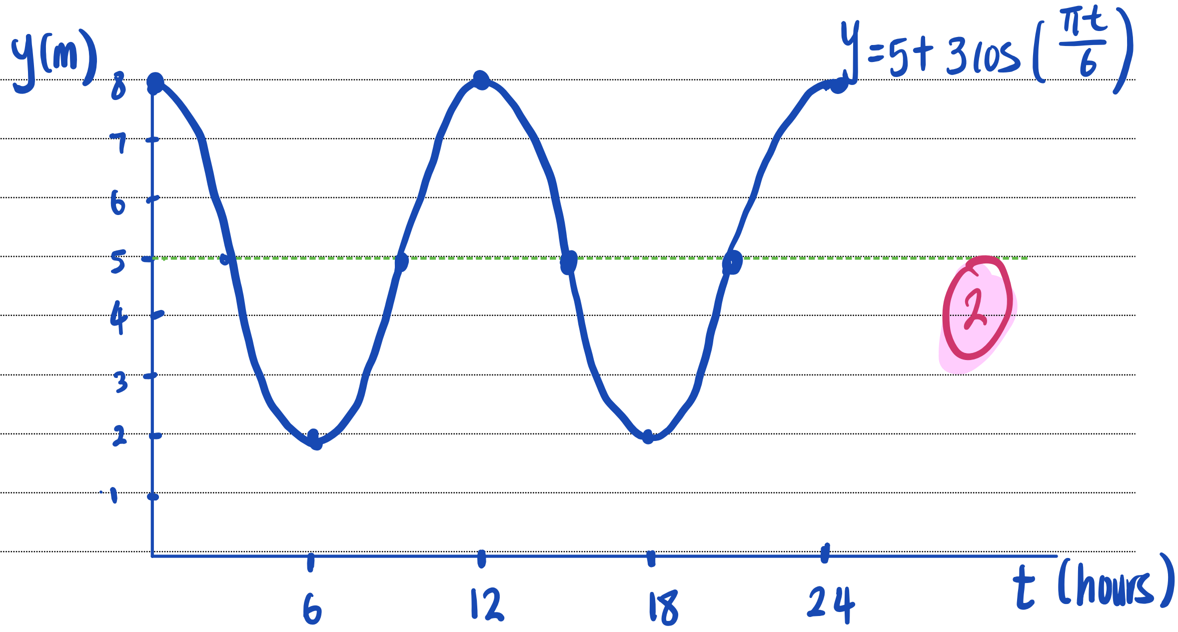
Period $\Rightarrow \dfrac{2\pi}{\frac{\pi}{6}}$

$= 2\pi \times \dfrac{6}{\pi} = 12$ hrs 1

max $= 5+3 = 8$ 1

min $= 5-3 = 2$ 1

(b) Sketch the graph of $y(t)$ for $0 \le t \le 24$. ~~3~~ 2



(c) A boat is to sail from a dock to a canal and then out to the main lake. An overhead bridge 7 metres above the bottom of the lake obstructs the boat's exit from the dock. The boat itself has a height of three metres from the waterline to its highest point. 3

What is the earliest time that the boat can leave the dock? Give your answer correct to the nearest minute.

$t = ?$

$y = 7-3$

$y = 4$

$4 = 5 + 3\cos\left(\frac{\pi t}{6}\right)$ 1

$\cos\left(\frac{\pi t}{6}\right) = -\frac{1}{3}$

$\frac{\pi t}{6} = 1.910633236$

$t = 3.649040688$ 1

3 hrs 39 min from 6:00 am.

6 + 3 hr 39 min

= 9:39 am 1

Question 30 (4 marks)

(a) Show that the function $f(x) = \dfrac{\sin x}{1+\cos x}$ is increasing for $0 < x < \pi$. **2**

$u = \sin x \qquad v = 1 + \cos x$

$u' = \cos x \qquad v' = -\sin x$

$$f'(x) = \frac{\cos x(1+\cos x) + \sin^2 x}{(1+\cos x)^2}$$

$$f'(x) = \frac{\cos x + 1}{(1+\cos x)^{\cancel{2}}}$$

$$f'(x) = \frac{1}{1+\cos x} \quad (1)$$

increasing $f'(x) > 0$

For $0 < x < \pi$

$-1 < \cos x < 1$

$+1 \quad +1 \quad +1$

$\therefore \ 0 < 1+\cos x < 2 \quad (1)$

$\therefore \ f'(x) = \dfrac{1}{1+\cos x} > 0$

increasing for $0 < x < \pi$.

(b) Find in simplest exact form $\displaystyle\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{\sin x}{1+\cos x}\,dx$. **2**

$$\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{\sin x}{1+\cos x}\,dx$$

$$= -\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{-\sin x}{1+\cos x}\,dx$$

$f'(x)$

$f(x)$

$$= -\Big[\ln|1+\cos x|\Big]_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \quad (1)$$

$$= -\left[\ln\left|1+\cos\tfrac{2\pi}{3}\right| - \ln\left|1+\cos\tfrac{\pi}{3}\right|\right]$$

$$= -\left[\ln\tfrac{1}{2} - \ln\tfrac{3}{2}\right]$$

$$= -\ln\tfrac{1}{2} + \ln\tfrac{3}{2}$$

$$= \ln\left(\tfrac{3}{2} \div \tfrac{1}{2}\right) = \ln 3 \quad (1)$$

Question 31 (8 marks)

Consider the function $f(x) = x^3\sqrt{2x+4}$.

(a) Find the domain of $f(x)$. **1**

D: $2x+4 \geq 0$

$2x \geq -4$

$x \geq -2$ 1

(b) Show that $f'(x) = \dfrac{x^2(7x+12)}{\sqrt{2x+4}}$. **3**

$u = x^3$ $\quad v = (2x+4)^{\frac{1}{2}}$

$u' = 3x^2$ $\quad v' = \frac{1}{2}(2x+4)^{-\frac{1}{2}} \times 2$

$v' = \dfrac{1}{\sqrt{2x+4}}$

$f'(x) = \dfrac{x^3}{\sqrt{2x+4}} + 3x^2\sqrt{2x+4}$ 1

$f'(x) = \dfrac{x^3 + 3x^2(2x+4)}{\sqrt{2x+4}}$

$f'(x) = \dfrac{x^3 + 6x^3 + 12x^2}{\sqrt{2x+4}}$

1

$= \dfrac{7x^3 + 12x^2}{\sqrt{2x+4}} = \dfrac{x^2(7x+12)}{\sqrt{2x+4}}$ 1

(c) State the coordinates and nature of the stationary points of *f*. **2**

$f'(x)=0 \rightarrow$ stat pts

$$0 = \frac{x^3(7x+12)}{\sqrt{2x+4}}$$

$$0 = x^3(7x+12)$$

$\begin{pmatrix} x=0 \\ y= \end{pmatrix}$ $\begin{pmatrix} x=-\frac{12}{7} \\ y=\frac{-3456\sqrt{7}}{2401} \\ y \approx -3.808 \end{pmatrix}$

hor inf (1)

max tp. (1)

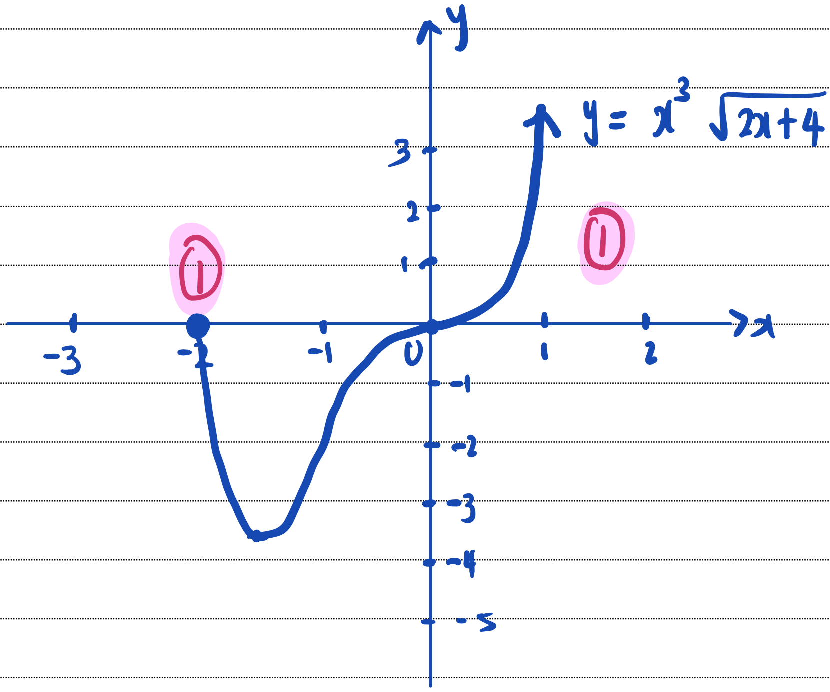
x	-1	0	1
$f'(x)$	$+$	0	$+$

hor inf

x	1	$-\frac{12}{7}$	2
$f'(x)$	$+$	0	$-$

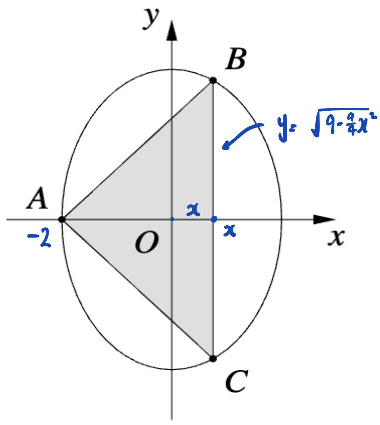
max

(d) Sketch the graph of $y=f(x)$. Indicate coordinates of intercepts and stationary points. **2**



Question 32 (5 marks)

The diagram below shows the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$. The point A has coordinates $(-2,0)$. Points B (in the first quadrant) and C both lie on the ellipse such that the interval BC is vertical. Triangle ABC is shaded.



(a) Show that for points in the top half of the ellipse, $y=\sqrt{9-\frac{9}{4}x^2}$. **1**

$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

$$\frac{y^2}{9}=1-\frac{x^2}{4}$$

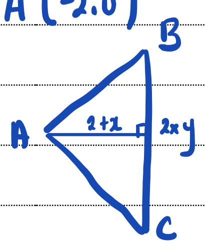
$$y^2=9-\frac{9x^2}{4}$$

$$y=\sqrt{9-\frac{9}{4}x^2}$$

1

(b) Hence, find the x-coordinate of B that maximises the area of triangle ABC. Justify your answer. **4**

$A(-2,0)$



$$\text{Area } ABC=\frac{1}{2}bh$$

$$=\frac{1}{\cancel{2}}\cdot\cancel{2}\cdot\sqrt{9-\frac{9}{4}x^2}\cdot(x+2)$$

$$A(x)=(x+2)\sqrt{9-\frac{9}{4}x^2}$$ 1

$u=x+2$ $\quad v=(9-\frac{9}{4}x^2)^{\frac{1}{2}}$

$u'=1$ $\quad v'=\frac{1}{2}(9-\frac{9}{4}x^2)^{-\frac{1}{2}}\cdot(\cancel{2}\,\frac{9}{4_2}x)$

$$v'=-\frac{9}{4}x(9-\frac{9}{4}x^2)^{-\frac{1}{2}}$$

$$v'=-\frac{9}{4}x\,\frac{1}{\sqrt{9-\frac{9}{4}x^2}}$$

$$v'=\frac{-9x}{4\sqrt{9-\frac{9}{4}x^2}}$$

$$A'(x)=\frac{-9x(x+2)}{4\sqrt{9-\frac{9}{4}x^2}}+\frac{\sqrt{9-\frac{9}{4}x^2}}{1}$$

$$A'(x)=\frac{-9x(x+2)+4(9-\frac{9}{4}x^2)}{4\sqrt{9-\frac{9}{4}x^2}}$$

$A'(x)=0 \rightarrow$ stat pts 1

$$0=-9x(x+2)+4(9-\frac{9}{4}x^2)$$

$$0=-9x^2-18x+36-9x^2$$

$$0=18x^2+18x-36$$

$$0=x^2+x-2$$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x = -2$ or $x = 1$

not valid as $x > 0$ (1) $\therefore x = 1$

test

x	0.9	1	1.1
$A'(x)$	+	0	−

+ / 0 \ −

$\therefore$ max at $x = 1$ (1)

End of EXAM

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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