

Name _____

ASCHAM SCHOOL
MATHEMATICS TRIAL EXAMINATION 2013

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen. Black pen is preferred.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- Show all necessary working in Questions 11–16.

Total marks – 100**Section I 10 marks**

- Attempt Questions 1–10 using the Multiple Choice sheet.
- Allow about 15 minutes for this section.

Section II 90 marks

- Attempt Questions 11–16.
- Allow about 2 hours 45 minutes for this section.
- Do each question in a separate booklet.
- Write your name/number and your teacher's name on each booklet.
- Clearly label the front of each booklet with the number of the question.

Collection

- Start each question of Section II in a new booklet.
- If you use a second booklet for a question, place it inside the first.
Indicate on the outside of the first booklet that you have used two booklets for that question.
- Write your name/number, teacher's name and question number on each booklet.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet at the back of this exam paper for Questions 1 – 10

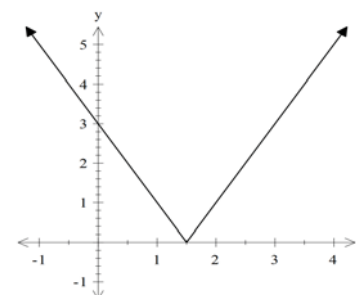
1 Evaluate $\sqrt[3]{\frac{3}{7}}$ to three significant figures.

- (A) 1.087
 (B) 1.09
 (C) 1.508
 (D) 1.51

2 The first and last terms of an arithmetic series are 10 and 60.
 If the sum of the series is 3535, how many terms are there in the series?

- (A) 11
 (B) 101
 (C) 110
 (D) 51

3 What is the equation of the graph drawn below?



- (A) $y = |2x + 3|$
 (B) $y = |2x - 3|$
 (C) $y = |x - 1.5|$
 (D) $y = |x + 1.5|$

4 The perimeter of a sector is 30cm. If the angle at the centre is 3 radians, what is the radius of the circle?

- (A) 10cm
- (B) 20cm
- (C) 6cm
- (D) 3cm

5 Find $\int \frac{1}{x^2} dx$

- (A) $\log(x^2) + c$
- (B) $\frac{-2}{x^3} + c$
- (C) $\frac{-1}{x} + c$
- (D) $2x \log(x^2) + c$

6 What is the equation of a parabola with focus (2, 3) and directrix $y = -5$?

- (A) $(x+1)^2 = 16(y-2)$
- (B) $(x-2)^2 = 16(y+1)$
- (C) $(x-2)^2 = 4(y-1)$
- (D) $(y+1)^2 = 4(x-2)$

7 What is the limiting sum of the series $-\frac{1}{27} + \frac{1}{9} - \frac{1}{3}$?

- (A) $\frac{-1}{108}$
- (B) $\frac{1}{54}$
- (C) Can't be found
- (D) $\frac{1}{108}$

8 What is the derivative of $\frac{4}{3x^3}$?

- (A) $-\frac{4}{x^4}$
- (B) $-\frac{2}{3x^2}$
- (C) $-\frac{4}{x^2}$
- (D) $-\frac{36}{x^4}$

9 $(2\sqrt{3}-5)^2$ is equal to

- (A) $1-20\sqrt{3}$
- (B) 37
- (C) $37-20\sqrt{3}$
- (D) $37-10\sqrt{3}$

- 10** What is the compound interest on \$1000 invested for 5 years at 6% per annum interest, compounded monthly? (to nearest dollar)
- (A) \$1338
- (B) \$349
- (C) \$1349
- (D) \$1025

End of Multiple Choice

Section II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet

- a) Factorise $2x^2 + 3x - 20$. (1)
- b) Find the exact value of $\cos 210^\circ$. (1)
- c) Find $\int \frac{2}{e^{2x}} dx$. (1)
- d) One of the roots of the equation $kx^2 - 2x - 3 = 0$ is -3 . Find the value of k . (1)
- e) Differentiate $\sin^2 2x$. (2)
- f) Solve $|2x - 3| \leq 5$. (2)
- g) Find $\int \frac{3x}{x^2 - 4} dx$. (2)
- h) Solve for x correct to 2 significant figures:
 $(\log_e 2x)^2 = 16$ (3)
- i) Is $f(x) = (x - x^5)^2$ odd or even? Show all working. (2)

End of question 11

Question 12 (15 marks) Use a new booklet

a) Differentiate with respect to x :

i) $\frac{e^x}{\log_e 2x}$ (3)

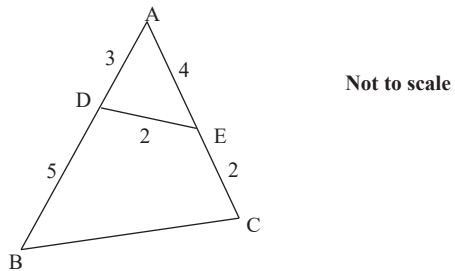
ii) $\log[x^4(x-1)^2]$ (2)

b) Find the primitive of $\cos(\frac{1-x}{-5})$. (1)

c) Find, in general form, the equation of the tangent to the curve $y = x \ln x$ at the x intercept. (3)

d) If α and β are the roots of the equation $2x^2 - 3x + 4 = 0$, find the value of $\alpha^2 + \beta^2$. (3)

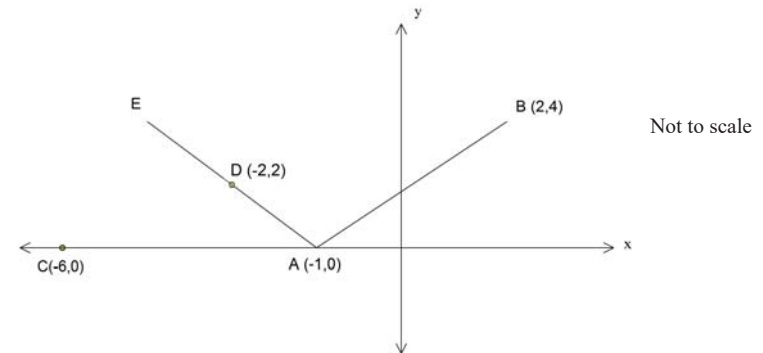
e) By proving 2 triangles similar, find the length of BC. (3)



End of question 12

Question 13 (15 marks) Start a new booklet.

a) A, B and C are the points $(-1,0)$, $(2,4)$ and $(-6,0)$ respectively. D is the point $(-2,2)$ and is the midpoint of AE.



i) Find the length of the interval AB (2)

ii) Find the midpoint of BC (1)

iii) Find the coordinates of E (2)

iv) What type of quadrilateral is ABEC? Give a clear explanation for your answer. (2)

b) Find the equation of the locus of the point $P(x,y)$ which moves so that it is twice the distance from $R(-3,4)$ as it is from $S(-1,2)$. (4)

c) The slope at any point on a curve is given by $3\sec^2 2x$. Find the equation of the curve if it passes through the point $(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$. (4)

End of question 13

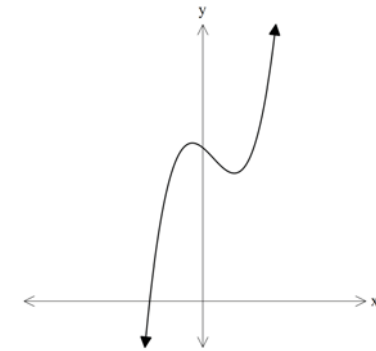
Question 14 (15 marks) Start a new booklet.

- a) For what values of k will the equation $9x^2 - kx + 1 = 0$ have real and different roots? (3)
- b) Evaluate $\int_0^{\pi} (1 + \sin 2\pi x) dx$. (3)
- c) i) Sketch the function $y = 4\cos 2x$ for $0 \leq x \leq \pi$. (2)
- ii) Find the area between the curve $y = 4\cos 2x$ and the x axis from $x = 0$ to $x = \frac{\pi}{2}$. (3)
- d) A tank contains 50 litres of water. A tap at the base of the tank allows water to flow out at a rate proportional to the quantity of water still in the tank at that time. After 2 minutes, 10 litres have run out.
- Use the equation $W = W_0 e^{-kt}$, where W is the amount of water in the tank and t is time in minutes.
- i) Show that $k = -\frac{1}{2} \ln \frac{4}{5}$. (2)
- ii) How much water has run out after 10 minutes? (To the nearest litre) (2)

End of question 14

Question 15 (15 marks) Start a new booklet

- a) The graph of $y = x^3 - x^2 - x + 6$ is sketched below.



Not to scale

- i) Find the coordinates of the stationary points. (3)
- ii) Find any point(s) of inflexion (2)
- iii) For what values of x is the curve decreasing and concave up? (1)
- iv) For what values of p has the equation $x^3 - x^2 - x + 6 = p$ exactly two real solutions? (2)
- b) Use Simpson's Rule with 5 function values to evaluate, to 2 decimal places, $\int_1^3 \log_e x dx$. (3)
- c) A particle moves in a straight line such that its position at time t seconds is given by $x = t - \log_e t$.
- i) When is the particle at rest? (1)
- ii) Find the exact distance travelled by the particle between $t = \frac{1}{2}$ and $t = 1\frac{1}{2}$. (3)

End of Question 15

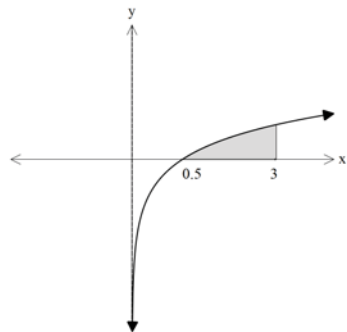
Question 16 (15 marks) Start a new booklet

- a) Diana borrows \$10 000 and arranges to pay it back with interest in 20 equal instalments every three months over 5 years. She is charged 6% per annum interest compounded monthly.

Let A_n be the amount owing after n months and let M be the instalment.

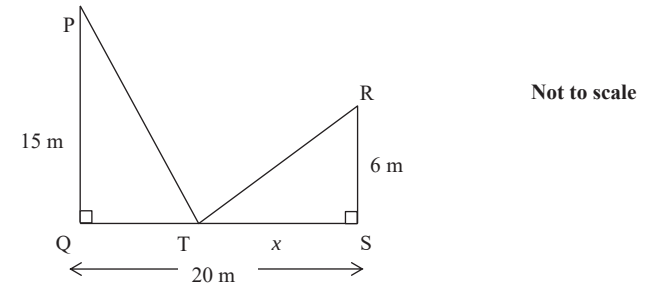
- Find the amount owing after the first three months, just after she has made her first payment. (1)
- Show that $M = \frac{10000(1.005^{63} - 1.005^{60})}{1.005^{60} - 1}$. (3)
- Find the size of the instalment she pays. (1)
- How much does Diana pay in interest? (1)

- b) The graph of $y = \log_e 2x$ is given below.



Find the volume when the shaded region between the curve, the x axis and the line $x = 3$ is rotated about the y axis. (3)

- c)



Two poles, PQ and RS are 20 metres apart. PQ is 15 metres high and RS is 6 metres high. A length of wire is attached to the top of each pole and also staked to the ground at T somewhere between the two poles.

Let $TS = x$ metres.

- Show that the length of wire $L = \sqrt{36 + x^2} + \sqrt{625 - 40x + x^2}$. (2)
- Find the shortest length of wire that can be used. (4)

End of exam

Standard Integrals

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

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Student Number: _____

Name: _____

SECTION I Mathematics Multiple Choice Answer Sheet

10 Marks

This sheet must be handed in separately. Detach it from the question paper.

Shade the correct answer:

- | | | | | |
|-----|-------------------------|-------------------------|-------------------------|-------------------------|
| 1. | A ☐ | B ☐ | C ☐ | D ☐ |
| 2. | A ☐ | B ☐ | C ☐ | D ☐ |
| 3. | A ☐ | B ☐ | C ☐ | D ☐ |
| 4. | A ☐ | B ☐ | C ☐ | D ☐ |
| 5. | A ☐ | B ☐ | C ☐ | D ☐ |
| 6. | A ☐ | B ☐ | C ☐ | D ☐ |
| 7. | A ☐ | B ☐ | C ☐ | D ☐ |
| 8. | A ☐ | B ☐ | C ☐ | D ☐ |
| 9. | A ☐ | B ☐ | C ☐ | D ☐ |
| 10. | A ☐ | B ☐ | C ☐ | D ☐ |

2013 Y12 MATHEMATICS TRIALSECTION IMULTIPLE CHOICE

1. D
2. B
3. B
4. C
5. C
6. B
7. C
8. A
9. C
10. B

QUESTION 11

a) $2x^2 + 3x - 20 = (2x - 5)(x + 4)$ ①

b) $\cos 210^\circ = -\cos 30^\circ$
 $= -\frac{\sqrt{3}}{2}$ ①

c) $\int \frac{2}{e^{2x}} dx = \int 2e^{-2x} dx$
 $= \frac{2e^{-2x}}{-2} + C$
 $= -\frac{1}{e^{2x}} + C$ ①

d) $kx^2 - 2x - 3 = 0$

$x = 3:$

$9k + 6 - 3 = 0$

$9k = -3$

$k = -\frac{1}{3}$ ①

e) $\frac{d}{dx} \sin^2 2x = \frac{d}{dx} (\sin 2x)^2$

$= 2 \sin 2x \cos 2x \times 2$

$= 4 \sin 2x \cos 2x$ ②

f) $|2x - 3| \leq 5$

$-5 \leq 2x - 3 \leq 5$

$-2 \leq 2x \leq 8$

$-1 \leq x \leq 4$ ②

g) $\int \frac{3x}{x^2 - 4} dx = \frac{3}{2} \int \frac{2x}{x^2 - 4} dx$ ②
 $= \frac{3}{2} \log_e (x^2 - 4) + C$

h) $(\log_e 2x)^2 = 16$
 $\log_e 2x = \pm 4$
 $2x = e^4$ or $2x = e^{-4}$
 $x = \frac{e^4}{2}$ or $\frac{e^{-4}}{2} = \frac{1}{2e^4}$ ③
 $= 27$ or 9.2×10^{-3} (2sf)

i) $f(x) = (x - x^5)^2$
 $f(-x) = (-x - (-x)^5)^2$
 $= (-x + x^5)^2$ ②
 $f(x) = f(-x)$ (b/c squared)
 $\therefore$ even

QUESTION 12

a) i) $\frac{d}{dx} \frac{e^x}{\log_e 2x}$

$$= \frac{(\log_e 2x)(e^x) - e^x \cdot \frac{1}{2x} \times 2}{(\log_e 2x)^2}$$

$$= \frac{e^x \log_e 2x - \frac{e^x}{x}}{(\log_e 2x)^2}$$

$$= \frac{e^x (x \log_e 2x - 1)}{x (\log_e 2x)^2} \quad (3)$$

ii) $\frac{d}{dx} \log [x^4 (x-1)^2]$

$$= \frac{d}{dx} (\log x^4 + \log (x-1)^2)$$

$$= \frac{d}{dx} (4 \log x + 2 \log (x-1))$$

$$= \frac{4}{x} + \frac{2}{x-1} \quad (2)$$

b) $\int \cos\left(\frac{1-x}{5}\right) dx$

$$= \frac{\sin\left(\frac{1-x}{5}\right)}{-\frac{1}{5}} + C$$

$$= -5 \sin\left(\frac{1-x}{5}\right) + C \quad (1)$$

c) $y = x \ln x$

$$y' = x \cdot \frac{1}{x} + \ln x$$

$$= 1 + \ln x$$

x int: $y = 0 \therefore x \ln x = 0$

$$x = 0 \text{ or } \log_e x = 0$$

$$x = 0 \text{ not valid} \therefore e^0 = x$$

$$x = 1$$

$$y = 0$$

(1, 0)

grad of tangent = $1 + \ln 1$

$$= 1$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 1(x - 1)$$

$$y = x - 1$$

$$x - y - 1 = 0 \quad (3)$$

d) $2x^2 - 3x + 4 = 0$

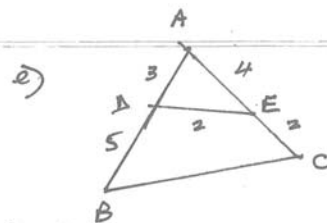
$$\alpha + \beta = \frac{3}{2} \quad \alpha\beta = 2$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{3}{2}\right)^2 - 2 \times 2$$

$$= \frac{9}{4} - 4$$

$$= -\frac{7}{4} \quad (2)$$



2 Δ SADE, ABC

$$\frac{AD}{AC} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{AE}{AB} = \frac{4}{8} = \frac{1}{2}$$

$\angle A$ is common

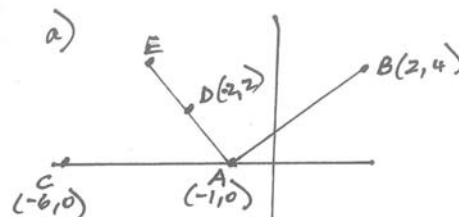
$\therefore \Delta ADE \parallel \Delta ACB$ (sides around = $\angle$ in same ratio)

$$\frac{AD}{AC} = \frac{DE}{CB} = \frac{AE}{AB} \text{ (matching sides of similar } \Delta\text{s)}$$

$$\frac{2}{BC} = \frac{4}{8}$$

$$\therefore BC = 4 \quad (3)$$

QUESTION 13



i) $AB = \sqrt{3^2 + 4^2}$

$$= 5 \quad (2)$$

ii) $M = \left(\frac{2-6}{2}, \frac{4-0}{2}\right)$

$$= (-2, 2)$$

$\therefore M$ is same pt as D. (1)

iii) $(-2, 2) = \left(\frac{x-1}{2}, \frac{y+0}{2}\right)$

$$-2 = \frac{x-1}{2} \quad \frac{y}{2} = 2$$

$$x-1 = -4 \quad y = 4$$

$$x = -3$$

$$\therefore E(-3, 4) \quad (2)$$

iv) Diagonals bisect each other: $\therefore$ parallelogram $\checkmark$
But $AC = AB = 5$
 $\therefore$ Rhombus. $\checkmark \quad (2)$

b) $PR = 2PS$

$$\sqrt{(x+3)^2 + (y-4)^2} = 2\sqrt{(x+1)^2 + (y-2)^2}$$

$$x^2 + 6x + 9 + y^2 - 8y + 16 = 4(x^2 + 2x + 1 + y^2 - 4y + 4)$$

$$4(x^2 + 2x + 1 + y^2 - 4y + 4)$$

$$x^2 + 6x + y^2 - 8y + 25 = 4x^2 + 8x + 4y^2 - 16y + 16$$

$$3x^2 + 2x + 3y^2 - 8y - 5 = 0 \quad (4)$$

c) $y' = 3 \sec^2 2x$

$$y = \frac{3 \tan 2x}{2} + C$$

Sub $\left(\frac{\pi}{3}, \sqrt{3}\right)$

$$\frac{\sqrt{3}}{2} = \frac{3}{2} \tan \frac{2\pi}{3} + C$$

$$\frac{\sqrt{3}}{2} = -\frac{3}{2} \times \sqrt{3} + C$$

$$\frac{\sqrt{3}}{2} = -\frac{3\sqrt{3}}{2} + C$$

$$C = \frac{\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} = 2\sqrt{3}$$

$$y = \frac{3}{2} \tan 2x + 2\sqrt{3} \quad (4)$$

QUESTION 14

a) $9x^2 - kx + 1 = 0$

$\Delta = k^2 - 36$

For real & different roots $\Delta > 0$

$k^2 - 36 > 0$

$k < -6$ or $k > 6$ (3)

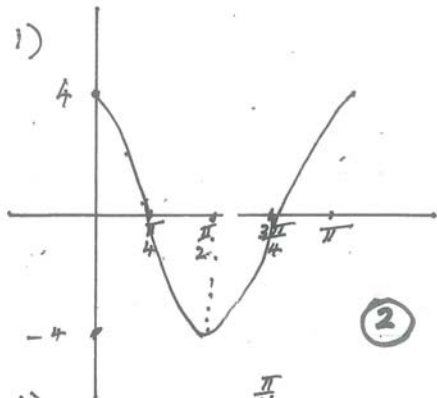
b) $\int_0^n (1 + \sin 2\pi x) dx$
 $= \left[x - \frac{\cos 2\pi x}{2\pi} \right]_0^n$

$= n - \frac{\cos 2\pi n}{2\pi} - 0 + \frac{\cos 0}{2\pi}$

$= n - \frac{1}{2\pi} + \frac{1}{2\pi}$

$= n$ (3)

c) $y = 4 \cos 2x$ $0 \leq x \leq \pi$



ii) Area = $2 \int_0^{\pi/4} 4 \cos 2x dx$
 $= 2 \times 4 \left[\frac{\sin 2x}{2} \right]_0^{\pi/4}$

$= 4 \left[\sin \frac{\pi}{2} - \sin 0 \right]$
 $= 4 \times 1$
 $= 4$ (3)

d)

i) $W = W_0 e^{-kt}$

$W_0 = 50$

$W = 50 e^{-kt}$

$t = 2$ $W = 40$

$40 = 50 e^{-k \times 2}$

$\frac{4}{5} = e^{-2k}$

$k = \frac{\log \frac{4}{5}}{-2}$

$= \left(\log \frac{5}{4} \right)^{-1}$

$= -\frac{\log \frac{5}{4}}{2}$

$= \frac{1}{2} \log \frac{5}{4}$ (2)

ii) $W = 50 e^{-kt}$

$t = 10$ $W = 50 e^{-5 \ln \frac{5}{4} \times 10}$

$= 50 e^{-5 \ln \frac{5}{4}}$

$= 16.384$

$\therefore 16.384$ litres are in tank

ie. 33.616 litres has run out

$= 34$ l (to n litre) (2)

QUESTION 15

a) $y = x^3 - x^2 - x + 6$

i) For stationary points $y' = 0$

$y' = 3x^2 - 2x - 1$

$(3x+1)(x-1) = 0$

$x = -\frac{1}{3}$ $x = 1$

$y = 6\frac{1}{27}$ $y = 5$

$\therefore$ stationary points are

$(-\frac{1}{3}, 6\frac{1}{27})$, $(1, 5)$ (3)

ii) For points of inflexion $y'' = 0$

$6x - 2 = 0$

$x = \frac{1}{3}$

$y = 5\frac{16}{27} = \frac{151}{27}$ ✓

$\therefore (\frac{1}{3}, 5\frac{16}{27})$ is a possible point of inflexion

Check concavity

x	0	$\frac{1}{3}$	1
y''	-2	0	4

$\therefore$ change in concavity (2)

$\therefore (\frac{1}{3}, 5\frac{16}{27})$ is a point of inflexion

[if state there is change in concavity on diagram, that is OK for the mark]

iii) $\frac{1}{3} < x < 1$ (1)

iv) $p = 5$ and $p = 6\frac{5}{27}$ (2)

b)

x	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
log x	log 1	log $1\frac{1}{2}$	log 2	log $2\frac{1}{2}$	log 3

 $\int_1^3 \log x dx \div \frac{1}{3} \{y_0 + y_4 + 4(y_1 + y_3) + 2y_2\}$
 $= \frac{1}{3} \{0 + \log 3 + 4(\log 1.5 + \log 2.5) + 2 \log 2\}$
 $= 1.29532...$
 $= 1.30$ (2 dp) (3)

c) $x = t - \log e^t$

i) $v = 1 - \frac{1}{t}$

Particle is at rest when $v = 0$

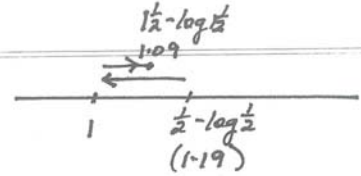
$1 - \frac{1}{t} = 0$

$\frac{1}{t} = 1$

$t = 1$

At rest at 1 sec. (1)

ii) $t = \frac{1}{2}$ $x = \frac{1}{2} - \log \frac{1}{2}$ ✓
 $t = 1\frac{1}{2}$ $x = 1\frac{1}{2} - \log 1\frac{1}{2}$ ✓
 $t = 1$ $x = 1 - \log 1$ ✓
 $= 1$ ✓



$\therefore$ Distance

$= \frac{1}{2} - \log \frac{1}{2} - 1 + 1\frac{1}{2} - \log 1\frac{1}{2} - 1$ ✓
 $= -\log \frac{1}{2} - \log 1\frac{1}{2}$ ✓
 $= -(\log \frac{1}{2} + \log 1\frac{1}{2})$ ✓
 $= -\log \frac{3}{4}$ ✓
 $= \log \frac{4}{3}$ (3)

Question 16

a) \$10000

Every 3 months 5 years

∴ 20 payments

6% pa = 0.005 p/month

$$i) A_1 = 10000(1.005) \\ A_3 = 10000(1.005)^3 - M \quad (1)$$

$$ii) A_6 = A_3(1.005)^3 - M \\ = [10000(1.005)^3 - M]1.005^3 - M \\ = 10000(1.005^6) - M(1 + 1.005^3)$$

$$A_{60} = 10000(1.005)^{60} - M(1 + 1.005^3 + \dots + 1.005^{57})$$

$$A_{60} = 10000(1.005)^{60} - M \frac{(1.005^60 - 1)}{1.005^3 - 1}$$

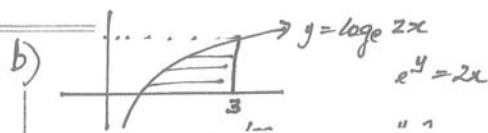
$$A_{60} = 0 \\ 10000(1.005)^{60} = \frac{M(1.005^60 - 1)}{1.005^3 - 1}$$

$$\frac{10000(1.005)^{60}(1.005^3 - 1)}{1.005^60 - 1} = M$$

$$M = \frac{10000(1.005^63 - 1005^60)}{1.005^60 - 1} \quad (2)$$

$$iii) M = \$582.89 \quad (1)$$

$$iv) \text{Interest} = 582.89 \times 20 - 10000 \\ = \$1657.80 \quad (1)$$



$$\text{Volume} = \pi \int_0^{\log 6} 3^2 - \left(\frac{e^y}{2}\right)^2 dy$$

$$= \pi \int_0^{\log 6} 9 - \frac{e^{2y}}{4} dy$$

$$= \pi \left[9y - \frac{e^{2y}}{8} \right]_0^{\log 6}$$

$$= \pi \left[9 \log 6 - \frac{e^{2 \log 6}}{8} - \left(0 - \frac{e^0}{8} \right) \right]$$

$$= \pi \left[9 \log 6 - \frac{e^{\log 36}}{8} + \frac{1}{8} \right]$$

$$= \pi \left[9 \log 6 - \frac{36}{8} + \frac{1}{8} \right]$$

$$= \pi \left(9 \log 6 - \frac{35}{8} \right) u^3$$

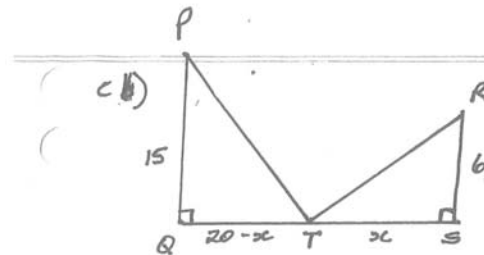
OR

$$4.37 \pi u^3 \quad 11.75 \pi u^3$$

OR

$$13.73 u^3 \quad 36.92 u^3 \quad (2dp)$$

-1 if no subtraction



$$i) PT^2 = 15^2 + (20-x)^2 \\ = 225 + 400 - 40x + x^2 \\ = 625 - 40x + x^2$$

$$RS^2 = 6^2 + x^2 \\ = 36 + x^2 \quad (2)$$

$$\therefore L = \sqrt{36 + x^2} + \sqrt{625 - 40x + x^2}$$

$$ii) \text{For min } L, L' = 0 \\ L' = \frac{1}{2}(36 + x^2)^{-\frac{1}{2}} \cdot 2x + \frac{1}{2}(625 - 40x + x^2)^{-\frac{1}{2}}(2x - 40)$$

$$L' = \frac{x}{\sqrt{x^2 + 36}} + \frac{x - 20}{\sqrt{625 - 40x + x^2}} \quad \checkmark$$

$$\frac{-x}{\sqrt{x^2 + 36}} = \frac{x - 20}{\sqrt{625 - 40x + x^2}}$$

$$\frac{x^2}{x^2 + 36} = \frac{x^2 - 40x + 400}{625 - 40x + x^2} \quad \checkmark$$

$$625x^2 - 40x^3 + x^4 = x^4 - 40x^3 + 400x^2 + 36x^2 - 1440x + 14400$$

$$625x^2 = 400x^2 + 36x^2 - 1440x + 14400$$

189

$$189x^2 + 1440x - 14400 = 0 \quad \checkmark$$

$$x = \frac{-1440 \pm \sqrt{1440^2 - 4(189)(-14400)}}{2 \times 189}$$

$$= \frac{40}{7} \quad \text{or } \log \text{ and } (not \text{ valid})$$

Check if min L

x	5	5.57	6
L'	-0.06692	0	0.02479

∴ if $x = \frac{40}{7}$, L is min.

✓ (4)

∴ shortest L

$$= \sqrt{36 + \left(\frac{40}{7}\right)^2} + \sqrt{625 - 40 \times \frac{40}{7} + \left(\frac{40}{7}\right)^2}$$

$$= 29 \text{ m.} \quad \checkmark$$