Student Number

Barker College

2019 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Staff Involved:

AM FRIDAY 9TH AUGUST

BHC* ARP*
DZP PJR
GPR ARM
AYG JGD
ESP JZT
RAS

140 copies

General Instructions:

- Reading Time - 5 minutes
- Working time - 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided separately
- In Questions 11 - 16 show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I - 10 marks (pages 2 - 4)
- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II - 90 marks (pages 5 - 10)
- Attempt Questions 11 - 16
- Allow about 2 hours and 45 minutes for this section

Section I – Multiple Choice

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. What is the value of $f'(2)$ if $f(x)=3x^4-x$?

A. 46
B. 94
C. 95
D. 13823

2. What is the minimum value of the function $f(x)=4+2\sin 2x$?

A. 0
B. 2
C. 4
D. 6

3. In an arithmetic sequence $T_{10}=0$ and $T_{12}=8$. What is the value of T_1?

A. -48
B. -44
C. -40
D. -36

4. If $\log_a 5=1.2$ then, correct to one decimal place, what is the value of a?

A. 3.7
B. 3.8
C. 3.9
D. 4.0

5. If $\cos\theta > 0$ and cosec $\theta = -3$ what is the exact value of $\cot\theta$?

A. $\frac{1}{\sqrt{8}}$

B. $\frac{-1}{\sqrt{8}}$

C. $\sqrt{8}$

D. $-\sqrt{8}$

6. What is the value of $\sum_{r=1}^{3} r3^{-r}$?

A. $\frac{2}{3}$

B. $\frac{19}{27}$

C. $\frac{20}{27}$

D. $\frac{7}{9}$

7. What is the equation of the directrix of the parabola $(y-2)^2 = -12(x+4)$?

A. $x = -7$

B. $y = -7$

C. $x = -1$

D. $y = -1$

8. Over the interval $a \le x \le b$ a function $y = f(x)$ is described as "increasing at a decreasing rate". This means that, for all values of x such that $a \le x \le b$,

A. $f'(x) > 0$ and $f''(x) < 0$

B. $f'(x) < 0$ and $f''(x) > 0$

C. $f'(x) > 0$ and $f''(x) > 0$

D. $f'(x) < 0$ and $f''(x) < 0$

9. If $\frac{-\pi}{4} < \theta < \frac{\pi}{4}$, find the value of the limiting sum of $1 - \tan^2\theta + \tan^4\theta - \tan^6\theta + \ldots$

A. $\sec^2\theta$

B. $\text{cosec}^2\theta$

C. $\cos^2\theta$

D. $\sin^2\theta$

10. What is $\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} -x^3 \sin x \, dx$ equal to?

A. 0

B. $2\int_{0}^{\frac{\pi}{3}} -x^3 \sin x \, dx$

C. $2\left|\int_{0}^{\frac{\pi}{3}} -x^3 \sin x \, dx\right|$

D. $\left|\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} -x^3 \sin x \, dx\right|$

End of Section I

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) [START A NEW BOOKLET]

(a) Solve $-x-1=\dfrac{4x+11}{3}$. **2**

(b) Express $\dfrac{6}{3-2\sqrt{5}}$ with a rational denominator. **2**

(c) Find the values of x for which $|2x+1|\le 2-x$. **2**

(d) Differentiate $y=2x\cos(2x)$. **2**

(e) Evaluate $\lim\limits_{x\to 2}\left(\dfrac{x^2-4}{2-x}\right)$. **2**

(f) Find $\int e^{4x+1}\,dx$. **1**

(g) Find $\int \dfrac{x}{4+x^2}\,dx$. **2**

(h) Find $\int \tan x\,dx$. **2**

End of Question 11

Question 12 (15 marks) [START A NEW BOOKLET]

(a) Evaluate $\dfrac{\ln\left(\pi^2\right)}{e^{\pi}}$ correct to 2 significant figures. **1**

(b) Find the **exact** values of k for which the equation $2x^2+kx+25=0$ has two equal roots. **2**

(c) Given that $\log_a b=1.6$ and $\log_a c=0.4$, find the value of $\log_a\left[(abc)^2\right]$. **2**

(d) The diagram shows triangle XYZ with sides $XY=7\text{cm}$, $XZ=10\text{cm}$ and $\angle XZY=32^o$. **3**

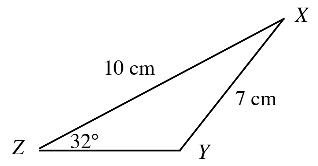


Diagram not drawn to scale

Note that $\angle XYZ>90^o$

Calculate $\angle XYZ$ correct to the nearest degree.

(e)

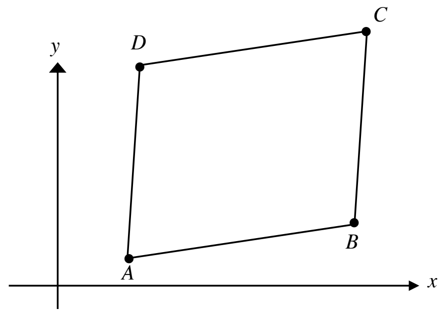


Diagram is not drawn to scale

In the diagram A, B, C and D are the points (2, 1), (7, 4), (12, 11) and (5, 6) respectively.

(i) Find the distance BD. **1**

(ii) Find the midpoint of BD. **1**

(iii) Show that AC is perpendicular to BD. **2**

(iv) Find the midpoint of AC and hence give the most specific name of quadrilateral $ABCD$. **2**

(v) Hence, or otherwise, find the area $ABCD$. **1**

End of Question 12

Question 13 (15 marks) [START A NEW BOOKLET]

(a) 108 tagged fish were released into a dam known to contain a lot of fish. **2**
Later a sample of 62 fish was netted from this dam and then released.
Of these 62 fish in the sample it was recorded that 8 were tagged.

Estimate the total number of fish in the dam now.

(b) (i) Prove that the line $y = x - 2$ is a tangent to the parabola $y = x^2 - 7x + 14$. **2**

(ii) Let T be the point where $y = x - 2$ touches the parabola $y = x^2 - 7x + 14$. **1**
Show that the equation of the normal to the parabola at T is $y = -x + 6$.

(iii) Find the area of the region enclosed between the parabola and the line $y = -x + 6$. **3**

(c) Mr Peattie brought to class a box of 30 chocolates which all appeared to be exactly alike. However, 22 chocolates were rambutan flavoured and 8 were durian flavoured. Lucy ate 3 chocolates chosen at random from the box. Find the probability that

(i) the first chocolate Lucy eat was rambutan flavoured. **1**

(ii) Lucy ate 3 rambutan flavoured chocolates. **1**

(iii) Lucy ate at least one rambutan chocolate. **1**

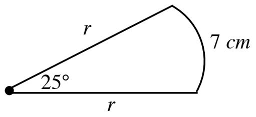
(iv) Lucy ate exactly one rambutan chocolate. **1**

(d) Find the **exact** solutions of the equation $\ln x - \dfrac{3}{\ln x} = 2$. **3**

End of Question 13

Question 14 (15 marks) [START A NEW BOOKLET]

(a) Find the radius of this sector of a circle. **2**
Give your answer correct to the nearest *cm*.



(b) (i) Draw the graphs of $y = 5\cos x$ and $y = 2 - x$ on the same set of axes for $-2\pi \le x \le 2\pi$. **2**

(ii) Using your graph in (i), how many solutions are there to the equation $5\cos x = 2 - x$? **1**

(c) Sean is arranging rows of centicubes flat on a table. The centicubes are stacked in horizontal rows, with each row containing 2 less centicubes than the row below it.

There are 16 centicubes in the top row (1^{st} row), 18 in the row below that (2^{nd} row), and 20 in the row below that (3^{rd} row), and so on.

There are n rows altogether and Sean always completes each row.

(i) Write down an expression for the number of centicubes in the nth-row. **1**

(ii) Show that there are $n^2 + 15n$ centicubes altogether in the construction. **1**

(iii) If Sean says that there is a total of 448 centicubes in the construction, explain why he must have miscounted. **1**

(iv) Sean has only 1100 centicubes from which to build the construction. What is the maximum number of complete rows the construction can contain? **2**

(d) Sketch the curve $y = 4^{x-2}$ between $x = 2$ and $x = 4$.

(i) Use the Trapezoidal Rule with 3 function values to estimate the area between the curve, the x-axis and the lines $x = 2$ and $x = 4$. **2**

(ii) If this area is rotated about the x-axis, use Simpson's Rule with 3 function values to approximate the volume of the solid that is formed. **3**

End of Question 14

Question 15 (15 marks) [START A NEW BOOKLET]

(a) A function is given by $f(x) = 3x^4 + 4x^3 - 12x^2$.

(i) Find the coordinates of the stationary points and determine their nature. **3**

(ii) Hence, sketch the graph of the function. **2**

(iii) For what values of x is $f(x)$ increasing? **1**

(iv) For what values of k will the equation $3x^4 + 4x^3 - 12x^2 + k = 0$ have no solution? **1**

(b) The gradient of a curve is given by $\frac{dy}{dx} = 1 - 6\sin(3x)$. **2**

The curve goes through the point (0, 10).

What is the equation of the curve?

(c) (i) Assuming that $a > 1$, simplify $\int_0^{\ln a} e^{-2x}\, dx$, leaving your answer in the form of a simplified single fraction. **3**

(ii) Find $\lim_{a\to\infty}\left[\int_0^{\ln a} e^{-2x} dx\right]$. **1**

(d) Find $\int \frac{e^{-x}}{e^{-x}(1+e^x)} dx$. **2**

End of Question 15

Question 16 (15 marks) [START A NEW BOOKLET]

(a) A "Barker Thirst Buster Water Bottle" is a closed cylinder of height h cm and radius R cm.

(i) The capacity of the bottle is 1 litre.
Find an expression for h in terms of R. **1**

(ii) Show that the surface area (S) of the bottle is given by $S = \frac{2}{R}\left(1000 + \pi R^3\right)$. **1**

(iii) If the area of the metal used to make the bottle is minimised, find the height and radius of the bottle (to the nearest mm). **3**

(b) Gen, Tia and Arjun each chose a number at random from 1 to 10 (inclusive).

Find the probability that:

(i) they each chose the same number. **1**

(ii) they each chose a different number. **1**

(iii) two of them chose the same number and one of them chose a different number. **1**

(iv) both the numbers chosen by Gen and Tia are greater than the number chosen by Arjun. **2**

(c) Triangle ABC is right angled at B.

D is a point on AC such that BD is perpendicular to AC. Let $\angle BAC$ be θ.

Draw a diagram showing this information. It is known that $\sqrt{3}BC + 6AD = 3AC$.

(i) Show that $\sqrt{3}\tan\theta + 6\cos\theta = 3\sec\theta$. **2**

(ii) Deduce that $2\sin^2\theta - \frac{1}{\sqrt{3}}\sin\theta - 1 = 0$. **2**

(iii) Find θ. **1**

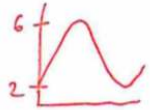
End of Question 16

End of Paper

1. (C) $f'(x) = 12x^3 - 1$

$f'(2) = 95$

2. (B)



3. (D) $0 = a + 9d$

$8 = a + 11d$

$\therefore 2d = 8$

$d = 4$

$a = -36$

$\therefore T_1 = -36 + 0$

$= -36$

4. (B) $a^{1.2} = 5$

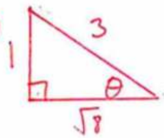
$\frac{\log 5}{\log a} = 1.2$

$\log a = \frac{1.2}{\log 5}$

$a = \sqrt[1.2]{5}$

$= 3.82$

5. (D)



S	A
T	C ✓

$\cot\theta = -\sqrt{8}$

6. (A) $\frac{1}{3} + \frac{2}{9} + \frac{3}{27} = \frac{2}{3}$

7. (C)

8. (A)

9. (C) $a = 1$

$r = -\tan^2\theta$

$\therefore S_\infty = \frac{1}{1+\tan^2\theta}$

$= \frac{1}{\sec^2\theta}$

$= \cos^2\theta$

10. (B) $-x^3 \sin x$ is even function

11. a) $-3x - 3 = 4x + 11$

$7x = -14$

$x = -2$

b) $\frac{6}{3-2\sqrt{5}} \times \frac{3+2\sqrt{5}}{3+2\sqrt{5}} = \frac{18+12\sqrt{5}}{9-20}$

$= \frac{18+12\sqrt{5}}{-11}$

$= \frac{-12\sqrt{5}-18}{11}$

c) $|2x+1| \le 2 - x$

$2x + 1 \le 2 - x$

$3x \le 1$

$x \le \frac{1}{3}$

$-2x - 1 \le 2 - x$

$x \ge -3$

$\therefore -3 \le x \le \frac{1}{3}$

d) $u = 2x \quad v = \cos 2x$

$u' = 2 \quad v' = -2\sin 2x$

$y' = 2\cos 2x - 4x\sin 2x$

e) $\lim_{x\to 2} \frac{(x+2)(x-2)}{2-x}$

$= \lim_{x \to 2} -(x+2)$

$= -4$

f) $\int e^{4x+1}\,dx = \frac{e^{4x+1}}{4} + C$

g) $\int \frac{x}{4+x^2}\,dx = \frac{1}{2}\int \frac{2x}{4+x^2}\,dx$

$= \frac{1}{2}\ln(4+x^2) + C$

h) $\int \frac{\sin x}{\cos x}\,dx = -\int \frac{-\sin x}{\cos x}\,dx$

$= -\ln\cos x + C$

2. a) $= 0.0989$

$= 0.099$ (2 sig fig)

b) $2x^2 + kx + 25 = 0$

$\Delta = k^2 - 200 = 0$

$k = \pm\sqrt{200}$

$= \pm 10\sqrt{2}$

c) $\log_a (abc)^2 = 2(\log_a a + \log_a b + \log_a c)$

$= 2(1 + 1.6 + 0.4)$

$= 6$

d) $\frac{\sin\theta}{10} = \frac{\sin 32}{7}$

$\sin\theta = \frac{10\sin 32}{7}$

$\theta = 49.2$

but θ obtuse

$\therefore \theta = 130.8^\circ$

$= 131^\circ$

e)(i) $BD = \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$

(ii) $M_{BD} = \frac{12}{2}, \frac{10}{2}$

$= 6, 5$

(iii) $m_{AC} = \frac{10}{10}$

$= 1$

$m_{BD} = \frac{2}{-2}$

$= -1$

$m_{AC} \times m_{BD} = -1 \quad \therefore AC \perp BD$

iv) $M_{AC} = (7, 6)$

diagonals intersect at 90° but not each other
∴ a kite

v) Area $= \frac{d_1 d_2}{2}$

$AC = \sqrt{10^2 + 10^2}$

$= \sqrt{200}$

∴ Area $= \frac{\sqrt{1600}}{2}$

$= 20u^2$

3.a) $\frac{8}{62} = \frac{108}{x}$

$x = 837$

b)(i) $x^2 - 7x + 14 = x - 2$

$x^2 - 8x + 16 = 0$

$(x-4)^2 = 0$

1 root ∴ $y = x - 2$ is a tangent

(ii) $x = 4, y = 2$

∴ $T(4, 2)$

$y' = 2x - 7$

at $x = 4$ $m = 1$ ∴ m normal is -1

∴ $y - 2 = -x + 4$

Eqn normal $y = -x + 6$

(iii) Find points of intersection

$x^2 - 7x + 14 = -x + 6$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$x = 4, 2$

$y = 2, 4$

∴ Area $= \int_2^4 (-x + 6 - x^2 + 7x - 14)\,dx$

$= \int_2^4 -x^2 + 6x - 8\,dx$

$= \left[\frac{-x^3}{3} + 3x^2 - 8x\right]_2^4$

$= -\frac{16}{3} + \frac{20}{3}$

$= \frac{4}{3}u^2$

c)(i) $\frac{22}{30} = \frac{11}{15}$

(ii) $P(3R) = \frac{22}{30} \times \frac{21}{29} \times \frac{20}{28}$

$= \frac{11}{29}$

(iii) $P(\text{at least }1) = 1 - P(\text{none})$

$= 1 - \left(\frac{8}{30} \times \frac{7}{29} \times \frac{6}{28}\right)$

$= \frac{143}{145}$

(iv) $P\binom{\text{exactly}}{1R} = \left(\frac{22}{30} \times \frac{8}{29} \times \frac{7}{28}\right) 3$

$= \frac{22}{435} \times 3$

$= \frac{22}{145}$

d.) $\ln x - \frac{3}{\ln x} = 2$

$(\ln x)^2 - 2\ln x - 3 = 0$

if $u = \ln x$

$u^2 - 2u - 3 = 0$

$(u-3)(u+1) = 0$

∴ $\ln x = 3$ or -1

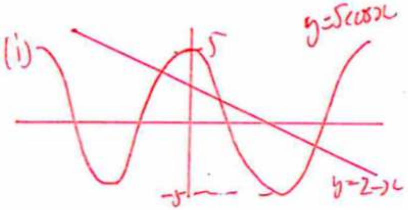
$x = e^3$ $x = e^{-1}$ or $\frac{1}{e}$

14. a) $7 = r\theta$

$\theta = 0.436$ rad

∴ $r = 16.04$

b)(i)



(ii) 3 solutions

c)(i) $T_1 = 16$ $T_2 = 18$

$a = 16$ $d = 2$

∴ $T_n = 16 + (n-1)2$

$= 14 + 2n$

(ii) $S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{n}{2}(32 + 2n - 2)$

$= \frac{n}{2}(30 + 2n)$

$= n^2 + 15n$

(iii) $n^2 + 15n = 448$

$n^2 + 15n - 448 = 0$

$n = \frac{-15 \pm \sqrt{2017}}{2}$

not a whole number ∴ 448 incorrect

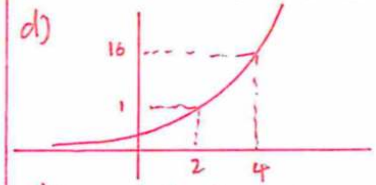
(iv) $15n + n^2 = 1100$

$n = \frac{-15 \pm \sqrt{4625}}{2}$

$= 26.5$ as $n > 0$

∴ 26 complete rows

d)



(i)

x	2	2.5	3	3.5	4
$f(x)$	1	2	4	8	16

$A = \frac{0.5}{2}\left(1 + 16 + 2(2 + 4 + 8)\right)$

$= 11.25\,u^2$

ii)

x	2	3	4
$f(x)$	1	4	16
$f(x)^2$	1	16	256

$V = \frac{\pi}{3}(1+256+4\times16)$

$= \frac{321\pi}{3}\ u^3$

$= 107\pi\ u^3$

5. a) (i) $f'(x) = 12x^3 + 12x^2 - 24x = 0$

$x(x^2+x-2)=0$

$x(x+2)(x-1)=0$

$x = 0, -2, 1$

$y = 0, -32, -5$

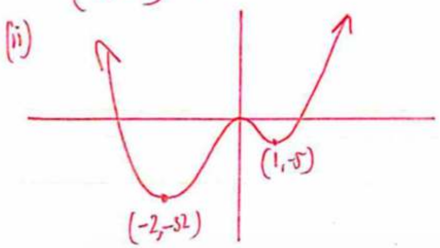
x	-3	-2	-1	0	½	1	2
$f'(x)$	-144	0	24	0	-7.5	0	96

$\therefore (-2,-32)$ min

$(0,0)$ max

$(1,-5)$ min

(ii)



(iii) Increasing $-2<x<0$

$x>1$

(iv) $k>32$ for no solution

b) $y = x + 2\cos 3x + C$

sub in $(0,10)$

$10 = 0+2+C$

$C = 8$

$\therefore y = x + 2\cos 3x + 8$

c) (i) $\left[\frac{e^{-2x}}{-2}\right]_0^{\ln a}$

$= \frac{-1}{2a^2} + \frac{1}{2}$

$= \frac{a^2-1}{2a^2}$

(ii) $\lim_{a\to\infty}\left[\frac{a^2-1}{2a^2}\right]$

$= \frac{a^2}{2a^2}$

$= \frac{1}{2}$

d) $\int \frac{e^{-x}}{e^{-x}(1+e^{x})}\,dx$

$= \int \frac{e^{-x}}{e^{-x}+1}\,dx$

$= -\ln(e^{-x}+1) + C$

6 a) (i) $V = \pi R^2 h$

$1000 = \pi R^2 h$

$h = \frac{1000}{\pi R^2}$

(ii) $SA = 2\pi R^2 + 2\pi R h$

$= 2\pi R^2 + \frac{2\pi R\times1000}{\pi R^2}$

$= 2\pi R^2 + \frac{2000}{R}$

$= \frac{2}{R}(1000+\pi R^3)$

(iii) $\frac{dS}{dr} = 4\pi R - \frac{2000}{R^2} = 0$ for max/min

$\frac{4\pi R^3 - 2000}{R^2} = 0$

$4\pi R^3 = 2000$

$R^3 = \frac{500}{\pi}$

$R = \sqrt[3]{\frac{500}{\pi}}$

$= 5.42$ cm

$= 54$ mm

$R=5$ $\frac{dS}{dr} = -17$

$R=6$ $\frac{dS}{dr} = 19$

$\therefore$ 54mm is a min

b) (i) $P(\text{same}) = 1 \times \frac{1}{10} \times \frac{1}{10}$

$= \frac{1}{100}$

(ii) $P(\text{diff}) = 1 \times \frac{9}{10} \times \frac{8}{10}$

$= \frac{72}{100} = \frac{18}{25}$

(iii) $\left(1\times\frac{1}{10}\times\frac{9}{10}\right)\times 3 = \frac{27}{100}$

(iv)

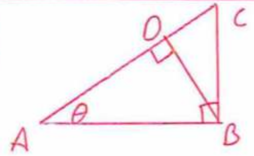
A	G	T
1	0.81	= 0.081
2	0.64	= 0.064
3	0.49	= 0.049
4	0.36	= 0.036
5	0.25	= 0.025
6	0.16	= 0.016
7	0.09	= 0.009
8	0.04	= 0.004
9	0.01	= 0.001

Sum

0.285

or $\frac{57}{200}$

:)



$\sqrt{3}BC + 6AD = 3AC$

(i) $\tan\theta = \frac{BC}{AB}$

$BC = AB\tan\theta$

$\cos\theta = \frac{AD}{AB}$

$AD = AB\cos\theta$

$\cos\theta = \frac{AB}{AC}$

$AC = AB\sec\theta$

sub in BC, AD, AC

$\therefore \sqrt{3}\,AB\tan\theta + 6AB\cos\theta = 3AB\sec\theta$

$\sqrt{3}\tan\theta + 6\cos\theta = 3\sec\theta$

(ii) $\frac{\sqrt{3}\sin\theta}{\cos\theta} + 6\cos\theta = \frac{3}{\cos\theta}$

$\sqrt{3}\sin\theta + 6\cos^2\theta = 3$

$6(1-\sin^2\theta) + \sqrt{3}\sin\theta - 3 = 0$

$3 - 6\sin^2\theta + \sqrt{3}\sin\theta = 0$

$2\sin^2\theta - \frac{1}{\sqrt{3}}\sin\theta - 1 = 0$

(iii)

$\sin\theta = \dfrac{\frac{1}{\sqrt{3}} \pm \sqrt{\frac{1}{3} + 8}}{4}$

$= \dfrac{\frac{1}{\sqrt{3}} \pm \frac{5}{\sqrt{3}}}{4}$

$= \frac{3}{2\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

$\theta = 60°$ as $\theta > 0$