Barker College

2021	YEAR 12 TRIAL EXAMINATION

Mathematics Advanced

Staff Involved:

Friday August 20

- ARP* • AYG • LZM • LAK • DXC
- PJR • RJW • PDJ • DZP

130 copies

General Instructions:

- Time – 3 hours, 10 minutes (10 minutes recommended reading time)
- Write using black pen
- Calculators approved by NESA may be used
- Reference sheet is provided separately and at the end of this paper
- For questions in Section II, show relevant mathematical reasoning and / or calculations
- Marks may not be awarded for careless or poorly arranged working
- Diagrams are not to scale unless specifically stated

Total marks: 100

Section I - 10 marks (pages 2 - 5)
- Use Canvas to select your preferred response for Questions 1-10
- Allow about 15 minutes for this section

Section II - 90 marks (pages 6 - 27)
- Attempt Questions 11 - 33
- Allow about 2 hours and 45 minutes for this section
- Answer the questions on your own lined paper, clearly indicating which question you are answering
- Show all necessary working
- Write your student number on all answer pages
- Section II will be uploaded in 4 sections: Q11-19, Q20-24, Q25-29, Q30-33
- Start each section on a new page

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use Canvas to select your preferred response for Questions 1–10.

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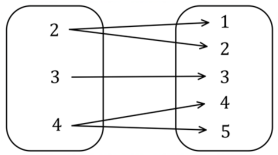
1 What type of relation is shown below?



A. Many-to-many

B. One-to-many

C. One-to-one

D. Many-to-one

2 What is the derivative of $\cos^2 3x$ with respect to x?

A. $-6\sin 3x\cos 3x$

B. $-2\sin 3x\cos 3x$

C. $2\sin 3x\cos 3x$

D. $6\sin 3x\cos 3x$

3 What is the amplitude and period for the function $f(x)=4\sin\left(\dfrac{x+\pi}{3}\right)$?

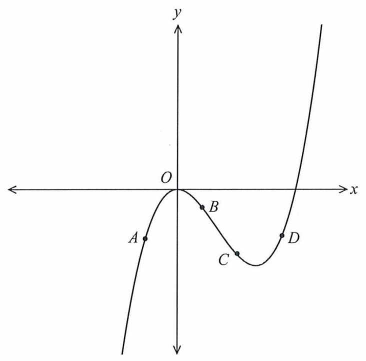
A. Amplitude 3 and period $\dfrac{\pi}{2}$

B. Amplitude 3 and period 6π

C. Amplitude 4 and period $\dfrac{\pi}{2}$

D. Amplitude 4 and period 6π

4 At which point on this curve are both the first and second derivatives positive?



A. A

B. B

C. C

D. D

5 The probability distribution of a discrete random variable X is shown below.

x	−3	−2	−1	0	1	2	3
$P(X=x)$	0.05	0.05	a	0.20	0.15	a	0.05

What is the value of a if no other values of X are possible?

A. 0.15

B. 0.20

C. 0.25

D. 0.30

6 4 red socks and 4 blue socks are in a drawer. If 2 socks are selected at random at the same time what is the probability that both are the same colour?

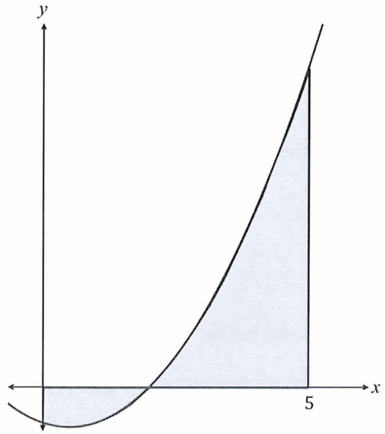
A. $\frac{1}{2}$

B. $\frac{3}{7}$

C. $\frac{4}{7}$

D. $\frac{3}{14}$

7 Which expression will give the area of the shaded region below bounded by the curve $y=x^2-x-2$, the x-axis and the lines $x=0$ and $x=5$?



A. $A=\left|\int_0^1 (x^2-x-2)\,dx\right|+\int_1^5 (x^2-x-2)\,dx$

B. $A=\int_0^1 (x^2-x-2)\,dx+\left|\int_1^5 (x^2-x-2)\,dx\right|$

C. $A=\left|\int_0^2 (x^2-x-2)\,dx\right|+\int_2^5 (x^2-x-2)\,dx$

D. $A=\int_0^2 (x^2-x-2)\,dx+\left|\int_2^5 (x^2-x-2)\,dx\right|$

8 What is the value of $\int \tan^2 x\,dx$?

A. $\tan x - x + c$

B. $-\ln(\cos^2 x) + c$

C. $\ln(\cos^2 x) + c$

D. $\dfrac{\tan^3 x}{3} + c$

9 A particle is travelling to the left with a positive acceleration. Which of the following statements must be true?

A. The particle will eventually become stationary.

B. The particle is slowing down.

C. The particle is speeding up.

D. The particle will never go through the origin.

10 Consider the functions $f(x) = \ln x$ and $g(x) = e^{-x}$. How many solutions are there to the equation $f'(x) = f(g(x)) - 3$?

A. 0

B. 1

C. 2

D. 3

End of Section I

Section II

90 marks
Attempt Questions 11 – 33
Allow about 2 hours and 45 minutes for this section.

Answer the questions on your own lined paper, clearly indicating which question you are answering. Spaces provide guidance for the expected length of response.

Write your student number on all answer pages.

In Questions 11 – 33, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (1 mark)

Solve $|2x-1| = 5$. **1**

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Question 12 (1 mark)

Differentiate $x\tan x$. **1**

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Question 13 (2 marks)

Evaluate $\int_0^{\frac{\pi}{2}} \sin\frac{x}{2}\,dx$. **2**

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Question 14 (3 marks)

(a) Sketch the curve of $y = \ln(x+1)$ indicating intercepts and asymptotes. **1**

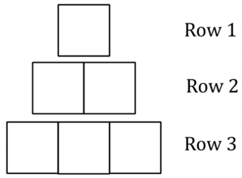
(b) Using the trapezoidal rule with 4 sub intervals, estimate $\int_1^3 \ln(x+1)\,dx$ correct to 3 decimal places. **2**

Question 15 (2 marks)

Prove that $\sec\theta - \cos\theta = \sin\theta\tan\theta$. **2**

Question 16 (6 marks)

Bruce has some sticks that are all of the same length. He arranges them in squares and has made the following 3 rows of patterns:



He notices that 4 sticks are required to make the single square in the first row, 7 sticks to make 2 squares in the second row and 10 to make 3 squares in the third row.

(a) Find a simplified expression, in terms of n, for the number of sticks required to make an arrangement of n squares in the nth row. **1**

(b) Bruce continues to make squares following the same pattern. He makes 4 squares in the 4^{th} row and so on until he has completed 10 rows. Find the total number of sticks Bruce uses in making these 10 rows. **2**

(c) Bruce started with 1750 sticks. Given that he continues the pattern to complete k rows but does not have sufficient sticks to complete the $(k+1)$th row, show that k satisfies $(3k-100)(k+35) < 0$. **2**

(d) What is the value of k? **1**

Question 17 (3 marks)

The circle $x^2+8x+y^2-4y-29=0$ is translated up by 1 unit and reflected in the y-axis. **3**
Sketch the new circle, showing the coordinates of the centre and the radius.
(x and y intercepts are not required)

Question 18 (2 marks)

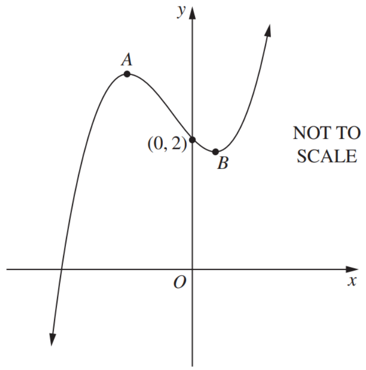
Solve $4^{2x+1}=\frac{1}{8}$ **2**

Question 19 (2 marks)

Find p given that $\int_2^3 \frac{-1}{x-1}\,dx=\ln p$. **2**

Question 20 (6 marks)

The graph $y = x^3 + x^2 - x + 2$ is sketched below. The points A and B are turning points.



(a) Find the coordinates of A and B. **3**

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(b) For what values of x is the curve concave up? **2**

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(c) For what values of k has the equation $x^3 + x^2 - x + 2 = k$ three real solutions? **1**

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Question 21 (7 marks)

A particle moves such that its velocity (m/s) at a given time t is:

$$\dot{x} = 8 - 16\sin t$$

(a) What is the initial acceleration of the particle? **2**

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(b) When is the particle first at rest? **1**

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(c) Given that the particle is initially at the origin, find an equation for the displacement (x) at a given time t. **2**

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(d) Find, correct to 2 decimal places, the distance travelled between the first two times when the particle is at rest. **2**

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Question 22 (2 marks)

(a) Find $\dfrac{d}{dx}(7x-2)^7$. **1**

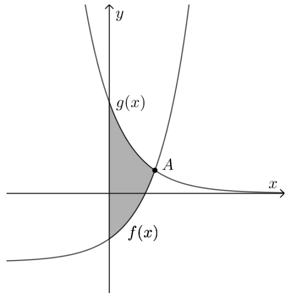
(b) Hence or otherwise find $\int 7(7x-2)^6\,dx$. **1**

Question 23 (3 marks)

Solve $2\cos\left(x-\dfrac{\pi}{3}\right)=1$ for $[0,2\pi]$. **3**

Question 24 (6 marks)

The graph below shows the functions $f(x)=2^x-3$ and $g(x)=4\times 2^{-x}$.



(a) By creating a quadratic equation show that the coordinates of A are $(2,1)$. **3**

(b) Find the exact value of the shaded area in simplest form. **3**

START A NEW PAGE SINCE QN 25-29 WILL BE UPLOADED SEPARATELY

Question 25 (3 marks)

Sketch the curve of $y = 1 - 2\sin\frac{x}{2}$ for $0 \le x \le 2\pi$. **3**

Ensure you indicate any x −intercepts.

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Question 26 (5 marks)

A rare species of bird lives on a remote island. In 2020 a model predicts that the bird population, P, will be:

$P = 150 + 300e^{-0.05t}$.

where t is the number of years after observations began at the start of 2020.

(a) According to the model, how many birds were there when observations began in 2020? **1**

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(b) Sketch a graph of P against t for $t \ge 0$ clearly indicating the limiting value of the bird population. **2**

(c) The species will become eligible for inclusion in the endangered species list when the population falls below 200. In what year does the model predict this will occur? **2**

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Question 27 (5 marks)

The length of time, in minutes, that a customer queues to buy beverages is a random variable, X, with probability density function:

$$f(x)=\begin{cases}k\left(64-x^2\right) & \textit{for } 0\le x\le 8\\ 0 & \text{otherwise}\end{cases}$$

where k is a constant.

(a) Show that the value of k is $\frac{3}{1024}$. **2**

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(b) Find the cumulative distribution function. **1**

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Question 27 continues on the next page

(c) Find the probability that a customer will queue for longer than five minutes. Round your answer to 3 significant figures. **2**

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End of Question 27

Question 28 (4 marks)

Consider the curve $y = \dfrac{\sin x}{1 + \cos x}$.

(a) Find $\dfrac{dy}{dx}$ in simplest form. **2**

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(b) Find the equation of the tangent to the curve at the point on the curve where $x = \dfrac{\pi}{3}$. **2**

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Question 29 (4 marks)

A soccer coach notices that her team has a probability of 0.6 of winning their game if it is raining but a probability of 0.3 if it is dry. The probability that it will rain when they play is 0.25.

(a) Find the probability that they will win the game. **2**

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(b) Given that the team has won the game, calculate the probability that it rained on the day of the match. **2**

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START A NEW PAGE SINCE QN 30-33 WILL BE UPLOADED SEPARATELY

Question 30 (8 marks)
A company borrows $\$800000$ to update its car fleet. The interest rate is 12% p.a. compounded monthly. It pays off the loan by 24 equal monthly instalments. The first instalment is paid one month after the loan is taken out.

Let A_n be the amount owing after n instalments are paid. Let M be the amount of each instalment.

(a) Show that the amount owing after two months is $A_2 = 816080 - M(2.01)$. **2**

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(b) Show that $M = \dfrac{8000 \times 1.01^{24}}{1.01^{24} - 1}$. **2**

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Question 30 continues on the next page

Question 30 (continued)

(c) Show that after paying two instalments the balance owing is $\$740386$ to the nearest dollar. **1**

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(d) After paying two instalments, the company decides to increase its repayments to $\$50000$ each month. Find the reduction in the number of months it takes the company to pay off its debt, remembering that the original loan was for 24 months. Express your answer to the nearest month. **3**

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End of Question 30

Question 31 (3 marks)

Find the value(s) of w that satisfy the infinite series: **3**

$\frac{6}{w}+6+6w+\ldots=27.$

Question 32 (7 marks)

A cement mixer is tipping cement onto a pile. The rate, R kg/s, at which the cement is flowing is given by the expression $R=100t-t^3$, for $0 \le t \le T$, where t is the time in seconds after the cement begins to flow.

(a) Determine the rate of flow at $t=8$. **1**

(b) What is the largest value of T for which the expression for R is reasonable? **2**

(c) Determine the maximum rate of flow of cement. **2**

Question 32 continues on the next page

Question 32 (continued)

(d) When the cement begins to flow the pile already contains 100 kg of cement. Find an expression for the amount of cement in the pile at time t. **2**

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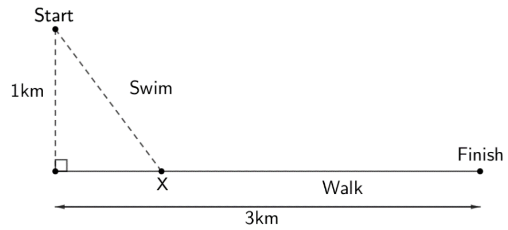
End of Question 32

Question 33 (5 marks)

Deliah is in the water 1 km off the coast and wants to get to town 3 km down the coast. She needs to swim to shore and then walk to town.

Deliah can swim at 2 km/hour and walk at 4 km/hour.



(a) How long will it take Deliah to swim the 1 km to shore and walk the 3 km to town? **1**

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(b) Instead, Deliah saves time by swimming to the point X and then walking to town. By first determining the location of point X, calculate to the nearest minute, the maximum amount of time Deliah can save. **4**

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Question 33 (continued)

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End of Paper

1) B

2) $y = \cos^2 3x$

$\frac{dy}{dx} = -6\sin 3x \cos 3x$ A

3) D

4) D

5) $1 = 0.005 + 0.005 + a + 0.2 + 0.15 + a + 0.15$

$1 = 2a + 0.5$

$a = 0.25$ C

6) B

7) $y = (x-2)(x+1)$

$\therefore x = 2$ or -1 C

8) $\int \sec^2 x - 1 \, dx$

$= \tan x - x + C$ A

9) B

10) $f'(x) = \frac{1}{x}$

$x > 0$

$f'(x) = f(g(x)) - 3$

$\frac{1}{x} = \ln e^{-x} - 3$

$1 = -x^2 - 3x$

$0 = x^2 + 3x + 1$

$x = \frac{-3 \pm \sqrt{9 - 4(1)(1)}}{2}$

$x = -0.38\ldots$ or $-2.61\ldots$ A

11) $2x - 1 = 5$ or $2x - 1 = -5$

$x = -2$ $\quad$ $x = 3$

12) $x \tan x$

$u = x \quad v = \tan x$

$u' = 1 \quad v' = \sec^2 x$

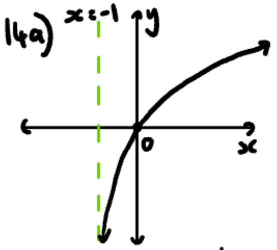
$\tan x + x \sec^2 x$

13) $\left[-2\cos\frac{x}{2}\right]_0^{\frac{\pi}{2}}$

$= -2\cos\frac{\pi}{4} - -2\cos\frac{0}{2}$

$= \frac{-2}{\sqrt{2}} + 2 = -\sqrt{2} + 2$

14a)



14b)

x	1	1.5	2	2.5	3
y	ln2	ln2.5	ln3	ln3.5	ln 4

$A \approx \frac{0.5}{2}\left(\ln 2 + \ln 4 + 2(\ln 2.5 + \ln 3 + \ln 3.5)\right)$

$A \approx 2.154 \text{ units}^2$ (3dp)

15)

$RHS = \sin\theta \tan\theta$

$= \frac{\sin^2\theta}{\cos\theta}$

$= \frac{1 - \cos^2\theta}{\cos\theta}$

$= \sec\theta - \cos\theta$

$= LHS$

$LHS = \frac{1}{\cos\theta} - \cos\theta$

$= \frac{1 - \cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\sin\theta}{1} \times \frac{\sin\theta}{\cos\theta}$

$= \sin\theta \tan\theta$

$= RHS$

16a) $S = 3n+1$

b) $S_{10} = 31$

$Sum_{10} = \frac{10}{2}(4+31)$

$= 175$

c) $\frac{n}{2}\left(2a+(n-1)d\right) < 1750$

$\frac{n}{2}(8+3n-3) < 1750$

$\frac{5n}{2} + \frac{3n^2}{2} < 1750$

$3n^2 + 5n - 3500 < 0$

$(3n-100)(n+35) < 0$

at row k $n = k$

$(3k-100)(n+35) < 0$

d) $k < 33\frac{1}{3}$, 33 rows complete

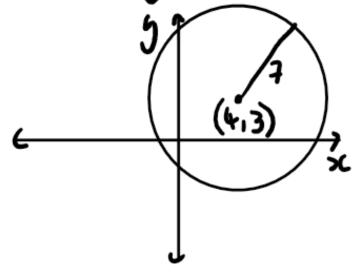
17) $x^2 + 8x + 16 + y^2 - 4y + 4 = 49$

$(x+4)^2 + (y-2)^2 = 49$

translate up 1

reflect around y

$(x-4)^2 + (y-3)^2 = 49$



18) $2^{4x+2} = 2^{-3}$

$\therefore 4x+2 = -3$

$4x = -5$

$x = \frac{-5}{4}$

19) $-\int_2^3 \frac{1}{x-1}\,dx$

$-\left[\ln|x-1|\right]_2^3$

$-(\ln 2 - \ln 1)$

$-\ln 2$

$\ln 2^{-1}$

$\ln \frac{1}{2}$

$\therefore p = \frac{1}{2}$

20a) $y' = 3x^2 + 2x - 1$

for stat points $y' = 0$

$0 = 3x^2 + 3x - x - 1$

$0 = 3x(x+1) - 1(x+1)$

$0 = (3x-1)(x+1)$

$\therefore x = -1$ or $\frac{1}{3}$

A B

$y_A = (-1)^3 + (-1)^2 - (-1) + 2$

$= 3$

$A(-1,3)$

$y_B = \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right) + 2$

$y = \frac{49}{27}$

$B\left(\frac{1}{3}, \frac{49}{27}\right)$

20b) $y'' = 6x + 2$

$y'' > 0$ when concave up

$6x + 2 > 0$

$x > -\frac{1}{3}$ $\left(-\frac{1}{3}, \infty\right)$

c) $\left(\frac{49}{27}, 3\right)$

21a) $\ddot{x} = -16\cos t$

$\ddot{x} = -16\ m/s^2$ $(t=0)$

b) $\dot{x} = 0$

$8 = 16\sin t$

$t = \frac{\pi}{6}$ seconds

c) $x = 8t + 16\cos t + c$

when $t=0$ $x=0$

$0 = 8(0) + 16\cos(0) + c$

$c = -16$

$\therefore x = 8t + 16\cos t - 16$

d) stops at $t = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

$x_{\frac{\pi}{6}} = 8\left(\frac{\pi}{6}\right) + 16\cos\left(\frac{\pi}{6}\right) - 16$

$= 2.045\ldots$

$x_{\frac{5\pi}{6}} = 8\left(\frac{5\pi}{6}\right) + 16\cos\left(\frac{5\pi}{6}\right) - 16$

$= -8.912\ldots$

$\therefore$ distance $= 2.045\ldots - -8.912$

$= 10.96$ (2dp)

22a) $49(7x-2)^6$

b) $\frac{1}{7}\int 49(7x-2)^6\,dx$

$= \frac{1}{7}(7x-2)^7 + c$

23) $\cos\left(x - \frac{\pi}{3}\right) = \frac{1}{2}$

$x - \frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}, -\frac{\pi}{3}$

$x = \frac{2\pi}{3}, 2\pi, 0$

24a) $2^x - 3 = \frac{4}{2^x}$

$2^{2x} - 3\times 2^x - 4 = 0$

Let $2^x = u$

$u^2 - 3u - 4 = 0$

$(u-4)(u+1) = 0$

$2^x = 4$ or $2^x = -1$

$x = 2$ (no solution)

$f(x) = 2^2 - 3$

$= 1$

$A(2,1)$

b) $\int_0^2 4\times 2^{-x} - 2^x + 3\,dx$

$= \left[\frac{-4\times 2^{-x}}{\ln 2} - \frac{2^x}{\ln 2} + 3x\right]_0^2$

$= \left(\frac{-1}{\ln 2} - \frac{4}{\ln 2} + 6\right) - \left(\frac{-4}{\ln 2} - \frac{1}{\ln 2} + 0\right)$

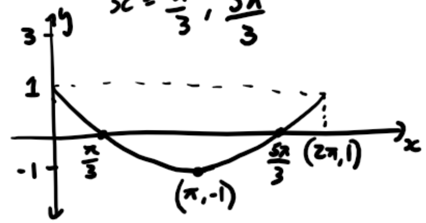
$= \frac{-5}{\ln 2} + \frac{5}{\ln 2} + 6$

$= 6$ units^2

25a) $\sin\frac{x}{2} = \frac{1}{2}$

$\frac{x}{2} = \frac{\pi}{6}, \frac{5\pi}{6}$

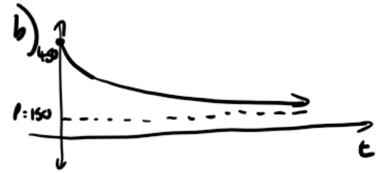
$x = \frac{\pi}{3}, \frac{5\pi}{3}$



26a) $P = 150 + 300e^{-0.05(0)}$

$P = 450$

b)



c) $200 = 150 + 300e^{-0.05t}$

$e^{-0.05t} = \frac{1}{6}$

$t = \frac{\ln\frac{1}{6}}{-0.05}$

$t = 35.835$ $\therefore$ during 2055

27a) $k\int_0^8 64 - x^2\,dx$

$k\left[64x - \frac{x^3}{3}\right]_0^8 = 1$

$k = \frac{3}{1024}$

b) $\frac{3}{1024}\int_0^x 64 - x^2\,dx = CDF$

$\frac{3x}{16} - \frac{x^3}{1024} = CDF$

c) $P(X > 5) = 1 - \left(\frac{3(5)}{16} - \frac{5^3}{1024}\right)$

$= 0.185$ (3 sig fig)

28a) $u = \sin x$ $\quad v = 1 + \cos x$

$u' = \cos x$ $\quad v' = -\sin x$

$\frac{dy}{dx} = \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$

$= \frac{1}{1+\cos x}$

b) $m = \frac{1}{1+\cos\frac{\pi}{3}}$

$= \frac{2}{3}$

when $x = \frac{\pi}{3}$

$y = \frac{\sin\frac{\pi}{3}}{1+\cos\frac{\pi}{3}}$

$= \frac{\sqrt{3}}{3}$

$\therefore$ tangent

$y - \frac{\sqrt{3}}{2} = \frac{2}{3}\left(x - \frac{\pi}{3}\right)$

$y = \frac{2x}{3} - \frac{2\pi}{3} + \frac{\sqrt{3}}{3}$

29a) $P(win) = (0.25)(0.6) + (0.75)(0.3)$

$= 0.375$

b) $P(rain|win) = \frac{P(rain \cap win)}{P(win)}$

$= \frac{(0.25)(0.6)}{0.375}$

$= 0.4$

30a) $A_1 = 800\,000(1.01) - M$

$A_2 = A_1(1.01) - M$

$= 800\,000(1.01)^2 - M(1.01) - M$

$= 800\,000(1.01)^2 - M(1.01+1)$

$= 816080 - M(2.01)$

b)

$A_{24} = 800\,000(1.01)^{24} - M\left(\frac{(1.01)^{24} - 1}{1.01 - 1}\right)$

$A_{24} = 0$

$\frac{M((1.01)^{24} - 1)}{0.01} = 800000(1.01)^{24}$

$M = \frac{8000(1.01)^{24}}{1.01^{24} - 1}$

c) from b) $M = 37658.77\ldots$

$A_2 = 816070 - (37658.77\ldots)(2.01)$

$= 740386$ (nearest \$)

d) $A_n = 740386\,(1.01)^n - 50000\left(\frac{1.01^n - 1}{0.01}\right)$

$5000000(1.01)^n - 740386\,(1.01)^n = 5000000$

$(1.01)^n(4259614) = 5000000$

$1.01^n = 1.1738\ldots$

$n = \frac{\ln 1.1738\ldots}{\ln 1.01}$

$n = 16.10569\ldots$

$\doteqdot 17$ months

total repayments will be 19 months

$\therefore$ reduction of 5 months.

31) $27 = \frac{6}{\frac{w}{1-w}}$

$0 = 27w^2 - 27w + 6$

$0 = 9w^2 - 9w + 2$

$0 = (3w-1)(3w-2)$

$\therefore w = \frac{1}{3}$ or $\frac{2}{3}$

32a) $R = 100(8) - (8)^3$

$= 288$ kg/s

b) when $T = 0$ $\quad R = 0$

$T = 11$ $\quad R = -231$

(not reasonable to have -ive rate)

$\therefore T = 10$ largest reasonable answer

c) $R' = 100 - 3t^2$

$0 = 100 - 3t^2$

$t = \pm\frac{10}{\sqrt{3}}$ (discard $\ominus$)

$t \approx 5.77\ldots$

$R'' = -6t$ $\quad$ when $t = 5.77$

$R'' = -34.62\ldots$ $\therefore t = 5.77$ is max.

$R = 100\left(\frac{10}{\sqrt{3}}\right) - \left(\frac{10}{\sqrt{3}}\right)^3$

$= 384.9$ kg/s (1dp)

d) amount $= 50t^2 - \frac{t^4}{4} + 100$

33a) time = swim + walk

$= 1 \div 2 + 3 \div 4$

$= 1.25$ hours.

b) $\text{time} = \dfrac{\sqrt{1+x^2}}{2} + \dfrac{3-x}{4}$

$\text{time}' = \dfrac{x(1+x^2)^{-\frac{1}{2}}}{2} - \dfrac{1}{4}$

Let $\text{time}' = 0$

$\dfrac{1}{4} = \dfrac{x}{2\sqrt{1+x^2}}$

$\dfrac{1}{4} = \dfrac{x^2}{1+x^2}$

$3x^2 = 1$

$x = \pm \dfrac{1}{\sqrt{3}}$ discard $\ominus$

$x \approx 0.577\ldots$

x	0.5	$\frac{1}{\sqrt{3}}$	0.6
t'	-0.03	0	0.007
	-	0	+

$\therefore x = \dfrac{1}{\sqrt{3}}$ is min turning point.

$\text{time} = \dfrac{\sqrt{1+\left(\frac{1}{\sqrt{3}}\right)^2}}{2} + \dfrac{3-\frac{1}{\sqrt{3}}}{4}$

$= 1.183$ hours

$= 0.066\ldots$ hours saved

4 minutes saved.