Student Number

Barker College

2022 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

Staff Involved:

Friday August 5th, 2022

- ALY • PCB • RJW • RAS • DXC*
- ESP • AXD • GPF • JZT

160 copies

General Instructions:

- Reading time - 10 minutes
- Writing time - 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided separately
- For questions in Section II, show relevant mathematical reasoning and / or calculations
- Marks may not be awarded for careless or poorly arranged working
- Diagrams are not to scale unless specifically stated

Total marks: 100

Section I - 10 marks (pages 2 - 6)
- Attempt Multiple Choice Questions 1 - 10
- Allow about 20 minutes for this section

Section II - 90 marks (pages 7 - 27)
- Attempt Questions 11 - 33
- Show all necessary working
- Allow about 2 hours and 40 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 20 minutes for this section

1 What is domain of the function $y = \ln(2x - 3)$?

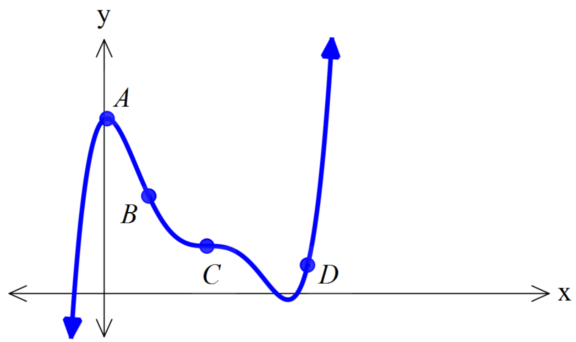
A. All real x

B. $(0, \infty)$

C. $\left[\frac{3}{2}, \infty\right)$

D. $\left(\frac{3}{2}, \infty\right)$

2 At which point on this curve are **both** the first derivative and second derivative equal to zero?



A. A

B. B

C. C

D. D

3 Evaluate the following integral:

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (\cos x + 1)\, dx$$

A. $\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{\pi}{2}$

B. $\frac{\sqrt{3}}{2} - \frac{1}{2} + \frac{\pi}{2}$

C. $\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{\pi}{6}$

D. $\frac{\sqrt{3}}{2} - \frac{1}{2} - \frac{\pi}{6}$

4 Which of the following equations is the reflection of the curve $y = 3^x + 2$ in the x-axis, then translated 2 units downwards?

A. $y = 3^{-x}$

B. $y = 3^{-x} - 4$

C. $y = -3^x$

D. $y = -3^x - 4$

5 What values of x satisfy the equation $|3x - 4| - 2 = 3$?

A. $x = -3$ and $x = \frac{1}{3}$

B. $x = -\frac{1}{3}$ and $x = 3$

C. $x = 1$ and $x = 3$

D. $x = -1$ and $x = \frac{1}{3}$

6 There are 16 T-shirts in a basket. 10 of the T-shirts are blue and 8 of the T-shirts have stripes. 4 of the T-shirts are neither blue nor have stripes. What is the probability of taking a T-shirt at random from the basket that is blue and has stripes?

A. $\frac{1}{8}$

B. $\frac{1}{4}$

C. $\frac{3}{8}$

D. $\frac{1}{2}$

7 Which of the following is equal to $\frac{\ln 8^x + 3\ln 3^x}{2\ln 3 + \ln 8 - \ln 2}$?

A. x

B. $\frac{3x}{2}$

C. $3x$

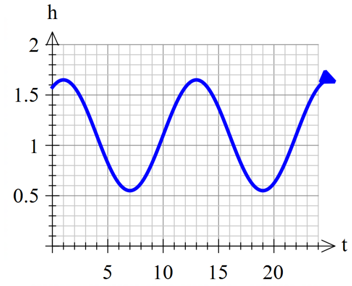
D. $6x$

8 The height of the water in a harbor is modelled by the following function:

$$h(t) = 0.55\cos\left(\frac{\pi}{6}(t-1)\right) + 1.1$$

Which of the following is the graph of this function?

A.



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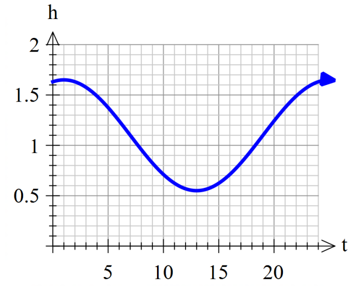

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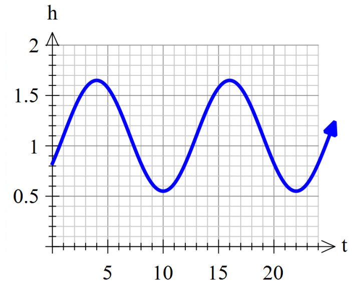
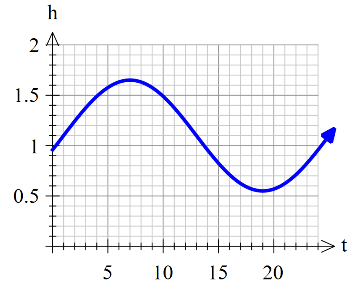


D.



9 The probability density function for the continuous random variable *X* is:

$$f(x) = \begin{cases} 2xe^{-x^2} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

What is the mode of the continuous random variable *X*?

A. $\ln 2$

B. $\sqrt{\ln 2}$

C. $\sqrt{2}$

D. $\frac{1}{\sqrt{2}}$

10 $f(x)$ is an odd function defined over all real x.

It is known that $\int_0^1 f(x)\,dx = 2$ and $\int_{-1}^0 f(x)\,dx = \int_1^3 f(x)\,dx$.

What is the value of $\int_0^4 f(x-1)\,dx$?

A. -2

B. 0

C. 2

D. 4

End of Section I

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Student Number

Section II

90 marks
Attempt Questions 11 – 33
Allow about 2 hours and 40 minutes for this section.
Write your student number on all answer pages.

In Questions 11 – 33, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (1 mark)

Differentiate $e^x \cos x$. **1**

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Question 12 (1 mark)

Find $\int 3^x \, dx$ **1**

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Question 13 (2 marks)

Evaluate the arithmetic series $3 + 6.5 + 10 + 13.5 + \cdots + 66$ **2**

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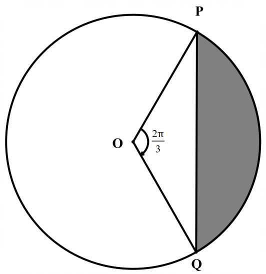
Question 14 (2 marks)

The diagram shows a circle with centre O. **2**
The points P and Q are lying on the circle such that $\angle POQ = \frac{2\pi}{3}$.
If the triangle POQ has an area of $8\sqrt{3}$ cm^2, find the exact area of the shaded minor segment.



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Question 15 (3 marks)

Find the equation of the normal to the curve $y = x^4 - 6x - 3$ at the point on the curve where $x = 2$. **3**

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Question 16 (4 marks)

In the mobile game *Desmos Royale*, players earn 0, 1, 2, or 3 star points after completing every level of the game. Let the discrete random variable X be the number of star points that a player earns from completing a level in the game.

x	0	1	2	3
$p(X=x)$	0.1	$2a$	$5a$	a

(a) Find the value of a. **1**

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(b) Calculate the expected value of X. **2**

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(c) Unlocking a feature in the game costs 50 star points. How many levels should a player be expected to complete in order to earn enough star points to unlock a feature? **1**

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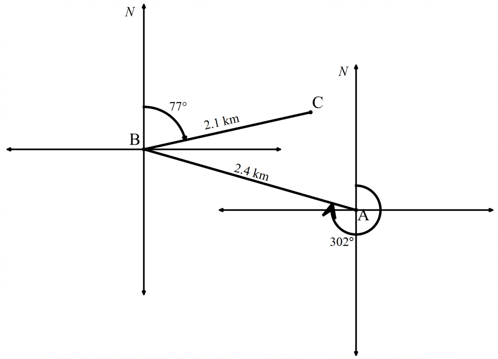
Question 17 (3 marks)

Ms Kalnins flies a drone for 2.4 km on a bearing of 302° from location A to location B. She then flies the drone for 2.1 km on a bearing of 077° to location C.

The information is shown in the diagram below.



(a) What is the size of $\angle ABC$? **1**

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(b) What is the distance from point A to point C?
Give your answer correct to 1 decimal place. **2**

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Question 18 (2 marks)

A pencil case contains 7 blue pens and 3 red pens. Two pens are selected at random without replacement. What is the probability of selecting at least one red pen? **2**

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Question 19 (5 marks)

The number N of E. coli bacteria that is present in a petri dish after t minutes is given by the following equation:

$$N = 6e^{kt}$$

The population of E. coli bacteria will double in number in 20 minutes.

(a) Find the exact value of k. **2**

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(b) Determine the exact number of bacteria after 6 hours. **1**

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(c) At what rate is the number of bacteria increasing after 6 hours? **2**
Give your answer correct to the nearest whole number.

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Question 20 (4 marks)

Consider the graph of $f(x) = 3\sin\left(2x - \frac{\pi}{3}\right)$

(a) State the amplitude and period for the graph. **2**

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(b) Solve the equation $3\sin\left(2x - \frac{\pi}{3}\right) = 3$ over the domain $0 \leq x \leq \pi$. **2**

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Question 21 (9 marks)

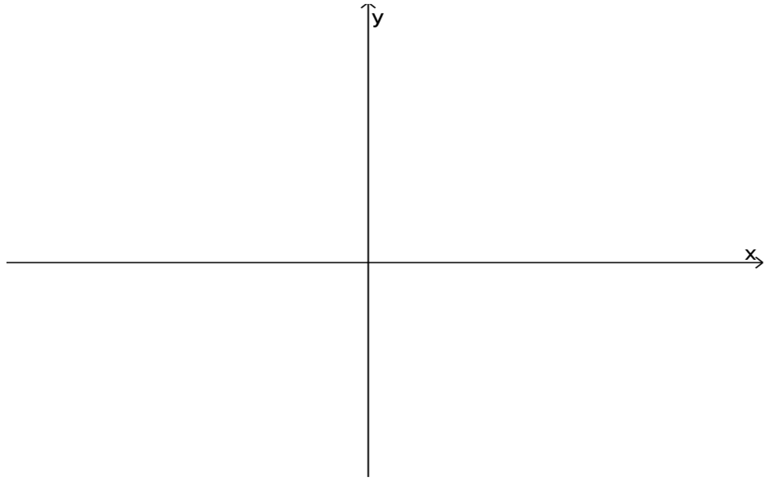
Let $f(x) = x^3 + 5x^2 - 8x - 40$.

(a) State the x-intercepts of $f(x)$. **2**

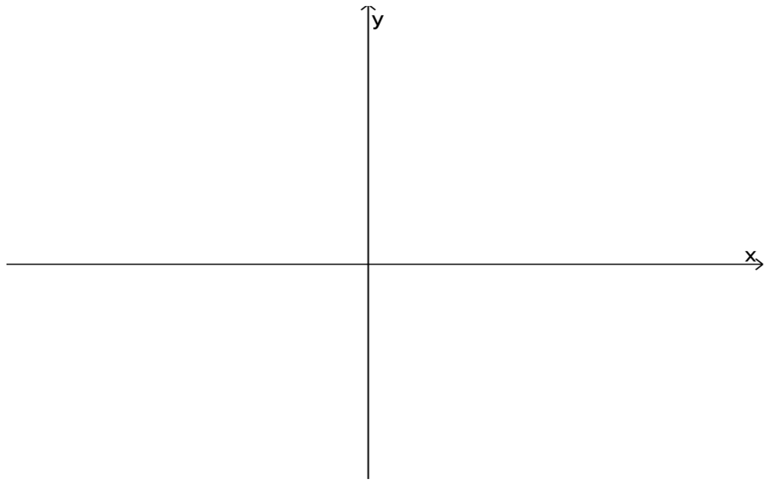
(b) Find the coordinates of the stationary points of $f(x)$ and determine their nature. **3**

Question 21 continues on the next page

(c) Hence, sketch the graph of $y = f(x)$ showing all stationary points and intercepts. **2**



(d) Sketch the graph of $y = f\left(-\frac{x}{2}\right)$ showing all stationary points and intercepts. **2**



End of Question 21

Student Number

Question 22 (2 marks)

If $f'(x) = 3xe^{x^2}$, find $f(x)$ if the graph $y = f(x)$ has a y-intercept of 4. **2**

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Question 23 (2 marks)

A fuel pump connected to a power generator operates at a varying rate, $R(t)$, for an hour, where $R(t)$ is in L/min. The rate is recorded every 10 minutes and is given in the table below. **2**

t	0	10	20	30	40	50	60
$R(t)$	264	295	264	250	268	273	286

The number of litres of fuel pumped over this time frame is given by

$$\int_0^{60} R(t)\, dt$$

Using the trapezoidal rule with 7 function values, find the approximate number of litres of fuel pumped over the course of the hour.

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Question 24 (5 marks)

A film director invites 24 film critics to rate his latest film out of 5 stars. The table below shows the ratings given by the film critics:

Number of stars	*Frequency*
0	3
1	4
2	5
3	7
4	3
5	2

(a) Calculate the mean and median for this data set. **2**

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The director decided to record the gender of each film critic along with their review score in a two-way table:

	Male	Female	Total
3 or more stars			
Less than 3 stars		9	
Total	11		

(b) Using the information stated previously, complete the rest of the above two-way table. **2**

(c) What is the probability a film critic is male, given the critic gave the film a score of 3 or more stars? **1**

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Question 25 (2 marks)

Find the point(s) of intersection of the graphs of $y = 9^x + 4$ and $y = 13 - 8 \times 3^x$. **2**

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Question 26 (3 marks)

2.4L of organic material is added to a compost bin every day. Each day, bacteria in the compost bin break down 12.5% of the total material in the compost bin.
Initially, the compost bin is empty.

(a) How much organic material is in the compost bin immediately after material is added on the second day? **1**

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(b) The maximum capacity of the compost bin is 18L. Is the capacity of the bin sufficient for its current usage? Give justification as to why or why not. **2**

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Question 27 (4 marks)

The average Sydney daily temperature, $T(x)$, in degrees Celsius, is modelled by the equation

$$T(x) = 22 + 4\cos\left(\frac{2\pi x}{366}\right) \qquad \text{for } 0 \leq x \leq 365$$

where x is the number of days after 1st January in a leap year.

(a) State the average Sydney daily temperature to 2 decimal places for February 25th. **1**

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(b) Find the values of x for when the average Sydney daily temperature is 20°C. **3**

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Question 28 (9 marks)

Waymond borrows \$532 000 to purchase a house. The loan is to be repaid in equal monthly repayments of \$$M$ over 25 years. The interest, which is 4.8% p.a., is reducible and calculated immediately before each repayment at the end of the month.

(a) Show that the amount owing after n months, immediately after the monthly repayment is $A_n = 532\,000(1.004)^n - 250M(1.004^n - 1)$. **4**

(b) Show that $M = \$3048.34$ to the nearest cent. **1**

Question 28 continues on the next page

Question 28 (continued)

(c) Waymond can pay an upfront deposit of 20% of the principal of his loan to reduce the amount of his regular monthly repayment. If the interest rate (4.8% p.a.) and time frame for the loan remains the same (25 years), calculate how much interest he saves by paying the 20% deposit compared to his original repayment plan. **4**

End of Question 28

Student Number

Question 29 (5 marks)

(a) Show that $\frac{d}{dx}\left(\ln\left(\frac{x}{x+1}\right)\right) = \frac{1}{x^2+x}$ **2**

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(b) Hence, or otherwise, find the exact value of k if $\int_1^3 \frac{2}{x^2+x}\,dx = \ln k$. **3**

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Question 30 (4 marks)

A particle is initially at the origin and its velocity v m/sec after t seconds is given by:

$$v = 2t(2t-1)(t-1)$$

(a) Find the displacement function, x, and the times when the particle is at the origin. **2**

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(b) Calculate the total distance the particle has travelled in the first second. **2**

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Question 31 (9 marks)

An election is held where the proportion of the vote that goes to a certain politician is modelled by the continuous random variable X with probability density function $f(x)$ given by:

$$f(x) = \begin{cases} kx(1-x)^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = 12$. **2**

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(b) The cumulative distribution function of the random variable X is given by the piecewise function $F(x)$. Fill in the blank spaces to complete the definition for $F(x)$: **2**

$$F(x) = \begin{cases} \underline{\qquad\qquad\qquad\qquad}, & \text{for } x < 0 \\ \underline{\qquad\qquad\qquad\qquad}, & \text{for } 0 \leq x \leq 1 \\ \underline{\qquad\qquad\qquad\qquad}, & \text{for } x > 1 \end{cases}$$

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Question 31 continues on the next page

(c) What is the probability that the politician will win at least 50% of the vote? **2**

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(d) For any continuous random variable X with a probability density function $f(x)$ over the domain $[a, b]$, the values of $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$ are given by: **3**

$\mathrm{E}(X) = \int_a^b x f(x)\, dx$ $\qquad\qquad$ $\mathrm{E}(X^2) = \int_a^b x^2 f(x)\, dx$

Hence, calculate the mean and standard deviation of the probability distribution.

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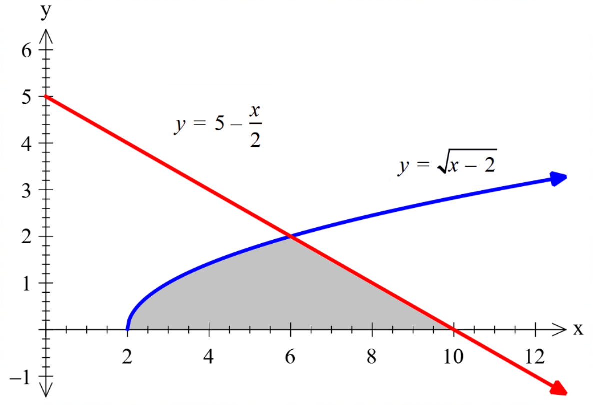
End of Question 31

Student Number

Question 32 (3 marks)

The diagram below shows the graphs of $y = 5 - \frac{x}{2}$ and $y = \sqrt{x-2}$.



(a) Show that there is a point of intersection when $x = 6$. **1**

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(b) Find the exact value of the shaded area in simplest form. **2**

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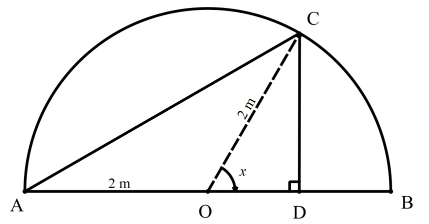
Question 33 (6 marks)

Let A and B form the diameter of a semicircle of radius 2 metres and centre at O.
From a point D lying on the line AB, a perpendicular is drawn meeting the circumference of the semicircle at C.
Let $\angle COD = x$.



(a) Show that the area, $A(x)$, of triangle ACD can be given by the equation: **2**

$$A(x) = 2\sin x\,(1 + \cos x)$$

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(b) Hence, find the value of x that gives triangle ACD the greatest area. **4**

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Question 33 (continued)

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End of Paper

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Student Number

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If you use this space, clearly indicate what question you are answering.

2022 Mathematics Advanced Trial Student Solutions

1.) $2x-3>0$

$2x>3$

$x>\frac{3}{2}$

$\therefore x \in (\frac{3}{2}, \infty)$

(D)

2.) Point C is the only point where the gradient of the tangent $=0$ $(y'=0)$ AND there is a point of inflexion. $(y''=0)$

(C)

3.) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (\cos x+1)\,dx = \left[\sin x + x\right]_{-\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= \left(\sin\frac{\pi}{3}+\frac{\pi}{3}\right) - \left(\sin\left(-\frac{\pi}{6}\right)+ -\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2}+\frac{\pi}{3}+\frac{1}{2}+\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}+\frac{1}{2}+\frac{\pi}{2}$

(A)

4) $y=3^x+2$

Reflection about x-axis $\rightarrow y=-(3^x+2)$

$=-3^x-2$

Translation 2 units down $\rightarrow y=-3^x-2-2$

$=-3^x-4$

(D)

5.) $|3x-4|-2=3$

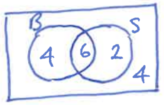
$|3x-4| = 5$

$3x-4=5$ or $3x-4=-5$

$3x=9$ $\quad$ $3x=-1$

$x=3$ $\quad$ $x=-\frac{1}{3}$

(B)

6.) 

Blue or striped $=16-4$

$=12$

Blue and Striped $=10+8-12$

$=6$

P(Blue and Striped) $=\frac{6}{16}$

$=\frac{3}{8}$

(C)

7.) $\frac{\ln(8^x)+3\ln(3^x)}{2\ln 3+\ln 8-\ln 2} = \frac{\ln(2^{3x})+3x\ln 3}{2\ln 3+\ln 2^3-\ln 2}$

$=\frac{3x\ln 2+3x\ln 3}{2\ln 3+3\ln 2-\ln 2}$

$=\frac{3x(\ln 2+\ln 3)}{2\ln 3+2\ln 2}$

$=\frac{3x(\ln 2+\ln 3)}{2(\ln 3+\ln 2)}$

$=\frac{3x}{2}$

(B)

8.) Period $=\frac{2\pi}{\frac{\pi}{6}}$

$=12$

Only (A) and (C) have a period of 12.

Horizontal shift 1 to the right.

Cosine curve is symmetrical about y-axis, so a 1 unit shift to right results in (A)

9.) Mode occurs when $f(x)$ peaks $\rightarrow f'(x)=0$

$f'(x)=2e^{-x^2}-4x^2e^{-x^2}$

when $2e^{-x^2}-4x^2e^{-x^2}=0$

$2e^{-x^2}(1-2x^2)=0$

$1-2x^2=0$

$2x^2=1$

$x^2=\frac{1}{2}$

$x=\pm\frac{1}{\sqrt{2}}$, but $x\geq 0$

$\therefore$ (D)

10.) $\int_0^4 f(x-1)\,dx = \int_{-1}^3 f(x)\,dx$

$= \int_{-1}^0 f(x)\,dx + \int_0^1 f(x)\,dx + \int_1^3 f(x)\,dx$

Since $f(x)$ is odd: $\int_{-1}^0 f(x)\,dx = -\int_0^1 f(x)\,dx = -2$

So integral $= -2 + 2 - 2$

$= -2$

A

11.) $\frac{d}{dx}(e^x \cos x) = e^x \cos x + e^x(-\sin x)$

$= e^x \cos x - e^x \sin x$

12.) From the reference sheet: $3^x = e^{x\ln 3}$

$\therefore \int 3^x\,dx = \int e^{x\ln 3}\,dx$

$= \frac{e^{x\ln 3}}{\ln 3} + C$

$= \frac{3^x}{\ln 3} + C$

13.) Arithmetic Series:

$a = 3 \quad d = 3.5$

$T_n = 3 + 3.5(n-1)$

when $T_n = 66$

$66 = 3 + 3.5(n-1)$

$63 = 3.5(n-1)$

$18 = n - 1$

$\therefore n = 19$

$\therefore S_{19} = \frac{19}{2}(3 + 66)$

$= 655.5$

14.) Area $\Delta POQ = \frac{1}{2}r^2 \sin\theta$

$8\sqrt{3} = \frac{1}{2}r^2 \sin\frac{2\pi}{3}$

$8\sqrt{3} = \frac{1}{2}r^2\left(\frac{\sqrt{3}}{2}\right)$

$\therefore r^2 = 32$

Area of segment $= \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2} \times 32\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$

$= 16\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= \left(\frac{32\pi}{3} - 8\sqrt{3}\right)$ cm^2

15.) $y' = 4x^3 - 6$

when $x = 2$ grad. of tangent $= 4 \times 2^3 - 6$

$= 26$

$\therefore$ grad. of normal $= -\frac{1}{26}$

Eq^n of normal

$m = -\frac{1}{26}$ $\quad$ $x = 2$

$y = 2^4 - 6 \times 2 - 3$

$= 1$

$y - 1 = -\frac{1}{26}(x - 2)$

$y - 1 = -\frac{x}{26} + \frac{1}{13}$

$y = -\frac{x}{26} + \frac{14}{13}$ OR $x + 26y - 28 = 0$

16a.) $0.1 + 2a + 5a + a = 1$

$8a = 0.9$

$a = \frac{0.9}{8}$

$= \frac{9}{80}$

b.) $E(X) = 0 \times 0.1 + 1 \times 2a + 2 \times 5a + 3 \times a$

$= 2a + 10a + 3a$

$= 15a$

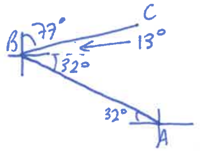
$= 15 \times \frac{9}{80}$

$= 1.6875$

c.) $\frac{50}{1.6875} = 29.6\ldots$

$\therefore$ need to complete 30 levels

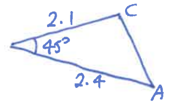
17a.)



$\angle ABC = 32° + 13°$

$= 45°$

b.)



$AC^2 = 2.1^2 + 2.4^2 - 2(2.1)(2.4)\cos 45°$

$\therefore AC = 1.7$ km (1 d.p.)

18.) P(at least 1 red) = 1 − P(no red)

$= 1 - \frac{7}{10} \times \frac{6}{9}$

$= \frac{8}{15}$

19.) a.) when $t = 0$

$N = 6$

when $t = 20$

$N = 12$ $\rightarrow$ sub into $N = 6e^{kt}$

$12 = 6e^{20k}$

$2 = e^{20k}$

$20k = \ln 2$

$k = \frac{\ln 2}{20}$

b.) doubles every 20 min.

6 hours = 360 min.

So no. of bacteria doubles $\frac{360}{20} = 18$ times.

After 6 hours, there are:

$6 \times 2^{18} = 1\,572\,864$

19c.) $\frac{dN}{dt} = 6ke^{kt}$

$= k \times 6e^{kt}$

when $t = 360$, $6e^{kt} = 1\,572\,864$

$\therefore \frac{dN}{dt} = \frac{\ln 2}{20} \times 1\,572\,864$

$= 54\,511$ bacteria/min. (nearest whole)

20a.) $A = 3$

$T = \frac{2\pi}{2}$

$= \pi$

b.) $3\sin\left(2x - \frac{\pi}{3}\right) = 3$, $\quad 0 \le x \le \pi$

$0 \le 2x \le 2\pi$

$\underbrace{-\frac{\pi}{3} \le 2x - \frac{\pi}{3} \le 2\pi - \frac{\pi}{3}}$

$\sin\left(2x - \frac{\pi}{3}\right) = 1$

$2x - \frac{\pi}{3} = \frac{\pi}{2}$ $\quad$ In this domain, only solution is $\frac{\pi}{2}$

$2x = \frac{\pi}{2} + \frac{\pi}{3}$

$2x = \frac{5\pi}{6}$

$x = \frac{5\pi}{12}$

21a.) $f(x) = x^3 + 5x^2 - 8x - 40$

when $f(x) = 0$

$x^3 + 5x^2 - 8x - 40 = 0$

$x^2(x + 5) - 8(x + 5) = 0$

$(x + 5)(x^2 - 8) = 0$

$x = -5 \quad x^2 = 8$

$x = \pm 2\sqrt{2}$

$\therefore$ x-int's are $x = -5, 2\sqrt{2}, -2\sqrt{2}$

b.) $f'(x) = 3x^2 + 10x - 8$

when $f'(x) = 0$

$3x^2 + 10x - 8 = 0$

$(3x - 2)(x + 4) = 0$

$x = \frac{2}{3}$, $\quad x = -4$

$y = \frac{-1156}{27}$, $\quad y = 8$

21b.) (continued)

Suspected stationary points are:

$\left(\frac{2}{3}, -\frac{1156}{27}\right)$ and $(-4, 8)$

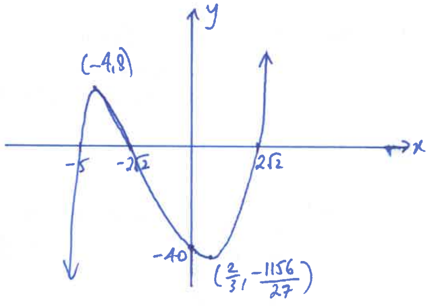
testing using $f''(x)$:

$f''(x) = 6x + 10$

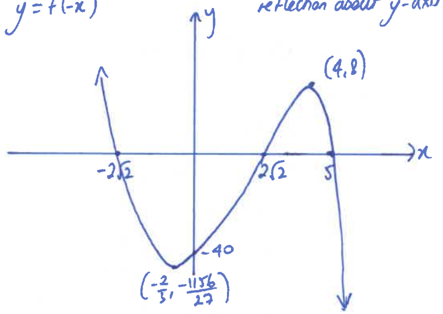
$f''\left(\frac{2}{3}\right) = 14 > 0 \quad \therefore \left(\frac{2}{3}, -\frac{1156}{27}\right)$ is a local min T.P.

$f''(-4) = -14 < 0 \quad \therefore (-4, 8)$ is a local max T.P.

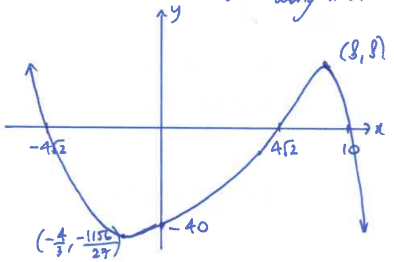
21c.)



21d.) $y = f(-x)$ reflection about y-axis



$y = f\left(-\frac{x}{2}\right) \longrightarrow$ dilation by a factor of 2 along x-axis



22.) $f(x) = \int 3x e^{x^2} dx$

$= \frac{3}{2}\int 2x e^{x^2} dx$

$= \frac{3}{2} e^{x^2} + C$

when $x = 0, y = 4$

$4 = \frac{3}{2} e^0 + C$

$4 = \frac{3}{2} + C$

$\therefore C = \frac{5}{2}$

$\therefore f(x) = \frac{3}{2} e^{x^2} + \frac{5}{2}$

23.)

$\int_0^{60} R(t)\,dt \approx \frac{10}{2}\left[264 + 286 + 2(295 + 264 + 250 + 268 + 273)\right]$

$= 16\,250\text{L}$

24.)a)

$\bar{x} = \frac{0\times3 + 1\times4 + 2\times5 + 3\times7 + 4\times3 + 5\times2}{24}$

$= 2.375$

median = halfway between 12th & 13th score

$= 2.5$

b.)

	M	F	Total
≥3	d	b	c
<3	e	9	f
Total	11	a	g

$11 + a = 24$
$a = 13$

$b + 9 = 13$
$b = 4$

c = frequency of three or more stars $= 12$

$d + 4 = 12$
$d = 8$

$8 + e = 11$
$e = 3$

$f = 3 + 9 = 12$

$g = 12 + 12 = 24$

24c.) $P(\text{male} \mid 3 \text{ or more stars}) = \frac{8}{12}$

$= \frac{2}{3}$

25.) when $9^x + 4 = 13 - 8(3^x)$

$(3^2)^x + 4 = 13 - 8(3^x)$

$(3^x)^2 + 4 = 13 - 8(3^x)$

$(3^x)^2 + 8(3^x) - 9 = 0$

$(3^x - 1)(3^x + 9) = 0$

$3^x = 1, \quad 3^x = -9$ reject

$x = 0$

when $x = 0$

$y = 9^0 + 4$

$= 5$

$(0, 5)$

26a.) Let A_n = amount in bin after n^{th} addition

$A_1 = 2.4$

$A_2 = 0.875 \times A_1 + 2.4$

$= 0.875 \times 2.4 + 2.4$

$= 4.5\text{L}$

b.) $A_3 = 0.875 \times A_2 + 2.4$

$= 0.875^2 \times 2.4 + 0.875 \times 2.4 + 2.4$

$\vdots$

$A_n = 2.4 \times 0.875^{n-1} + \dots + 2.4$

since $|r| < 1$

$A_\infty = \frac{a}{1-r}$

$= \frac{2.4}{0.125}$

$= 19.2\text{L}$

$\therefore$ The capacity of the bin is NOT sufficient for how much compost it would eventually need to contain.

27.) a.) no. of days passed $= 30 + 25 = 55$

when $x = 55$ $\quad T(55) = 22 + 4\cos\left(\frac{2\pi \times 55}{366}\right)$

$= 24.35°\text{C}$ (2 d.p.)

b.) want x when $T(x) = 20$

$20 = 22 + 4\cos\left(\frac{2\pi x}{366}\right), \quad 0 \le \frac{2\pi x}{366} \le \frac{2\pi \times 365}{366}$

$-2 = 4\cos\left(\frac{2\pi x}{366}\right)$

$\cos\left(\frac{2\pi x}{366}\right) = -\frac{1}{2}$

Looking for solutions in 2nd and 3rd quad.

the upper limit of the domain is close to 2π

$\frac{2\pi x}{366} = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\frac{x}{366} = \frac{1}{3}, \frac{2}{3}$

$x = 122, 244$

28a.) monthly rate $= \frac{4.8\%}{12} = 0.4\% = 0.004$

$A_1 = 532\,000(1.004)^1 - M$

$A_2 = 1.004 \times A_1 - M$

$= 532\,000(1.004)^2 - M(1.004) - M$

$\vdots$

$A_n = 532\,000(1.004)^n - M(1.004)^{n-1} - \dots - M$

$= 532\,000(1.004)^n - M(1.004^{n-1} + \dots + 1)$

$= 532\,000(1.004)^n - M\left(\frac{1.004^n - 1}{1.004 - 1}\right)$

$= 532\,000(1.004)^n - M\left(\frac{1.004^n - 1}{0.004}\right)$

$= 532\,000(1.004)^n - 250M(1.004^n - 1)$ //

b.) when $n = 25 \times 12 = 300$, $A_{300} = 0$

$0 = 532\,000(1.004)^{300} - 250M(1.004^{300} - 1)$

$250M(1.004^{300} - 1) = 532\,000(1.004)^{300}$

$M = \frac{532\,000(1.004)^{300}}{250(1.004^{300} - 1)}$

$= \$3048.34$

28c.) Without 20% deposit:

Total repayment $= 3048.34 \times 300$
$= 914\,502$

Total interest paid $= 914\,502 - 532\,000$
$= 382\,502$

With 20% deposit:

Amount borrowed $= 0.8 \times 532\,000$
$= 425\,600$

Monthly repayment $= 0.8 \times 3048.34$
$= 2438.672$

$\therefore$ Total repayment $= 2438.672 \times 300$
$= 731\,601.60$

Total interest paid $= 731\,601.60 - 425\,600$
$= 306\,001.60$

Saving in interest $= 382\,502 - 306\,001.60$
$= \$76\,500.40$

29a.) Derivative of inner function:

$$\frac{d\left(\frac{x}{x+1}\right)}{dx} = \frac{(x+1)(1) - x(1)}{(x+1)^2}$$

$$= \frac{1}{(x+1)^2}$$

$$\frac{d(\ln(f(x)))}{dx} = \frac{f'(x)}{f(x)}$$

$$\therefore \frac{d\left(\ln\left(\frac{x}{x+1}\right)\right)}{dx} = \frac{1}{(x+1)^2} \div \frac{x}{(x+1)}$$

$$= \frac{1}{(x+1)^2} \times \frac{(x+1)}{x}$$

$$= \frac{1}{x(x+1)}$$

$$= \frac{1}{x^2+x}$$

29b.)

$$\int_1^3 \frac{2}{x^2+x}\,dx = \ln k$$

$$2\int_1^3 \frac{1}{x^2+x}\,dx = \ln k$$

$$2\left[\ln\left(\frac{x}{x+1}\right)\right]_1^3 = \ln k$$

$$2\left[\ln\left(\tfrac{3}{4}\right) - \ln\left(\tfrac{1}{2}\right)\right] = \ln k$$

$$2\left(\ln\left(\tfrac{3}{4} \div \tfrac{1}{2}\right)\right) = \ln k$$

$$2\left(\ln\left(\tfrac{3}{2}\right)\right) = \ln k$$

$$\ln\left(\tfrac{9}{4}\right) = \ln k$$

$$\therefore k = \tfrac{9}{4}$$

30.) a.)

$$v = 2t(2t-1)(t-1)$$
$$= (4t^2 - 2t)(t-1)$$
$$= 4t^3 - 6t^2 + 2t$$

$$x = \int (4t^3 - 6t^2 + 2t)\,dt$$
$$= t^4 - 2t^3 + t^2 + c$$

when $t=0$, $x=0$ $\quad \therefore c = 0$

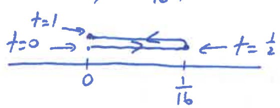
$$\therefore x = t^4 - 2t^3 + t^2$$

when $x=0$

$$t^4 - 2t^3 + t^2 = 0$$
$$t^2(t^2 - 2t + 1) = 0$$
$$t^2(t-1)^2 = 0$$
$$t=0,\ t=1$$

30b.) when $v=0$

$$2t(2t-1)(t-1) = 0$$
$$t=0,\ t=\tfrac{1}{2},\ t=1$$
$$x=0,\ x=\tfrac{1}{16},\ x=0$$



$\therefore$ total dist. travelled $= 2 \times \frac{1}{16}$
$= \frac{1}{8}$ metre

31.) a.)

$$\int_0^1 f(x)\,dx = 1$$

$$k\int_0^1 x(1-x)^2\,dx = 1$$

$$k\int_0^1 x(1-2x+x^2)\,dx = 1$$

$$k\int_0^1 (x - 2x^2 + x^3)\,dx = 1$$

$$k\left[\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4}\right]_0^1 = 1$$

$$k\left(\tfrac{1}{2} - \tfrac{2}{3} + \tfrac{1}{4} - 0\right) = 1$$

$$k \times \tfrac{1}{12} = 1$$

$$k = 12$$

b.)

$$F(x) = \int_0^x 12t(1-t)^2\,dt$$

$$= 12\left(\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4}\right) \quad \text{(from previous part)}$$

$F(0) = 0$

$F(1) = 1$

$$\therefore F(x) = \begin{cases} 0 & x<0 \\ 12\left(\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4}\right) & 0 \le x \le 1 \\ 1 & x>1 \end{cases}$$

31c.)

$$P(X \ge 0.5) = 1 - P(X \le 0.5)$$
$$= 1 - F(0.5)$$
$$= 1 - 12\left(\frac{0.5^2}{2} - \frac{2\times 0.5^3}{3} + \frac{0.5^4}{4}\right)$$
$$= 0.3125 \text{ or } 31.25\%$$

31d.)

$$E(X) = 12\int_0^1 x^2(1-x)^2\,dx$$
$$= 12\int_0^1 (x^2 - 2x^3 + x^4)\,dx$$
$$= 12\left[\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5}\right]_0^1$$
$$= 12\left(\tfrac{1}{3} - \tfrac{1}{2} + \tfrac{1}{5} - 0\right) = \tfrac{2}{5} \text{ or } 0.4$$

$$E(X^2) = 12\int_0^1 x^3(1-x)^2\,dx$$
$$= 12\int_0^1 (x^5 - 2x^4 + x^3)\,dx$$
$$= 12\left[\frac{x^4}{4} - \frac{2x^5}{5} + \frac{x^6}{6}\right]_0^1$$
$$= 12\left(\tfrac{1}{4} - \tfrac{2}{5} + \tfrac{1}{6}\right)$$
$$= 0.2 \text{ or } \tfrac{1}{5}$$

$$\mathrm{var}(X) = E(X^2) - [E(X)]^2$$
$$= 0.2 - (0.4)^2$$
$$= 0.04$$

$$\therefore \text{s.d.} = \sqrt{0.04}$$
$$= 0.2$$

32a.) when $x=6$

$$y_1 = 5 - \tfrac{6}{2} = 2 \qquad y_2 = \sqrt{6-2} = 2$$

$\therefore y_1 = y_2$ and point of int. occurs at $x=6$

b.)

$$\text{Area} = \int_2^6 \sqrt{x-2}\,dx + \text{area of a triangle}$$
$$= \int_2^6 \sqrt{x-2}\,dx + \tfrac{1}{2}(10-6)\times 2$$
$$= \int_2^6 (x-2)^{\frac{1}{2}}\,dx + 4$$
$$= \left[\tfrac{2}{3}(x-2)^{3/2}\right]_2^6 + 4$$
$$= \tfrac{2}{3}\left(4^{3/2} - 0\right) + 4$$
$$= \tfrac{2}{3} \times 8 + 4$$
$$= \tfrac{28}{3}\ u^2$$

33a.) In ΔOCD:

$\cos x = \frac{OD}{2} \Rightarrow OD = 2\cos x$

$\sin x = \frac{CD}{2} \Rightarrow CD = 2\sin x$

$A(x) = \frac{1}{2} \times AD \times CD$

$= \frac{1}{2}(2+2\cos x)(2\sin x)$

$= 2\sin x(1+\cos x)$

b.) $A'(x) = 2\cos x(1+\cos x) + 2\sin x(-\sin x)$

$= 2\cos x + 2\cos^2 x - 2\sin^2 x$

$= 2\cos x + 2\cos^2 x - 2(1-\cos^2 x)$

$= 4\cos^2 x + 2\cos x - 2$

when $A'(x) = 0$

$4\cos^2 x + 2\cos x - 2 = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = -1$

$x = \frac{\pi}{3}$ $\quad$ no solution since $x < \frac{\pi}{2}$

test:

x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$A'(x)$	+ve	–	-ve

$A'(0) = 4+2-2$
$= 4$
> 0

$A'(\frac{\pi}{2}) = 0+0-2$
< 0

$\therefore$ max at $x = \frac{\pi}{3}$

$\therefore A(x)$ is greatest when $x = \frac{\pi}{3}$