Student Number

Barker College

2023 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

Staff Involved:
- DXC - D. Chua
- JAI - J. Iles
- LAK - L. Kalnins
- PDJ - P. Jouliany
- LMD - L. de Gorter
- DZB - D. Batchen
- ARM - A. Mallam
- ARP - A. Perkins
- RJW - R. Williams
- ALY - A. Young
- RAS - R. Smith*

Friday, August 4th 2023
150 copies

General Instructions:
- Reading time - 10 minutes
- Writing time - 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided separately
- For questions in Section II, show relevant mathematical reasoning and / or calculations
- Marks may not be awarded for careless or poorly arranged working
- Diagrams are not to scale unless specifically stated

Total marks: 100

Section I - 10 marks (pages 2 - 6)
- Attempt Multiple Choice Questions 1 - 10
- Allow about 20 minutes for this section

Section II - 90 marks (pages 7 - 28)
- Attempt Questions 11 - 37
- Show all necessary working
- Allow about 2 hours and 40 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 20 minutes for this section
Use the Multiple-Choice answer sheet for Questions 1-10.

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1. What is the solution to the equation $2\cos x = \sqrt{3}$ for $0 \le x \le 2\pi$?

A. $x = \frac{\pi}{3}$ and $\frac{5\pi}{3}$

B. $x = \frac{\pi}{3}$ and $\frac{2\pi}{3}$

C. $x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

D. $x = \frac{\pi}{6}$ and $\frac{11\pi}{6}$

2. What is the range of the function $y = 3^{-x} + 2$?

A. $(3, \infty)$

B. $(-\infty, 2)$

C. $(2, \infty)$

D. $(-\infty, 1)$

3. What is $\int e + e^{4x}\, dx$?

A. $ex + 4e^{4x} + C$

B. $ex + \frac{1}{4}e^{4x} + C$

C. $e + 4e^{4x} + C$

D. $e + \frac{1}{4}e^{4x} + C$

4. There are 15 ties in a cupboard. 8 of the ties are silk and 9 of the ties are striped.
3 ties are neither silk nor striped.

If one tie is selected at random, what is the probability that it is silk but not striped?

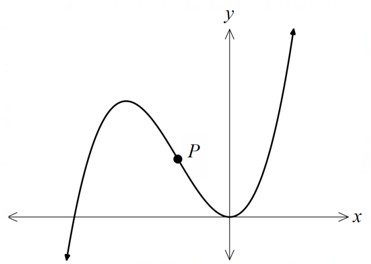
A. $\frac{8}{17}$

B. $\frac{4}{15}$

C. $\frac{1}{5}$

D. $\frac{2}{17}$

5. Which of the following statements are true about the point of inflection, *P,* on the curve below?



A. $f'(x) < 0, f''(x) = 0$

B. $f'(x) < 0, f''(x) > 0$

C. $f'(x) > 0, f''(x) = 0$

D. $f'(x) > 0, f''(x) < 0$

6. The probability distribution of a random variable X is shown below:

$\boldsymbol{x}$	0	1	2	3	4
$\boldsymbol{P(X = x)}$	$\frac{1}{5}$	k	$2k$	k	$\frac{2}{5}$

What is the value of k?

A. $\frac{1}{4}$

B. $\frac{3}{5}$

C. $\frac{3}{20}$

D. $\frac{1}{10}$

7. An infinite geometric series has a first term of 12 and a limiting sum of 18.
What is the common ratio?

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

8. What is the period of the function $f(x) = -3\cos\left(\frac{\pi x}{5}\right)$?

A. $\frac{1}{5}$

B. $\frac{\pi}{5}$

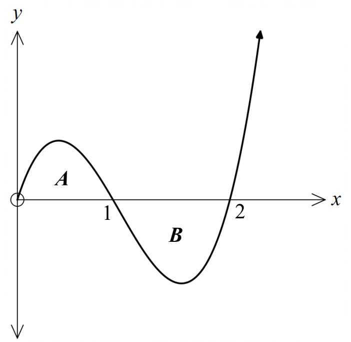
C. 10

D. 10π

9. The function $g(x)$ is even and part of the graph $y = g(x)$ is shown below.

It is known that the area of region *A* is 3 units^2 and that $\int_{-2}^{2} g(x)\,dx = -2$.

What is the area bound by the function $y = g(x)$ and the *x*-axis from $x = -2$ to $x = 2$?



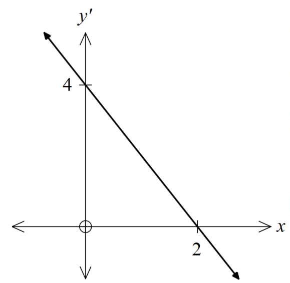
A. 2 units^2

B. 8 units^2

C. 14 units^2

D. 16 units^2

10. The graph of $y' = f'(x)$ is shown below.



The curve $y = f(x)$ has a maximum value of 20. What is the equation of the curve?

A. $y = x^2 - 4x + 16$

B. $y = x^2 - 4x + 20$

C. $y = 20 + 4x - x^2$

D. $y = 16 + 4x - x^2$

End of Section I

Student Number

Section II

90 marks
Attempt Questions 11 – 37
Allow about 2 hours and 40 minutes for this section.
Write your student number on all answer pages.

In Questions 11 – 37, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (1 mark)

Differentiate $\frac{sinx}{x}$ **1**

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Question 12 (3 marks)

Given that $g(x) = 2x + 1$ and $h(x) = x^2 - x$, find:

(a) $h\big(g(-2)\big)$ **1**

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(b) $g(h(x))$ **1**

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(c) x when $h(x) = 0$ **1**

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Question 13 (2 marks)

A biologist found that the frequency F (in beats per minute) that a bird beats its wings is inversely proportional to the length L (in cm) of its wings.
He observes that a bird with wings of length 14 cm beats them at a frequency of 9 beats per minute.

Find an equation relating F and L. **2**

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Question 14 (2 marks)

The first term of an arithmetic series is 34. The sum of the first 14 terms is 2023.
What is the common difference of the series? **2**

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Question 15 (2 marks)

Evaluate $\int_0^{\frac{\pi}{6}} \sec^2 2x \, dx$ **2**

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Question 16 (3 marks)

Find the equation of the normal to the curve $y = 2(4x - 3)^4$ at the point on the curve where $x = 1$. **Leave your answer in general form.** **3**

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Question 17 (3 marks)

$$\int_1^7 \frac{8}{x+1}\,dx$$

Use the trapezoidal rule with 3 sub-intervals to approximate the above integral.
Leave your answer in exact form. **3**

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Question 18 (2 marks)

Barry and Doreen are taking a driving test. There is a 70% chance of Barry passing his test and there is an 80% chance of Doreen passing her test.
What is the probability that at least one of them passes their test? **2**

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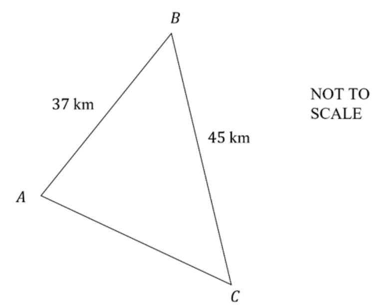
Question 19 (3 marks)

There are three towns in a regional area. Town *B* is 37 km from Town *A* on a bearing of 050ºT.
Town *C* is 45 km from Town *B* on a bearing of 165ºT.



(i) Find the size of $\angle ABC$. **1**

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(ii) Find the distance from Town *A* to Town *C*. Round your answer to 1 decimal place. **2**

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Question 20 (2 marks)

A Barker student visits *Sultan's Kebab House* every afternoon after school.
Let the discrete random variable X be the number of kebabs they order on a particular afternoon.

x	0	1	2	3
$P(X = x)$	0.1	0.35	0.4	0.15

(i) Calculate $E(X)$. **1**

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(ii) The kebab house has a deal for loyal customers.

Once you purchase a cumulative total of 24 kebabs, you get an exclusive *Sultan's* hoodie.

After how many visits would a student expect to have ordered enough kebabs to win the hoodie? **1**

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Question 21 (5 marks)

The population, P, of penguins in an island colony is decreasing at a rate proportional to P:

$$\frac{dP}{dt} = -kP \qquad \text{where } t \text{ is the time in years since 2013.}$$

In 2013 there was a population of 5000 penguins. By 2018 the population was 4250.

(i) Show that $P = Ae^{-kt}$ is a solution to $\frac{dP}{dt} = -kP$. **1**

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(ii) Find the value of k, rounding your answer to four decimal places. **2**

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(iii) What is the rate of change of the penguin population in 2023? **2**

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Question 22 (2 marks)

Consider the following function:

$$f(x) = \begin{cases} x^2, & x < 0 \\ 2x - 3, & x \geq 0 \end{cases}$$

Find the value of x if $f(x) = 25$. **2**

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Question 23 (3 marks)

The graph of the function $f(x) = x^2$ is translated a units to the left, dilated vertically by a factor of b and then translated 3 units up.

The equation of the transformed function is now $g(x) = 2x^2 + 16x + 35$.

Find the values of a and b. **3**

Question 24 (2 marks)

A probability density function is defined as $f(x) = \frac{4}{81}(4x^2 - x^3)$ for $1 \leq x \leq 4$.
What is the mode of this function? **2**

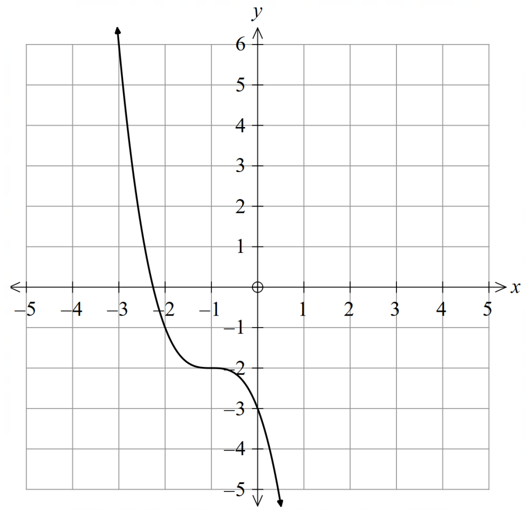
Question 25 (3 marks)

(i) Find $\frac{d}{dx}\, 2x \ln x$. **1**

(ii) Hence, or otherwise, find $\int 1 + \ln x \, dx$. **2**

Question 26 (2 marks)

Consider the graph of $y = f(x)$ as shown.



On the above number plane, draw a sketch of the function $y = 2f(x) + 6$, showing x and y intercepts and the coordinates of the point of inflection. **2**

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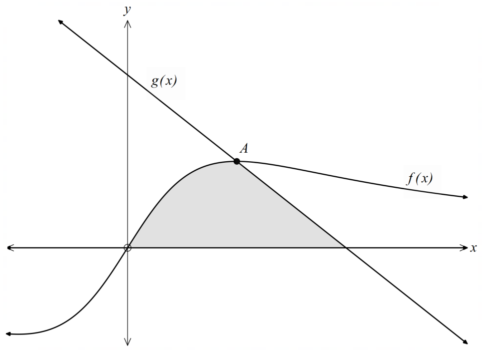
Question 27 (4 marks)

The graph below shows the functions $f(x) = \frac{2x}{x^2+1}$ and $g(x) = -x + 2$.



(i) Verify that the coordinates of A are (1, 1). **1**

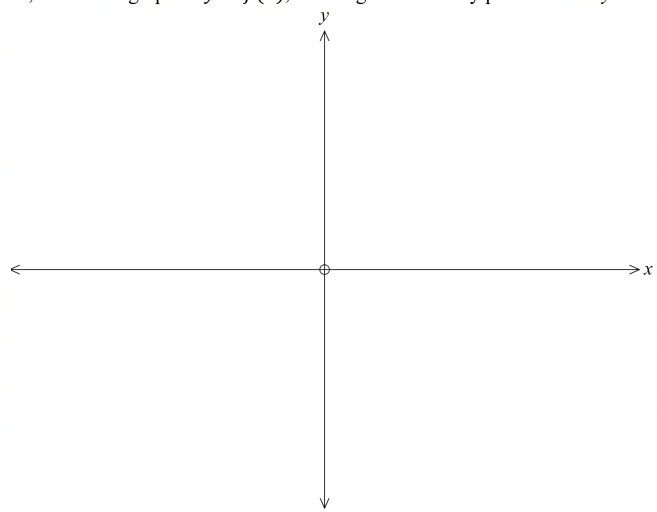
(ii) Find the exact value of the shaded area in simplest form. **3**

Question 28 (5 marks)

Let $f(x) = x^4 + 2x^3 + 1$.

(i) Find the coordinates of the stationary points of $f(x)$ and determine their nature. **3**

(ii) Hence, sketch the graph of $y = f(x)$, showing all stationary points and its y-intercept. **2**



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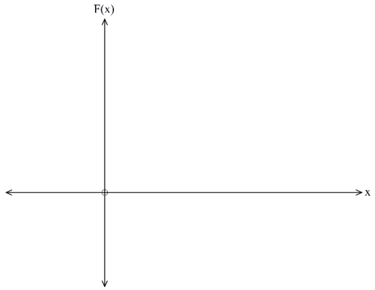
Question 29 (6 marks)

The duration, in hours, that a suburban train trip takes is a random variable *X*, which is modelled by the probability density function:

$$f(x)=\begin{cases}\cos x, & 0\le x\le k\\ 0, & \text{otherwise}\end{cases}$$

(i) Find the value of k. **2**

(ii) On the axes below, graph the cumulative density function. **2**



(iii) Find the probability that the train trip takes longer than one hour. **2**

Question 30 (4 marks)

In a local basketball competition, a team can either win or lose a match.
A certain team has a probability of 0.45 of winning a match if the coach's son is playing in that match or a probability of 0.75 if he is not. The probability of the coach's son playing is 0.4.

(i) Find the probability that the team will win a match. **2**

(ii) Given that the team has won a match, find the probability that the coach's son was playing on that day. **2**

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Question 31 (3 marks)

The moon has a lower gravity than Earth. This means that a ball will bounce higher on the moon than Earth.

When a ball is dropped on the moon, each bounce is 85% as high as the previous bounce.
When an identical ball is dropped on Earth, each bounce is 40% as high as the previous bounce.

Two identical balls are dropped on the moon and on Earth, each from a height of 2 metres.

Calculate the difference in total vertical distance travelled by these balls. **3**

Question 32 (3 marks)

Solve the equation $2\ln(x+4) - \ln x = \ln(2x+2)$. **3**

Question 33 (4 marks)

(i) On the axes below, sketch the graphs of $y = 1 + \cos x$ and $y = \sin x$ for the domain $0 \leq x \leq 2\pi$. **2**

(ii) Hence, or otherwise, solve $\sin^2 x + \cos^2 x + \cos x - \sin x = 0$, in the domain $0 \leq x \leq 2\pi$. **2**

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Questions 34 and 35 on other side.

Question 34 (7 marks)

Elon opened a savings account with an initial deposit of $30 000 on the 1st January 2023.
He withdraws $4080 at the end of each year to help pay for his daughter's university costs.

The account earned interest at a rate of 8% p.a., compounded six-monthly.
The first withdrawal was made one year after the account was set up.

(i) What was the account balance immediately after the first withdrawal? **1**

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(ii) Let A_n represent the amount of money in the account immediately after a withdrawal has been made, where n is the number of years since the account opened. **4**

Show that $A_n = 50\ 000 - 20\ 000(1.04)^{2n}$

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Question 34 continues on page 23

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(iii) For how many years will Elon's daughter be able to withdraw the full amount from the account? **2**

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Question 35 (4 marks)

Let $f(x) = e^{-kx} + 4x$, where k is a positive, rational number.

(i) Find, in terms of k, the x-coordinate of the stationary point of the graph $y = f(x)$. **3**

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(ii) State the values of k such that the x-coordinate of this stationary point is a positive number. **1**

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Questions 36 and 37 on other side.

Question 36 (3 marks)

The velocity of a particle is given by $\dot{x} = 6t^2(t^3 + 2)$, where $\dot{x}$ is velocity in metres per second and t is time in seconds.

If its initial displacement is 4 metres, show that the particle is never at the origin. **3**

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Question 37 (7 marks)

A closed rectangular prism has a height h cm, width x cm and length $1.5x$ cm. The surface area of the rectangular prism is 3240 cm^2.

(i) Find an expression for h in terms of x. **2**

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Question 37 continues on page 27

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(ii) If the volume of the rectangular prism is to be maximised, find the exact values of x and h. **5**

End of Paper

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Year 12 Advanced Trial - Student Solutions

1. $2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

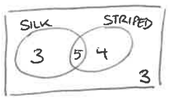
$x = \frac{\pi}{6}, \frac{11\pi}{6}$ $\boxed{D}$

2. Range of $y = 3^{-x}$ is $(0, \infty)$

Range of $y = 3^{-x} + 2$ is $(2, \infty)$ $\boxed{C}$

3. $\int e + e^{4x}\, dx$

$= ex + \frac{1}{4}e^{4x} + C$ $\boxed{B}$

4. 

$15 - 3 = 12$

$8 + 9 - 12 = 5$ (silk and striped)

P(silk not striped) $= \frac{3}{15} = \frac{1}{5}$ C

5. P is a point of inflection so $f''(x) = 0$

Gradient at P is negative, so $f'(x) < 0$ $\boxed{A}$

6. $\frac{1}{5} + k + 2k + k + \frac{2}{5} = 1$

$4k + \frac{3}{5} = 1$

$4k = \frac{2}{5}$

$k = \frac{1}{10}$ $\boxed{D}$

7. $S_\infty = 18$, $a = 12$, $r = ?$

$18 = \frac{12}{1-r}$

$1 - r = \frac{12}{18}$

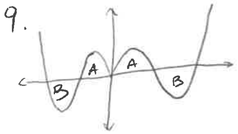
$r = 1 - \frac{12}{18}$

$r = \frac{1}{3}$ A

8. Period $= \frac{2\pi}{n}$

$= \frac{2\pi}{\frac{\pi}{5}}$

$= 10$ $\boxed{C}$

9. 

Since $g(x)$ is even, the graph is symmetrical about y-axis.

$-B + A + A + -B = -2$

$-B + 3 + 3 + -B = -2$

$-2B = -8$

$B = 4$

Area $= 2\times4 + 2\times3$

$= 14$ $\boxed{C}$

10. $f'(x) = -2x + 4$

$f(x) = -x^2 + 4x + C$

$-2x + 4 = 0$

$x = 2$

So Max at $(2, 20)$

$20 = -(2)^2 + 4(2) + c$

$c = 16$

$\therefore y = 16 + 4x - x^2$ $\boxed{D}$

Section II

11. $\frac{\sin x}{x}$

$u = \sin x \quad v = x$

$u' = \cos x \quad v' = 1$

$\frac{dy}{dx} = \frac{x\cos x - 1\times\sin x}{x^2}$

$= \frac{x\cos x - \sin x}{x^2}$

12. a) $g(-2) = 2(-2) + 1$

$= -3$

$h(-3) = (-3)^2 - (-3)$

$= 12$

b) $2(x^2 - x) + 1$

$= 2x^2 - 2x + 1$

c) $x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

13. $F = \frac{k}{L}$

$9 = \frac{k}{14}$

$k = 9\times14$

$= 126$

$\therefore F = \frac{126}{L}$

14. $S_{14} = 2023$

$n = 14$

$a = 34$

$d = ?$

$2023 = \frac{14}{2}(2\times34 + (14-1)d)$

$2023 = 7(68 + 13d)$

$289 = 68 + 13d$

$221 = 13d$

$d = 17$

15. $\int_0^{\frac{\pi}{6}} \sec^2 2x\, dx$

$= \left[\frac{1}{2}\tan(2x)\right]_0^{\frac{\pi}{6}}$

$= \frac{1}{2}\tan(2(\frac{\pi}{6})) - \frac{1}{2}\tan(2(0))$

$= \frac{1}{2}\times\sqrt{3} - \frac{1}{2}\times0$

$= \frac{\sqrt{3}}{2}$

16. $y = 2(4x-3)^4$

$\frac{dy}{dx} = 8(4x-3)^3\times4$

$= 32(4x-3)^3$

Let $x = 1$

$M_{tangent} = 32(4(1)-3)^3$

$= 32$

$M_{normal} = -\frac{1}{32}$

Let $x = 1$

$y = 2(4(1)-3)^4$

$y = 2$

$y - 2 = -\frac{1}{32}(x-1)$

$32y - 64 = -x + 1$

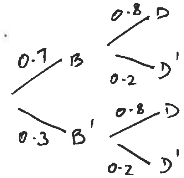
$x + 32y - 65 = 0$

17. $\int_1^7 \frac{8}{x+1}\, dx$

x	1	3	5	7
y	4	2	$\frac{4}{3}$	1

$\approx \frac{2}{2}\left[4 + 1 + 2(2 + \frac{4}{3})\right]$

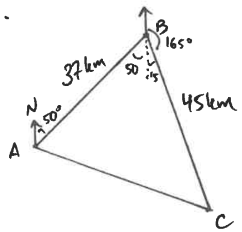
$\approx \frac{35}{3}$

18. 

P(at least one passing) $= 1 - $ P(both failing)

$= 1 - 0.3\times0.2$

$= 0.94$

19. 

i) $50 + 15 = 65°$

ii) $AC^2 = 37^2 + 45^2 - 2\times37\times45\times\cos65$

$AC^2 = 1986.68\ldots$

$AC^2 = 44.6km$

20. i) $E(x) = 0\times0.1 + 1\times0.35 + 2\times0.4 + 3\times0.15$

$= 1.6$

ii) $24 \div 1.6$

$= 15$ visits

21. i) $P = Ae^{-kt}$

$\frac{dP}{dt} = -k \times Ae^{-kt}$

$= -kP$

ii) $A = 5000$

$4250 = 5000e^{-5k}$

$\frac{17}{20} = e^{-5k}$

$\ln\left(\frac{17}{20}\right) = -5k$

$k = 0.0325$

iii) $\frac{dP}{dt} = -0.0325 \times 5000e^{-0.0325\times10}$

$= -117.4$

22. $x^2 = 25$

$x = \pm 5$

since $x < 0$ for $f(x) = x^2$

$\therefore x = -5$ only

$2x - 3 = 25$

$2x = 28$

$x = 14$

since $x \geq 0$, for $f(x) = 2x-3$

then $x = 14$ is included

$\therefore x = -5, 14$

23. $b(x+a)^2 + 3 = 2x^2 + 16x + 35$

$b = 2$

$2(x^2 + 2ax + a^2) + 3 = 2x^2 + 16x + 35$

$2x^2 + 4ax + 2a^2 + 3 = 2x^2 + 16x + 35$

$2a^2 + 3 = 35$

$2a^2 = 32$

$a^2 = 16$

$a = \pm 4$

$4ax = 16$

$4a = 16$

$a = 4$

$a = 4$ satisfies both.

24. $f(x) = \frac{4}{81}(4x^2 - x^3) \quad 1 \leq x \leq 4$

$f'(x) = \frac{4}{81}(8x - 3x^2)$

$0 = \frac{4}{81}(8x - 3x^2)$

$0 = x(8 - 3x)$

$x = 0$ (not in domain) $x = \frac{8}{3}$

$f''(x) = \frac{4}{81}(8 - 6x)$

$f''\left(\frac{8}{3}\right) = -\frac{32}{81} < 0 \therefore$ MAX

$\therefore$ mode is $x = \frac{8}{3}$

25. i) $\frac{d}{dx} 2x\ln x$

$u = 2x \quad v = \ln x$

$u' = 2 \quad v' = \frac{1}{x}$

$\frac{dy}{dx} = 2\ln x + 2$

ii) $\int 1 + \ln x \, dx$

$\frac{1}{2}\int 2 + 2\ln x \, dx$

$\frac{1}{2}(2x\ln x) + C$

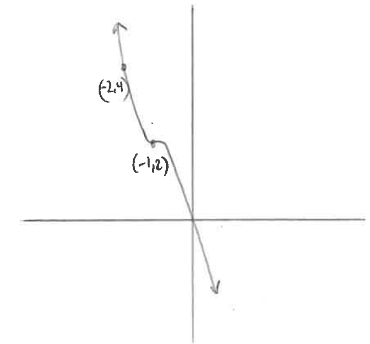
$x\ln x + C$

26. $y = 2f(x) + 6$

y-coordinates $\times 2$ then $+6$

$(0,-3) \quad (0,-6) \quad (0,0)$

$(-1,-2) \quad (-1,-4) \quad (-1,2)$



27. i) $f(1) = \frac{2(1)}{1^2+1}$

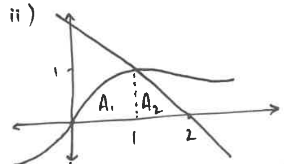
$= 1$

$g(1) = -1 + 2$

$= 1$

$\therefore (1,1)$ satisfies both equations.

ii)



$A_1 = \int_0^1 \frac{2x}{x^2+1} dx$

$= \left[\ln|x^2+1|\right]_0^1$

$= \ln|1^2+1| - \ln|0^2+1|$

$= \ln 2$

$A_2 = \frac{1}{2} \times 1 \times 1$

$= \frac{1}{2}$

$\therefore$ Area $= \frac{1}{2} + \ln 2$

28. i) $f(x) = x^4 + 2x^3 + 1$

$f'(x) = 4x^3 + 6x^2 \qquad f''(x) = 12x^2 + 12x$

$0 = 4x^3 + 6x^2$

$0 = 2x^2(2x+3)$

$x = 0, \; x = -\frac{3}{2}$

$f(0) = 1 \quad f\left(-\frac{3}{2}\right) = \left(-\frac{3}{2}\right)^4 + 2\left(-\frac{3}{2}\right)^3 + 1$

$= -\frac{11}{16}$

$f''\left(-\frac{3}{2}\right) = 12\left(-\frac{3}{2}\right)^2 + 12\left(-\frac{3}{2}\right)$

$= 9 > 0 \quad \therefore$ min

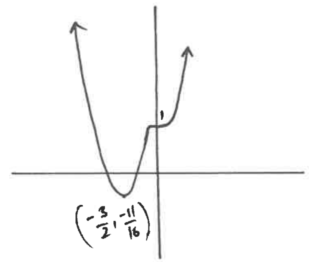
$\left(-\frac{3}{2}, -\frac{11}{16}\right)$ is a minimum

$f''(0) = 12(0)^2 + 12(0)$

$= 0 \quad \therefore$ point of inflexion

x	-1	0	1
$f'(x)$	2	0	10

ii)



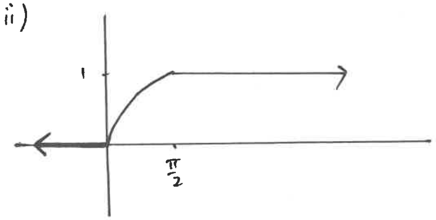
29. i) $\int_0^k \cos x \, dx = 1$

$\left[\sin x\right]_0^k = 1$

$\sin k - \sin 0 = 1$

$k = \frac{\pi}{2}$

ii)

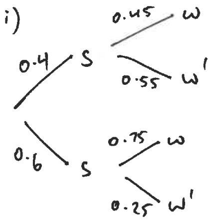


iii) $P(X > 1) = 1 - P(X < 1)$

$= 1 - \sin 1$

$= 0.159$

30. i)



$= 0.4 \times 0.45 + 0.6 \times 0.75$

$= 0.63$

ii) $P(S|W) = \frac{P(S \cap W)}{P(W)}$

$= \frac{0.4 \times 0.45}{0.63}$

$= \frac{2}{7}$

31. Moon

2m ↓ ↑1.7 ↓ ↑1.445 ↓ ...

Total distance Moon

$= 2 + 2(1.7 + 1.445 + \ldots)$

$= 2 + 2\left(\frac{1.7}{1-0.85}\right)$

$= \frac{74}{3}$

Earth

2m ↓ ↑0.8 ↓ ↑0.32 ↓

Total distance Earth

$2 + 2(0.8 + 0.32 + \ldots)$

$2 + 2\left(\frac{0.8}{1-0.4}\right)$

$= \frac{14}{3}$

$\frac{74}{3} - \frac{14}{3} = 20$m

32. $2\ln(x+4) - \ln x = \ln(2x+2)$

$\ln(x+4)^2 - \ln x = \ln(2x+2)$

$\ln\left(\frac{(x+4)^2}{x}\right) = \ln(2x+2)$

$\frac{(x+4)^2}{x} = 2x+2$

$x^2+8x+16 = 2x^2+2x$

$x^2-6x-16=0$

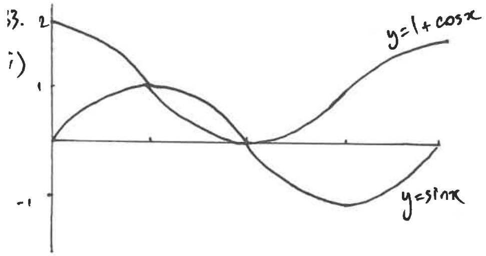
$(x-8)(x+2)=0$

$x=8,\ x=-2$

↳ does not satisfy equation

$\therefore x=8$ only

33. i)



ii) $\sin^2 x + \cos^2 x + \cos x - \sin x = 0$

$1 + \cos x = \sin x$

$\therefore$ points of intersection

$x = \frac{\pi}{2}, \pi$

34. i) $8\% \div 2 = 4\%$

$= 30000(1.04)^2 - 4080$

$= \$28368$

ii) $A_1 = 30000(1.04)^2 - 4080$

$A_2 = A_1(1.04)^2 - 4080$

$= 30000(1.04)^4 - 4080(1.04)^2 - 4080$

$A_3 = A_2(1.04)^2 - 4080$

$= 30000(1.04)^6 - 4080(1.04)^4 - 4080(1.04)^2 - 4080$

$\vdots$

$A_n = 30000(1.04)^{2n} - 4080(1 + 1.04^2 + 1.04^4 + \dots)$

$= 30000(1.04)^{2n} - 4080\left(\frac{1((1.04)^{2n}-1)}{1.04^2-1}\right)$

$= 30000(1.04)^{2n} - 50000(1.04^{2n} - 1)$

$= 50000 - 20000(1.04)^{2n}$

iii) $A_n = 0$

$0 = 50000 - 20000(1.04)^{2n}$

$50000 = 20000(1.04)^{2n}$

$\frac{5}{2} = 1.04^{2n}$

$\ln\left(\frac{5}{2}\right) = \ln(1.04^{2n})$

$\ln\left(\frac{5}{2}\right) = 2n\ln(1.04)$

$2n = \frac{\ln\left(\frac{5}{2}\right)}{\ln(1.04)}$

$= 23.36\ldots$

$n = 11.68\ldots$

$\therefore$ 11 years.

35 i) $f(x) = e^{-kx} + 4x$

$f'(x) = -ke^{-kx} + 4$

$0 = -ke^{-kx} + 4$

$ke^{-kx} = 4$

$e^{-kx} = \frac{4}{k}$

$-kx = \ln\left(\frac{4}{k}\right)$

$x = -\frac{1}{k}\ln\left(\frac{4}{k}\right)$

$x = \frac{1}{k}\ln\left(\frac{k}{4}\right)$

ii) $\frac{k}{4} > 1$

$k > 4$

36. $x = \int 6t^2(t^3+2)\,dt$

$x = (t^3+2)^2 + c$

let $t=0, x=4$

$4 = (0^3+2)^2 + c$

$c = 0$

$\therefore x = (t^3+2)^2$

OR $x = \int 6t^5 + 12t^2\,dt$

$x = t^6 + 4t^3 + c$

let $t=0, x=4$

$4 = 0^6 + 4(0)^3 + c$

$c = 4$

$\therefore x = t^6 + 4t^3 + 4$

$x = (t^3+2)^2$

let $x=0$

$0 = (t^3+2)^2$

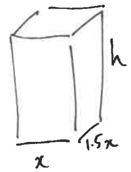
$-2 = t^3$

$\sqrt[3]{2} = t$

$t \geq 0$

$\therefore x$ can never equal zero so particle never at the origin

37.



SA=3240

i) $SA = 2(x \times 1.5x + xh + 1.5xh)$

$= 3x^2 + 5hx$

$3240 = 3x^2 + 5hx$

$3240 - 3x^2 = 5hx$

$h = \frac{3240 - 3x^2}{5x}$

$h = \frac{648}{x} - \frac{3x}{5}$

ii) $V = 1.5x^2h$

$V = 1.5x^2\left(\frac{648}{x} - \frac{3x}{5}\right)$

$V = 972x - \frac{9}{10}x^3$

$\frac{dV}{dx} = 972 - \frac{27}{10}x^2$

$0 = 972 - \frac{27}{10}x^2$

$\frac{27}{10}x^2 = 972$

$x^2 = 360$

$x = \pm\sqrt{360}$

$x = 6\sqrt{10}$ $(x > 0)$

$\frac{d^2V}{dx^2} = -\frac{54}{10}x$

let $x = 6\sqrt{10}$

$\frac{d^2V}{dx^2} = -\frac{162\sqrt{10}}{5} < 0$ $\therefore$ MAX

$h = \frac{648}{6\sqrt{10}} - \frac{3 \times 6\sqrt{10}}{5}$

$= \frac{108}{\sqrt{10}} - \frac{18\sqrt{10}}{5}$

$= \frac{108\sqrt{10}}{10} - \frac{18\sqrt{10}}{5}$

$= \frac{54\sqrt{10}}{5} - \frac{18\sqrt{10}}{5}$

$= \frac{36\sqrt{10}}{5}$

$x = 6\sqrt{10}$ $\quad h = \frac{36\sqrt{10}}{5}$