
Student Number

Barker College

2024 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

Staff Involved: **THURSDAY 15TH AUGUST, 2024**

- PDJ – P. Jouliany
  JZT – J. Thomas
  RAS – R. Smith
  DOB – D. O'Byrne
  PCB – P. Boyages
  ECB – E. Barton
  EJK – E. Koorey
  SJB – S. Byun
  JAI – J. Iles
  GPF – G. Fitzmaurice
  AXD – A. Davis

Topics Covered **150 copies**

Discrete Probability Distributions	Curve Sketching and Using the Derivative	Exponential and Logarithmic Functions	Continuous Probability Distributions
Sequences and Series	Integration	Motion and Rates	Year 11 topics
Graphs and Equations	Trigonometric Functions	Displaying and Interpreting Data	

General Instructions:

- Write your Student Number at the top of this page and all answer pages
- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A separate reference booklet is provided
- Marks may not be awarded for careless or poorly arranged working
- Diagrams are not to scale
- For questions in Section II, show relevant mathematical reasoning and / or calculations

Total marks:
100

Section I – 10 marks (pages 2 - 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7 - 32)

- Attempt Questions 11 - 32
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

---

1 What is the domain of $y = \sqrt{16 - x^2}$?

A. $(0\,,4)$

B. $(-4\,,4)$

C. $[-4\,,4]$

D. $[0\,,4]$

2 The graph of the function $y = f(x)$ is known to have a minimum turning point at $P(-4, -8)$. What will be the coordinates of the maximum turning point on the graph of $y = -f(2x)$?

A. $(-2\,,8)$

B. $(-8\,,8)$

C. $(-4\,,8)$

D. $(2\,,-8)$

3 Which of the following statements is NOT true?

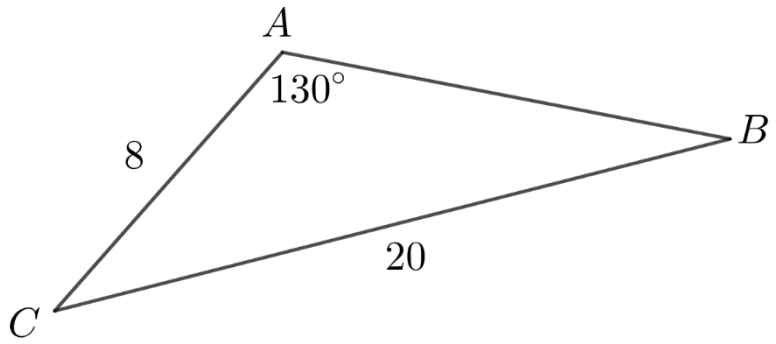
A. $\int_{-3}^{3} x^3\,dx = 0$

B. $\int_{-3}^{3} x^2\,dx = 2\int_{0}^{3} x^2\,dx$

C. $\int_{0}^{3} x\,dx = -\int_{-3}^{0} x\,dx$

D. $\int_{0}^{3} x^2\,dx = 2\int_{0}^{1.5} x^2\,dx$

4 In $\Delta ABC,\ AC = 8, BC = 20$ and $\angle BAC = 130°$ as shown.



NOT TO SCALE

Which of the following statements is correct?

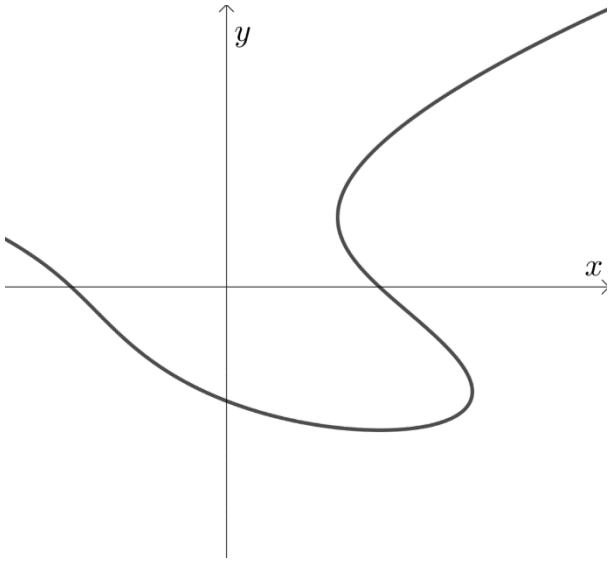
A. $\sin B = 0.4 \sin 130°$

B. $\sin B = 2.5 \sin 130°$

C. $\sin C = 0.4 \sin 130°$

D. $\sin C = 2.5 \sin 130°$

5 Consider the graph shown below.



What type of relation is this?

A. One-to-many

B. Many-to-one

C. One-to-one

D. Many-to-many

6 A table of future value interest factors for an annuity of \$1 is shown.

Future value of \$1						
Period	**1%**	**4%**	**8%**	**12%**	**16%**	**20%**
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0400	2.0800	2.1200	2.1600	2.2000
3	3.0301	3.1216	3.2464	3.3744	3.5056	3.6400
4	4.0604	4.2465	4.5061	4.7793	5.0665	5.3680
5	5.1010	5.4163	5.8666	6.3528	6.8771	7.4416
6	6.1520	6.6330	7.3359	8.1152	8.9775	9.9299

Sophie wants to save \$10 000 over the next 2 years by investing equal payments every 6 months into an account that earns interest of 8% p.a. compounded half-yearly.

What should be the value of each payment?

A. \$4807.69

B. \$2354.88

C. \$4901.96

D. \$2219.21

7 Which of the following is equivalent to $\int \frac{t^2 - 2t}{t^3} dt$?

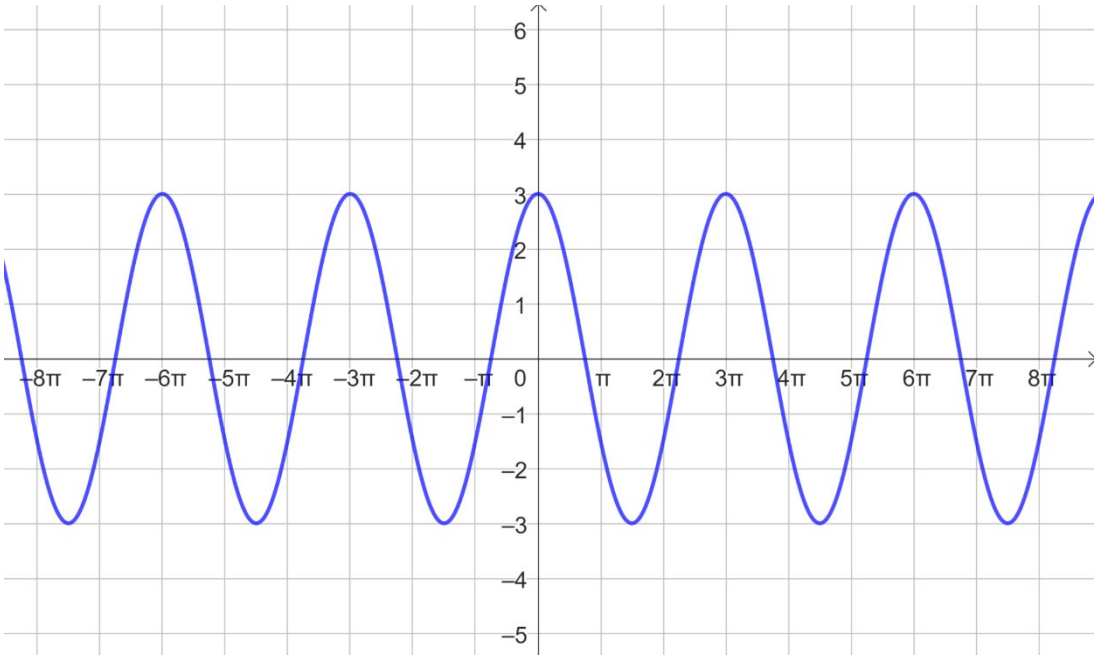
A. $-\frac{1}{t^2} + \frac{4}{t^3} + C$

B. $\ln|t| + \frac{4}{t} + C$

C. $\ln|t| + \frac{2}{t} + C$

D. $\ln|t| + \frac{2}{t^2} + C$

8 The graph of $y = k\cos(nx)$ is shown.



What are the values of k and n?

A. $k = 3, n = \frac{3}{2}$

B. $k = -3, n = \frac{3}{2}$

C. $k = 3, n = \frac{2}{3}$

D. $k = -3, n = \frac{2}{3}$

9 Let $f(x) = \frac{1}{\sqrt{x}}$ and $g(x) = 1 - x$. What is the range of $g(f(x))$?

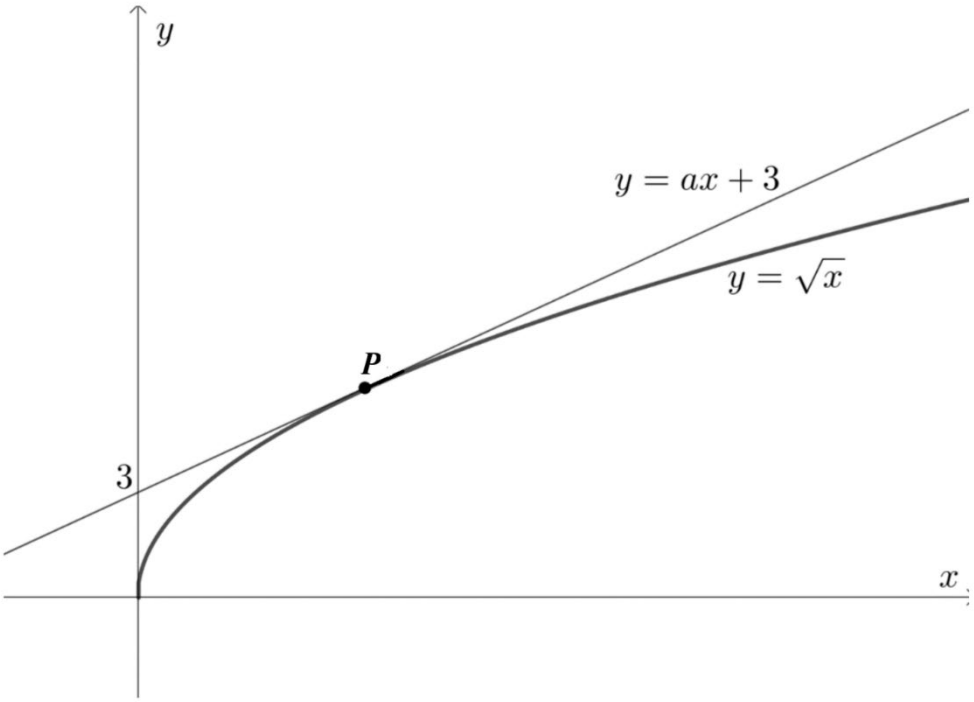
A. $y > 0$

B. $y \neq 0$

C. $y > 1$

D. $y < 1$

10 The graph of $y = \sqrt{x}$ has a tangent line drawn at point ***P*** shown. The equation of the tangent line at this point is $y = ax + 3$.



What is the value of a?

A. $a = \frac{1}{2}$

B. $a = \frac{1}{6}$

C. $a = \frac{1}{8}$

D. $a = \frac{1}{12}$

End of Section I

Student Number

Barker College

Mathematics Advanced
Section II Answer Booklet

90 marks
Attempt Questions 11 – 32
Allow about 2 hours and 45 minutes for this section

Instructions:

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing paper is available.

Question 11 (2 marks)

If $\log p = a$ and $\log q = b$, express $\log(pq^2)$ in terms of a and b. **2**

……

……

……

Question 12 (2 marks)

Solve the equation $2\cos\theta = 1$ for $0 \leq \theta \leq 2\pi$ **2**

……

……

……

Question 13 (3 marks)

Differentiate with respect to x:

(a) $f(x) = x^4 - \frac{3}{x}$ **2**

……

……

(b) $y = \ln(7 - 3x)$ **1**

……

……

Question 14 (3 marks)

A weightlifter training for an event performs a series of consecutive lifts. In each go, she lifts 90% of the previous weight. In her first go, she lifts 200 kg.

(i) What weight will she lift on her fifth go? **1**

……

(ii) Theoretically, what would be the total weight lifted by the time she is totally exhausted? **2**

……

……

……

Student Number

Question 15 (2 marks)

Find the exact value of $\int_2^3 \frac{x}{1-x^2}\,dx$. **2**

……

……

……

……

……

……

……

Question 16 (3 marks)

Determine the equation of the tangent to $y = e^{2x}$ at $x = \ln 2$. **3**

……

……

……

……

……

……

Question 17 (2 marks)

Find the primitive function of $7^x + \sec^2(3x)$. **2**

……

……

……

……

Question 18 (3 marks)

Alf plays archery competitively as part of a team. The number of attempts (X) that he needs to hit the target is given by the probability distribution table.

x	1	2	3	4	5
$P(X = x)$	0.12	0.35	0.32	0.17	0.04

(i) Find the expected number of attempts Alf needs to hit the target. **1**

……

……

(ii) The team's coach would like to select someone from the team to be the captain. Another archer in the team, Lena, has the same expected number of attempts as Alf, and a standard deviation of 1.2.

If the coach bases their decision on performance, are they more likely to choose Alf or Lena? Justify your answer with appropriate statistical reasoning. **2**

……

……

……

……

……

……

Student Number

Question 19 (5 marks)

(i) Show that $\dfrac{1}{\sec\theta - \tan\theta} - \dfrac{1}{\sec\theta + \tan\theta} = 2\tan\theta$ **3**

……

……

……

……

……

……

……

……

(ii) Hence, or otherwise, find $\int \left(\dfrac{1}{\sec\theta - \tan\theta} - \dfrac{1}{\sec\theta + \tan\theta}\right) d\theta$. **2**

……

……

……

……

……

Question 20 (5 marks)

A block of ice is removed from the refrigerator. The amount M grams of ice remaining after t minutes is given by the equation $M = M_0 e^{-kt}$, where M_0 is the initial amount of ice.

(i) Show that $M = M_0 e^{-kt}$ is a solution to the equation $\frac{dM}{dt} = -kM$. **1**

……

……

(ii) After 35 minutes, only half of the ice remains. Find the value of k to 3 significant figures. **2**

……

……

……

……

……

(iii) After how many hours and minutes will 5% of the original amount of ice remain? **2**

……

……

……

……

……

……

……

……

Student Number

BLANK PAGE

Question 21 (10 marks)

Consider the graph of the function $y = 3x^2 - x^3 + 9x - 3$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ **2**

……

……

(ii) Find the coordinates of the stationary points and determine their nature. **3**

……

……

……

……

……

……

……

……

……

(iii) Find the coordinates of the point of inflexion. **2**

……

……

……

……

……

……

Question 21 continues on the next page.

Student Number

Question 21 (continued)

(iv) Sketch the graph of the function in the domain $-2 \leq x \leq 5$, clearly identifying the stationary points, point of inflexion and y-intercept. **3**

……

……

……

……

……

……

……

……

……

……

……

BLANK PAGE

Student Number

Question 22 (4 marks)

The 4^{th} term of an Arithmetic sequence is 30 and its 10^{th} term is 54.

(i) Find the 20^{th} term in the sequence. **3**

……

……

……

……

……

……

……

……

……

……

……

……

……

……

(ii) Find the sum of the first 20 terms. **1**

……

……

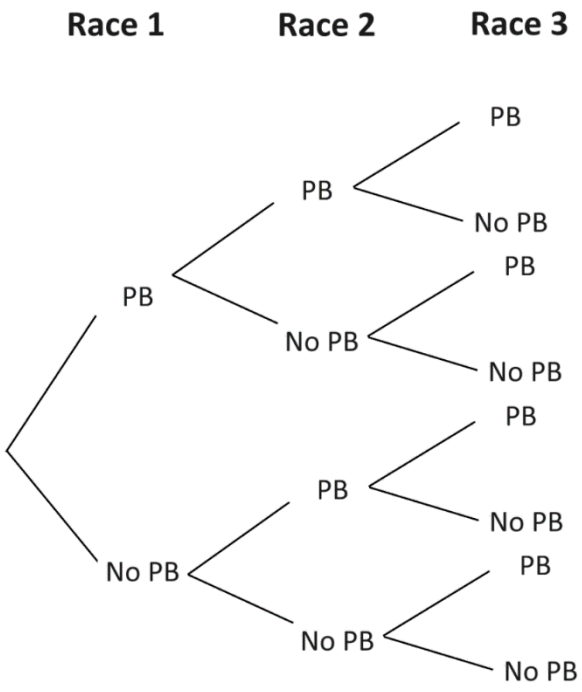
……

……

……

Question 23 (3 marks)

A sprinter knows that the probability of achieving a personal best time in any race is 0.2. The tree diagram below shows the possible outcomes for the sprinter across 3 successive races in achieving a personal best time (PB) or not achieving a personal best time (No PB).



(i) Find the probability that the sprinter will not achieve a personal best time in any of the three races. **1**

……..……

……..……

(ii) Find the probability that the sprinter will achieve a personal best time in exactly one of the three races.

2

……..……

……..……

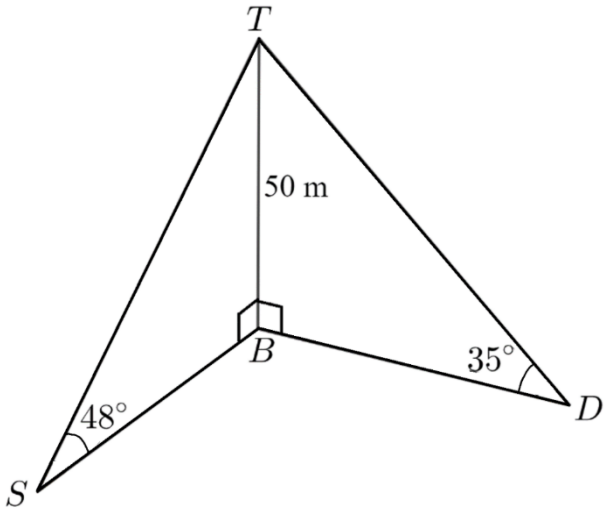
……..……

……..……

Student Number

Question 24 (3 marks)

The diagram below shows two wire cables, ST and DT, that are used to secure an electricity tower 50 metres in height.



NOT TO SCALE

From position S which is due South of B, the angle of elevation to the top of the tower (T) is $48°$.

From position D which is on a true bearing of $085°$ from B, the angle of elevation to the top of the tower (T) is $35°$.

(i) Find the distance SB. **1**

……

……

……

……

(ii) Find the distance SD, correct to the nearest metre. **2**

……

……

……

……

……

……

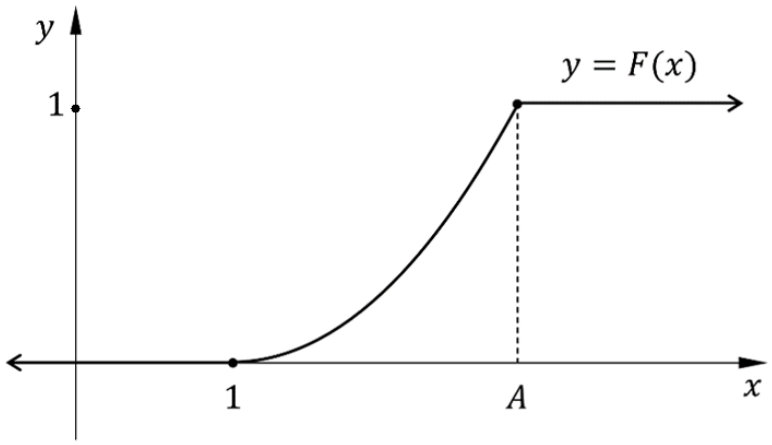
……

BLANK PAGE

Student Number

Question 25 (4 marks)

The graph of a cumulative distribution function $F(x)$ for a continuous random variable X is shown below.



NOT TO SCALE

The curved section of the graph is part of the function $y = \frac{(x-1)^3}{6}$.

(i) Find the exact value of A. **2**

……

……

……

……

……

(ii) Find $P(1.5 < X < 2.5)$, correct to 2 decimal places. **2**

……

……

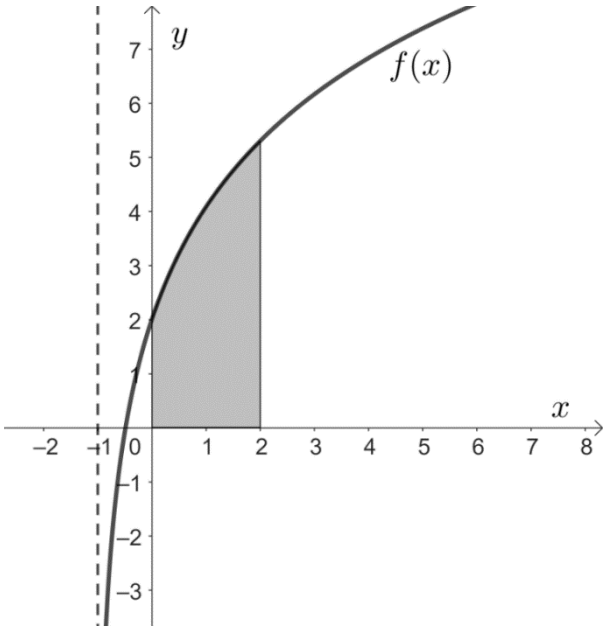
……

……

……

Question 26 (6 marks)

The graph of $f(x) = 3\ln(x+1) + 2$ is shown.



(i) Use the Trapezoidal rule with two subintervals to find the approximate shaded area bound by $f(x)$ and the x-axis between $x = 0$ and $x = 2$. Round your answer to 2 decimal places. **2**

……

……

……

……

……

……

……

(ii) Does your answer in part (ii) overestimate or underestimate the true area? Give reasons for your answer. **1**

……

……

……

Question 26 continues on the next page.

Student Number

Question 26 (continued)

(iii) The function $f(x) = 3\ln(x + 1) + 2$ undergoes the following transformations in the order given to form a new function $g(x)$:

1) Horizontal translation 2 units to the right
2) Vertical translation 3 units up
3) Vertical dilation by a factor of 2

State the equation of $g(x)$. **3**

……

……

……

……

……

……

……

……

BLANK PAGE

Student Number

Question 27 (3 marks)

A university is made up of students who hold a scholarship and students who do not.
85% of students do not hold a scholarship. 25% of students without a scholarship have changed courses.

Of the students who hold a scholarship, 45% have changed courses.

(i) Find the probability that a randomly chosen student has changed courses. **2**

……

……

……

……

……

……

……

……

(ii) Hence or otherwise, find the probability that a student holds a scholarship given they have changed courses. Round your answer to 2 decimal places. **1**

……

……

……

……

Question 28 (7 marks)

Alba joins a superannuation fund when she turns 20 years old and deposits $3000 in the fund. She continues to deposit the same amount every 6 months until she retires on the day she turns 60, with the last deposit made on this day.

The fund pays an interest rate of 4% per annum compounded half-yearly. Let V_n be the value of the fund immediately after the n^{th} deposit.

(i) Show that $V_3 = 150\,000(1.02^3 - 1)$. **2**

……

……

……

……

……

……

……

……

……

(ii) Calculate the value of the fund at retirement, to the nearest dollar. **2**

……

……

……

……

……

……

……

……

……

Question 28 continues on the next page.

Student Number

Question 28 (continued)

(iii) At retirement, Alba stops depositing money into the fund, but the balance of the fund continues to accrue at 4% p.a. compounded half-yearly. To cover living expenses, she begins to withdraw $30 000 from the fund every half year with the first withdrawal occurring 6 months after retirement.

How many $30 000 withdrawals will she be able to make from the fund? **3**

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

BLANK PAGE

Student Number

Question 29 (4 marks)

The amount of insulin (y units) present in your blood stream t hours after eating a certain food can be approximately modelled by the function:

$$y = \frac{100t}{t + e^t}$$

(i) Find the amount of time it takes for the amount of insulin present to reach a maximum. **3**

……

……

……

……

……

……

……

……

……

……

……

……

(ii) Find the maximum amount of insulin reached, rounding to the nearest whole number. **1**

……

……

……

……

……

Question 30 (6 marks)

Water flows into and out of a tank at a rate given by $\frac{dV}{dt} = 2\pi\sin(\pi t)$ litres/hour, where V represents the volume at time t. The tank is initially empty.

(i) Find an expression for the volume (V litres) of water in the tank after t hours. **2**

……

……

……

……

……

……

(ii) Find the maximum volume of water in the tank. **1**

……

……

……

……

……

……

(iii) Find the first time when the tank is filling at its fastest rate. **3**

……

……

……

……

……

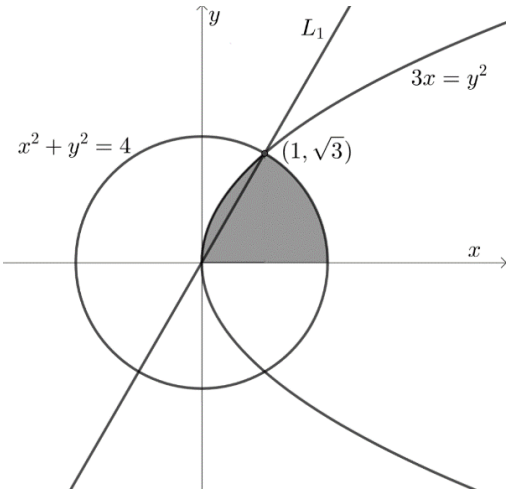
……

……

Student Number

Question 31 (5 marks)

The graphs of the curve $3x = y^2$, the circle $x^2 + y^2 = 4$, and the line L_1 are shown. The curve $3x = y^2$ and the line L_1 intersect at $(0,0)$ and $\left(1, \sqrt{3}\right)$.



(i) Find the acute angle of inclination of the line L_1 with the x-axis. **1**

……

……

(ii) Find the equation of line L_1. **1**

……

(iii) Hence or otherwise, calculate the exact shaded area bound by the two curves and the x-axis. **3**

……

……

……

……

……

……

……

……

……

Question 32 (5 marks)

A particle is undergoing straight line motion. At time t seconds it has displacement of x metres from a fixed point O on the line. The particle's velocity is given by $v = 3\sin t - \sqrt{3}\cos t$. Initially, the particle is 5 metres to the right of O.

Find the exact total distance travelled by the particle in the first π seconds. **5**

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

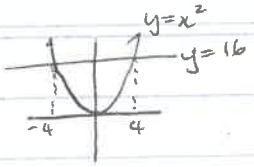
End of Paper

Maths Advanced Trial 2024 Student Solutions.

1. $16 - x^2 \geq 0$

$x^2 \leq 16$

$\therefore -4 \leq x \leq 4$ or $[-4, 4]$



(C)

2. $f(2x) \rightarrow 2x = -4$ $\quad -f(x) = -(-8)$

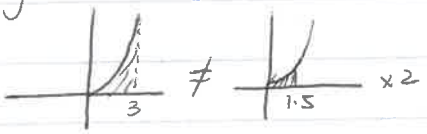
$x = -2$ $\quad = 8$

$\therefore (-2, 8)$ (A)

3. A. x^3 is odd $\therefore$ symm about origin T

B. integrals are equivalent T

C. $y = x$ is odd $\therefore$ T

D. 

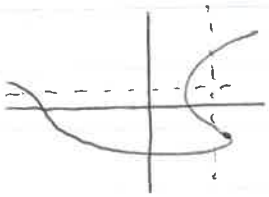
$\neq$ F

(D)

4. $\frac{\sin B}{8} = \frac{\sin 130}{20}$

$\sin B = \frac{8}{20} \sin 130^\circ$

$= 0.4 \sin 130^\circ$ (A)

5. 

fails horizontal and vertical line tests.

(D)

6. $\frac{10,000}{4.2465} = \$2354.88$ (B)

7. $\int \frac{t^2 - 2t}{t^3} dt = \int \frac{1}{t} - \frac{2}{t^2} dt$

$= \ln|t| + \frac{2}{t} + C$ (C)

8. $k = 3$ (amplitude) $\quad$ period $= 3\pi$

$3\pi = \frac{2\pi}{n} \quad \therefore n = \frac{2}{3}$ (C)

9. $g(f(x)) = g\left(\frac{1}{\sqrt{x}}\right) = 1 - \frac{1}{\sqrt{x}}$

$= 1 - (\text{a value})$

$\therefore y < 1$ (D)

10. Intersect at $ax + 3 = \sqrt{x}$

let $u = \sqrt{x}$

$au^2 - u + 3 = 0$

$\Delta = 0$ for 1 soln.

$= (-1)^2 - 4 \times a \times 3$

$\therefore 1 - 12a = 0$ so $a = \frac{1}{12}$ (D)

11. $\log p + \log q^2$

$= \log p + 2\log q = a + 2b$

12. $\cos\theta = \frac{1}{2}$ $\quad$ related acute $= \frac{\pi}{3}$ Quads 1, 2

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

13. a) $f'(x) = 4x^3 + \frac{3}{x^2}$

b) $\frac{dy}{dx} = \frac{-3}{7 - 3x}$

14. a) $a = 200 \quad r = 0.9$ GP

$T_n = 200 \times 0.9^4$

$= 131.22 \text{ kg}$

b) $S_\infty = \frac{200}{1 - 0.9} = 2000 \text{ kg}$

15. $\int_2^3 \frac{x}{1 - x^2} dx = -\frac{1}{2} \int_2^3 \frac{-2x}{1 - x^2} dx$

$= -\frac{1}{2} \left[\ln|1 - x^2|\right]_2^3$

$= -\frac{1}{2} (\ln 8 - \ln 3)$

$= -\frac{1}{2} \ln \frac{8}{3}$

16/ $\frac{dy}{dx} = 2e^{2x}$ at $x = \ln 2$

$= 2e^{2\ln 2}$

$= 2e^{\ln 4}$

$= 2 \times 4 = 8$

at $x = \ln 2$ $y = e^{\ln 4} = 4$.

$\therefore$ Eqn tangent $y - 4 = 8(x - \ln 2)$

$y = 8x - 8\ln 2 + 4$

17/ $\int 7^x + \sec^2 3x \, dx = \int e^{x\ln 7} + \sec^2 3x \, dx$

$= \frac{e^{x\ln 7}}{\ln 7} + \frac{1}{3}\tan 3x + C$

$= \frac{7^x}{\ln 7} + \frac{1}{3}\tan 3x + C$

18(i) $E(X) = 1 \times 0.12 + 2 \times 0.35 + 3 \times 0.32 + 4 \times 0.17 + 5 \times 0.04$

$= 2.66$

(ii) $\text{Var}(X) = 1^2(0.12) + 2^2(0.35) + 3^2(0.32) + 4^2(0.17) + 5^2(0.04) - 2.66^2$

$= 1.04$

$\sigma = 1.02$

	Alf	Lena
E(X)	2.66	2.66
σ	1.02	1.2

same mean, but Alf's SD is 1.02. Although similar on average, Alf's SD is lower, so more consistent.

$\therefore$ Choose Alf!

LHS =

19(i) $\frac{1}{\sec\theta - \tan\theta} - \frac{1}{\sec\theta + \tan\theta}$

$= \frac{\sec\theta + \tan\theta - (\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}$

$= \frac{\sec\theta - \sec\theta + \tan\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$

$= \frac{2\tan\theta}{1 + \tan^2\theta - \tan^2\theta}$

$= 2\tan\theta$ = RHS shown

(ii) $\int \frac{1}{\sec\theta - \tan\theta} - \frac{1}{\sec\theta + \tan\theta} \, d\theta$

$= \int 2\tan\theta \, d\theta$

$= -2\int \frac{-\sin\theta}{\cos\theta} d\theta$

$= -2\ln|\cos\theta| + C$

20 i) $\frac{dM}{dt} = -kM_0 e^{-kt}$

$= -kM$ $\therefore$ a solution

(ii) at $t = 35$ $M = \frac{M_0}{2}$

$\frac{M_0}{2} = M_0 e^{-35k}$

$\frac{1}{2} = e^{-35k}$

$-35k = \ln\frac{1}{2}$

$k = \frac{\ln\frac{1}{2}}{-35} = -0.0198$ (3sf)

(iii) $0.05 = e^{-0.0198t}$

$\ln 0.05 = -0.0198t$

$t = \frac{\ln 0.05}{-0.0198} = 151.2996$ min

$= 2$hr 31 min (nearest min)

21 (i) $\frac{dy}{dx} = 6x - 3x^2 + 9$

$\frac{d^2y}{dx^2} = 6 - 6x$

(ii) stat pts $\frac{dy}{dx} = 0$

$-3x^2 + 6x + 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = -1$ or 3

Test: at $x = -1$ $\frac{d^2y}{dx^2} = 12 > 0$ $\therefore$ min (concave up)

at $x = 3$ $\frac{d^2y}{dx^2} = -12 < 0$ $\therefore$ max (concave down)

$\therefore$ Local min $(-1, -8)$, local max $(3, 24)$

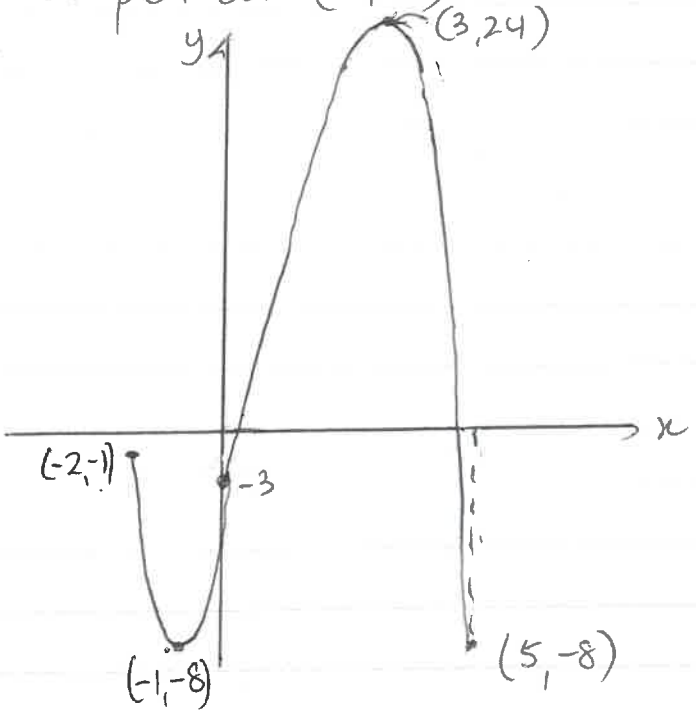
(iii) possible poi at $\frac{d^2y}{dx^2} = 0$ $\quad 6 - 6x = 0$

$x = 1$

x	0	1	2
y''	6	0	-6

$\therefore$ change in concavity

$\therefore$ poi at $(1, 8)$



22 (i) $30 = a + 3d$ —— ①

$54 = a + 9d$ —— ②

② − ①

$6d = 24$ so $\boxed{d = 4}$

$30 = a + 12$ so $\boxed{a = 18}$

$T_{20} = 18 + 19 \times 4 = 94$

(ii) $S_{20} = \frac{20}{2}[2 \times 18 + 19 \times 4] = 1120$

23 (i) $P(NNN) = (0.8)^3 = 0.512$

(ii) $P(NPN \text{ or } PNN \text{ or } NNP)$

$= 0.2 \times 0.8^2 \times 3$

$= 0.384$

24 (i) $\tan 48° = \frac{50}{SB}$

$SB = \frac{50}{\tan 48°}$

(ii) $SD^2 = SB^2 + BD^2 - 2 \times SB \times BD \times \cos 95$

and $BD = \frac{50}{\tan 35}$

$SD^2 = \left(\frac{50}{\tan 48}\right)^2 + \left(\frac{50}{\tan 35}\right)^2 - 2 \times \frac{50}{\tan 48} \times \frac{50}{\tan 35} \times \cos 95$

$SD^2 = 7686.207741...$

$SD = 87.67... = 88\text{m}$ (nearest metre)

25 (i) $\frac{(A-1)^3}{6} = 1$

$(A-1)^3 = 6$

$A = \sqrt[3]{6} + 1$

(ii) $F(2.5) - F(1.5)$

$= \frac{(2.5-1)^3}{6} - \frac{(1.5-1)^3}{6}$

$= 0.5416$

$= 0.54$ (2 dp)

26 (i)

x	0	1	2
y	2	$3\ln 2+2$	$3\ln 3+2$

$A = \frac{1}{2}\left[2 + 2(3\ln 2+2) + 3\ln 3 + 2\right]$

$= 7.7273...$

$= 7.73u^2$

(ii) underestimates as curve is concave down (trapezia lie below the curve.

(iii) 1) $3\ln(x+-2)+2$ — 2 units right

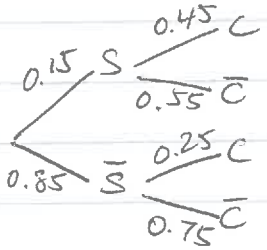
$= 3\ln(x-1)+2$

2) $3\ln(x-1)+5$ — 3 units up.

3) $6\ln(x-1)+5$ — vertical dil. factor 2

$= g(x)$

27 (i)



$P(\text{charged}) = 0.15 \times 0.45 + 0.85 \times 0.25$

$= 0.28$

(ii) $P(S|C) = \frac{0.0675}{0.28} = 0.24$ (2dp)

28 i) $V_1 = 3000$

$V_2 = V_1(1.02) + 3000 = 3000(1.02) + 3000$

$V_3 = V_2(1.02) + 3000 = 3000(1.02)^2 + 3000(1.02) + 3000$

$= 3000\left[1 + 1.02 + 1.02^2\right]$ — $a=1$, $r=1.02$, $n=3$

$= 3000\,\frac{(1.02^3-1)}{1.02-1}$

$= 150000(1.02^3-1)$

(ii) at retirement $n = (60-20)\times 2 + 1$

$= 81$

$V_{81} = 150000\left(1.02^{81}-1\right) = \$595942$ (nearest $)

cont'd

28 (iii) A_1 (1st withd. after 6mths)

$= 595942(1.02) - 30000$

$A_2 = 595942(1.02)^2 - 30000(1.02) - 30000$

$\vdots$

$A_n = 595942(1.02)^n - 30000(1.02)^{n-1} \cdots - 30000$

$0 = 595942(1.02)^n - 30000\left[1 + 1.02 + \ldots + 1.02^{n-1}\right]$

$595942(1.02)^n = 30000\left(\frac{1.02^n - 1}{0.02}\right)$

$11918.84(1.02)^n = 30000(1.02)^n - 30000$

$-18081.16(1.02)^n = -30000$

$1.02^n = 1.65918$

$n = \frac{\ln 1.65918}{\ln 1.02} = 25.568$

$\therefore$ 25 full withdrawals.

29 (i) $u = 100t \quad v = t + e^t$

$u' = 100 \quad v' = 1 + e^t$

$\frac{dy}{dt} = \frac{100(t+e^t) - 100t(1+e^t)}{(t+e^t)^2}$

$= \frac{100t + 100e^t - 100t - 100te^t}{(t+e^t)^2}$

$= \frac{100e^t - 100te^t}{(t+e^t)^2} = 0$

when $100e^t - 100te^t = 0$

$100e^t(1-t) = 0$

when $t = 1$

t	0	1	2
$\frac{dy}{dt}$	100	0	-8.38

$\therefore$ Max at 1 hr

(ii) $y = \frac{100t}{t+e^t} = \frac{100}{1+e}$ (at $t=1$)

$= 26.894...$

$= 27$ units (nearest whole)

30/(i) $V = 2\int \pi \sin \pi t \, dt$

$= -2\cos \pi t + C$

at $t=0$ $V=0$

$\therefore 0 = -2 + C$

$C = 2$

$\therefore V = -2\cos \pi t + 2$

(ii) Max vol $= -2(-1) + 2 = 4$ litres

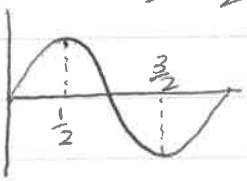
OR $2\pi \sin \pi t = 0$ (at $t=1$)

(iii) $\frac{d^2V}{dt^2} = 0$ for max rate change

$2\pi^2 \cos \pi t = 0$ so $\cos \pi t = 0$

$\pi t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$t = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$



$\therefore$ first max at $\frac{1}{2}$ hour

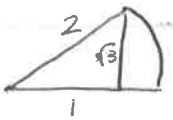
31/(i) $m = \tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$ (or $60°$)

(ii) $y = \sqrt{3x}$ as increasing line.

(iii) $A_{sector} = \frac{1}{2} \times 2^2 \times \frac{\pi}{3} = \frac{2\pi}{3}$

$A_{seg} = \int_0^1 \sqrt{3x}\, dx - \frac{1}{2} \times 1 \times \sqrt{3}$



$= \sqrt{3}\int_0^1 x^{1/2}\, dx - \frac{\sqrt{3}}{2}$

$= \frac{2\sqrt{3}}{3}\left[x^{3/2}\right]_0^1 - \frac{\sqrt{3}}{2}$

$= \frac{2\sqrt{3}}{3} - \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{6}$

$A_{TOTAL} = \frac{\sqrt{3}}{6} + \frac{2\pi}{3}$ units

32/ $V = 3\sin t - \sqrt{3}\cos t = 0$

for direction change

$3\tan t - \sqrt{3} = 0$

$\tan t = \frac{1}{\sqrt{3}}$

$\therefore t = \frac{\pi}{6}, \frac{7\pi}{6}$ (first 2 times)

dist $x = \int 3\sin t - \sqrt{3}\cos t \, dt$

$x = -3\cos t - \sqrt{3}\sin t + C$

at $t=0$ $x=5$

$5 = -3 + C$

$C = 8$

$\therefore x = -3\cos t - \sqrt{3}\sin t + 8$

at $t=0$ $x=5$

at $t=\frac{\pi}{6}$ $x = \frac{-3\sqrt{3}}{2} - \frac{\sqrt{3}}{2} + 8$

$= 8 - 2\sqrt{3}$ ($\doteq 4.5$)

at $t=\pi$ $x = 3 + 8 = 11$

Total dist $= 5 - (8 - 2\sqrt{3}) + 11 - (8 - 2\sqrt{3})$

$= 4\sqrt{3}$ metres

the end ☺