
Student Number

Barker College

2025 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

Staff Involved: **PM MONDAY 11th AUGUST**

- RJW – R. Williams DXC – D. Chua BHC – B. Carruthers ARP – A. Perkins
  DZB – D. Batchen SJB – S. Byun JZT – J. Thomas DZP – D. Peattie
  FHD – F. Deng PCB – P. Boyages*

Topics Covered **150 copies**

Discrete Probability Distributions	Curve Sketching and Using the Derivative	Exponential and Logarithmic Functions	Continuous Probability Distributions
Sequences and Series	Integration	Motion and Rates	Year 11 topics
Graphs and Equations	Trigonometric Functions		

General

Instructions:

- Write your Student Number at the top of this page and all answer pages
- Reading time - 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A separate reference booklet is provided
- Marks may not be awarded for careless or poorly arranged working
- Diagrams are not to scale
- For questions in Section II, show relevant mathematical reasoning and / or calculations

Total marks:

100

Section I – 10 marks (pages 3 - 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8 - 31)

- Attempt Questions 11 - 35
- Allow about 2 hours and 45 minutes for this section

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Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for Questions 1 – 10.

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1. Which of the following functions is neither odd nor even?

   A. $y = \tan x$
   B. $y = e^x$
   C. $y = x^6$
   D. $y = 4x^5 - x$

2. A table of future values for an annuity of \$1 invested at the start of each period is shown.

Future Value of \$1 invested at the start of each period						
Periods	**1%**	**2%**	**3%**	**4%**	**5%**	**6%**
5	5.1520	5.3081	5.4684	5.6330	5.8019	5.9753
10	10.5668	11.1687	11.8078	12.4864	13.2068	13.9716
15	16.2579	17.6393	19.1569	20.8245	22.6575	24.6725
20	22.2392	24.7833	27.6765	30.9692	34.7193	38.9927

   Mark made equal instalments of \$550 twice per year for 10 years with each investment earning 6% p.a., compounded twice per year.

   The future value of this investment is:

   A. \$6 494.28
   B. \$7 684.38
   C. \$15 222.08
   D. \$21 445.99

3. The table shows the function values for $y = f(x)$ and $y = g(x)$.

$\boldsymbol{x}$	-1	0	1	2	3
$\boldsymbol{f(x)}$	3	0	3	12	27
$\boldsymbol{g(x)}$	4	3	2	1	0

What is the value of $y = f\big(g(1)\big)$?

A. 0
B. 1
C. 6
D. 12

4. There are 73 teachers at a local school. Of these teachers:
   - 11 teach Mathematics
   - 10 teach Science
   - 54 teach neither Mathematics nor Science

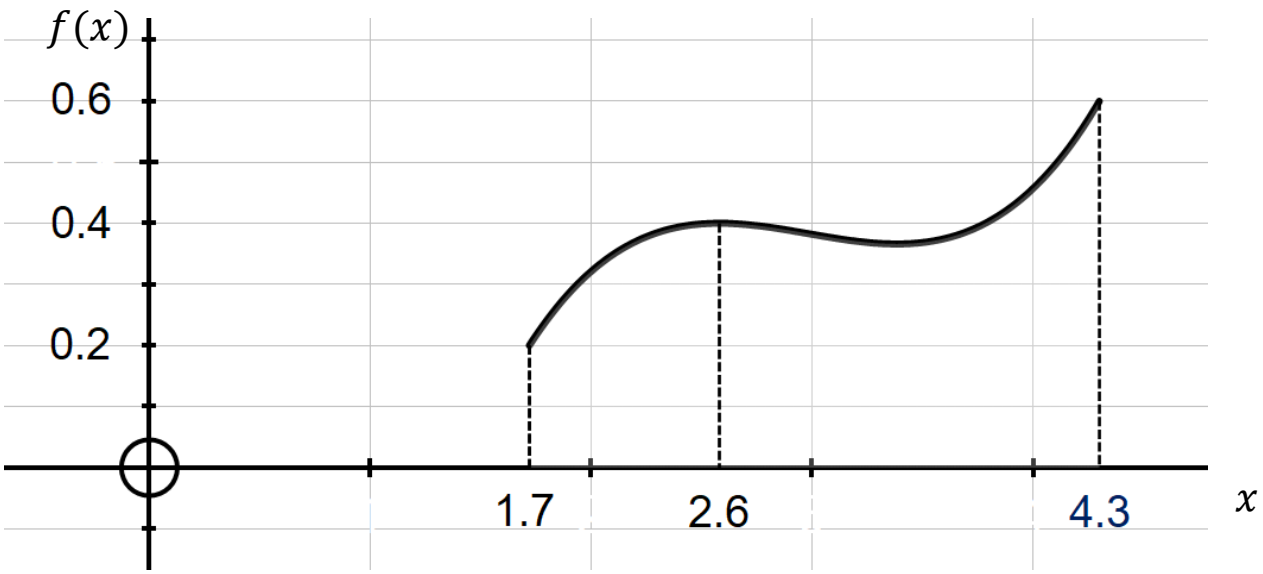
   What is the probability that a teacher chosen at random teaches Mathematics only or Science only?

A. $\frac{17}{73}$
B. $\frac{19}{73}$
C. $\frac{21}{73}$
D. $\frac{7}{18}$

5. Which of the following is true for $y = 5\cos\left(\frac{2\pi x}{3}\right) + 1$?

	Amplitude	Period	Maximum Value	Minimum Value
A.	$\frac{2\pi}{3}$	3	6	-4
B.	1	$\frac{2\pi}{3}$	5	-5
C.	5	$\frac{2\pi}{3}$	5	-5
D.	5	3	6	-4

6. The random variable X, has a probability density function of $y = f(x)$ as shown below. What is the mode of the random variable X.



A. 0.4
B. 0.6
C. 2.6
D. 4.3

7. The function $f(x) = \frac{1}{x}$ has a restricted domain of $[a,b)$, where a and b are positive real numbers.

The range of $f(x) = \frac{1}{x}$ is:

A. $\left[\frac{1}{a},\frac{1}{b}\right)$
B. $\left(\frac{1}{a},\frac{1}{b}\right]$
C. $\left[\frac{1}{b},\frac{1}{a}\right)$
D. $\left(\frac{1}{b},\frac{1}{a}\right]$

8. For the function $y = h(x)$ it is given that $\int_a^b h(x)\,dx < 0$.

Which of the following MUST always be true:

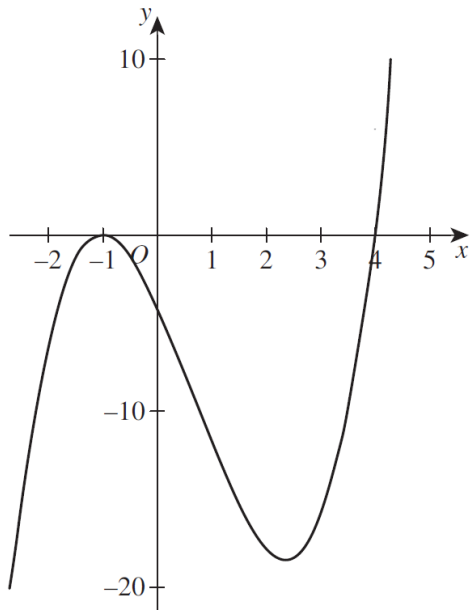
A. a and b are negative

B. All of the curve $y = h(x)$ is below the x-axis for $a \leq x \leq b$

C. The majority of the curve $y = h(x)$ is below the x-axis for $a \leq x \leq b$

D. The gradient of the curve $y = h(x)$ is negative for $a \leq x \leq b$.

9. The graph of $y = f'(x)$ is shown.



Which of the following describes the graph of $y = f(x)$ at the point $x = -1$?

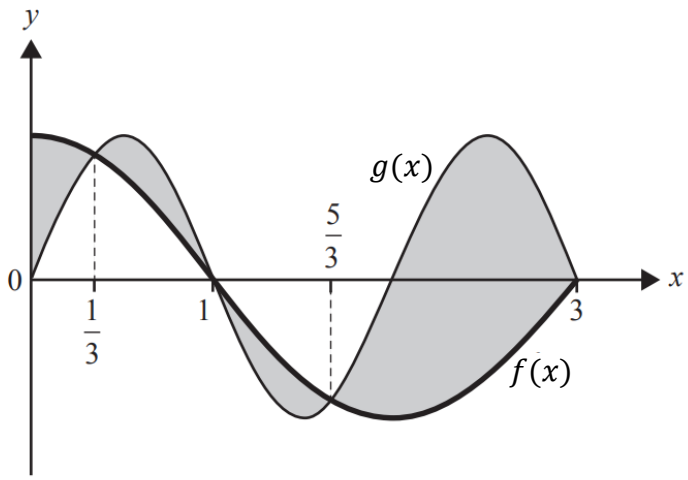
A. $x = -1$ is a vertical asymptote

B. $x = -1$ is a maximum turning point

C. $x = -1$ is a minimum turning point

D. $x = -1$ is a horizontal point of inflexion

10. The graphs of $f(x)=\cos\left(\frac{\pi x}{2}\right)$ and $g(x)=\sin(\pi x)$ are shown below.



An integral expression that gives the total area of the shaded regions is:

A. $2\int_{\frac{5}{3}}^{3}\sin(\pi x)-\cos\left(\frac{\pi x}{2}\right)dx$

B. $\int_{0}^{\frac{1}{3}}\cos\left(\frac{\pi x}{2}\right)-\sin(\pi x)\,dx-2\int_{\frac{1}{3}}^{1}\cos\left(\frac{\pi x}{2}\right)-\sin(\pi x)\,dx-\int_{\frac{5}{3}}^{3}\cos\left(\frac{\pi x}{2}\right)-\sin(\pi x)\,dx$

C. $2\int_{1}^{\frac{5}{3}}\cos\left(\frac{\pi x}{2}\right)-\sin\left(\pi x\right)dx-2\int_{\frac{5}{3}}^{3}\cos\left(\frac{\pi x}{2}\right)-\sin(\pi x)\,dx$

D. $\int_{0}^{\frac{1}{3}}\cos\left(\frac{\pi x}{2}\right)-\sin(\pi x)\,dx+2\int_{\frac{1}{3}}^{1}\sin(\pi x)-\cos\left(\frac{\pi x}{2}\right)dx+\int_{\frac{5}{3}}^{3}\cos\left(\frac{\pi x}{2}\right)-\sin(\pi x)\,dx$

End of Section I

Barker College

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11 – 35
Allow about 2 hours and 45 minutes for this section

Instructions:

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing paper is available.

Student Number

Question 11 (2 Marks)

Find the limiting sum of the series: **2**

$$96 + 72 + 54 + \cdots$$

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Question 12 (2 Marks)

Simplify $\dfrac{x^4 - 4x^2}{x^2 - 3x - 10}$ **2**

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Question 13 (3 Marks)
Solve the equation: **3**

$$\log_4(2x-1)-\log_4 x=-1$$

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Question 14 (3 Marks)

i. Find the length of x in the diagram below, correct to 2 decimal places. **2**

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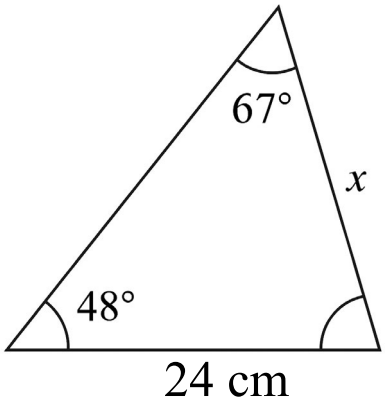

ii. Hence, find the area of the triangle, correct to 2 decimal places. **1**

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Student Number

Question 15 (2 Marks)
Find: **2**

$$\int \frac{3}{2x+1} + 12x^3 \, dx$$

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Question 16 (3 Marks)
Differentiate:

a) $y = \sqrt[3]{12x-11}$ **1**

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b) $y = \frac{\cos(5x)}{x}$ **2**

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Question 17 (4 Marks)
An arithmetic series is given below

$$118 + 105 + 92 + 79 + \ldots$$

i. Find the 10^{th} term of the series. **1**

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ii. Find the minimum number of terms needed for the series to be negative. **3**

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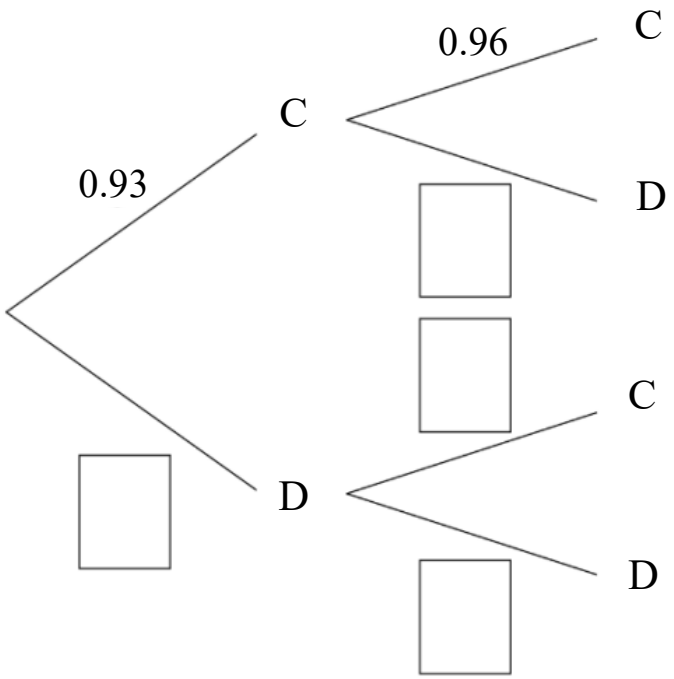
Student Number

Question 18 (4 Marks)

Alyssa enters a catching competition where she attempts to take two catches.
She either takes a catch (C) or drops a catch (D).
The probability Alyssa takes the first catch is 0.93.
If she catches the first attempt, the probability Alyssa takes the second catch is 0.96.
If she drops the first attempt, the probability Alyssa takes the second catch is 0.81.

i. Using the above information, complete the probability tree below **1**



ii. What is the probability that Alyssa takes both catches? **1**

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iii. Given that Alyssa takes at least one catch, find the probability that she takes both catches, as a decimal correct to 3 decimal places. **2**

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Question 19 (2 Marks)

Given that $\cos\theta = -\frac{2}{7}$ and $\frac{\pi}{2} \leq \theta \leq \pi$, find the exact value of cosec θ. **2**

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Question 20 (1 Mark) **1**

Given that $|t| = 2$ find the values of

$$|t-3| - |1+3t|$$

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Student Number

Question 21 (3 Marks)

Evaluate $\int_0^{\ln 2} e^{2x} - 6e^{3x}\, dx$ **3**

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Question 22 (2 Marks)

The graph of $y = \ln x$ is transformed to the graph $y = \ln\left(\frac{1}{x}\right)$. **2**

Describe the transformation.

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Question 23 (3 Marks)

The point P on the graph $y = g(x)$ has coordinates $(-6, 3)$. **3**

Find the coordinates of the image of point P on the graph $y = g(2x - 3) - 4$.

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Question 24 (3 Marks)
Prove that: **3**

$$\frac{\cos^2\theta}{\sin^2\theta+\sin\theta+\cos^2\theta} = 1-\sin\theta$$

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Question 25 (3 Marks)
Find the equation of the tangent, in simplest form, to the curve $y = x^2e^{-x}$ at $(1,\frac{1}{e})$. **3**

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Student Number

Question 26 (3 Marks)

Solve $3\sec\theta + 4 = 0$ for $0 \le \theta \le 2\pi$, correct to 2 decimal places. **3**

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Question 27 (2 Marks)

The probability distribution for the discrete random variable X is shown below. **2**

x	2	3	4	5
$P(X = x)$	0.35	0.44	0.1	0.11

Find $P(X > E(X))$

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Question 28 (5 Marks)
When hot cooking oil cools down, its temperate, T, in degrees Celsius, after t minutes is given by the formula:

$$T = 50 + 160e^{-0.1t}$$

i. What is the initial temperature of the oil? **1**

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ii. Ricky is cooking and at that time the temperature is cooling at a rate of $10°C$ per minute. Calculate the temperature of the oil at that time. **3**

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iii. What temperature does the oil eventually cool to? **1**

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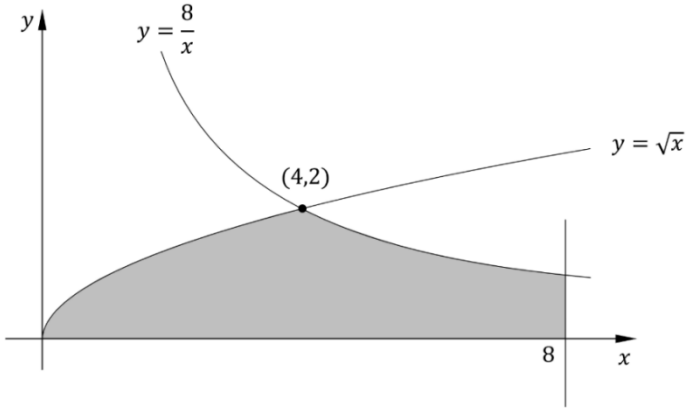
Student Number

Question 29 (3 Marks) **3**

The curve $y = \sqrt{x}$ intersects the curve $y = \frac{8}{x}$ at $(4,2)$.

The region bounded by the curve $y = \sqrt{x}$, the curve $y = \frac{8}{x}$, the line $x = 8$, and the x-axis is shaded in the diagram below.



Find the area of the shaded region in simplest exact form.

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Question 30 (9 Marks)

A curve has an equation $y = x^3 - \frac{1}{4}x^4$

i. Find all intercepts. **1**

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ii. Find the stationary points and determine their nature. **4**

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iii. Find any points of inflexion. **2**

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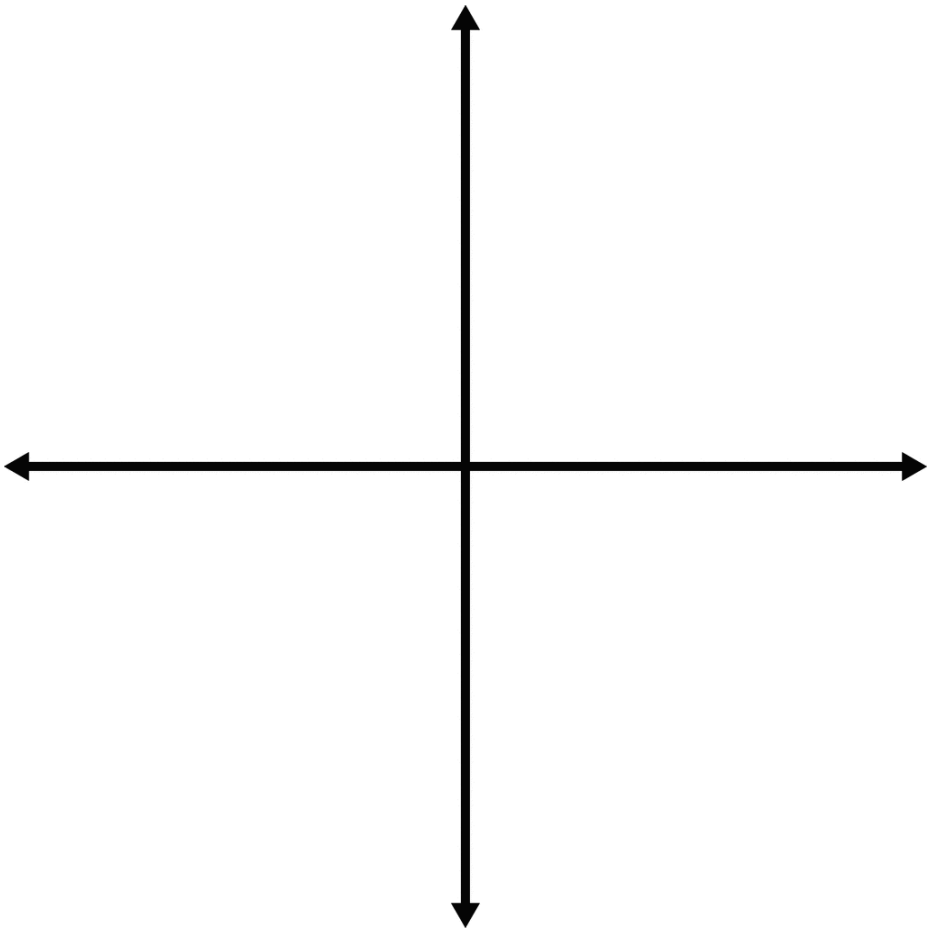
Student Number

Question 30 continues on page 21

Question 30 (continued) **2**

iv. Hence, sketch the graph, indicating all key features.



Question 31 (3 Marks)

i. Differentiate $y = 3x \sin x$ **1**

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ii. Hence, find $\int 3x \cos x \, dx$ **2**

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Question 32 (6 Marks)

The lifetime, X, of a battery has a **cumulative distribution function** $F(x)$ given by

$$F(x)=\begin{cases}0 & \text{for } x<10\\ \dfrac{1}{128}(x^2-12x+20) & \text{for } 10\le x\le 18\\ 1 & \text{for } x>18\end{cases}$$

The lifetime of a battery is guaranteed to be at least 10 hours but never more than 18 hours.

i. Sketch the cumulative distribution function. **2**

ii. An outdoor lantern requires three batteries, all of which must be working, to operate. Three new batteries are inserted into the lantern. **3**
Find the probability that the lantern will still be working after 15 hours.
Give your answer as a decimal, rounded to three decimal places.

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iii. Find the probability density function of the random variable X. **1**

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Question 33 (7 Marks)
A particle moves along the x-axis, and after t seconds its velocity, x metres per second, is given by the equation

$$v = \frac{5\pi}{3}\cos\left(\frac{\pi t}{3}\right)$$

i. Initially, the particle is 3 metres to the right of the origin. Find an equation for the displacement of the particle. **2**

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ii. Find when the particle is at rest for the **second** time and hence, find the total distance travelled at this time. **3**

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Question 33 continues on page 25

Student Number

Question 33 (continued) **2**

iii. Find the **exact** acceleration of the particle when $t = 0.5$.

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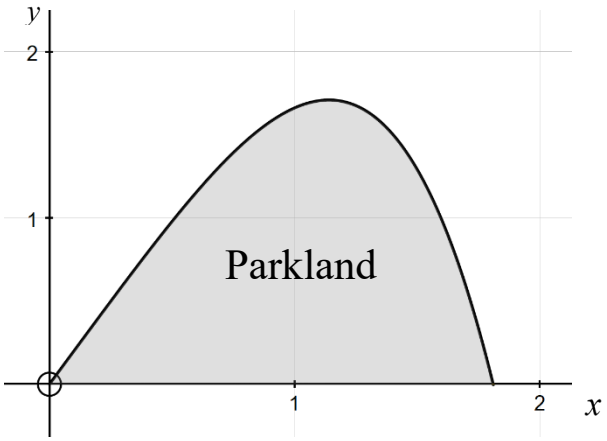
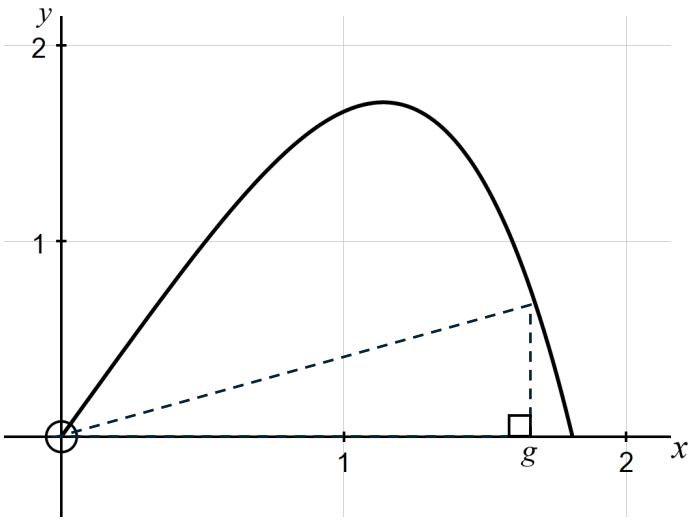
Question 34 (5 Marks)
Below is a map of a local parkland.

The curved boundary of the parkland is shown and has an equation of $y = -\frac{27}{80}x^4 + 2x$.



Meg, the park developer, wants to build a golf course in the shape of a right-angle triangle as shown below. The base length of the triangle is g kilometres.



i. Show that the area of the proposed golf course is: **1**

$$A = -\frac{27}{160}g^5 + g^2$$

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Question 34 continues on page 29

Question 34 (continued) **4**

ii. Find the maximum area of the golf course, clearly showing that it is a maximum.

Question 35 (7 Marks)

Steve borrows \$40 000 at a rate of 4.8% p.a., compounded monthly.
At the end of each month, he makes a repayment of \$$M$.

i. Let A_n be the amount in the account after n months. **2**

By building an expression, show that:

$A_3 = 40000(1.004)^3 - M(1+1.004+1.004^2)$

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ii. After 4 years, Steve doubles the monthly repayment. **5**
He pays off the loan in full at the end of 8 years in total.
Find the value of M.

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Student Number

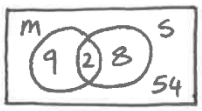
End of Paper

MULTIPLE CHOICE

1. $y = e^x$ [B]
- NO LINE OR ROTATIONAL SYMMETRY.

2. $6\% \div 2 = 3\%$
$10 \times 2 = 20$
550×27.6765
$= \$15\,222.08$ [C]

3. $g(1) = 2$
$f(2) = 12$ [D]

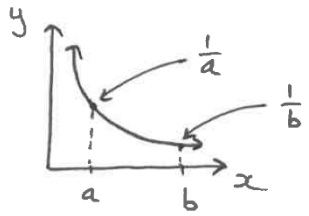
4. 

$= \frac{17}{73}$ [A]

5. [D]

6. MODE IS THE x-VALUE THAT PRODUCES MAX. $f(x)$
$\therefore x = 4.3$ [D]

7. THINK OF THE GRAPH IN Q1:



$\therefore \frac{1}{b} < y \leq \frac{1}{a}$

$\therefore \left(\frac{1}{b}, \frac{1}{a}\right]$ [D]

8. IF $\int_a^b h(x)\,dx < 0$,

THIS IMPLIES THERE IS MORE AREA BELOW THE GRAPH THAN ABOVE, AS THE TOTAL SUM IS -VE. [C]

9. AT $x = -1$, THE GRADIENT IS ZERO & EITHER SIDE IT IS NEGATIVE.
ie: 

$\therefore$ HORIZONTAL P.O.I [D]

10. [B]

EXTENDED RESPONSE

11. $a = 96$
$r = 3/4 \quad S_\infty = \frac{96}{1 - 3/4}$
$= 384$

12. $\frac{x^4 - 4x^2}{x^2 - 3x - 10} = \frac{x^2(x^2 - 4)}{(x-5)(x+2)}$
$= \frac{x^2(x+2)(x-2)}{(x-5)(x+2)}$
$= \frac{x^2(x-2)}{(x-5)}$

13. $\log_4(2x-1) - \log_4 x = -1$

$\log_4\left(\frac{2x-1}{x}\right) = -1$

$\therefore \frac{2x-1}{x} = 4^{-1}$

$\frac{2x-1}{x} = \frac{1}{4}$

$8x - 4 = x$

$7x = 4$

$x = 4/7$

14i. $\frac{x}{\sin 48^\circ} = \frac{24}{\sin 67^\circ}$

$x = \frac{24 \sin 48^\circ}{\sin 67^\circ}$

$= 19.38\text{cm}$

ii. $180^\circ - (48^\circ + 67^\circ) = 65^\circ$

AREA $= \frac{1}{2} \times 24 \times 19.38 \times \sin 65^\circ$

$= 210.77\text{cm}^2$

15. $\int \frac{3}{2x+1} + 12x^3\,dx$

$= \frac{3}{2}\int \frac{2}{2x+1}\,dx + \int 12x^3\,dx$

$= \frac{3}{2}\ln(2x+1) + 3x^4 + C$

16a. $y = (12x-11)^{1/3}$

$y' = 4(12x-11)^{-2/3}$

$= \frac{4}{\sqrt[3]{(12x-11)^2}}$

16b. oops! $y' = -\frac{5x\sin 5x - \cos 5x}{x^2}$

17i. $a = 118$, $d = -13$, $n = 10$

$T_{10} = 118 + (9 \times -13)$

$= 1$

ii. $S_n < 0$

$\therefore 0 > \frac{n}{2}\left(2 \times 118 + (n-1) \times -13\right)$

$0 > n(236 - 13n + 13)$

$0 > n(249 - 13n)$

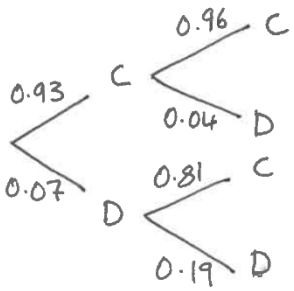
$\therefore n < 0$ OR $249 - 13n < 0$

$249 < 13n$

$19.1.. < n$

$\therefore n = 20$ TERMS

18i.



ii. $P(CC) = 0.93 \times 0.96$

$= 0.8928$

iii. $P(\text{AT LEAST 1}) = 1 - P(\text{NONE})$

$= 1 - (0.07 \times 0.19)$

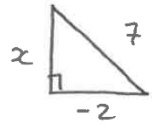
$= 0.9867$

$P(\text{BOTH} \mid \text{AT LEAST 1}) = \frac{0.8928}{0.9867}$

$= 0.9048\ldots$

$= 0.905$

19. USE A TRIANGLE:



$7^2 - (-2)^2 = x^2$

$45 = x^2$

$\sqrt{45} = x$

$\therefore \sin\theta = \frac{3\sqrt{5}}{7}$

$\text{cosec}\theta = \frac{7}{3\sqrt{5}}$

20. $|t-3| - |1+3t|$

$t = 2$: $= |2-3| - |1+6|$; $= |-1| - |7|$; $= -6$

$t = -2$: $= |-2-3| - |1-6|$; $= |-5| - |-5|$; $= 0$

21. $\int_0^{\ln 2} e^{2x} - 6e^{3x}\,dx$

$= \left[\frac{e^{2x}}{2} - 2e^{3x}\right]_0^{\ln 2}$

$= \left(\frac{e^{2\ln 2}}{2} - 2e^{3\ln 2}\right) - \left(\frac{e^0}{2} - 2e^0\right)$

$= \left(\frac{4}{2} - 2\times 8\right) - \left(\frac{1}{2} - 2\right)$

$= -12.5$

22. $y = \ln\left(\frac{1}{x}\right)$

$= \ln x^{-1}$

$y = -\ln x$

$\therefore$ REFLECTED ABOUT x-AXIS.

23. UNIVERSAL FORMULA:

$y = g\left(2\left(x - \frac{3}{2}\right)\right) - 4$

HORIZ: $(-6, 3)$ → $(-3, 3)$ → $(-1.5, 3)$ ($\frac{1}{2}$ DILATE, $1\frac{1}{2}$ RIGHT)

VERT: $(-1.5, +1)$ (4 DOWN)

$\therefore (-1.5, -1)$

24. LHS $= \frac{\cos^2\theta}{\sin^2\theta + \sin\theta + \cos^2\theta}$

$= \frac{\cos^2\theta}{1 + \sin\theta}$

$= \frac{1 - \sin^2\theta}{1 + \sin\theta}$

$= \frac{(1+\sin\theta)(1-\sin\theta)}{1 + \sin\theta}$

$= 1 - \sin\theta$

$=$ RHS

25. $y = x^2 e^{-x}$ @ $(1, \frac{1}{e})$

$u = x^2 \quad v = e^{-x}$
$u' = 2x \quad v' = -e^{-x}$
$y' = 2xe^{-x} - x^2 e^{-x}$

$m = 2.e^{-1} - e^{-1}$
$= \frac{2}{e} - \frac{1}{e}$
$= \frac{1}{e}$

$y - \frac{1}{e} = \frac{1}{e}(x-1)$

$y - \frac{1}{e} = \frac{x}{e} - \frac{1}{e}$

$\therefore y = \frac{x}{e}$

26. $\sec\theta = \frac{-4}{3}$

$\therefore \cos\theta = -\frac{3}{4}$

COSθ -VE IN Q2 & Q3.
Q1: $\theta = 0.723$ RAD
Q2: $\theta = \pi - 0.723...$
$= 2.42$ RAD
Q3: $\theta = \pi + 0.723...$
$= 3.86$ RAD

$\therefore \theta = 2.42, 3.86$ RAD.

27.

x	2	3	4	5
$p(x)$	0.35	0.44	0.1	0.11
$xp(x)$	0.7	1.32	0.4	0.55

$\therefore E(x) = 2.97$

$P(X > 2.97) = 0.44 + 0.1 + 0.11$
$= 0.65$

28i. $t=0 \quad T = 50 + 160e^0$
$= 210°C$

ii. $\frac{dT}{dt} = -16e^{-0.1t}$

$-10 = -16e^{-0.1t}$

$\frac{10}{16} = e^{-0.1t}$

$-0.1t = \frac{\ln\left(\frac{10}{16}\right)}{\ln e}$

$-0.1t = -0.4700...$
$t = 4.7$ MINS

$T = 50 + 160e^{-0.1 \times 4.7}$
$= 150°C$

iii. AS $t \to \infty$,
$T \to 50 + 160^{-\infty}$
$= 50 + 0$
$= 50°C$

29. $\text{AREA} = \int_0^4 x^{1/2}\,dx + \int_4^8 \frac{8}{x}\,dx$

$= \left[\frac{2x^{3/2}}{3}\right]_0^4 + \left[8\ln x\right]_4^8$

$= \left(\frac{16}{3} - 0\right) - \left(8\ln 8 - 8\ln 4\right)$

$= \frac{16}{3} + 8\ln 2 \text{ UNITS}^2$

30i. y-INT: 0

x-INT: $0 = x^3 - \frac{1}{4}x^4$

$0 = x^3\left(1 - \frac{1}{4}x\right)$

$\therefore x = 0, 4$

ii. $y' = 3x^2 - x^3$

$0 = 3x^2 - x^3$

$0 = x^2(3 - x)$

$\therefore x = 0, 3$

TEST NATURE:

$y'' = 6x - 3x^2$

$x = 3 \quad y'' = -9 \therefore$ MAX @ (3, 6.75)

$y = 0 \quad y'' = 0 \therefore$ POSS P.O.I

x	-1	0	1
y''	-5/4	0	3/4

$\therefore$ P.O.I @ (0, 0)

iii. $y'' = 6x - 3x^2$

$0 = 6x - 3x^2$

$0 = 3x(2 - x)$

$\therefore x = 0, 2$

CONT'D

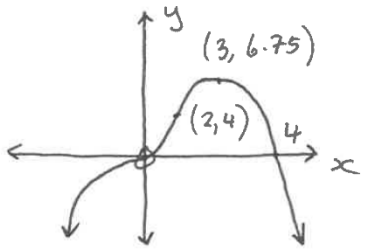
P.O.I @ (0, 0) IN (ii)

$x = 2$

x	1	2	3
y''	3/4	0	-9

$\therefore$ P.O.I @ (2, 4)

iv.



31i. $y = 3x\sin x$

$u = 3x \quad v = \sin x$

$u' = 3 \quad v' = \cos x$

$\therefore y' = 3x\cos x + 3\sin x$

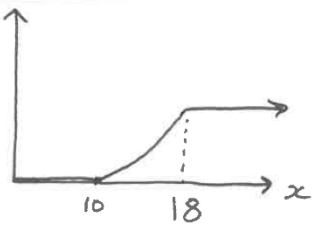
ii. $\int 3x\cos x\,dx$

$= \int 3x\cos x + 3\sin x - 3\sin x\,dx$

$= 3x\sin x - \int 3\sin x\,dx$

$= 3x\sin x + 3\cos x + C$

32 i)



ii) FOR 1 BATTERY:

$P(x>15) = 1 - P(x<15)$

$= 1 - P(15)$

$= 1 - \frac{1}{128}(15^2 - 12\times15 + 20)$

$= \frac{63}{128}$

FOR 3 BATTERIES:

$\left(\frac{63}{128}\right)^3 = 0.11923...$

$= 0.119$

iii) $F(x) = \frac{1}{128}(x^2 - 12x + 20)$

$F'(x) = \frac{1}{128}(2x - 12)$

33 i) $x = \int \frac{5\pi}{3}\cos\left(\frac{\pi t}{3}\right) dt$

$= \frac{3}{\pi}\cdot\frac{5\pi}{3}\sin\left(\frac{\pi t}{3}\right) + C$

$= 5\sin\left(\frac{\pi t}{3}\right) + C$

$t=0, x=3$: $3 = 5\sin\left(\frac{0.t}{3}\right) + C$

$\therefore 3 = C$

CONT'D

$\therefore x = 5\sin\left(\frac{\pi t}{3}\right) + 3$

ii) PARTICLE WILL CHANGE DIRECTION AT $v = 0$.

$0 = \frac{5\pi}{3}\cos\left(\frac{\pi t}{3}\right)$

$0 = \cos\left(\frac{\pi t}{3}\right)$

$0 = \cos\theta$ AT $\pi/2$ & $3\pi/2$

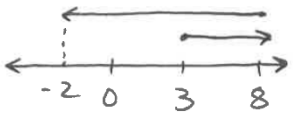
$\therefore \frac{\pi t}{3} = \pi/2$	$\frac{\pi t}{3} = \frac{3\pi}{2}$
$2\pi t = 3\pi$	$2\pi t = 9\pi$
$t = \frac{3}{2}$	$t = \frac{9}{2}$

AT $t = 3/2$ $x = 5\sin\left(\frac{\pi}{3}\times\frac{3}{2}\right)+3$

$= 8$ m

AT $t = 9/2$ $x = 5\sin\left(\pi/3\times\frac{9}{2}\right)+3$

$= -2$m



TOTAL DIST = $5 + 10$

$= 15$m

iii) $V = \frac{5\pi}{3}\cos\left(\frac{\pi t}{3}\right)$

$a = -\frac{5\pi}{3}\cdot\frac{\pi}{3}\sin\left(\frac{\pi t}{3}\right)$

$= -\frac{5\pi^2}{9}\sin\left(\frac{\pi t}{3}\right)$

AT $t = 0.5$, $a = -\frac{5\pi^2}{9}\sin\left(\frac{0.5\pi}{3}\right)$

$= -\frac{5\pi^2}{9}\sin\left(\frac{\pi}{6}\right)$

$= -\frac{5\pi^2}{18}\,ms^{-2}$

34i BASE $= g$

HEIGHT $= -\frac{27g^4}{80} + 2g$

AREA $= \frac{1}{2} \times g \times \left(-\frac{27g^4}{80} + 2g\right)$

$= -\frac{27g^5}{160} + g^2$

ii. $A' = -\frac{135g^4}{160} + 2g$

STAT PTS:

$0 = -\frac{135g^4}{160} + 2g$

$= g\left(-\frac{135g^3}{160} + 2\right)$

$\therefore g = \cancel{0}, \frac{4}{3}$

DETERMINE NATURE:

$A'' = -\frac{540g^3}{160} + 2$

$= -\frac{27g^3}{8} + 2$

AT $g = \frac{4}{3}$ $A'' = \frac{-27\left(\frac{4}{3}\right)^3}{8} + 2$

$= -6 \therefore$ MAX @ $g = \frac{4}{3}$

MAX AREA $= -\frac{127}{160}\left(\frac{4}{3}\right)^5 + \left(\frac{4}{3}\right)^2$

$= \frac{16}{15}$ km^2

OR 1·067 km^2

35i 4·8% ÷ 12 = 0·4%

$A_1 = 40\,000(1.004) - m$

$A_2 = A_1(1.004) - m$

$= (40\,000(1.004) - m)(1.004) - m$

$= 40\,000(1.004^2) - 1.004m - m$

$= 40\,000(1.004^2) - m(1.004 + 1)$

$A_3 = A_2(1.004) - m$

$= (40\,000(1.004^2) - m(1.004 + 1))(1.004) - m$

$= 40\,000(1.004^3) - m(1.004^2 + 1.004) - m$

$= 40\,000(1.004^3) - m(1.004^2 + 1.004 + 1)$

ii. SEE NEXT PAGE!

ii. $A_{48} = 40\,000(1.004)^{48} - m\left(1.004^{47} + 1.004^{46} + \ldots + 1\right)$

$A_{49} = A_{48}(1.004) - 2m$

$= 40\,000(1.004)^{49} - m\left(1.004^{48} + 1.004^{46} + \ldots + 1.004\right) - 2m$

$A_{50} = A_{49}(1.004) - 2m$

$= 40\,000(1.004)^{50} - m\left(1.004^{49} + \ldots + 1.004^{2}\right) - 2m(1.004) - 2m$

$= 40\,000(1.004)^{50} - m\left(1.004^{49} + \ldots + 1.004^{2}\right) - 2m(1.004 + 1)$

$A_{51} = 40\,000(1.004)^{51} - m\left(1.004^{50} + \ldots + 1.004^{3}\right) - 2m\left(1.004^{2} + 1.004 + 1\right)$

$A_{96} = 0 = 40\,000(1.004)^{96} - m\left(1.004^{95} + \ldots + 1.004^{48}\right) - 2m\left(1.004^{47} + \ldots + 1\right)$

$m\left(1.004^{95} + \ldots + 1.004^{48}\right) + 2m\left(1.004^{47} + \ldots + 1\right) = 40\,000(1.004)^{96}$

$a = 1.004^{48}$, $r = 1.004$, $n = 48$: $m\left(\dfrac{1.004^{48}\left(1.004^{48} - 1\right)}{0.004}\right)$

$a = 1$, $r = 1.004$, $n = 48$: $m\left(\dfrac{2\left(1.004^{48} - 1\right)}{0.004}\right)$

$m(63.95\ldots + 105.60\ldots) = 40\,000(1.004)^{96}$

$\therefore m = \dfrac{40\,000(1.004)^{96}}{169.55\ldots}$

$= \$346.08$