

QUESTION 1 [12 marks]**Marks**

- (a.) Evaluate, correct to three significant figures

2

$$\sqrt{\frac{(2.044)^3}{35.5 - 1.2^2}}$$

- (b.) Solve for x : $x^3 = 6x^2$

2

- (c.) Differentiate with respect to x

2

$$y = e^{x^2 - 4x}$$

- (d.) Solve the pair of simultaneous equations

2

$$2x + 3y = -1$$

$$3x - y = 15$$

- (e.) Find a primitive of $\sin \frac{x}{2}$

1

- (f.) Solve $|7 - 3x| \geq 2$ and graph your solution on the number line

3**QUESTION 2** [12 marks]

- (a.) If $\cos \theta = \frac{2}{5}$ and $\tan \theta < 0$, find the exact value of $\sin \theta$

2

- (b.) Differentiate

i.) $x^2 \ln x$

2

ii.) $\frac{\sqrt{x}}{3x - 2}$

2

- (c.) Find $\int \frac{x^2}{x^3 + 5} dx$

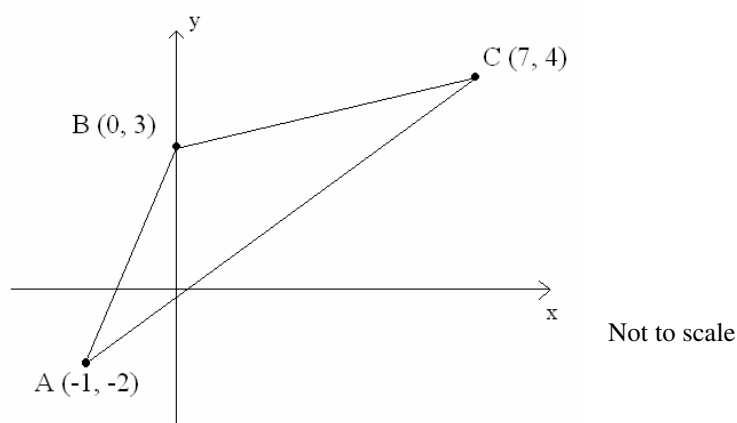
2

- (d.) Evaluate $\int_0^{\ln 3} e^{2x} dx$

2

- (e.) Solve $e^{\log_e x^3} = 27$

2

QUESTION 3 [12 marks]**Marks**

The diagram shows $\triangle ABC$ with vertices $A(-1, -2)$, $B(0, 3)$ and $C(7, 4)$. Copy the diagram onto your answer sheet.

- | | | |
|------|--|---|
| (a.) | E is the midpoint of AC. Find the coordinates of E | 1 |
| (b.) | Find the gradient of AC | 1 |
| (c.) | A line l is drawn through B, perpendicular to AC.
Show that the equation of line l is $4x + 3y - 9 = 0$ | 2 |
| (d.) | What is the angle that l makes with the positive x axis? | 1 |
| (e.) | Find the perpendicular distance of B from AC | 2 |
| (f.) | Find the area of $\triangle ABC$ | 2 |
| (g.) | AC is the diameter of a circle | |
| | i.) Calculate the radius of the circle | 1 |
| | ii.) Hence find the equation of the circle | 2 |

QUESTION 4 [12 marks]

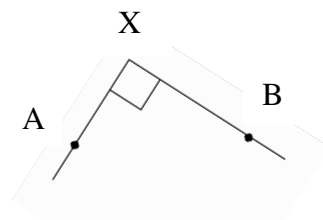
- | | | |
|------|--|---|
| (a.) | Given the parabola $y = 2x^2 - 8x + 1$, find: | |
| | i.) focal length | 1 |
| | ii.) vertex | 1 |
| | iii.) focus | 1 |
| | iv.) directrix | 1 |

QUESTION 4 (continued)**Marks**

- (b.) The third term and the tenth term of an arithmetic series are 10 and 31 respectively. Find the:
- i.) first term and the common difference 2
 - ii.) sum of the first ten terms of the series 1
- (c.) Using Simpson's Rule with three function values, find an approximate value for the area represented by the definite integral 3
- $$\int_2^3 \cos^2 x \, dx$$
- (d.) One hundred tickets are sold in a raffle. Two different tickets are to be drawn for first and second prizes. Bianca buys 15 tickets. What is the probability that she:
- i.) wins first prize 1
 - ii.) wins at least one prize 1

QUESTION 5 [12 marks]

- (a.) Find the equation of the tangent to the curve $y = \ln(x^2 + 2)$ at the point where $x = 1$. Answer in general form 4
- (b.) The gradient of the curve $y = f(x)$ is given by $f'(x) = \frac{2x^2 + 1}{x}$. 3
Find the equation of the curve if it passes through the point (1, 5)
- (c.)



Two athletes are jogging on separate roads which meet at right angles at town X. Athlete A is 8km from X and is travelling at 5km/h away from X. Athlete B is 10km from X and is travelling at 6km/h and travelling towards X.

- i.) Show that the distance apart, P km, after t hours is given by 2
$$P = \sqrt{61t^2 - 40t + 164}$$
- ii.) Hence find their minimum distance apart. 3

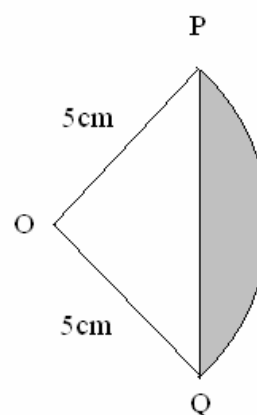
QUESTION 6 [12 marks]**Marks**

- (a.) Consider the curve $y = x(x - 3)^3$
- i.) Find the coordinates of any stationary points and determine their nature **4**
- ii.) Find the point(s) of inflexion **2**
- iii.) Sketch the curve, showing clearly all features **1**
- (b.) If α , β are roots of the quadratic equation $6x^2 - x + 5 = 0$, find:
- i.) $\alpha + \beta$ **1**
- ii.) $\alpha \times \beta$ **1**
- iii.) $\alpha^2 + \beta^2$ **1**
- (c.) Solve $\log_{27} 32 = x \log_3 2$ without the aid of a calculator. **2**
Show all working

QUESTION 7 [12 marks]

- (a.) i.) Draw a neat sketch of the function $y = 1 - 2\sin x$ for $0 \leq x \leq 2\pi$ **2**
- ii.) Determine the exact values where the graph cuts the x -axis in the given domain **2**
- iii.) Calculate the area bounded by the curve $y = 1 - 2\sin x$, the x -axis and the lines $x = 0$ and $x = \frac{\pi}{2}$ **2**

- (b.) In the diagram PQ is an arc of a circle with centre O and radius 5cm. The chord PQ has length 6cm.



- i.) Find the size of $\angle POQ$ in radians **2**
- ii.) Calculate the shaded area **2**

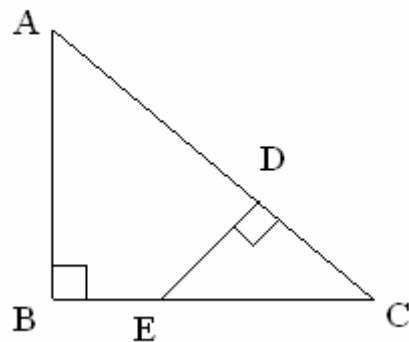
QUESTION 7 (continued)**Marks**

- (c.) Find the values of K for which the equation $(2 - K)x^2 - 4(K - 2)x - 5 = 0$, has two real distinct roots 2

QUESTION 8 [12 marks]

- (a.) A particle, initially at rest at the origin, moves in a straight line with velocity V m/s so that $V = 5t(4 - t)$ where t is the time in seconds. Find:
- i.) the acceleration of the particle after 4 seconds 2
 - ii.) an expression for the displacement x metres of the particle in terms of t 2
 - iii.) the total distance travelled in the first 6 seconds 2

(b.)



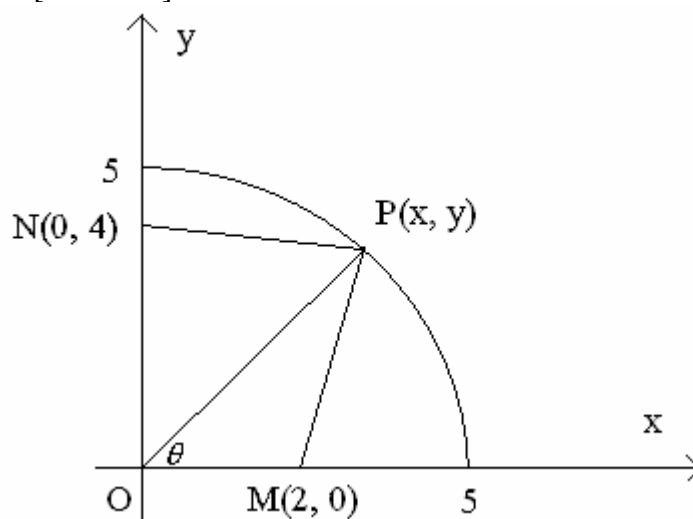
Not to scale

ABC is right angled at B and DE is perpendicular to AC.

- i.) Prove that $\triangle ABC$ and $\triangle CDE$ are similar 2
- ii.) Prove that $BC \times CE = AC \times CD$ 2
- iii.) Prove that $DE^2 = AD \times DC - BE \times EC$ 2

QUESTION 9 [12 marks]**Marks**

- (a.) A bottle of solvent is open and the solvent evaporates in such a way that the amount remaining, V ml, in the bottle is given by $V = 2000e^{-0.005t}$, where t is time in hours.
- i.) How much solvent is in the bottle initially? **1**
- ii.) How much solvent has evaporated out of the bottle after 30 hours? **1**
- iii.) How long is it before half the initial amount of solvent has evaporated from the bottle? **1**
- iv.) If the solvent continues to evaporate will the bottle ever become empty? Explain. **1**
- (b.) Boxes in a storeroom are stacked in a pile such that there are 25 on the bottom row, 22 on the next, 19 on the next, and so on until 117 boxes are on the pile altogether.
- i.) How many rows of boxes are there? **2**
- ii.) How many boxes are there on the top row? **1**
- (c.) By expressing $1.2\dot{5}$ as the sum of an infinite geometric series, find the simple equivalent fraction for $1.2\dot{5}$. **2**
- (d.) The graph $y = e^x - 1$ is rotated about the x -axis from $x = 1$ to $x = 3$. Find the volume generated. **3**

QUESTION 10 [12 marks]**Marks**

- (a.) The diagram shows a part of the circle $x^2 + y^2 = 25$. The point $P(x, y)$ is on the circle and O is the origin. Given $M(2, 0)$ and $N(0, 4)$ and $\angle MOP$ is θ radians.
- i.) Show that the area A , of the quadrilateral $OMPN$ is given by 2
 $A = 5\sin\theta + 10\cos\theta$
- ii.) Find the value of $\tan\theta$ for which A is maximum 3
- iii.) Hence find (in surd form) the coordinates of P 2
for which A is maximum
- (b.) Alice is retiring tomorrow and her Super Fund contains \$450,000. The fund is earning 15% p.a. compound interest, compounded monthly. Alice wishes to withdraw a regular amount of \$6000 per month to cover her expenses.
- i.) Show that after 1 month she will have an amount A in her account 1
where $A_1 = 450000 \times 1.0125 - 6000$
- ii.) Find an expression for the amount remaining after n months 1
- iii.) How many years will the money last? 2
- iv.) If she wishes to withdraw \$6000 per month from her account for 1
30 years, use your calculator to approximate the required interest rate.

End of Paper

Yr. 12 TRIAL 2007 - Answers - TOTAL (120)

Question 1 (12)

(a) $0.5007 = 0.501$ (3s.f.) (1)

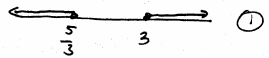
b) $x^3 = 6x^2$
 $x^3 - 6x^2 = 0$
 $x^2(x - 6) = 0$
 $x = 0$ $x = 6$
 (1) (1)

c) $y_1 = e^{x^2 - 4x}$
 $y_1 = (2x - 4) \cdot e^{x^2 - 4x}$
 (1) (1)

d) $2x + 3y = -1$
 $3x - y = 15$
 $x = 4$ $y = -3$
 (1) (1)

e) $\int \sin \frac{x}{2} dx = -\frac{1}{\frac{1}{2}} \cos \frac{x}{2} + C$
 $= -2 \cos \frac{x}{2} + C$
 (1)

f) $|7 - 3x| \geq 2$
 $7 - 3x \geq 2$ $7 - 3x \leq -2$
 $5 \geq 3x$ $9 \leq 3x$
 $\frac{5}{3} \geq x$ $3 \leq x$



answer $\therefore x \leq \frac{5}{3}, x \geq 3$ (2)

Question 2 (12)

(a) $\cos \theta = \frac{2}{5}$ $\tan \theta < 0$

$\frac{\sqrt{21}}{2} \cdot \frac{5}{2} \therefore \sin \theta = \frac{\sqrt{21}}{5}$ (1)

(b) i) $\frac{d}{dx}(x^2 \ln x^2) = 2x \cdot \ln x^2 + x^2 \cdot \frac{2x}{x^2}$
 $= 2x \ln x^2 + 2x$
 (1) (1)

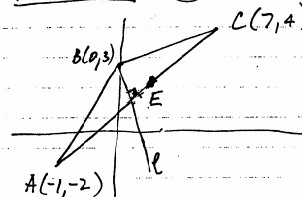
ii) $\left(\frac{\sqrt{x}}{3x-2}\right)' = \frac{\frac{1}{2}x^{-\frac{1}{2}}(3x-2) - x^{\frac{1}{2}}(3)}{(3x-2)^2}$
 (1) (1)

(c) $\int \frac{x^2}{x^3+5} dx = \frac{1}{3} \ln(x^3+5) + C$
 (1) (1)

(d) $\int_0^{\ln 3} e^{2x} dx = \frac{1}{2} [e^{2x}]_0^{\ln 3} = 4$
 (1) (1)

e) $e^{\log_e x^3} = 27$
 $x^3 = 27$ (1)
 $x = 3$ (1)

Question 3 (12)



(a) $E(3, 1)$ (1)

(b) $m_{AC} = \frac{6}{8} = \frac{3}{4}$ (1)

(c) $m_{\perp} = -\frac{4}{3}$ (1)
 $\therefore$ line $y - 3 = -\frac{4}{3}(x - 0)$ (1)
 $y = -\frac{4}{3}x + 3$
 $4x + 3y - 9 = 0$ shown

(d) $\tan \theta = m_{\ell} = -\frac{4}{3}$
 $\therefore \theta = 126^\circ 52'$ (1)

e) $B(0, 3)$
 line AC $\therefore y - 4 = \frac{3}{4}(x - 7)$
 $3x - 4y - 5 = 0$ (1)
 $d = \frac{|3 \times 0 - 4 \times 3 - 5|}{\sqrt{3^2 + (-4)^2}} = \frac{17}{5}$ (1)

f) $\text{Area}_\Delta = \frac{1}{2} AC \times d$
 $AC = \sqrt{100} = 10$ (1)
 $\text{Area} = \frac{1}{2} \times 10 \times \frac{17}{5} = 17$ units² (1)

g) i) $r = \frac{1}{2} AC = 5$ (1)
 ii) $E(3, 1)$ centre (1) $r = 5$
 $\therefore (x - 3)^2 + (y - 1)^2 = 5^2$ (1)

Question 4 (12)

(a) $y = 2x^2 - 8x + 1$
 $(x - m)^2 = 4a(y - n)$
 $(x - 2)^2 = \frac{1}{2}(y + 7)$
 i) focal length $= a = \frac{1}{8}$ (1)
 ii) vertex $(2, -7)$ (1)
 iii) focus $(2, -6\frac{7}{8})$ (1)
 iv) directrix $y = -7\frac{1}{8}$ (1)

(b) $T_3 = a + 2d = 10$
 i) $T_{10} = a + 9d = 31$
 $d = 3$ (1) $a = 4$ (1)

ii) $S_{10} = 175$ (1)
 (c) $\int \cos^2 x dx$

x	$\frac{\pi}{2}$	$\frac{\pi}{2}$	2.5	3
$\cos^2 x$	0.17318	0.64183	0.98009	

 (1)

$A \approx \frac{0.5}{3} [0.17318 + 4 \times 0.64183 + 0.98009]$
 $= 0.62009 \approx 0.62$ (1)

(d) i) $P = \frac{15}{100}$ (1)
 ii) $P = 1 - P(\text{unpinned})$
 $= 1 - \frac{85}{100} \times \frac{84}{99}$
 $P = 0.278$ or $\frac{143}{500}$ (1)

Question 5 (12)

(a) $y = \ln(x^2 + 2)$

$y' = \frac{2x}{x^2 + 2}$ ①

$m = \frac{2}{3}$ ① $y = \ln 3$ ①

$\therefore y - \ln 3 = \frac{2}{3}(x - 1)$

$0 = 2x - 3y - 2 + 3 \ln 3$ ①

or $0 = 2x - 3y - 2 + \ln 27$

(b) $f'(x) = \frac{2x+1}{x}$

$f(x) = \int \frac{2x+1}{x} dx$ ①
 $= \int 2x + \frac{1}{x} dx$

$f(x) = x^2 + \ln x + C$ ①

$5 = 1 + 0 + C \therefore C = 4$ ①

$\therefore f(x) = x^2 + \ln x + 4$

(c) $d = \sqrt{(8+5t)^2 + (10+6t)^2}$ ①

$= d = \sqrt{64 + 80t + 25t^2 + 100 + 120t + 36t^2}$ ①

$P = \sqrt{61t^2 - 40t + 164}$ ①

(ii) $P' = \frac{1}{2}(61t^2 - 40t + 164)^{-\frac{1}{2}} \times (122t - 40)$

$0 = \frac{122t - 40}{2(61t^2 - 40t + 164)^{\frac{1}{2}}}$

$0 = 122t - 40$

$\therefore t = 0.327 \dots$ ①
 ≈ 0.33 hours

showing min. ①

$\frac{t}{P} \begin{array}{|c|c|c|} \hline 0.3 & 0.33 & 0.4 \\ \hline -1.7 & 0 & 4.4 \\ \hline \end{array}$

(ii) Distance Apart $= \sqrt{61(0.33)^2 - 40(0.33) + 164}$

① $= 12.5$ km.

Question 6 (12)

(a) $y = x(x-3)^3$

(i) $y' = 1(x-3)^3 + x \cdot 3(x-3)^2$ ①

$0 = (x-3)^2 [x-3+3x]$

$x = 3$ or $x = \frac{3}{4}$ ① (both)

$y = 0$ ① $\rightarrow y = -\frac{2187}{256} = -8.54$

$y'' = 2(x-3)(4x-3) + (x-3)^2 \cdot 4$

$y'' = 2(x-3)[6x-9]$ ① concavity

$y''(3) = 0 \therefore (3,0)$ horizontal pt. of inf.

$y''(\frac{3}{4}) = \frac{81}{4} > 0 \therefore (\frac{3}{4}, -8.54)$ Min. turning

(ii) $y'' = 2(x-3)[6x-9] = 0$

both $\begin{cases} x = 3 \\ y = 0 \end{cases}$ $\begin{cases} x = \frac{3}{2} \\ y = -\frac{81}{16} \end{cases}$

(3,0) horiz. pt. of inflexion

$\frac{x}{y''} \begin{array}{|c|c|c|} \hline 1 & \frac{3}{2} & 2 \\ \hline 12 & 0 & -6 \\ \hline \end{array}$

① concavity changes $\therefore (\frac{3}{2}, -\frac{81}{16})$ pt. of inf.

(iii) $\frac{x}{y''} \begin{array}{|c|c|c|} \hline 1 & \frac{3}{2} & 2 \\ \hline 12 & 0 & -6 \\ \hline \end{array}$

① shape

$\frac{x}{y''} \begin{array}{|c|c|c|} \hline 1 & \frac{3}{2} & 2 \\ \hline 12 & 0 & -6 \\ \hline \end{array}$

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Question 6-cont.

(b) $6x^2 - x + 5 = 0$

(i) $\alpha + \beta = -\frac{b}{a} = \frac{1}{6}$ ①

(ii) $\alpha \cdot \beta = \frac{c}{a} = \frac{5}{6}$ ①

(iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
 $= (\frac{1}{6})^2 - 2 \times \frac{5}{6} = -\frac{59}{36}$ ①

(c) $\log_{27} 32 = x \cdot \log_3 2$

① $\frac{\log_3 32}{\log_3 27} = \log_3 2^x$

$\frac{\log_3 32}{3} = \log_3 2^x$

$\frac{1}{3} \log_3 32 = \log_3 2^x$

$\log_3 32^{\frac{1}{3}} = \log_3 2^x$

$32^{\frac{1}{3}} = 2^x$

$(2^5)^{\frac{1}{3}} = 2^x$

$\frac{5}{3} = x$ ①

Question 7 (12)

(i) $y = 1 - 2\sin x$

$0 = 1 - 2\sin x$

$2\sin x = 1 \therefore \sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ ① ①

(iii) Area $= \int_0^{\frac{\pi}{6}} 1 - 2\sin x dx + \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 1 - 2\sin x dx$

$= \left[x + 2\cos x \right]_0^{\frac{\pi}{6}} + \left[x + 2\cos x \right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$

$= \frac{\pi}{6} + 2\cos \frac{\pi}{6} - 0 - 2\cos 0 + \frac{5\pi}{6} + 2\cos \frac{5\pi}{6} - \frac{\pi}{6} - 2\cos \frac{\pi}{6}$

$= 0.2556 + 9 - 0.68485$

$= 0.94$ ①

① the correct answer includes the knowledge of the abs. value

(b) i) $\cos(\angle POQ) = \frac{5^2 + 5^2 - 6^2}{2 \times 5 \times 5}$ ①

$\theta = \angle POQ = 1.287$ radians ①

ii) $A = \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \angle POQ$ ①

$= \frac{1}{2} \times 5^2 \theta - \frac{1}{2} \times 5^2 \sin \theta = 4.0875$

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Q.7 cont.

$$(2-k)x^2 - 4(k-2)x - 5 = 0$$

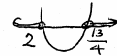
$$\Delta = b^2 - 4ac > 0$$

$$[4(k-2)]^2 - 4 \cdot (2-k) \cdot (-5) > 0$$

$$16(k-2)^2 - 20(2-k) > 0$$

$$4(k-2)[4(k-2) - 5] > 0$$

$$4(k-2)[4k - 13] > 0$$



$$k < 2 \text{ or } k > \frac{13}{4} \quad (1)$$

Question 8 (12)

a) $v = 5t(4-t) = 20t - 5t^2$
 $t=0 \quad v=0 \quad x=0$

i) $a = \frac{dv}{dt} = 20 - 10t \quad (1)$

$a(t=4) = -20 \text{ m/s}^2 \quad (1)$

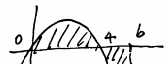
ii) $x = \int v dt = \int 20t - 5t^2 dt$

$x = 10t^2 - \frac{5}{3}t^3 + C \quad (1)$

$0 = 0 - 0 + C \therefore C = 0 \quad (1)$

$\therefore x = 10t^2 - \frac{5}{3}t^3$

iii) $v = 20t - 5t^2 = 5t(4-t)$



$d = \int_0^4 (20t - 5t^2) dt + \int_4^6 20t - 5t^2 dt \quad (1)$

$$\therefore d = \left[10t^2 - \frac{5}{3}t^3 \right]_0^4 + \left[10t^2 - \frac{5}{3}t^3 \right]_4^6$$

$$d = \frac{320}{3} = 106.6 \text{ m} \quad (1)$$

(b) $\angle C$ in common

$\angle ABC = \angle EDC = 90^\circ$ (given)

$\therefore \triangle ABC \sim \triangle EDC$ (matching $\angle$'s) (1)

(ii) $\frac{EC}{AC} = \frac{CD}{BC}$ (matching sides in the same ratio) (1)

$\therefore BC \times CE = AC \times CD$

(iii) using (i) (2)

$BC \times CE = AC \times CD$

$(BE + EC) \times CE = (AD + DC) \times CD$

$BE \cdot EC + EC^2 = AD \cdot DC + DC^2$

$EC^2 - DC^2 = AD \cdot DC - BE \cdot EC$

$\frac{EC^2 - DC^2}{DE} = \frac{AD \cdot DC - BE \cdot EC}{DE}$

$DE = AD \cdot DC - BE \cdot EC$ (shown)

Question 9 (12)

(a) $V = 2000e^{-0.005t}$

i) $t=0 \therefore V = 2000 \text{ mL} \quad (1)$

ii) $V = 2000e^{-0.005 \times 30} = 1721.41$

$\therefore$ evaporated $2000 - 1721.41 = 278.58 \text{ mL} \quad (1)$

iii) $1000 = 2000e^{-0.005t}$

$\ln \frac{1}{2} = -0.005t$

$t = 138.629 \text{ hours} \quad (1)$

Q.9 cont.

(a) iv) since



$v \neq 0 \therefore$ the bottle will never become empty

b) $T_1 = a = 25$
 $T_2 = 2a$
 $T_3 = 19$

$A \cdot P \quad a = 25$
 $d = -3$

$\therefore S_n = 117 = \frac{n}{2} (2 \times 25 - (n-1) \times 3)$
 or $117 = 25n - \frac{3n(n-1)}{2}$
 $4 + 1 \therefore 9 \text{ rows} \quad (1)$

ii) 1 box only (1)

(c) $1.25 = 1.2 + \frac{5}{100} + \frac{5}{1000} + \dots$
 or $1 + \frac{2}{10} + \frac{5}{100} + \frac{5}{1000} + \dots$

$\therefore 1.25 = 1.2 + \frac{5}{100} \quad a = \frac{5}{100}$
 $= 1.2 + \frac{a}{1-r} \quad r = \frac{1}{10}$

$1.25 = 1.2 + \frac{50}{900} = \frac{113}{90} \quad (1)$

(d) $V = \pi \int_0^3 (e^x - 1)^2 dx$

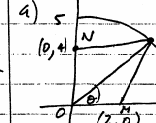
$V = \pi \int_0^3 e^{2x} - 2e^x + 1 dx \quad (1)$

$= \pi \left[\frac{1}{2} e^{2x} - 2e^x + x \right]_0^3 \quad (1)$

$= \pi \times 165.285 = 519.26 \quad (1)$

Question 10 (12)

a) 5



i) $A_{\triangle OHP} = \frac{1}{2} OH \times OP \times \sin \theta$
 $= \frac{1}{2} \times 2 \times 5 \times \sin \theta$
 $= 5 \sin \theta \quad (1)$

$A_{\triangle OPN} = \frac{1}{2} ON \times OP \times \sin(90^\circ - \theta)$
 $= \frac{1}{2} \times 4 \times 5 \times \cos \theta$
 $= 10 \cos \theta$

$\therefore A = 5 \sin \theta + 10 \cos \theta$

ii) $A' = 5 \cos \theta - 10 \sin \theta \quad (1)$

$0 = 5 \cos \theta - 10 \sin \theta$

$10 \sin \theta = 5 \cos \theta$

$\tan \theta = \frac{1}{2}$

showing Max: $\tan \theta = \frac{1}{2} \therefore \theta = 26.56^\circ$

$A'' = -5 \sin \theta - 10 \cos \theta \quad (1)$

$A''(\theta = 26.56^\circ) = -11.18 < 0 \therefore \text{Max.}$

iii) since $\tan \theta = \frac{1}{2} = m_{OP}$

$\therefore \frac{1}{2} = \frac{y}{x}$

and $PO = 5 = \sqrt{x^2 + y^2}$

$\therefore \frac{1}{2} = \frac{y}{x}$

$25 = x^2 + y^2$

$x = \sqrt{20} = 2\sqrt{5}$

$y = \frac{\sqrt{20}}{2} = \sqrt{5}$

$P(2\sqrt{5}, \sqrt{5}) \quad (1)$

8.10 cont.

$$b) i) A_1 = 450000 \times \left(1 + \frac{1.25}{100}\right) - 6000 \quad \text{①}$$

$$= 450000 \times 1.0125 - 6000$$

$$(ii) A_2 = A_1 \times 1.0125 - 6000$$

$$= 450000 \times 1.0125^2 - 6000 \times 1.0125 - 6000$$

$$\therefore A_n = 450000 \times 1.0125^n - 6000 \times 1.0125^{n-1} - \dots - 6000 \quad \text{①}$$

$$\text{OR } A_n = 450000 \times 1.0125^n - 6000 [1.0125^{n-1} + \dots + 1]$$

$$iii) 0 = 450000 \times 1.0125^n - 6000 \times \frac{(1.0125^n - 1)}{1.0125 - 1} \quad \text{①}$$

$$0 = 450000 \times 1.0125^n - 480000 (1.0125^n - 1)$$

$$0 = 450000 \times 1.0125^n - 480000 \times 1.0125^n + 480000$$

$$\therefore 30000 \times 1.0125^n = 480000$$

$$1.0125^n = 16$$

$$n = \frac{\ln 16}{\ln 1.0125}$$

$$\text{① } \begin{cases} n = 223.19 \text{ months} \\ = 18.59 \text{ yrs (18 yrs 7.2 months)} \end{cases}$$

$$(iv) \text{ Try } r = 16\% \therefore r = \frac{16}{100}$$

$$\therefore 450000 \times (1.013)^{360} = 6000 \frac{(1.013)^{360} - 1}{(1.013 - 1)}$$

$$\therefore 450000 \times (1.013)^{360} = 6000 \times (8753. \dots)$$

$$\frac{450000 \times (1.013)^{360}}{8753. \dots} = 6052 \div 6000 \quad \text{①}$$