BAULKHAM HILLS HIGH SCHOOL
YEAR 12 ASSESSMENT TASK 4 2025
MATHEMATICS ADVANCED

Nesa Number: _____

Teacher: _____

Mathematics Advanced

General Instructions

- Reading time - 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet and z-tables are provided separately
- In Questions 11 - 33, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks: 100

Section I - 10 marks (pages 4-8)

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II - 90 marks (pages 9 - 35)

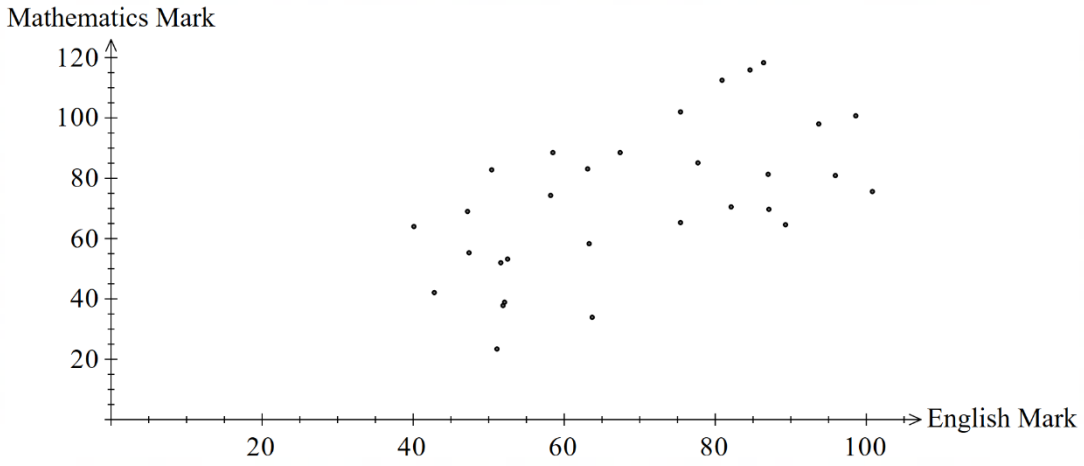
- Attempt Questions 11 - 33
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

1 The scatterplot below shows a linear relationship between the marks a class receives on a Mathematics examination and on an English examination.



Which of the following would be closest to Pearson's correlation coefficient for the set of data?

(A) 0.95

(B) 0.5

(C) –0.5

(D) –0.95

2 There are 25 seats in the front row of an amphitheatre, 30 in the second row, 35 in the 3rd row and so on such that the number of seats in each row is 5 more than the row in front. How many seats are in the 50th row?

(A) 90

(B) 125

(C) 270

(D) 7375

3 Which of the following correctly describes the order of the transformations of $y = \sin x$ to $y = 3\sin(2x - 1)$?

(A) Vertical dilation by a factor of 3, horizontal dilation by a factor of 2, horizontal translation right 1.

(B) Horizontal translation right 1, horizontal dilation by a factor of 2, vertical dilation by a factor 3.

(C) Horizontal dilation by a factor of $\frac{1}{2}$, horizontal translation right $\frac{1}{2}$, vertical dilation by a factor of 3.

(D) Horizontal dilation by a factor of $\frac{1}{2}$, horizontal translation right 2, vertical dilation by a factor of $\frac{1}{3}$.

4 The weights of an Australian Cattle Dog are normally distributed with mean 25 kg and standard deviation 4 kg. Approximately what percentage of dogs will weigh between 29 kg and 33 kg?

(A) 2.35%

(B) 2.5%

(C) 13.5%

(D) 27%

5 When a drug enters the bloodstream, its concentration (y) decreases over time (x) such that the rate of decrease slows down. Which of the following best describes $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$?

(A) $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} < 0$

(B) $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} > 0$

(C) $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$

(D) $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} > 0$

6 Which of the following could be a continuous probability density function?

(A) $f(x) = \frac{1}{2}x$

(B) $f(x) = \begin{cases} 1-x & 0 \le x \le 2 \\ 0 & \text{for all other values of } x \end{cases}$

(C) $f(x) = \begin{cases} \frac{1}{4}x & 0 \le x < 2 \\ 1-\frac{1}{4}x & 2 \le x \le 4 \\ 0 & \text{for all other values of } x \end{cases}$

(D) $f(x) = \begin{cases} 1-x^2 & 0 \le x \le 1 \\ 0 & \text{for all other values of } x \end{cases}$

7 Which of the following is the derivative of $y = \sqrt[3]{(9x-1)^2}$?

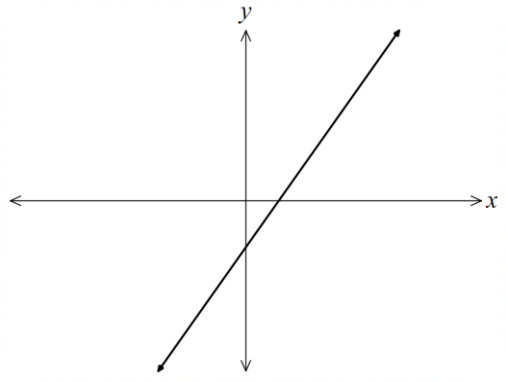
(A) $\frac{6}{\sqrt[3]{9x-1}}$

(B) $6\sqrt[3]{9x-1}$

(C) $\frac{2}{3\sqrt[3]{9x-1}}$

(D) $\frac{9\sqrt{9x-1}}{2}$

8 The diagram below shows the graph of a straight line $y = mx + b$.



Which of the following statements is true about m and b?

(A) $0 < m < b$

(B) $0 < b < m$

(C) $b < 0 < m$

(D) $m < 0 < b$

9 Which of the following is the correct expression for the indefinite integral of 5^{2x+3}?

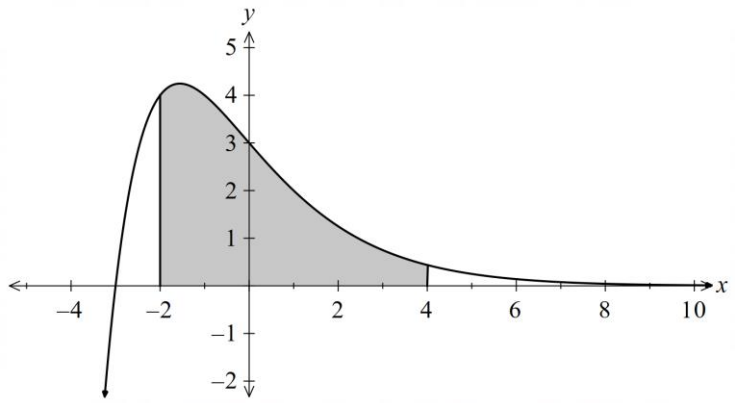
(A) $2 \times 5^{2x+3} + c$

(B) $2 \ln 5 \times 5^{2x+3} + c$

(C) $\dfrac{5^{2x+3}}{2} + c$

(D) $\dfrac{5^{2x+3}}{2 \ln 5} + c$

10 The area bounded by the graph of $y = f(x)$, the line $x = -2$, the line $x = 4$ and the x-axis, as shaded in the diagram below is A units^2.



Which of the following integrals below is correct for $f(2x)$?

(A) $\int_{-4}^{8} f(2x)dx = 2A$

(B) $\int_{-4}^{8} f(2x)dx = A$

(C) $\int_{-1}^{2} f(2x)dx = A$

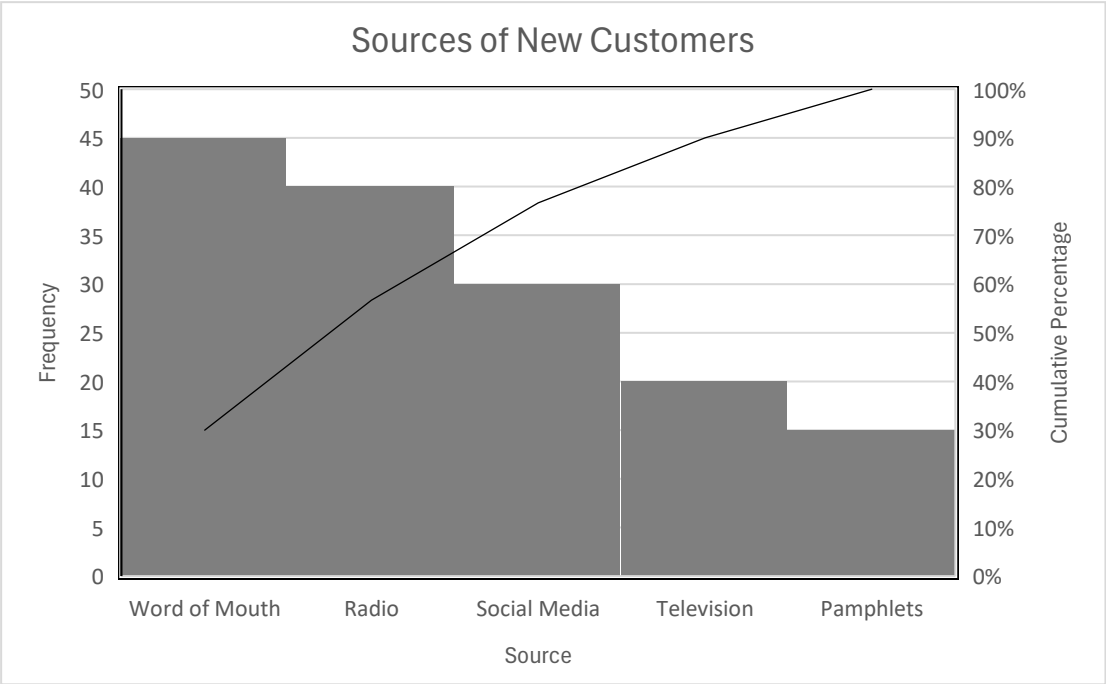
(D) $\int_{-1}^{2} f(2x)dx = \frac{A}{2}$

Section II

90 marks
Attempt Questions 11 - 33
Allow about 2 hours and 45 minutes for this section
Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Your responses should include relevant mathematical reasoning.

Question 11 (2 marks)

The graph below shows an incomplete Pareto Chart for the sources of new customers to a business.



a) How many new sources were acquired through social media? **1**

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b) What percentage of new sources were acquired through pamplets? **1**

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Question 12 (2 marks)

Use three applications of the trapezoidal rule to estimate the value of $\int_1^4 \sqrt{x-1}\, dx$. Give your answer correct to 4 significant figures. **2**

Question 13 (3 marks)

Eric is organising a gala dinner. The function centre will charge him \$2 000 to hire the room and he needs to pay a catering company \$80 per person for food.

a) Write an equation for the cost \$$C$ of hosting the event for n people. **1**

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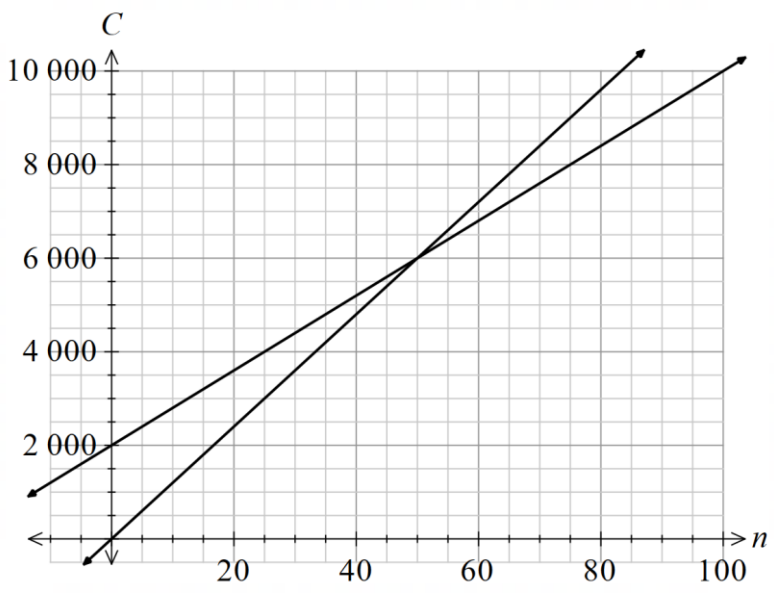
The graph below shows the planned income and costs if Eric charges \$120 per ticket.



b) Estimate the number of people required to break even. **1**

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c) What is the profit if 75 people attend the dinner? **1**

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Question 14 (4 marks)

A saucepan of hot water at a temperature of $T°$ Celsius loses heat when taken off the stove. It cools according to the equation $T = T_0 + Ae^{kt}$ where t is the time elapsed in minutes, T_0 is the room temperature measured in degrees Celsius and k is a constant.

a) A saucepan of hot water of temperature $95°$ is removed from the stove. It cools to a temperature of $90°$ five minutes later in a room with a temperature of $27°$. By finding the value of the constants T_0 and A, write an equation for T in terms of t. **3**

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b) How hot will the water be after 10 minutes? **1**

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Question 15 (3 marks)

Nathan's marks for a recent fitness test are shown below. The scores for each test are normally distributed.

	Nathan's Score	Population Mean	Population Standard Deviation
Muscular Strength	93	84	6
Agility	78	70	5

You may assume for the following questions that a higher number gives a better result.

a) In which test did Nathan perform best in relative to the population? Justify your answer with suitable calculations. **2**

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b) Students who score above 93 or above for muscular strength will be recommended for an elite training program. By using the *z*-tables provided or otherwise, find how many people you would expect to be recommended for the program if 200 people are tested? **1**

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Question 16 (4 marks)

Chloe manages a coffee shop which sells ice creams and coffee. The monthly sales for May to October are given below.

	May	June	July	August	September	October
Ice Cream Sales (S) ($'000)	6	3	2	4	8	14
Coffee Sales (C) ($'000)	55	60	62	58	52	45

a) Calculate Pearson's Correlation Coefficient for this set of data using ice cream sales as the independent variable. Give your answer correct to 3 decimal places. **1**

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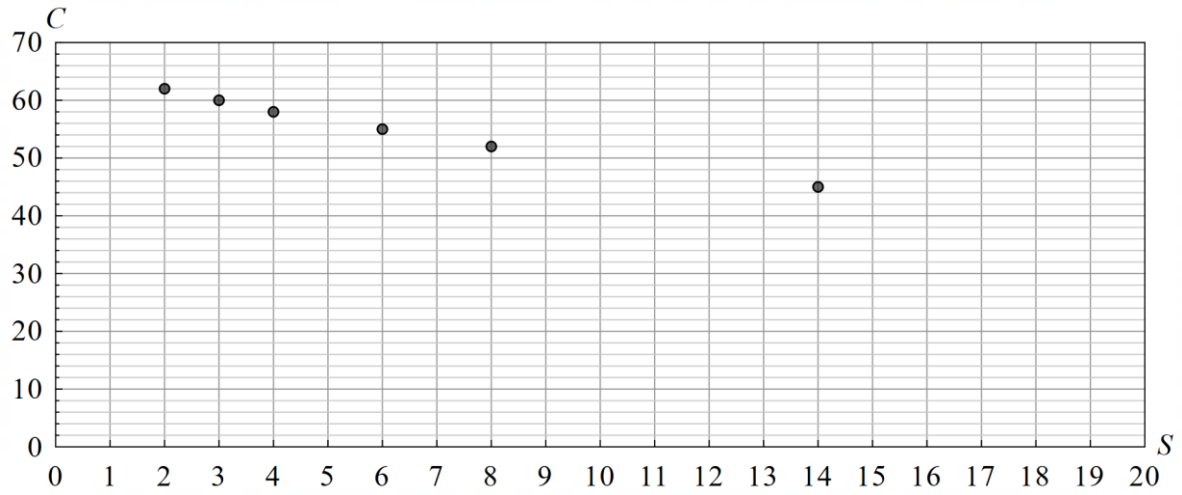
b) The diagram below shows a scatter plot of the data. **1**

The equation of the least squares regression line can be given by $C = 64 - 1.4S$.
Draw the least squares regression line on the scatter plot below.



c) By using the equation for the least squares regression line, estimate the value of coffee sales if Chloe sells $12 000 worth of ice cream. **1**

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d) Chloe is hoping that she can make $70 000 in sales from coffee next January. Explain the validity for Chloe to estimate her ice cream sales using this model. **1**

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Question 17 (2 marks)

Find the gradient of the normal to the curve $y = \frac{x}{\tan x}$ at the point where $x = \frac{\pi}{4}$. Give your answer correct to 3 decimal places. **2**

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Question 18 (8 marks)

a) Find the points of intersection of $y = 4x + 7$ and $y = x^3 - 3x^2 + 7$. **2**

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b) Sketch the function $y = x^3 - 3x^2 + 7$, by first finding all stationary points and determining their nature. Do NOT find the x-intercepts and points of inflection. **4**

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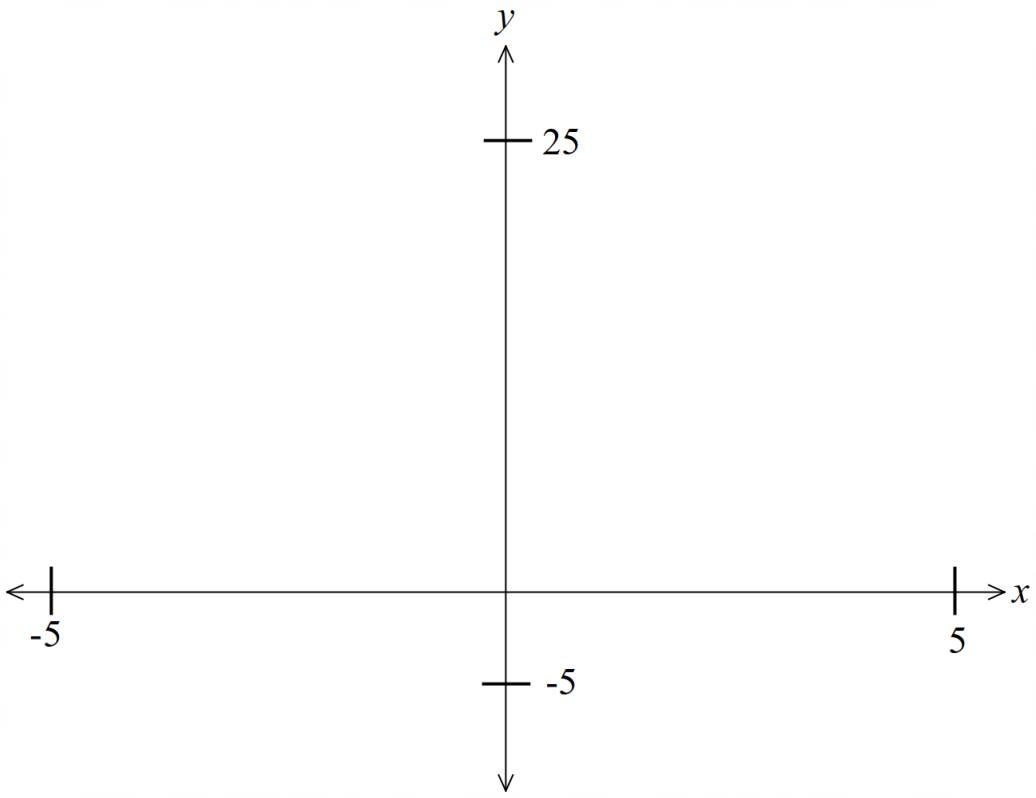

c) On the same number plane, graph the line $y = 4x + 7$. **1**

d) Hence or otherwise solve the inequality $4x + 7 < x^3 - 3x^2 + 7$. **1**

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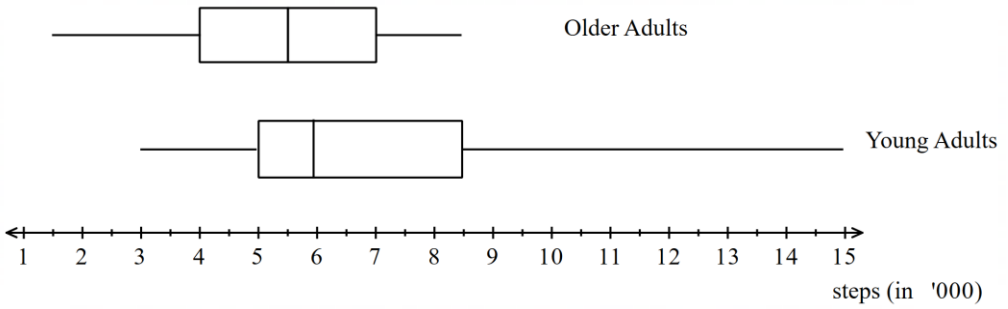
Question 19 (4 marks)

A study records the number of steps taken by young and older adults on a particular day. The results are shown in the box and whisker plot below.



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Compare and contrast the sets of data with reference to the context given.

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Question 20 (5 marks)

The acceleration of a particle is given by $a = 5e^{-2t}$. Initially the particle is at rest.

a) Write an equation for the velocity. **2**

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b) Explain why the particle is always speeding up for $t > 0$. **1**

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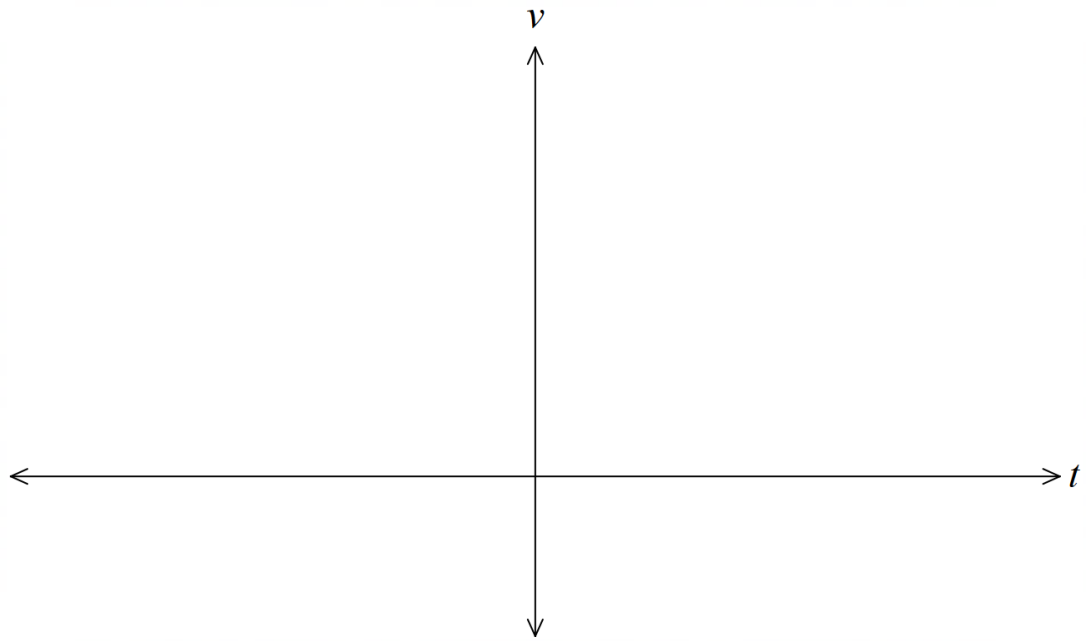
c) On the number plane below, sketch the graph of the particle's velocity as a function of time. **2**



Question 21 (4 marks)

The function $y = e^x \cos 2x$ has a stationary point at $x = 0.23$ and $x = 1.80$ (correct to 2 decimal places). Find the point of inflection of the function $y = e^x \cos 2x$ for $0.23 < x < 1.80$. **4**

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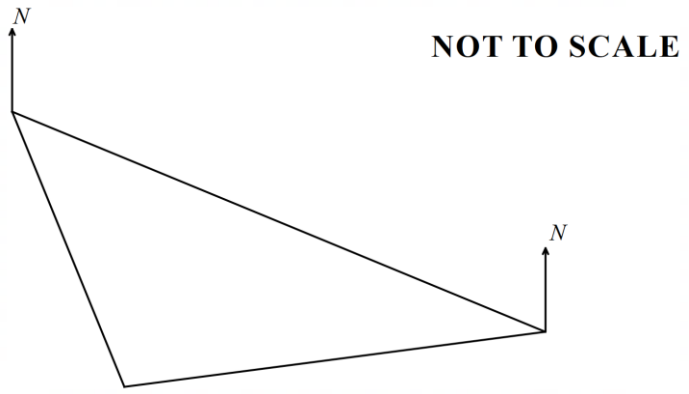
Question 22 (6 marks)

Joshua and Fahim leave base camp (C) together and walk on a bearing of 100°T for 10 km to a waterfall (W). Joshua stays at the waterfall and Fahim continues his journey by walking 6 km on a bearing of 258°T to a lookout (L).

a) Complete the diagram by showing all information. **1**



b) Find the distance between the lookout and base camp. Give your answer correct to 2 decimal places. **2**

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c) Hence, or otherwise find the bearing that Fahim must walk on, to return to the base camp. **3**

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Question 23 (3 marks)

Solve $\log_8 x + \log_4 x = 5$. **3**

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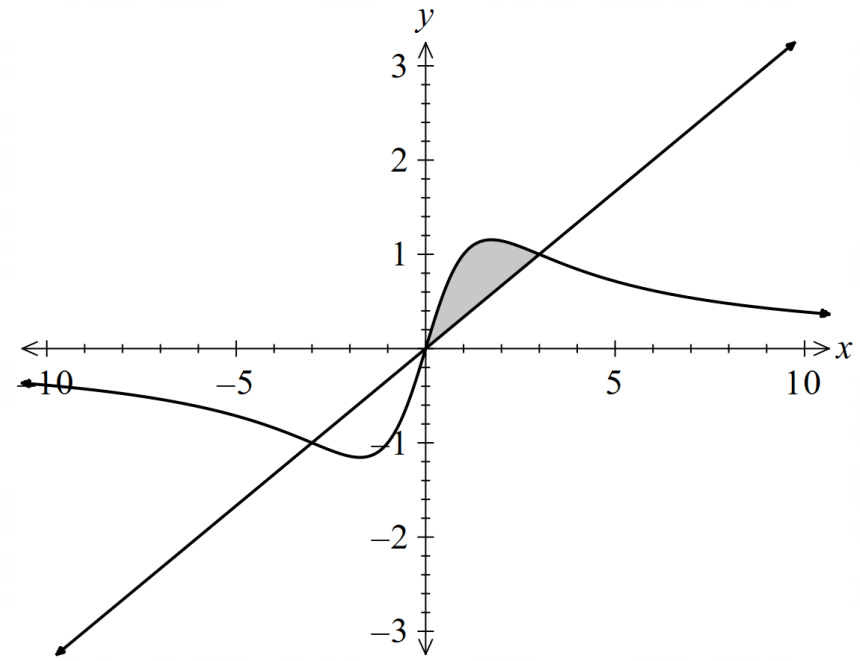
Question 24 (2 marks)

The curve $y = \dfrac{4x}{x^2+3}$ intersects the line $y = \dfrac{x}{3}$ at the points $(0,0)$, $(3,1)$ and $(-3,1)$. The region bounded by the curve $y = \dfrac{4x}{x^2+3}$ and the line $y = \dfrac{x}{3}$ between $x = 0$ and $x = 3$ is shaded in the diagram below.



Find the exact area of the shaded region. **2**

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Question 25 (6 marks)

The number of hours of daylight (H) in Sydney can be modelled by the trigonometric function $H = 2.3\sin\left(0.005(\pi t - 265)\right) + 13.1$ where t is the number of days after 1st January, 2025.

a) What is the maximum number of hours of daylight in Sydney in a year? **1**

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b) How quickly are the number of hours of daylight increasing when $t = 10$? **2**

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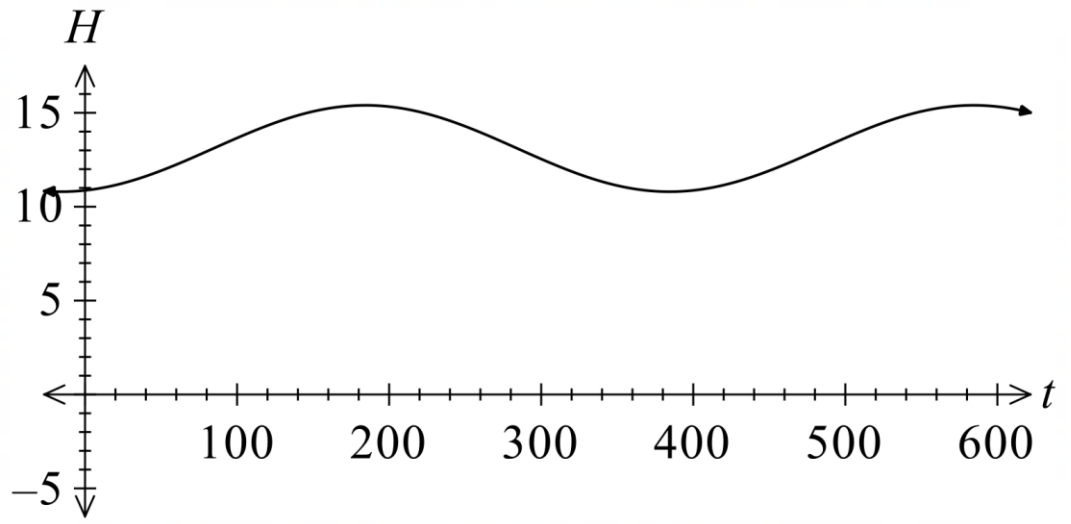
c) The graph of $H = 2.3\sin\left(0.005(\pi t - 265)\right) + 13.1$ is shown below. **3**



Catherine is organising a festival in 2025. She would like to have it on a day that has more than 12 hours of daylight. Between which 2 values of t should the festival be held? Give your answer to the nearest whole number.

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Question 26 (6 marks)

The continuous random variable X has probability density function given by

$$f(x)=\begin{cases}\dfrac{a}{x^2} & 2\le x\le 10\\ 0 & \text{for all other values of } x\end{cases}$$

a) Find the value of a such that $f(x)$ is a probability density function. **2**

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b) Find $P(X>8)$ **1**

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c) Find the median of X. **3**

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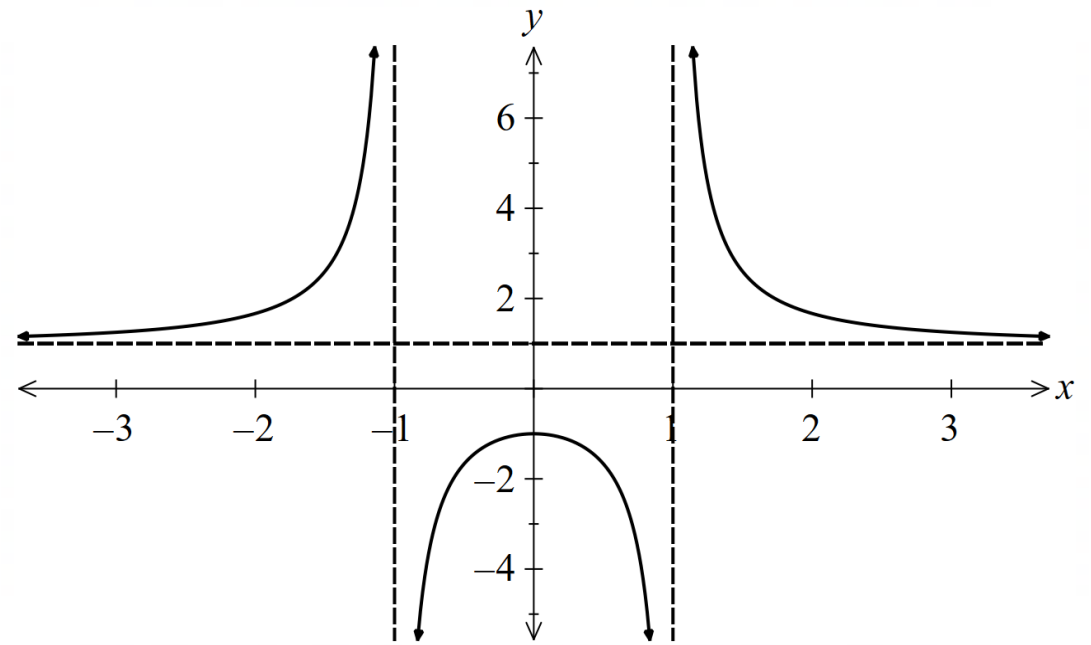
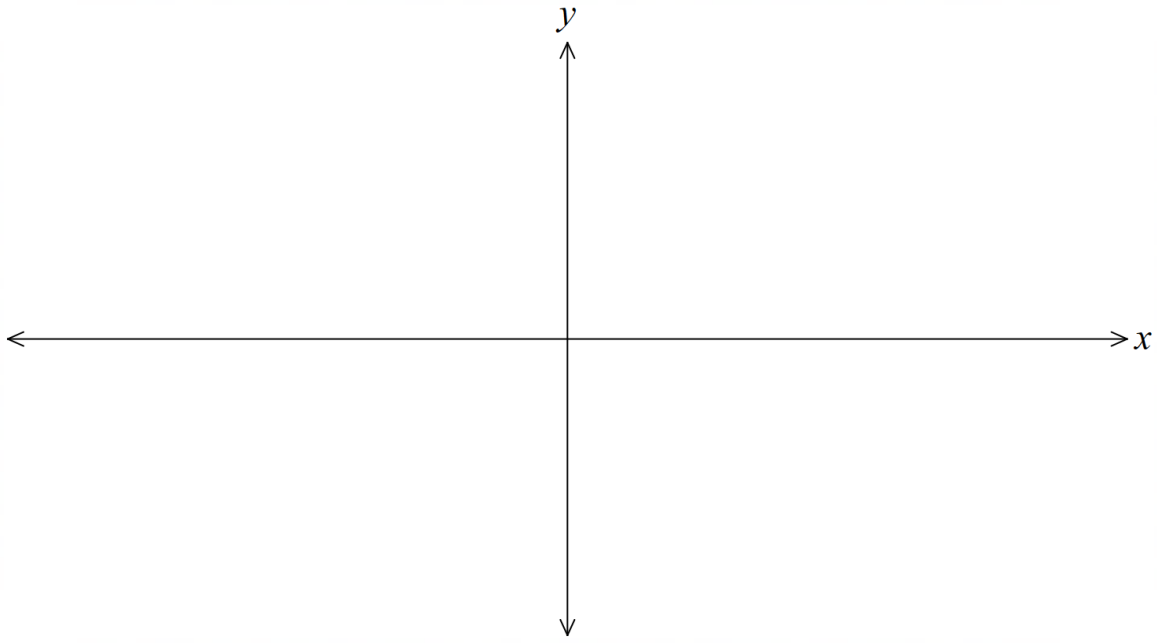
Question 27 (2 marks)

The number plane below shows the graph of $y = f(x)$. **2**

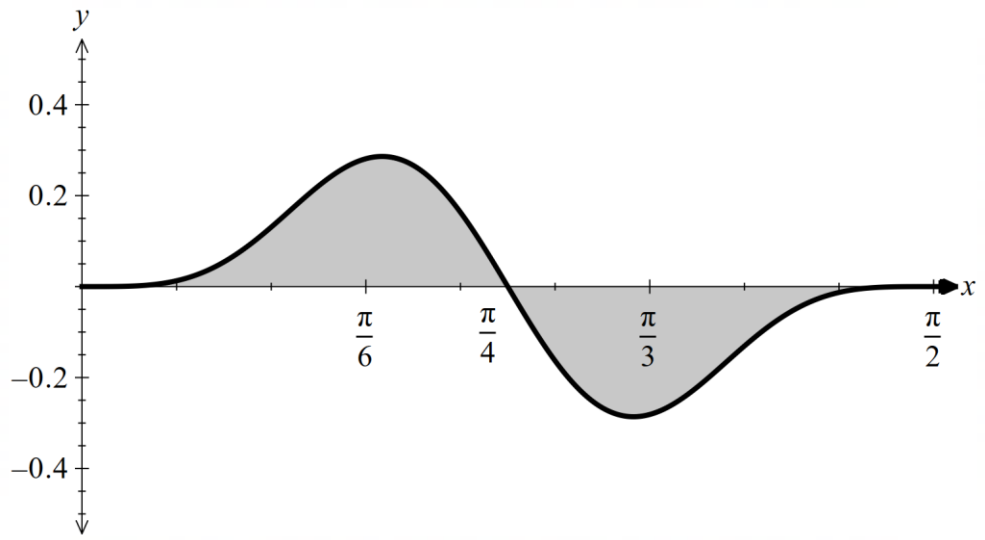


On the number plane below, sketch $y = 1 - 2f(x)$. Clearly label the y-intercept and asymptotes.



Question 28 (3 marks)

The diagram below shows the graph of $y = \cos 2x \sin^4 2x$.



Find the shaded area bounded by the curve and the x-axis between $x = \frac{\pi}{2}$ and $x = 0$. **3**

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Question 29 (3 marks)

Nila builds furniture to order. The amount of time that it takes between placing an order and it leaving the warehouse follows a normal distribution.

Nila knows that 2.5% of all her orders are made and leave her warehouse within 10 days of being placed. In total, last year, Nila received 35 orders and customers waited a combined total of 1610 days for their orders to leave the warehouse.

Nila decides that she will give a discount to at most 5% of her customers who have to wait a long time for their orders and would like to advertise this.

What is the minimum waiting time a customer must exceed in order to qualify for the discount? You may use the *z*-tables provided. **3**

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Question 30 (4 marks)

Sunya and Jonathan are playing a game. The spinner below is spun. If the hand lands on red then Sunya wins (and the game is over). If the hand lands on blue then Jonathan wins (and the game is over). If the hand lands on green then another spin is taken.

a) What is the probability that Jonathan wins on the first spin? **1**

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b) What is the probability that Jonathan wins on the third spin? **1**

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c) What is the probability that Jonathan eventually wins the game? **2**

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Question 31 (3 marks)

The common difference of an arithmetic sequence is -4. The sum of the first 30 terms is 23940 and the sum of the last 30 terms is -1980. Find how many terms are in the sequence. **3**

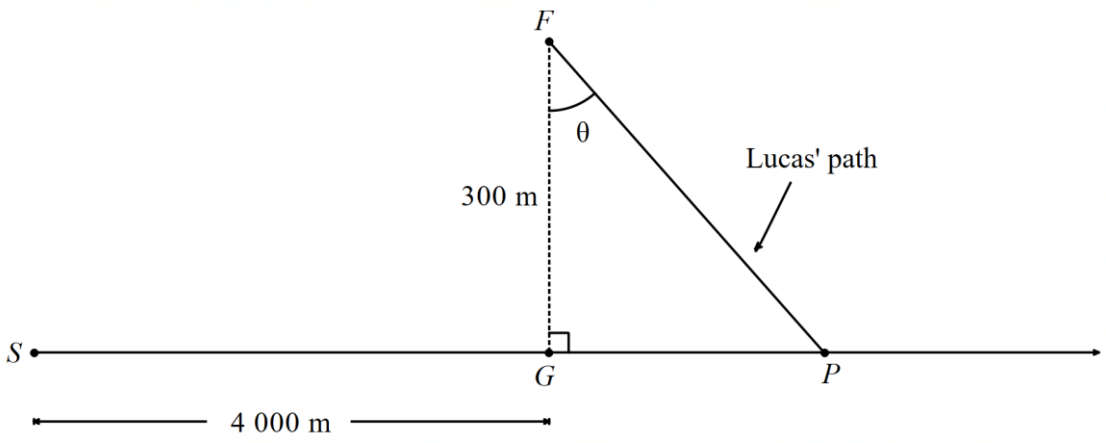
Question 32 (5 marks)

Ella borrows $820 000 to purchase a house. She is being charged 8.4% per annum and plans to making equal monthly repayments of $6248 over 30 years.

a) Show that the amount owing after 10 years is $724 960.57 **3**

b) After paying the home loan off for 10 years, Ella gets a pay rise. Immediately after making the 120th payment, Ella makes a one-off payment to reduce the balance owing to $720 000. She then increases her monthly repayments so that the loan is paid off 25 years after the initial loan. Calculate the amount of her new monthly payment. **2**

Question 33 (6 marks)



The diagram shows a farmhouse F that is located 300 metres from a straight section of road. The road begins at the shops S, which is situated 4 000 metres from the front gate G of the farmhouse. The school bus leaves the shops at 8 am and travels along the road at a speed of 20m/s. Lucas lives in the farmhouse and he can cycle across the open paddock between the house and the road at a speed of 8 m/s. The bus will stop for Lucas anywhere on the road but will not wait for him.

Assume that Lucas catches the bus at the point P on the road where $\angle GFP = \theta$.

a) Find two expressions in terms of θ, one expression for the time taken for the bus to travel from S to P and the other expression for the time taken by Lucas to cycle from F to P. **2**

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b) By considering the difference in the time taken for the bus and Lucas to travel to *P*, find the latest time that Lucas can leave home in order to catch the bus? Give your answer to the nearest minute. **4**

End of Exam

2025 Advanced Maths

Trial Examination

Marking Guidelines

Question Number	1	2	3	4	5	6	7	8	9	10
Answer	B	C	C	C	B	C	A	C	D	D

Question 1

(B)

The data has a positive moderate correlation.

$\therefore$ Pearson's correlation coefficient would be closest to 0.5

Question 2

(C)

25, 30, 35, ….

AP where $a = 25$, $n = 50$, $d = 5$

$$\begin{aligned} T_n &= a + (n-1)d \\ &= 25 + (50-1) \times 5 \\ &= 270 \end{aligned}$$

Question 3

(C)

Order of transformations: for $y = f(x)$ to $y = kf(a(x+b))+c$

- Horizontal dilation by a factor of $\frac{1}{a}$
- Translate b units left
- Dilate vertically by a factor of k
- Translate up c units

$$y = 3\sin(2x-1)$$
$$= 3\sin\left(2\left(x-\frac{1}{2}\right)\right)$$

$\therefore$ Horizontal dilation by a factor of $\frac{1}{2}$

Horizontal translation of $\frac{1}{2}$

Vertical dilation by a factor of 3

Question 4

(C)

One standard deviation above/below mean = 25 ± 4

$$= 21 \text{ and } 29$$

ie 68% of cattle dogs will weigh between 21kg and 29kg (Empirical Rule)

Two standard deviations above/below mean = $25 \pm 2 \times 4$

$$= 17 \text{ and } 33$$

ie 95% of cattle dogs will weigh between 17kg and 33kg

$\therefore \frac{95\% - 68\%}{2} = 13.5\%$ of cattle dogs will weigh between 29kg and 33kg.

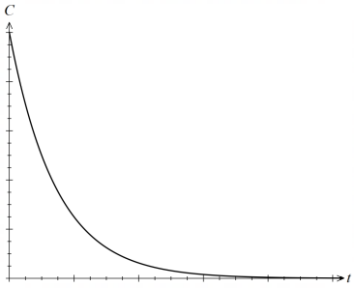
Question 5

(B)

Since the concentration is decreasing over time then $\frac{dy}{dx} < 0$.

Since the rate of decrease is slowing down then the curve is concave up and $\frac{d^2y}{dx^2} > 0$

Possible graph of time *vs* concentration



Question 6

(C)

Test each option:

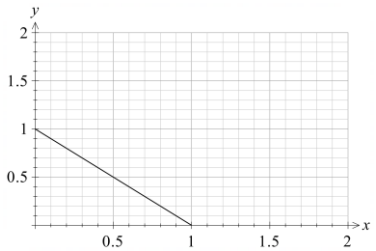
(A) Since the range of $f(x)$ is all real y, $f(x)$ could have negative values and cannot be a probability distribution function

(B) Area between the probability density function and the axes = $\frac{1}{2} \times 1 \times 1$

$$= \frac{1}{2}$$

$\therefore$ Sum of all probabilities $\neq$ 1

$\therefore$ not a probability density function



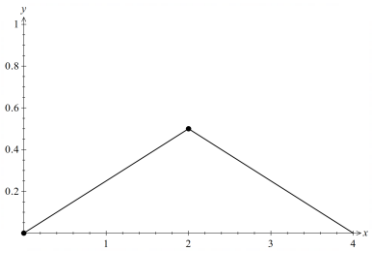
(C) Area between the probability density function and the axes = $\frac{1}{2} \times 4 \times \frac{1}{2}$

$= 1$

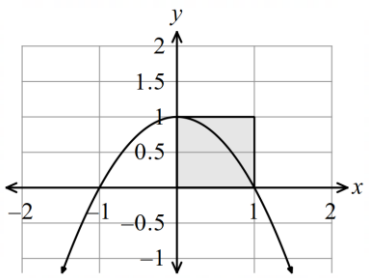
$\therefore$ Sum of all probabilities = 1

$f(x) \geq 0$ for all x

$\therefore$ is a probability density function



(D)



Since the area of the shaded rectangle is 1, $\int_0^1 1 - x^2 \, dx < 1$

$\therefore$ not a probability density function.

Question 7

(A)

$$y = \sqrt[3]{(9x-1)^2}$$

$$= (9x-1)^{\frac{2}{3}}$$

$$\frac{dy}{dx} = \frac{2}{3}(9x-1)^{\frac{-1}{3}} \times 9$$

$$= 6(9x-1)^{\frac{-1}{3}}$$

$$= \frac{6}{\sqrt[3]{9x-1}}$$

Question 8

(C)

Since the line has positive gradient ($m > 0$) and negative y-intercept ($b < 0$), then (C) is the correct statement.

Question 9

(D)

From reference sheet: $\int f'(x)a^{f(x)}\,dx = \dfrac{a^{f(x)}}{\ln a} + c$

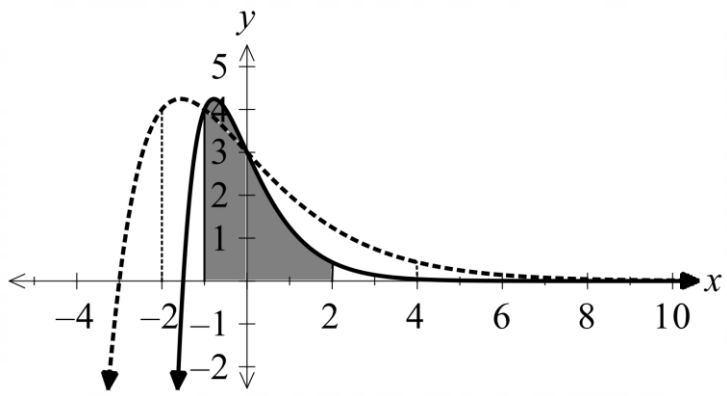
For 5^{2x+3}, $a = 5, f(x) = 2x+3, f'(x) = 2$

$$\int 5^{2x+3}\,dx = \frac{5^{2x+3}}{2\ln 5} + c$$

Question 10

(D)

$f(2x)$ is a horizontal dilation of $f(x)$ by a factor of $\dfrac{1}{2}$.



$\therefore$ The shaded area will be half of the original area

$$\therefore \int_{-1}^{2} f(2x)dx = \frac{A}{2}$$

Section II

Question 11(a)

Criteria	Marks
• Correct solution	1

30

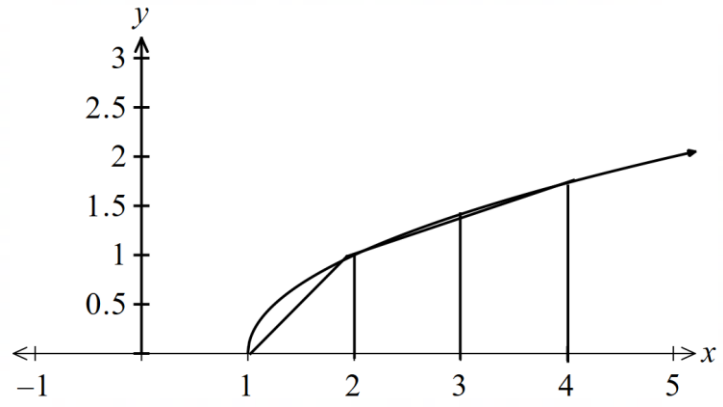
Question 11(b)

Criteria	Marks
• Correct solution	1

$100\% - 90\% = 10\%$

Question 12

Criteria	Marks
• Correct solution	2
• Uses 3 function values to correctly estimate the area • Attempts to use the trapezoidal rule with 3 applications with up to 2 incorrect function values or equivalent merit	1



x	1	2	3	4
$f(x)$	0	1	$\sqrt{2}$	$\sqrt{3}$

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\{f(a)+f(b)+2[f(x_1)+f(x_2)]\}$$

$$\int_1^4 \sqrt{x-1}dx \approx \frac{4-1}{2\times 3}\{0+\sqrt{3}+2(1+\sqrt{2})\}$$

$$\approx 3.280238966..$$

$$\approx 3.280\,(4 \textit{ significant figures})$$

Markers Note: A significant number of students integrated the function instead of using the Trapezoidal Rule

Question 13(a)

Criteria	Marks
• Correct solution (in terms of n and C)	1

$C = 80n + 2000$

Question 13(b)

Criteria	Marks
• Correct solution	1

The graphs intersect when $n = 50$ (and $C = \$6000$)

$\therefore 50$ people required to break even

Question 13(c)

Criteria	Marks
• Correct solution	1

From the graph, when 75 people attend, \$9 000 is received in revenue, \$8 000 is spent in costs.

$\therefore 9000 - 8000 = \$1000$ profit when 75 people atten**d**

Question 14(a)

Criteria	Marks
• Correct solution	3
• Correctly finds A and k	2
• Writes an exponential equation in the form $T = 27 + 68e^{5k}$ or writes an equation to find k in terms of A.	1

$$T = T_0 + Ae^{kt}$$

when $t = 0, T = 95, T_0 = 27$

$$95 = 27 + Ae^{k(0)}$$

$$A = 68$$

when $t = 5, T = 90$

$$90 = 27 + 68e^{5k}$$

$$63 = 68e^{5k}$$

$$e^{5k} = \frac{63}{68}$$

$$5k = \ln\frac{63}{68}$$

$$k = \frac{1}{5}\ln\frac{63}{68}$$

$$= -0.01527459576$$

$$= -0.0153 \text{ (3 significant figures)}$$

$$T = 27 + 68e^{-0.153t} \text{ (or equivalent)}$$

Question 14(b)

Criteria	Marks
• Correct solution	1

Sample answer:

when $t = 10$,

$$T = 27 + 68e^{-10k} \text{ where } k = -0.01527459576$$

$$= 85.36764706$$

$$= 85°$$

Question 15(a)

Criteria	Marks
• Correct solution	2
• Correctly finds one z-score	1

Sample answer:

$$z = \frac{x - \mu}{\sigma}$$

For Muscular Strength:

$$z = \frac{93 - 84}{6}$$
$$= 1.5$$

For Agility:

$$z = \frac{78 - 70}{5}$$
$$= 1.6$$

Since Agility has a higher z-score, Nathan is better in agility than Muscular Strength.

Question 15(b)

Criteria	Marks
• Correct solution	1

Sample answer:

From z-table 0.9332 of all people will have a z-score below 1.5.

$(1 - 0.9332) \times 200 = 13.36$

$\therefore$ Approximately 13 people will be recommended for the program

Question 16(a)

Criteria	Marks
• Correct solution	1

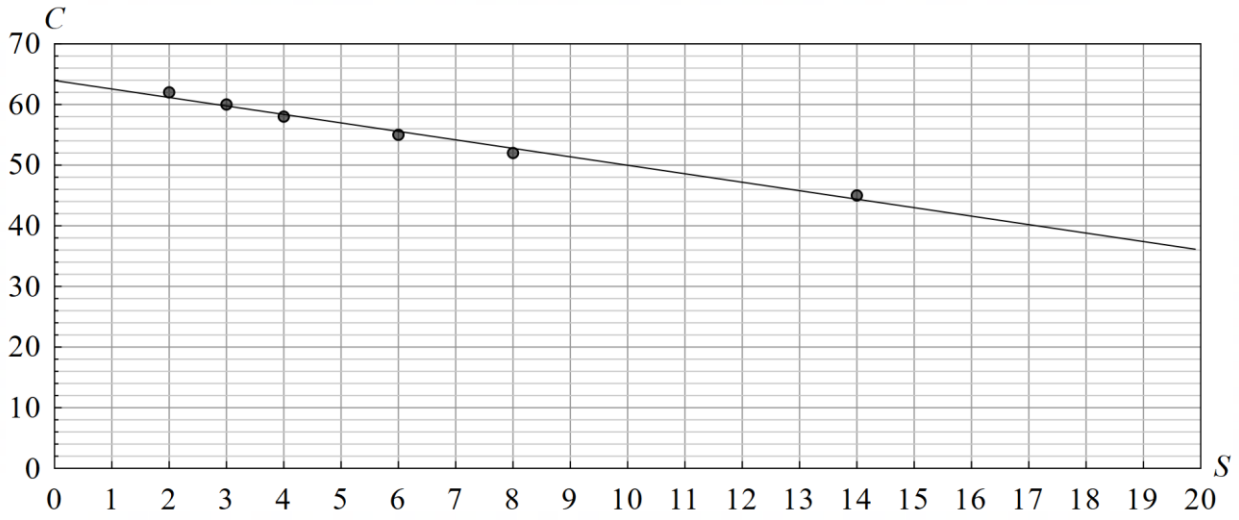
Sample answer:

$-0.9942532494 \approx -0.994 \ (3dp)$

Question 16(b)

Criteria	Marks
• Correct solution	1

Sample answer:



Question 16(c)

Criteria	Marks
• Correct solution	1

Sample answer:

$$C = 64 - 1.4S$$
$$= 64 - 1.4 \times 12$$
$$= 47.2$$

$\therefore$ It is predicted that Chloe will make \$47 200 in coffee sales

Question 16(d)

Criteria	Marks
• Correct solution	1

Sample answer:

As Chloe is extrapolating the prediction for coffee sales would not be as reliable ie she has not collected data for coffee sales over \$62 000 prior. We do not know what the pattern is beyond \$62 000 in coffee sales eg sales could plummet beyond the linear pattern, or they could increase due to the sale of iced coffees as the weather gets warmer and more people are out and about.

OR

when $C = 70\,000$,

$$70 = 64 - 1.4x$$
$$6 = -1.4x$$
$$x = -\frac{30}{7}$$

Since $x > 0$, it would not be appropriate to use the model.

Question 17

Criteria	Marks
• Correct solution	2
• Correctly applies the quotient rule or equivalent merit	1

Sample answer:

$$\text{if } y = \frac{u}{v} \text{ then } \frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2} \text{ (from reference sheet)}$$

$$u = x \qquad v = \tan x$$

$$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \sec^2 x$$

$$\frac{dy}{dx} = \frac{\tan x - x\sec^2 x}{\tan^2 x}$$

$$\text{when } x = \frac{\pi}{4}, \frac{dy}{dx} = m_T = \frac{\tan\dfrac{\pi}{4} - \dfrac{\pi}{4}\sec^2\dfrac{\pi}{4}}{\tan^2\left(\dfrac{\pi}{4}\right)}$$

$$= \frac{1 - \dfrac{\pi}{4} \times 2}{1}$$

$$= 1 - \frac{\pi}{2}$$

$$m_N = \frac{-1}{1 - \dfrac{\pi}{2}}$$

$$= \frac{2}{\pi - 2}$$

$$= 1.751938394$$

$$= 1.752$$

Question 18(a)

Criteria	Marks
• Correct solution	2
• Correctly finds all x values or, equivalent merit	1

Sample answer:

$$x^3 - 3x^2 + 7 = 4x + 7$$
$$x^3 - 3x^2 - 4x = 0$$
$$x(x^2 - 3x - 4) = 0$$
$$x(x-4)(x+1) = 0$$
$$x = 0, -1 \text{ or } 4$$

when $x = 0, y = 7$

when $x = -1, y = (-1)^3 - 3(-1)^2 + 7$
$= 3$

when $x = 4, y = (4)^3 - 3(4)^2 + 7$
$= 23$

Question 18(b)

Criteria	Marks
• Correct solution	4
• Finds the nature of the stationary points	3
• Finds the x values of both stationary points, or equivalent merit	2
• Finds the first derivative, or equivalent merit	1

Sample answer:

$$y = x^3 - 3x^2 + 7$$

Cuts the y-axis when $x = 0$

$\therefore$ y-intercept = 7

stationary points occur when $y' = 0$

$$y' = 3x^2 - 6x$$

$$3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$x = 0 \text{ or } x = 2$$

when $x = 2$,

$$y = 2^3 - 3 \times 2^2 + 7$$

$$= 3$$

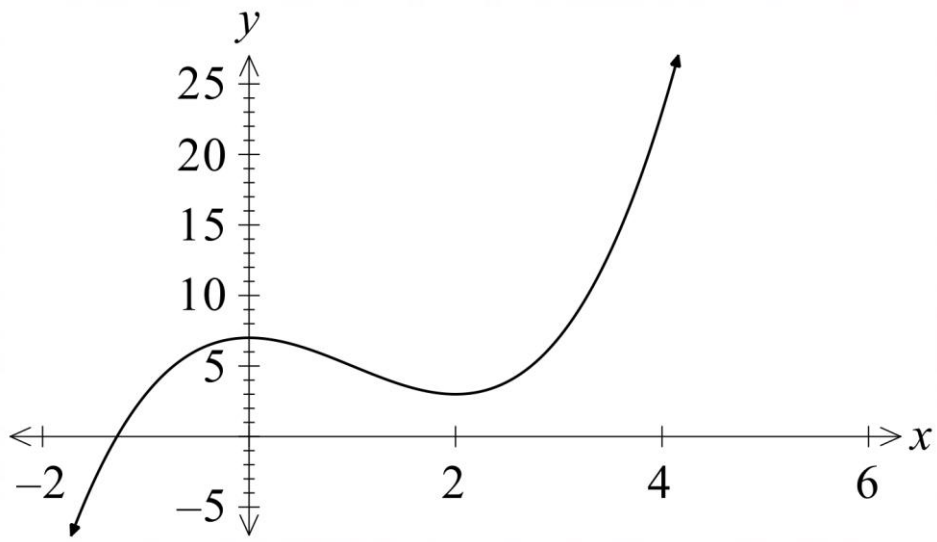
$$y'' = 6x - 6$$

when $x = 0, y'' = -6$

when $x = 2, y'' = 6$

$\therefore (0, 7)$ is a maximum turning point

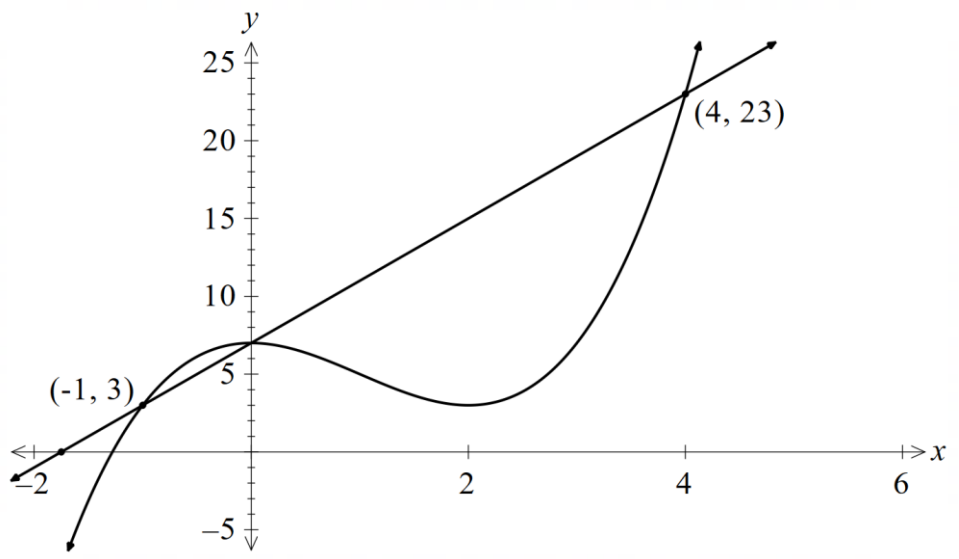
(2, 3) is a minimum turning point



Question 18(c)

Criteria	Marks
• Correct solution with 2 defining points	1

Sample answer:



Question 18(d)

Criteria	Marks
• Correct solution	1

Sample answer:

$-1 < x < 0$ or $x > 4$

Question 19

Criteria	**Marks**
• Thoroughly compares both groups of data including quoting all statistics.	4
• Compares all statistical measures and makes one further comparison of the two groups OR • States all statistical measures and makes one further comparison of the two groups or equivalent.	3
• States all statistical measures or compares all statistical measures	2
• States four statistical measures or compares 2 pairs of statistical measures	1
Note: All statistical measures refer to the following: *1. One of the range or highest and lowest score* *2. One of the interquartile range or lower and upper quartile* 3. *Median*	

Sample answer:

Statistical Measure	Older Adults	Younger Adults
Median	5 500	6 000
Interquartile Range	3 000	3 500
Range	7 000	12 000
Lowest Score	1 500	3 000
Q_1	4 000	5 000
Q_3	7 000	8 500
Highest Score	8 500	15 000

From the table above it can be seen that younger adults tend to take more steps (higher median) than older adults.

The data for the older adults is approximately symmetrical but the data for younger adults is positively skewed.

The scores for younger adults are less consistent than the older adults shown by the higher interquartile range and range.

25% of young adults take more steps than all of the older adults.

All statistical measures for older adults are lower than the same for younger adults.

Question 20(a)

Criteria	Marks
• Correct solution	2
• Correctly integrates $5e^{-2t}$	1

Sample answer:

$$a = 5e^{-2t}$$

$$v = -\frac{5}{2}e^{-2t} + c$$

when $t = 0, v = 0$

$$0 = \frac{-5}{2}e^{-2(0)} + c$$

$$c = \frac{5}{2}$$

$$v = -\frac{5}{2}e^{-2t} + \frac{5}{2}$$

Question 20(b)

Criteria	Marks
• Correct solution	1

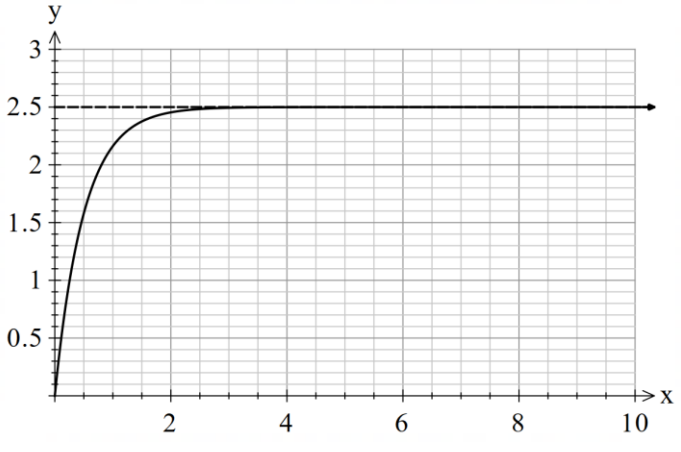
Sample answer:

Since $e^{-2t} > 0$ for all t, then both the velocity and acceleration will be positive for all t. Hence the particle will be speeding up.

Question 20(c)

Criteria	Marks
• Correct solution	2
• Correct shape or Correct solution from previous error where constant was ignored	1

Sample answer:



Question 21

Criteria	Marks
• Correct solution	4
• Correctly finds the reference angle	3
• Correctly finds second derivative and equates to zero	2
• Correctly finds first derivative	1

Sample answer:

$$y = e^x \cos 2x$$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$u = e^x \qquad v = \cos 2x$$

$$\frac{du}{dx} = e^x \qquad \frac{dv}{dx} = -2\sin 2x$$

$$\frac{dy}{dx} = -2e^x \sin 2x + e^x \cos 2x$$

$$u = -2e^x \qquad v = \sin 2x \qquad \textit{and} \qquad u = e^x \qquad v = \cos 2x$$

$$\frac{du}{dx} = -2e^x \qquad \frac{dv}{dx} = 2\cos 2x \qquad \frac{du}{dx} = e^x \qquad \frac{dv}{dx} = -2\sin 2x$$

$$\frac{d^2y}{dx^2} = -4e^x \cos 2x - 2e^x \sin 2x - 2e^x \sin 2x + e^x \cos 2x$$

$$= -3e^x \cos 2x - 4e^x \sin 2x$$

Possible points of inflection occur when $\frac{d^2y}{dx^2} = 0$

$$-3e^x \cos 2x - 4e^x \sin 2x = 0$$

$$-e^x(3\cos 2x + 4\sin 2x) = 0$$

$$3\cos 2x + 4\sin 2x = 0 \text{ since } e^x > 0$$

$$3\cos 2x = -4\sin 2x$$

$$\frac{\sin 2x}{\cos 2x} = \frac{-3}{4}$$

$$\tan 2x = \frac{-3}{4}$$

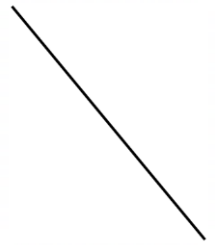
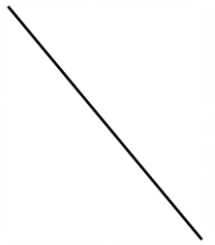
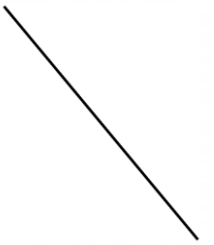
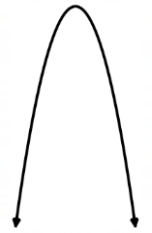
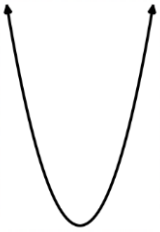
$$\text{reference angle} = 0.6435011088$$

$$2x = \pi - 0.6435011088$$

$$= 2.498091545$$

$$x = 1.249045772$$

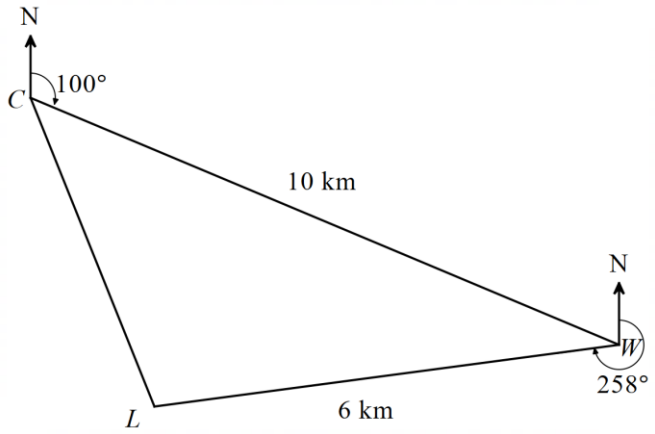
$$= 1.25 \text{ (2 decimal places)}$$

x	1	1.25	1.7
$\frac{dy}{dx}$	-6.07…	-6.97	-2.49…
			
$\frac{d^2y}{dx^2}$	-6.49…	0	21.47….
			

Question 22(a)

Criteria	**Marks**
• Correct solution	1

Sample answer:



Question 22(b)

Criteria	Marks
• Correct solution	2
• Correctly finds CL^2	1

Sample answer:

$$\angle NWL + 258 = 360°$$
$$\angle NWL = 102°$$
$$\angle CWL = 102 - 80$$
$$= 22°$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$CL^2 = 10^2 + 6^2 - 2 \times 10 \times 6\cos 22°$$
$$CL^2 = 24.73793745$$
$$CL = 4.973724706$$
$$= 4.97 \text{ (2 decimal places)}$$

Question 22(c)

Criteria	Marks
• Correct solution	3
• Finds $\angle WLC$ using the cosine rule or the sine rule.	2
• Finds an expression for cos $\angle WLC$ or sin $\angle WLC$	1

Sample answer:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$\cos C = \frac{4.97...^2 + 6^2 - 10^2}{2 \times 4.97... \times 6}$$
$$= -0.6578246162$$
$$C = 131.1341759$$
$$= 131°$$

$\angle NWL = 102°$ (angles at a point)
$\angle WLN = 78°$ (cointerior angles)
$$\angle NWL = 131 - 78$$
$$= 53°$$

Bearing is $360 - 53 = 307°T$

Question 23

Criteria	Marks
• Correct solution	3
• Makes $\log_a x$ the subject	2
• Uses change of base law	1

Sample answer:

$$\log_8 x + \log_4 x = 5$$

$$\frac{\log_2 x}{\log_2 8} + \frac{\log_2 x}{\log_2 4} = 5$$

$$\frac{\log_2 x}{3} + \frac{\log_2 x}{2} = 5$$

$$2\log_2 x + 3\log_2 x = 30$$

$$5\log_2 x = 30$$

$$\log_2 x = 6$$

$$x = 2^6$$

$$= 64$$

Question 24

Criteria	Marks
• Correct solution	2
• Correctly integrates $\frac{4x}{x^2+3}-\frac{x}{3}$	1

Sample answer:

$$
\begin{aligned}
A &= \int_0^3 \frac{4x}{x^2+3}-\frac{x}{3}\,dx \\
&= 2\int_0^3 \frac{2x}{x^2+3}\,dx-\int_0^3 \frac{x}{3}\,dx \\
&= \left[2\ln(x^2+3)-\frac{x^2}{6}\right]_0^3 \\
&= \left(2\ln(3^2+3)-\frac{3^2}{6}\right)-\left(2\ln(0^2+3)-\frac{0^2}{6}\right) \\
&= 2\ln 12-\frac{9}{6}-2\ln 3 \\
&= 2(\ln 12-\ln 3)-\frac{3}{2} \\
&= 2\ln 4-\frac{3}{2}
\end{aligned}
$$

Question 25(a)

Criteria	Marks
• Correct solution	1

Sample answer:

Maximum value of $H=2.3\sin\left(0.005(\pi t-265)\right)+13.1$ will occur when $\sin\left(0.005(\pi t-265)\right)=1$

$$
\begin{aligned}
\therefore \text{Maximum number of daylight hours} &= 2.3\times 1+13.1 \\
&= 15.4
\end{aligned}
$$

Question 25(b)

Criteria	Marks
• Correct solution	2
• Correctly finds the derivative of $H = 2.3\sin\left(0.005(\pi t - 265)\right) + 13.1$	1

Sample answer:

$$H = 2.3\sin(0.005(\pi t - 265)) + 13.1$$

$$\frac{dH}{dt} = 0.0115\pi\cos(0.005(\pi t - 265))$$

$$= 0.0115\pi\cos(0.005(\pi \times 10 - 265))$$

$$= 0.01416467195$$

$$= 0.0142 \text{ hours/day}$$

Question 25(c)

Criteria	Marks
• Correct solution	3
• Correctly finds two values of t	2
• Correctly find the reference angle	1

Sample answer:

$$2.3\sin(0.005(\pi t - 265)) + 13.1 = 12$$

$$2.3\sin(0.005(\pi t - 265)) = -1.1$$

$$\sin(0.005(\pi t - 265)) = \frac{-1.1}{2.3}$$

$$reference\,angle = 0.4986733471$$

$$0.005(\pi t - 265) = \pi + 0.498... \quad or \quad 0.005(\pi t - 265) = -0.498...$$

$$\pi t - 265 = 728.0532001 \quad or \quad \pi t - 265 = -99.73466941$$

$$\pi t = 993.0532001 \quad or \quad \pi t = 165.2653306$$

$$t = 316.0986511 \quad or \quad t = 52.60558857$$

$$ie \quad 53 \le t \le 316$$

Question 26(a)

Criteria	Marks
• Correct solution	2
• Correctly integrates $\frac{a}{x^2}$ with correct limits and equates to 1	1

Sample answer:

$$\int_2^{10} \frac{a}{x^2}\,dx = 1$$

$$\int_2^{10} ax^{-2}\,dx = 1$$

$$\left[-ax^{-1}\right]_2^{10} = 1$$

$$\left[\frac{-a}{x}\right]_2^{10} = 1$$

$$\frac{-a}{10} - \left(\frac{-a}{2}\right) = 1$$

$$-a + 5a = 10$$

$$4a = 10$$

$$a = \frac{5}{2}$$

Question 26(b)

Criteria	Marks
• Correct solution	1

Sample answer:

$$P(X > 8) = \int_8^{10} \frac{5}{2x^2}\,dx$$

$$= \left[\frac{-5}{2x}\right]_8^{10}$$

$$= \left(\frac{-5}{2\times 10}\right) - \left(\frac{-5}{2\times 8}\right)$$

$$= \frac{1}{16}$$

Question 26(c)

Criteria	Marks
• Correct solution	3
• Correctly solves for x	2
• Forms an expression for $F(x)-F(2)$	1

Sample answer:

$$\int_2^x \frac{5}{2x^2}\,dx = 0.5$$

$$\left[-\frac{5}{2x}\right]_2^x = 0.5$$

$$\left(-\frac{5}{2x}\right)-\left(-\frac{5}{2\times 2}\right) = 0.5$$

$$\frac{-5}{2x}+\frac{5}{4} = 0.5$$

$$-10+5x = 2x$$

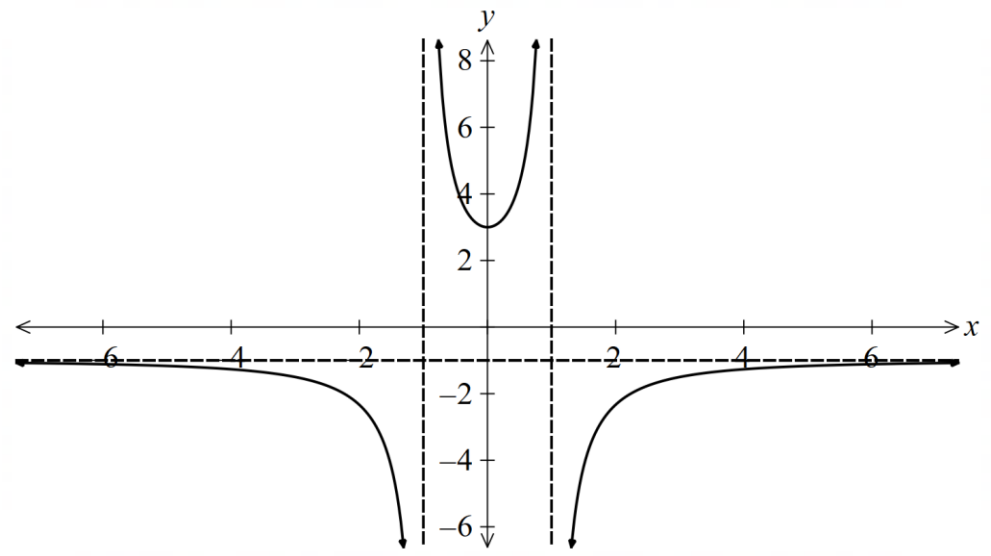
$$3x = 10$$

$$x = \frac{10}{3}$$

Question 27

Criteria	Marks
• Correct solution	2
• Correctly applies one transformation with labelled intercepts or asymptotes OR • Correctly applies three transformations without intercepts nor asymptotes	1

Sample answer:



Question 28

Criteria	Marks
• Correct solution	3
• Correctly integrates $\cos 2x \sin^2 2x$	2
• Attempts to find the area by finding two integrals	1

Sample answer:

$$A = \int_0^{\frac{\pi}{4}} \cos 2x \sin^4 2x \, dx + \left| \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \sin^4 2x \, dx \right|$$

$f(x) = \sin 2x$

$f'(x) = 2\cos 2x$

$n = 4$

$$\int f'(x)[f(x)]^n \, dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

$$A = \int_0^{\frac{\pi}{4}} \cos 2x \sin^4 2x \, dx + \left| \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \sin^4 2x \, dx \right|$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{4}} 2\cos 2x \sin^4 2x \, dx + \left| \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\cos 2x \sin^4 2x \, dx \right|$$

$$= \left[\frac{1}{10} \sin^5 2x \right]_0^{\frac{\pi}{4}} + \left| \left[\frac{1}{10} \sin^5 2x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \right|$$

$$= \frac{1}{10} \sin^5 \frac{2 \times \pi}{4} - \frac{1}{10} \sin^5 0 + \left| \frac{1}{10} \sin^5 \frac{2 \times \pi}{2} - \frac{1}{10} \sin^5 \frac{2 \times \pi}{4} \right|$$

$$= \frac{1}{10} + \frac{1}{10}$$

$$= \frac{1}{5}$$

Question 29

Criteria	Marks
• Correct solution	3
• Finds the mean and standard deviation	2
• Finds the mean	1

Sample answer:

$$\text{Mean} = \frac{1610}{35}$$
$$= 46$$

Since 2.5% of orders are despatched within 10 days, 10 has a z-score of -2

$$z = \frac{\bar{x} - \mu}{\sigma}$$
$$-2 = \frac{10 - 46}{\sigma}$$
$$-2\sigma = -36$$
$$\sigma = 18$$

Since the discount will be given to the 5% of customers who need to wait the longest, we need the score which correlates to a z-score of 1.65

$$1.65 = \frac{x - 46}{18}$$
$$29.7 = x - 46$$
$$x = 75.7$$

Customers who wait more than 76 days will receive a discount.

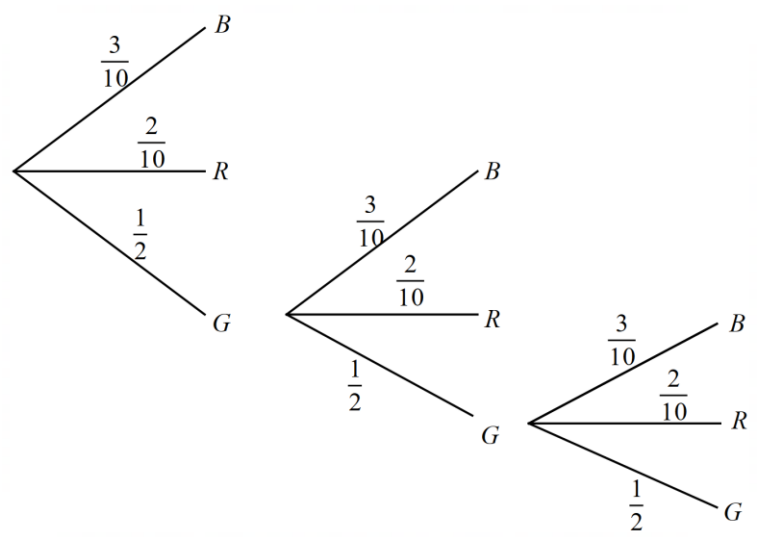
Question 30(a)

Criteria	Marks
• Correct solution	1

Sample answer: $P(B) = \frac{3}{10}$

Question 30(b)

Criteria	Marks
• Correct solution	1



Jonathan will win on the third spin if the first two spins are green (roll again)

$$P(Jonathan\ wins) = \frac{1}{2} \times \frac{1}{2} \times \frac{3}{10}$$

$$= \frac{3}{40}$$

Question 30(c)

Criteria	Marks
• Correct solution	**2**
• Correctly attempts to use the limiting sum	**1**

Sample Answer:

$$P(\text{Jonathan wins}) = \frac{3}{10} + \frac{1}{2} \times \frac{3}{10} + \frac{1}{2} \times \frac{1}{2} \times \frac{3}{10} + \ldots$$

We are required to find the limiting sum of a geometric series where $a = \frac{3}{10}$ and $r = \frac{1}{2}$.

$$\begin{aligned} S_\infty &= \frac{a}{1-r} \\ &= \frac{\frac{3}{10}}{1-\frac{1}{2}} \\ &= \frac{3}{5} \end{aligned}$$

Markers Note: a mark was deducted if students did not show working out

Question 31

Criteria	Marks
• Correct solution	**3**
• Writes a correct simplified expression (algebraic or numeric) for the sum of the last 30 terms.	**2**
• Finds the correct value for *a,* or equivalent merit	**1**

Sample Answer:

$$S_{30} = \frac{30}{2}\left(2a + (30-1)\times(-4)\right)$$

$$23940 = 15(2a - 116)$$

$$1596 = 2a - 116$$

$$1712 = 2a$$

$$a = 856$$

Last 30 terms of the sequence will be $a + (n-1)\times d, a + (n-2)d, a + (n-3)d, ..., a + (n-30)d$

$$\begin{aligned} Sum &= a + (n-1)d + a + (n-2)d + a + (n-3)d + ... + a + (n-30)d \\ &= 30a + 30dn - d(1 + 2 + 3 + ... + 30) \\ &= 30a + 30dn - d \times \frac{30}{2}(1 + 30) \\ &= 30a + 30dn - 465d \end{aligned}$$

$$-1980 = 30 \times 856 + 30 \times (-4)n - 465(-4)$$

$$-1980 = 27540 - 120n$$

$$120n = 29520$$

$$n = 246$$

Question 32(a)

Criteria	Marks
• Correct solution	**3**
• Writes an expression for A_{120}	**2**
• Establishes a pattern and writes an expression for A_2	**1**

Sample Answer:

Let A_n be the amount owing after the n^{th} month.

$$A_1 = 1.007 \times 820\,000 - 6248$$

$$A_2 = 1.007 \times A_1 - 6248$$

$$= 1.007 \times \left(1.007 \times 820\,000 - 6248\right) - 6248$$

$$= 1.007^2 \times 820\,000 - 1.007 \times 6248 - 6248$$

$$A_3 = 1.007 \times A_2 - 6248$$

$$= 1.007 \times \left(1.007^2 \times 820\,000 - 1.007 \times 6248 - 6248\right) - 6248$$

$$= 1.007^3 \times 820000 - 1.007^2 \times 6248 - 1.007 \times 6248 - 6248$$

...

$$A_n = 1.007^n \times 820000 - 1.007^{n-1} \times 6248 - 1.007^{n-2} \times 6248 - \ldots 1.007^2 \times 6248 - 1.007 \times 6248 - 6248$$

$$A_{120} = 1.007^{120} \times 820000 - \underbrace{1.007^{119} \times 6248 - 1.007^{118} \times 6248 - \ldots 1.007^2 \times 6248 - 1.007 \times 6248 - 6248}_{GP\ where\ a=6248, r=1.007, n=120,\ S_n = \frac{a(r^n-1)}{r-1}}$$

$$= 1.007^{120} \times 820000 - \frac{6248(1.007^{120} - 1)}{1.007 - 1}$$

$$= 724960.5746$$

$$= \$724960.57$$

Question 32(b)

Criteria	Marks
• Correct solution	**2**
• Writes an expression to find the monthly repayment	**1**

Sample Answer:

$$A_n = 1.007^n \times 720\,000 - M - M \times 1.007 - M \times 1.007^2 - \ldots - M \times 1.007^{n-1}$$

$$0 = 1.007^{180} \times 720\,000 - \frac{M(1.007^{180} - 1)}{1.007 - 1}$$

$$0 = 1.007^{180} \times 720\,000 - \frac{M(1.007^{180} - 1)}{0.007}$$

$$1.007^{180} \times 720\,000 = \frac{M(1.007^{180} - 1)}{0.007}$$

$$1.007^{180} \times 720\,000 \times 0.007 = M(1.007^{180} - 1)$$

$$M = \frac{1.007^{180} \times 720\,000 \times 0.007}{1.007^{180} - 1}$$

$$= 7047.983754$$

$$= \$7047.98$$

Question 33(a)

Criteria	Marks
• Correct solution	**2**
• Correctly finds the time taken for either the bus or Lucas to make their journey OR • Correctly writes an expression for *GP* and *FP*	**1**

Sample Answer:

$$\tan\theta = \frac{GP}{FG}$$
$$= \frac{GP}{300}$$
$$GP = 300\tan\theta$$
$$T_B = \frac{D}{S}$$
$$= \frac{4000 + 300\tan\theta}{20}$$
$$= 200 + 15\tan\theta$$

$$\cos\theta = \frac{FG}{FP}$$
$$= \frac{300}{FP}$$
$$FP = \frac{300}{\cos\theta}$$
$$T_L = \frac{300}{\cos\theta} \div 8$$
$$= \frac{75}{2\cos\theta}$$

Question 33(b)

Criteria	Marks
• Correct solution	**4**
• Recognises that $\sin\theta = \frac{2}{5}$ gives maximum value of T.	**3**
• Gives correct expression for $\frac{dT}{d\theta}$	**2**
• Recognises that $T = T_B - T_T$ has to be a maximum	**1**

We need to maximise difference between the time taken for the bus and Lucas to reach *P*.

$$Let\ T = T_B - T_L$$

$$\therefore T = 200 + 15\tan\theta - \frac{75}{2\cos\theta}$$

Stationary points occur when $\frac{dT}{d\theta} = 0$.

$$T = 200 + 15\tan\theta - \frac{75}{2\cos\theta}$$

$$= 200 + 15\tan\theta - \frac{75}{2}(\cos\theta)^{-1}$$

$$T' = 15\sec^2\theta + \frac{75}{2}(\cos\theta)^{-2} \times (-\sin\theta)$$

$$= \frac{15}{\cos^2\theta} - \frac{75\sin\theta}{2\cos^2\theta}$$

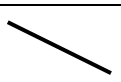
$$0 = \frac{15}{\cos^2\theta} - \frac{75\sin\theta}{2\cos^2\theta}$$

$$0 = 15 - \frac{75}{2}\sin\theta$$

$$\frac{75}{2}\sin\theta = 15$$

$$\sin\theta = \frac{30}{75}$$

$$\theta = 0.4115168461$$

θ	0	0.411…	0.5
T'	40	0	–47
			

$\therefore$ when $\theta = 0.411, T$ is a maximum

$$\text{When } \theta = 0.411..., T = 200 + 15\tan\theta - \frac{75}{2\cos\theta}$$
$$= 165.6306823 \text{ seconds}$$
$$= 2 \text{ minutes and } 45 \text{ seconds}$$

Lucas must leave at 8:02 am at the very latest to catch the bus.

Markers Note: 8:03am was not awarded as Lucas will miss the bus by 15 seconds