NAME: ____________________________

Blacktown Boys' High School

2025 Year 12

Trial Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided for the paper
- In Section II, show all relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (page 3 - 7)

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 - 38)

- Attempt Questions 11 - 34
- Allow about 2 hours 45 minutes for this section

Assessor: P Govender

Students are advised that this is a trial examination only and cannot in any way guarantee the content of format of the 2025 Higher School Certificate Examination.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

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1 Which of the following is an arithmetic sequence?

A. $6, 24, 96, \ldots$

B. $9, 14, 20, \ldots$

C. $27, 34, 41, \ldots$

D. $48, -24, 12, \ldots$

2 Which of the following is the domain of the function $y = \dfrac{1}{\sqrt{x^2 - 4}}$?

A. $[-2,2]$

B. $(-2,2)$

C. $(-\infty, -2] \cup [2, \infty)$

D. $(-\infty, -2) \cup (2, \infty)$

3 Find the 100^{th} term of the sequence $10, 6\frac{1}{2}, 3, \ldots$

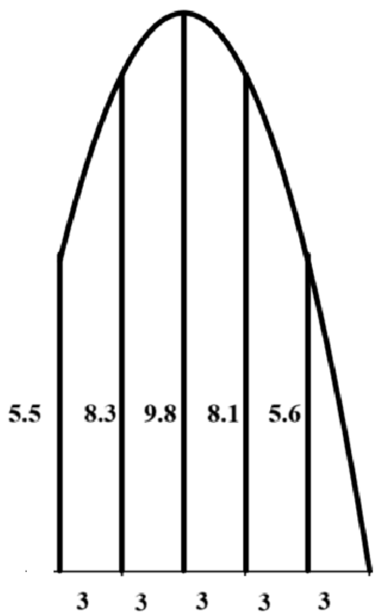
A. $356\frac{1}{2}$

B. $-336\frac{1}{2}$

C. -333

D. 353

4 The diagram below shows the section of a wall in a building. All measurements are in metres.



NOT TO SCALE

An approximate value of the area of the wall using the trapezoidal rule with five sub-intervals is:

A. $103.7\ m^2$

B. $41.3\ m^2$

C. $107.3\ m^2$

D. $86.4\ m^2$

5 Which expression is a term of the geometric series $2x - 4x^2 + 8x^3 - 16x^4$+……?

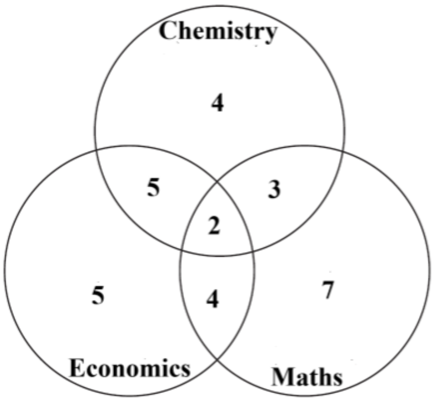
A. $-2048x^{11}$

B. $2048x^{11}$

C. $-2048x^{10}$

D. $2048x^{10}$

6 In a class of 30 students, students were asked what subjects they studied and the results are shown in the Venn diagram.



A student who studies Maths is chosen at random. What is the probability that the chosen student also studies Chemistry?

A. $\frac{1}{6}$

B. $\frac{16}{25}$

C. $\frac{3}{16}$

D. $\frac{5}{16}$

7 Find $\frac{d}{dx}(\ln(\tan x))$

A. $-\cot x$

B. $-\operatorname{cosec} x \sec x$

C. $\cot x$

D. $\operatorname{cosec} x \sec x$

8 The discreet random variable X has the following probability distribution.

x	0	1	2	3
$P(X = x)$	a	$4a$	$5a$	$2a$

Which of the following is the mean of X?

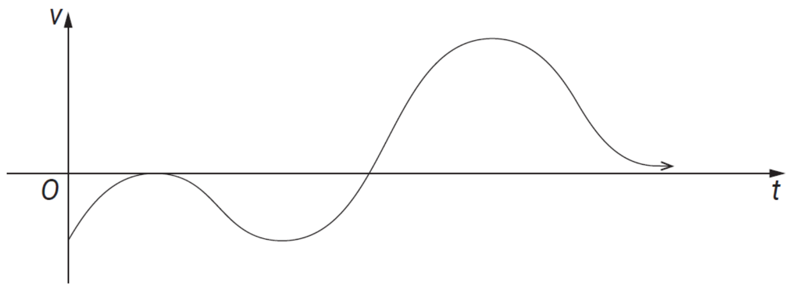
A. $\frac{1}{12}$

B. $\frac{5}{3}$

C. 1

D. $\frac{3}{5}$

9 A particle is moving along a straight line.
The graph shows the velocity, v, of the particle for the time $t \geq 0$.



How many times does the acceleration of the car equal zero?

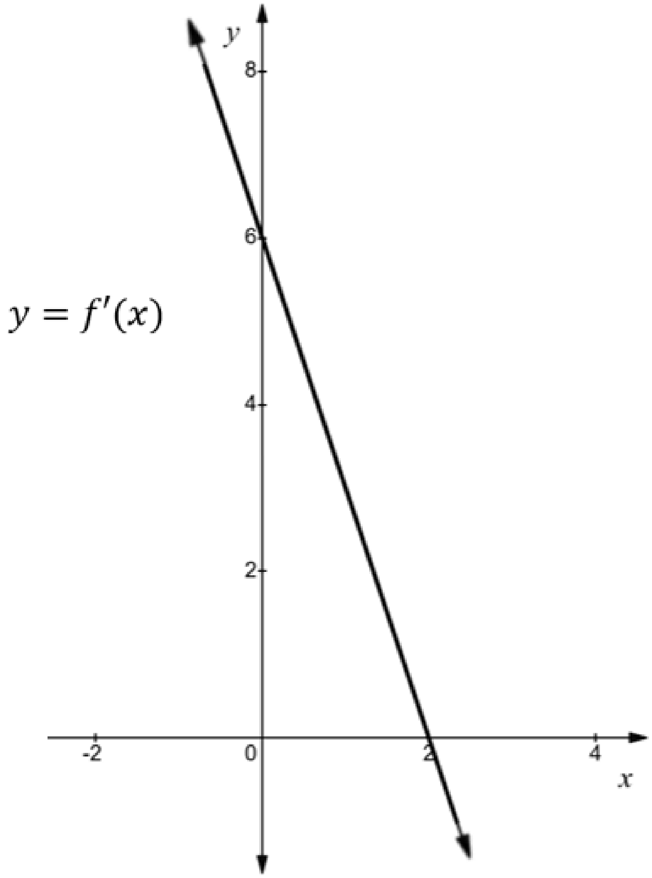
A. 1

B. 2

C. 3

D. 4

10 The graph of $y = f'(x)$ is show.



The curve $y = f(x)$ has a maximum value of 14.
What is the equation of the curve $y = f(x)$?

A. $y = -\frac{3}{2}x^2 + 6x + 6$

B. $y = \frac{3}{2}x^2 + 6x + 20$

C. $y = \frac{3}{2}x^2 + 6x + 14$

D. $y = -\frac{3}{2}x^2 + 6x + 8$

End of Section I

Mathematics Advanced
Section II Answer Booklet 1

NAME: ___________________________________

90 marks
Attempt Questions 11 – 33
Allow about 2 hour 45 minutes for this section

Booklet 1 – Attempt Questions 11 – 23 (45 marks)
Booklet 2 – Attempt Questions 24 – 33 (45 marks)

Instructions

- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your response should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet.
  If you use this space, clearly indicate which question you are answering.

Question 11 (3 marks)

For the geometric series given below.

$$18 + 6 + 2 + \cdots$$

(a) Find the common ratio. **1**

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(b) Find the 8^{th} term. **1**

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(c) Find the limiting sum of this series. **1**

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Question 12 (3 marks)

Given the function $f(x) = x\cos^2 3x$,

(a) Show that $f(x) = x\cos^2 3x$ is odd. **1**

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(b) Find $f'(x)$. **2**

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Question 13 (3 marks)

The queueing times, in minutes at a busy canteen is modelled by a Normal Distribution with a mean of 16 and a standard deviation of 4. Use the table below to answer the questions that follow.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441

(a) Use the probability table for a standard normal distribution to find the $P(z \leq 1.36)$. **1**

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(b) Determine the probability that a randomly chosen customer will have to queue between 13 and 21 minutes. **2**

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Question 14 (4 marks)

Given the function $y = |4x - 2|$

(a) Sketch $y = |4x - 2|$, showing all key features. **2**

(b) Hence evaluate $\int_{-1}^{3} |4x - 2| \, dx$. **2**

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Question 15 (4 marks)

Given the function $f(x) = \dfrac{3}{2-x} + 1$,

(a) Write down the domain of the function using interval notation. **1**

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(b) Sketch the function, showing all key features. **2**

(c) If $f(x)$ is translated to the left by 4 units and dilated vertically by a factor of 6, what is the equation of the final graph? **1**

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Question 16 (3 marks)

Solve the equation $2\sin^2 x + \cos x = 1$ for $0 \leq x \leq 2\pi$ **3**

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Question 17 (2 marks)

In an arithmetic sequence, the sixth term is 37 and the nineteenth term is 193. Find the value of the first terms, a, and the common difference, d. **2**

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Question 18 (4 marks)

The table shows the future value of a $1 annuity at different interest rates over different numbers of time periods.

Future values of a $1 annuity

Time Period	*Interest rate*				
	1%	*2%*	*3%*	*4%*	*5%*
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

(a) What would be the future value of a $7000 per year annuity at 4% per annum for 5 years, with interest compounding yearly? **1**

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(b) What is the value of an annuity that would provide a future value of $1020285 after 6 years at 5% per annum compound interest? **1**

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(c) An annuity of $4000 per six months is invested at 2% per annum, compounded every six months for 3 years. What will be the amount of interest earned? **2**

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Question 19 (4 marks)

Given that $f(x) = \dfrac{2x}{e^{5x-2}}$,

(a) Show that $f'(x) = \dfrac{2(1-5x)}{e^{5x-2}}$. **2**

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(b) Find the equation of the normal at $x = \dfrac{2}{5}$. **2**

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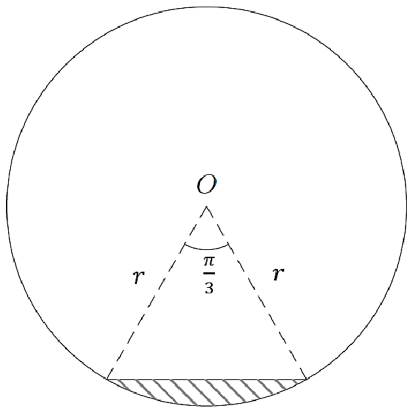
Question 20 (2 marks)

The area of a minor segment of the circle pictured below is $2.5\ m^2$.



Find the radius of the circle to 2 decimal places. **2**

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Question 21 (6 marks)

Consider the probability density function f for a random variable X given by

$$f(x) = \begin{cases} \dfrac{m}{\sqrt{25+3x}} & ;0 \le x \le 8 \\ 0 & ;otherwise \end{cases}$$

where m is a constant.

(a) Show that $m = \dfrac{3}{4}$. **2**

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(b) Find the cumulative distribution function, $F(x)$. **1**

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(c) Hence find the interquartile range. **3**

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Question 22 (2 marks)

Thomas is playing two games. He is equally likely to win each game. The probability that he will win at least one of the games is 90%. What is the probability that Thomas will win both games? **2**

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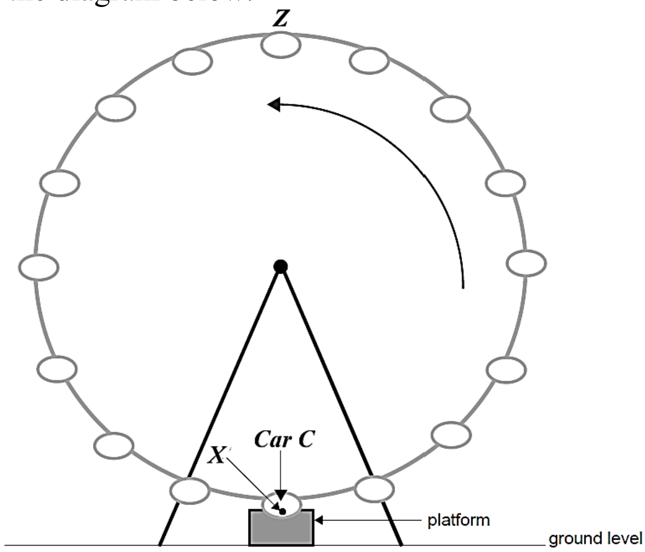
Question 23 (5 marks)

A Ferris wheel at a city rotates in an anticlockwise direction at a constant rate. Passengers enter the cars of the Ferris wheel from a platform which is above ground level. The Ferris wheel does not stop at any time and has 16 cars which is evenly spaced around the circular structure as shown in the diagram below.



A camera was attached to point X on the side of car C when point X was at its lowest point at 2:00pm.

The height, h in metres, of the point X above the ground level, at time t hours after 2:00pm is given by:

$$h(t) = 52 + 50\sin\left(\frac{(5t-1)\pi}{2}\right)$$

(a) What is the minimum height, in metres, of point X above the ground? **1**

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(b) At what time after 2:00pm does point X first return to its lowest point? **1**

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Question 23 continues on the next page.

Question 23 (continued)

(c) Find the number of minutes during one rotation that point X will be at least 77 m above ground level. **3**

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Mathematics Advanced
Section II Answer Booklet 2

NAME: ____________________________________

Booklet 2 – Attempt Questions 24 – 34 (45 marks)

Instructions

- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your response should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 24 (2 marks)

Evaluate $\int_0^2 \frac{3}{7x+2}\,dx$. **2**

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Question 25 (2 marks)

Find $\int \left(\sin 5x - \frac{3}{e^{2x}}\right) dx$. **2**

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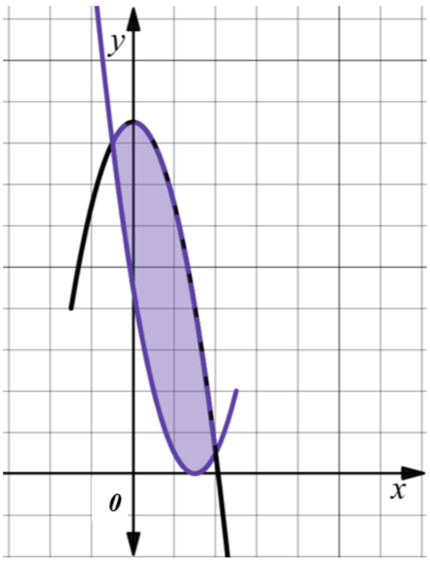
Question 26 (5 marks)

The curves $y = (x-3)^2$ and $y = 17 - x^2$ intersect at two points, as shown in the diagram.



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(a) Solve simultaneously to find the x-coordinates of the points of intersection of the two curves. **2**

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(b) Hence find the area enclosed by the two curves. **3**

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Question 27 (5 marks)

The velocity of a particle moving along a straight line is given by $v = 6 - 4t$, where t is time in seconds. Initially the particle is 8 units to the right of the origin.

(a) Show that the particle is initially moving to the right. **1**

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(b) Show that the displacement function is $x = 8 + 6t - 2t^2$. **2**

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(c) Find the distance the particle travelled in the first 3 seconds. **2**

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Question 28 (3 marks)

A small meteor passes through a dust shower. It picks up particles and gains weight (kg) in time *t* (hours) according to the expression $W_t = 1.7e^{0.05t}$.

(a) What was the initial weight of the meteor? **1**

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(b) How long will it take for the meteor to triple in weight? **2**

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Question 29 (2 marks)

Tom and Ricky play against each other in a chess match. The champion will be the first person to win two games. Tom's probability of winning each game is 0.7 and Ricky's probability of winning each game is 0.3.

What is the probability that John wins the match? **2**

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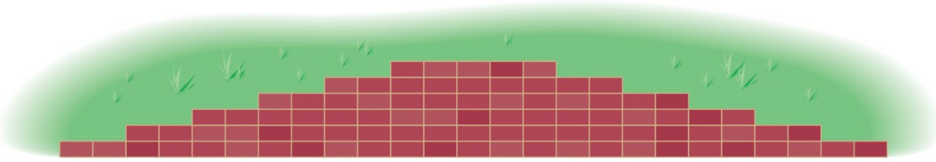
Question 30 (3 marks)

Mike is a building contractor who is planning to build a brick wall that approximates the shape of a trapezoid as shown in the diagram below. The shorter base of the trapezoid needs to start with a row of 5 bricks, and each row must increase by 2 bricks on each side until all of the 495 bricks that he has are used. How many rows will Mike be able to lay with the 495 bricks? **3**



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Question 31 (7 marks)

Consider the curve given by $y = x^3 - 12x + 2$.

(a) Find the coordinates of the stationary points and determine their nature. **3**

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(b) Determine any point of inflection. **1**

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Question 31 continues on the next page

Question 31 (continued)

(c) Sketch the curve for the domain $-4 \leq x \leq 3$. Clearly label endpoints, turning points, point of inflection, and *y*-intercept. **2**

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(d) What is the maximum value of $x^3 - 12x + 2$ in the domain $-4 \leq x \leq 3$. **1**

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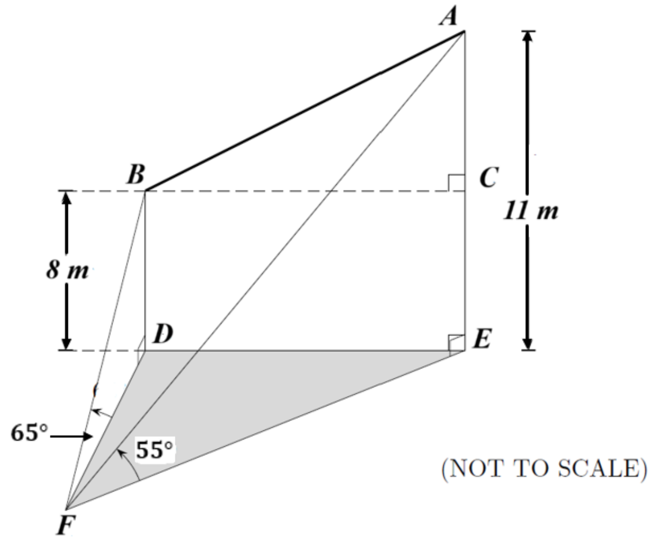
Question 32 (5 marks)

Ravi plans to better support a section of roof truss *ABC* which sits on top of a wall *CBDE,* by installing two steel ropes *AF* and *BF*, as shown in the diagram below. The *CBDE* sits on the ground which is a level surface. The apex *A* of the roof is *11 m* above the ground and the other end of the roof truss (Point *B*) is *8 m* above the ground.

Ravi installs a steel rope from Point *B* to a peg at Point *F* which is located at ground level and another steel rope from Point *A* to Point *F*. The angle of elevation from Point *F* to Point *B* is 65° and angle of elevation from Point *F* to Point *A* is 55°.



(a) Show that $DF = 8\cot 65°$. **1**

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(b) The area of $\triangle DFE$ is $12.34\ m^2$. Show that $\angle DFE = 59°12'$. **2**

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Question 32 continues next page.

Question 32 (continued)

(c) Hence find the slant length of the roof truss AB, correct to two decimal places. **2**

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Question 33 (5 marks)

$50000 is borrowed at 9% per annum reducible interest, calculated monthly. The loan is repaid in equal monthly instalments of $*M* over 6 years. Interest is charged before repayments are made every month.

Let A_n be the amount owing after n monthly repayments.

(a) Show that $A_n = 50\,000\left(\frac{403}{400}\right)^n \times -400M\left(\frac{\left(\frac{403}{400}\right)^n - 1}{3}\right)$. **3**

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Question 33 continues next page.

Question 33 (continued)

(b) Show that the monthly repayment is \$901.28. **1**

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(c) What is the balance owing after the 21^{st} payment ? **1**

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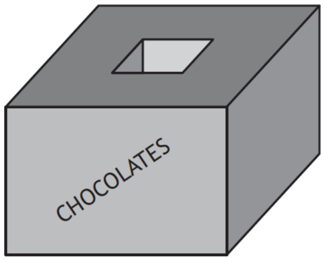
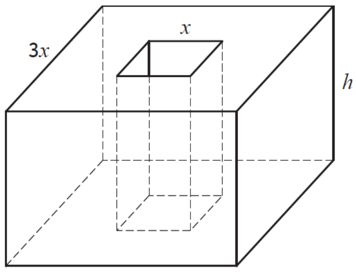
Question 34 (6 marks)

A manufacturer of chocolates is launching a new product in novelty shaped cardboard boxes.



The box is a rectangular prism with a different rectangular prism tunnel through it.

- The height of the box is h centimetres
- The top of the box is a square of side $3x$ centimetres
- The end of the tunnel is a square of side x centimetres
- The volume of the box is 3456 cm^3



(a) Show that $h = \dfrac{432}{x^2}$. **1**

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(b) Hence, show that the total surface area, $A\ cm^2$, of the box is given by: **2**

$$A = 16x^2 + \frac{6912}{x}$$

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Question 34 continues next page.

Question 34 (continued)

(c) To minimise the cost of production, the surface area, A, of the box should be as small as possible. **3**
Find the minimum value of A.

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End of Exam

NAME: Solutions

Blacktown Boys' High School

2025 Year 12

Trial Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided for the paper
- In Section II, show all relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (page 3 - 7)

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 - 38)

- Attempt Questions 11 - 34
- Allow about 2 hours 45 minutes for this section

Assessor: P Govender

Students are advised that this is a trial examination only and cannot in any way guarantee the content of format of the 2025 Higher School Certificate Examination.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

---

1 Which of the following is an arithmetic sequence?

A. $6, 24, 96, \ldots$

B. $9, 14, 20, \ldots$

C. $27, 34, 41, \ldots$

1 mark D. $48, -24, 12, \ldots$

2 Which of the following is the domain of the function $y = \dfrac{1}{\sqrt{x^2 - 4}}$?

A. $[-2,2]$

B. $(-2,2)$

C. $(-\infty,-2] \cup [2,\infty)$

D. $(-\infty,-2) \cup (2,\infty)$

1 mark

$x^2 - 4 > 0$
$x > 2$ or $x < -2$
$\therefore (-\infty, 2) \cup (2, \infty)$
$\therefore$ D

3 Find the 100^{th} term of the sequence $10, 6\frac{1}{2}, 3, \ldots$

A. $356\frac{1}{2}$

B. $-336\frac{1}{2}$

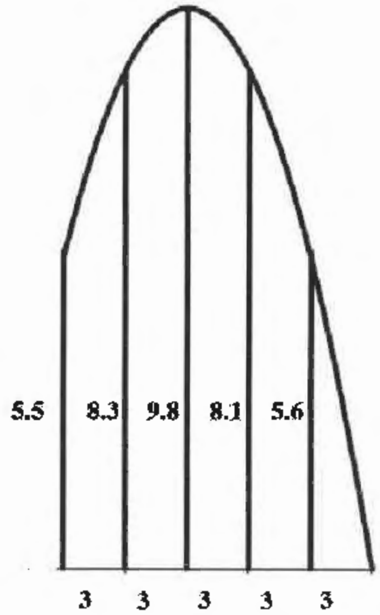
1 mark

C. -333

D. 353

$a = 10$, $d = -3\frac{1}{2}$ $n = 100$.
$T_{100} = 10 + (100-1)(-\frac{7}{2})$
$= 10 - 99 \times \frac{7}{2}$
$= -\frac{673}{2}$
$= -336\frac{1}{2}$.
$\therefore$ B

4 The diagram below shows the section of a wall in a building. All measurements are in metres.



NOT TO SCALE

An approximate value of the area of the wall using the trapezoidal rule with five sub-intervals is:

$Area \simeq \frac{b-a}{2n}\{f(a)+f(b)+2[f(x_1)+f(x_2)+f(x_3)+f(x_4)]\}$

1 mark A. $103.7\ m^2$ $= \frac{15-0}{2\times5}\{5.5+0+2[8.3+9.8+8.1+5.6]\}$

B. $41.3\ m^2$ $= \frac{3}{2}\{5.5+63.6\}$

C. $107.3\ m^2$ $= 103.7\ m^2$

D. $86.4\ m^2$

$\therefore$ A

5 Which expression is a term of the geometric series $2x - 4x^2 + 8x^3 - 16x^4 + \ldots\ldots$?

A. $-2048x^{11}$ $a = 2x$; $r = \frac{T_2}{T_1} = \frac{-4x^2}{2x} = -2x$

1 mark B. $2048x^{11}$ $T_n = ar^n$

C. $-2048x^{10}$ $T_n = (2x)(-2x)^n$

D. $2048x^{10}$ $T_n = 2\times(-2)^n \times x^{n+1}$

If $n=9$, $T_9 = 2(-2)^9 \times x^{10}$

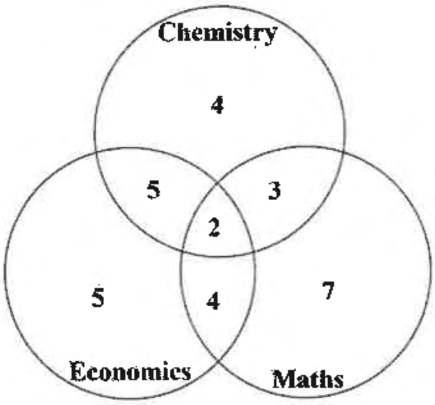
$T_9 = -1024x^{10}$

If $n=10$, $T_{10} = 2(-2)^{10}x^{11}$

$T_{10} = 2048x^{11}$

$\therefore$ B

6 In a class of 30 students, students were asked what subjects they studied and the results are shown in the Venn diagram.



A student who studies Maths is chosen at random. What is the probability that the chosen student also studies Chemistry?

A. $\frac{1}{6}$

B. $\frac{16}{25}$

C. $\frac{3}{16}$

D. $\frac{5}{16}$

1 mark

$P(Chem / Maths) = \frac{n(Chem \cap Maths)}{n(Maths)}$

$= \frac{2+3}{2+3+4+7}$

$= \frac{5}{16}$

$\therefore$ D

7 Find $\frac{d}{dx}(\ln(\tan x))$

A. $-\cot x$

B. $-\operatorname{cosec} x \sec x$

C. $\cot x$

D. $\operatorname{cosec} x \sec x$

1 mark

$\frac{d}{dx}(\ln(\tan x))$

$= \frac{1}{\tan x} \times \sec^2 x$

$= \frac{\cos x}{\sin x} \times \frac{1}{\cos^2 x}$

$= \frac{1}{\sin x \cdot \cos x}$

$= \operatorname{cosec} x \sec x$

$\therefore$ D

8 The discrete random variable X has the following probability distribution.

x	0	1	2	3
$P(X = x)$	a	$4a$	$5a$	$2a$

Which of the following is the mean of X?

A. $\frac{1}{12}$

B. $\frac{5}{3}$ ✓ 1mark

C. 1

D. $\frac{3}{5}$

$a + 4a + 5a + 2a = 1$

$12a = 1$

$a = \frac{1}{12}$

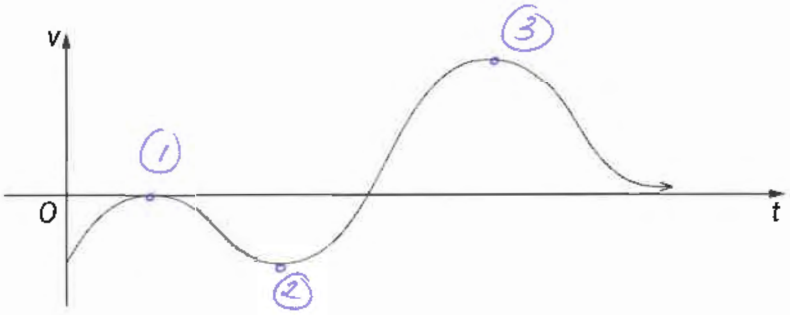
$X = 0\times\frac{1}{2} + 1\times\frac{4}{12} + 2\times\frac{5}{12} + 3\times\frac{2}{12}$

$= 0 + \frac{4}{12} + \frac{10}{12} + \frac{6}{12}$

$= \frac{5}{3}$

$\therefore$ B

9 A particle is moving along a straight line.
The graph shows the velocity, v, of the particle for the time $t \geq 0$.



How many times does the acceleration of the car equal zero?

A. 1

B. 2

C. 3 ✓ 1mark

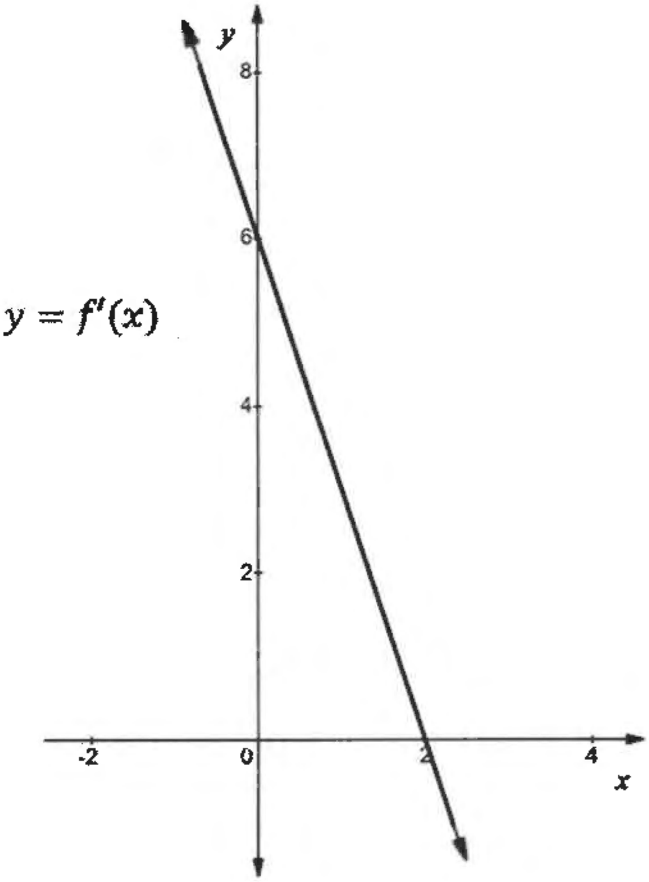
D. 4

The graph of velocity vs time has maximums and minimums on 3 occasions.

$\therefore$ 3 times.

C

10 The graph of $y = f'(x)$ is show.



The curve $y = f(x)$ has a maximum value of 14.
What is the equation of the curve $y = f(x)$?

A. $y = -\frac{3}{2}x^2 + 6x + 6$

B. $y = \frac{3}{2}x^2 + 6x + 20$

C. $y = \frac{3}{2}x^2 + 6x + 14$

D. $y = -\frac{3}{2}x^2 + 6x + 8$

1mark

$m = \frac{6-0}{0-2} = -3$

$(2,0)$: $y - 0 = -3(x-2)$

$y = -3x+6$

$\therefore f'(x) = -3x+6$

Maximum value when $f'(x) = 0$

$-3x+6 = 0$

$x = 2$

$f(x) = \int f'(x)\,dx$

$f(x) = \int (-3x+6)\,dx$

$f(x) = -\frac{3}{2}x^2 + 6x + C$

when $x = 2$, $f(x) = 14$

$14 = -\frac{3}{2}(2^2) + 6\times 2 + C$

$C = 8$

$\therefore f(x) = -\frac{3}{2}x^2 + 6x + 8$

End of Section I $\therefore$ D

Mathematics Advanced
Section II Answer Booklet 1

NAME: ______________________________

90 marks
Attempt Questions 11 – 33
Allow about 2 hour 45 minutes for this section

Booklet 1 – Attempt Questions 11 – 23 (45 marks)
Booklet 2 – Attempt Questions 24 – 33 (45 marks)

Instructions

- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your response should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (3 marks)

For the geometric series given below.

$$18 + 6 + 2 + \cdots$$

(a) Find the common ratio. **1**

$r = \frac{T_2}{T_1} = \frac{6}{18}$

$r = \frac{1}{3}$ ✓ 1 mark

(b) Find the 8^{th} term. $a = 18$, $r = \frac{1}{3}$. **1**

$T_n = ar^{n-1}$

$T_8 = 18 \times \left(\frac{1}{3}\right)^{8-1}$

$T_8 = \frac{2}{243}$ ✓ 1 mark.

(c) Find the limiting sum of this series. **1**

Since $r < 1$: $S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{18}{1-\frac{1}{3}}$

$S_\infty = 27$ ✓ 1 mark.

Question 12 (3 marks)

Given the function $f(x) = x\cos^2 3x$,

(a) Show that $f(x) = x\cos^2 3x$ is odd. **1**

Odd if $f(-x) = -f(x)$

$f(x) = x\cos^2 3x$

$-f(x) = -x\cos^2 3x$

$f(-x) = -x\cos^2(3x - x)$

$= -x\cos^2(-3x)$

$= -x\cos^2 3x$

$= -f(x)$.

$\therefore f(x)$ is ODD. ✓ 1 mark.

(b) Find $f'(x)$. **2**

$f(x) = x\cos^2 3x$.

Let $u = x$ $\quad v = \cos^2 3x$.

$u' = 1$ $\quad v' = 2(\cos 3x) \times 3 \times (-\sin 3x)$

$= -6\cos 3x \sin 3x$.

$f'(x) = uv' + u'v$

$= x \times (-6\cos 3x \sin 3x) + 1 \times \cos^2 3x$.

$= \underbrace{\cos^2 3x}_{\text{1 mark}} \underbrace{- 6x\cos 3x \sin 3x}_{\text{1 mark}}$

Question 13 (3 marks)

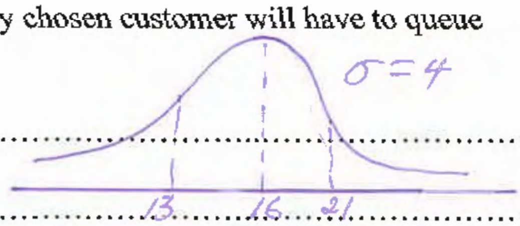
The queueing times, in minutes at a busy canteen is modelled by a Normal Distribution with a mean of 16 and a standard deviation of 4. Use the table below to answer the questions that follow.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441

(a) Use the probability table for a standard normal distribution to find the $P(z \leq 1.36)$. **1**

$P(z < 1.36) = 0.9131$ ✓ 1 mark

(b) Determine the probability that a randomly chosen customer will have to queue between 13 and 21 minutes. **2**



$P(13 < T < 21) = P(T < 21) - P(T < 13)$

$= P\left(z < \frac{21-16}{4}\right) - P\left(z < \frac{13-16}{4}\right)$

$= P(z < 1.25) - P(z < -0.75)$ ✓ 1 mark

$= P(z < 1.25) - [1 - P(z < 0.75)]$

$= 0.8944 - [1 - 0.7734]$

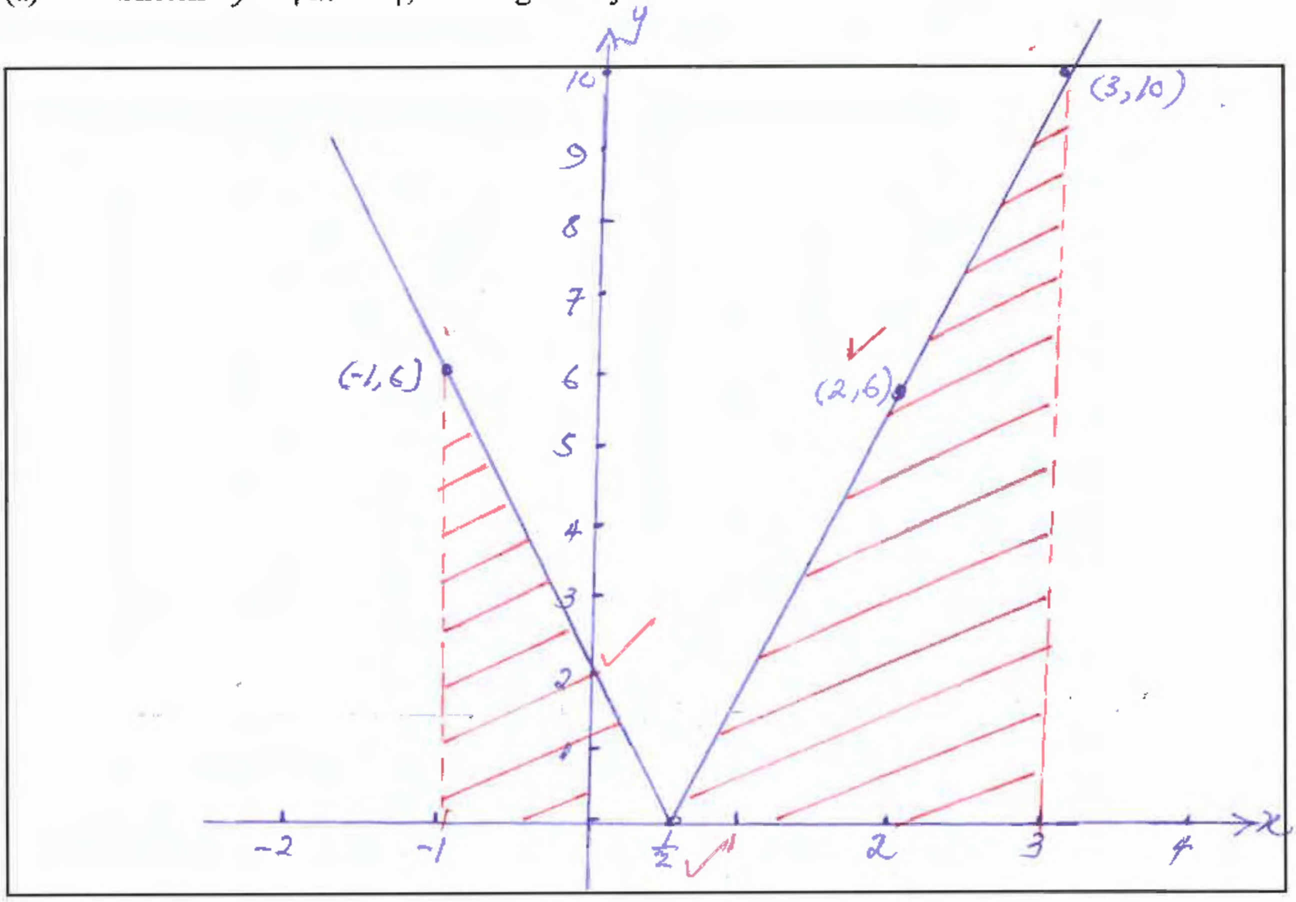
$= 0.6678$ ✓ 1 mark

Question 14 (4 marks)

Given the function $y = |4x - 2|$

1 mark for 2 out of 3 correct plots.
2 marks All correct.

(a) Sketch $y = |4x - 2|$, showing all key features. **2**



(b) Hence evaluate $\int_{-1}^{3} |4x - 2|\, dx$. **2**

$\int_{-1}^{3} |4x-2|\, dx = \int_{-1}^{\frac{1}{2}} (2-4x)\,dx + \int_{\frac{1}{2}}^{3} (4x-2)\,dx$

$= \left[\left(2\times\frac{1}{2} - 2\times\left(\frac{1}{2}\right)^2\right) - \left(2\times(-1) - 2\times(-1)^2\right)\right] + \left[\left(2\times3^2 - 2\times3\right) - \left(2\left(\frac{1}{2}\right)^2 - 2\times\frac{1}{2}\right)\right]$

$= \left[1 - \frac{1}{2} - (-2-2)\right] + \left[(18-6) - \left(\frac{1}{2} - 1\right)\right]$

$= \frac{9}{2} + \frac{25}{2}$ 1 mark 1 mark

$= \underline{17}$.

OR $\int_{-1}^{3} |4x-2|\, dx$

$= \left[\frac{1}{2}\times 1\frac{1}{2}\times 6\right] + \left[\frac{1}{2}\times 2\frac{1}{2}\times 10\right]$

$= \frac{9}{2} + \frac{25}{2}$

$= \underline{17}$

Question 15 (4 marks)

Given the function $f(x) = \dfrac{3}{2-x} + 1$,

(a) Write down the domain of the function using interval notation. **1**

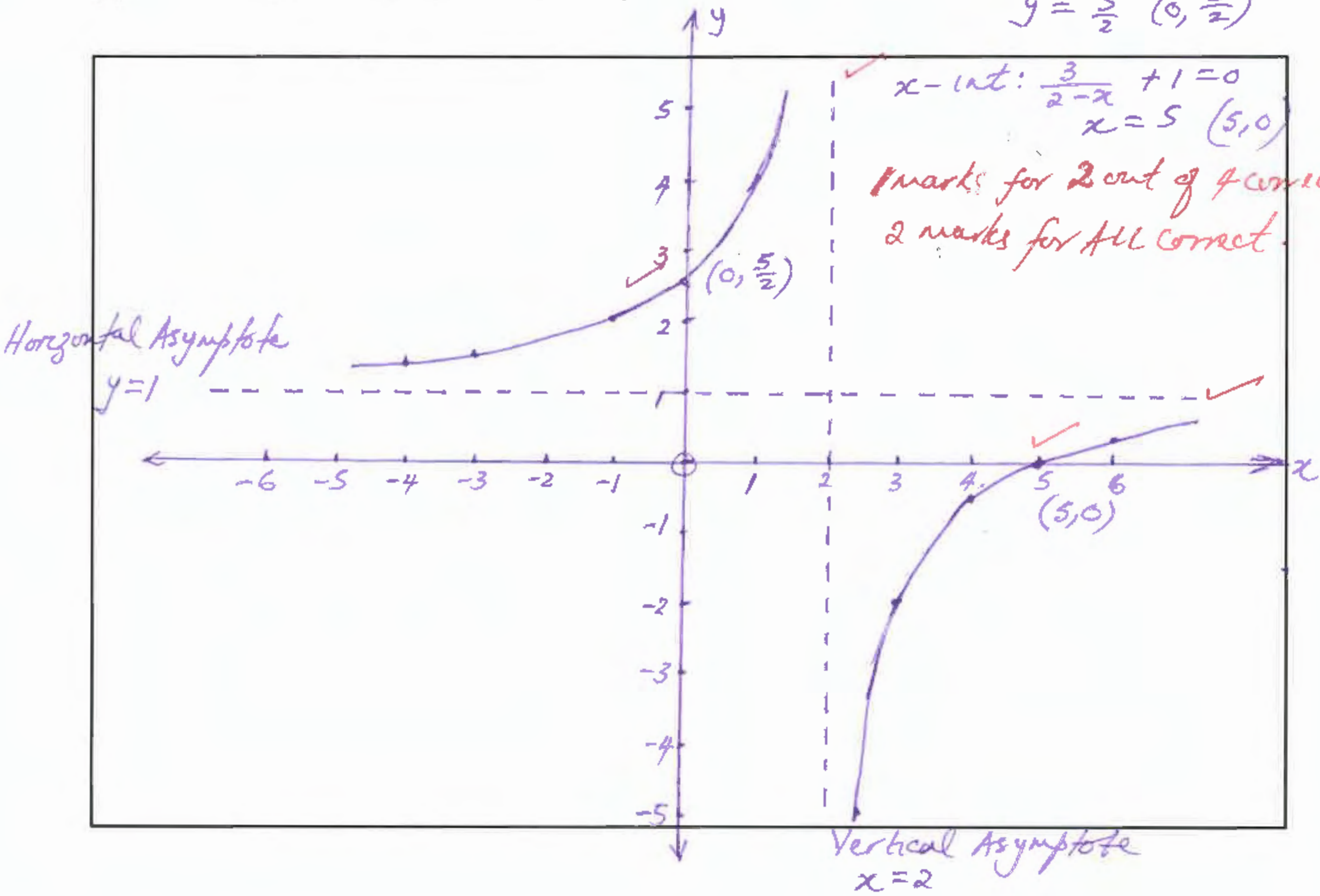
Domain : $2 - x \neq 0$, $x \neq 2$.

Domain : $(-\infty, 2) \cup (2, \infty)$ ✓ 1 mark

(b) Sketch the function, showing all key features. **2**

y-int: $y = \frac{3}{2-0} + 1$

$y = \frac{5}{2}$ $(0, \frac{5}{2})$



(c) If $f(x)$ is translated to the left by 4 units and dilated vertically by a factor of 6, what is the equation of the final graph? **1**

$f(x) = \frac{3}{2-x} + 1 \longrightarrow y = 6\left[\frac{3}{2-(x+4)} + 1\right]$

$= 6\left[\frac{3}{-x-2} + 1\right]$

$= \frac{18}{-x-2} + 6$ ✓ 1 mark .

Question 16 (3 marks)

Solve the equation $2\sin^2 x + \cos x = 1$ for $0 \le x \le 2\pi$ **3**

$2\sin^2 x + \cos x = 1$ for $0 \le x \le 2\pi$

$2(1-\cos^2 x) + \cos x = 1$

$2 - 2\cos^2 x + \cos x = 1$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$ ✓ 1mark

$2\cos x + 1 = 0$ or $\cos x - 1 = 0$

$\cos x = -\frac{1}{2}$ or $\cos x = 1$

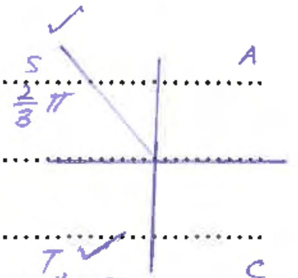
$x = \frac{2}{3}\pi, \frac{4}{3}\pi$ or $x = 0, 2\pi$ ✓ 1mark for either $\frac{2}{3}\pi, \frac{4}{3}\pi$ or $0, 2\pi$ correct

$x = 0, \frac{2}{3}\pi, \frac{4}{3}\pi, 2\pi$ ✓ 3 marks for All correct

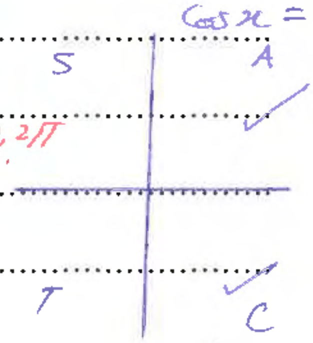
$\cos x = -\frac{1}{2}$

$\pi - \frac{\pi}{3} = \frac{2}{3}\pi$

$\pi + \frac{\pi}{3} = \frac{4}{3}\pi$



$\cos x = 1$



Question 17 (2 marks)

In an arithmetic sequence, the sixth term is 37 and the nineteenth term is 193. Find the value of the first terms, a, and the common difference, d. **2**

$T_6 = 37$, $T_{19} = 193$

$T_n = a + (n-1)d$

$37 = a + (6-1)d$

$37 = a + 5d$ ——(1)

$193 = a + 18d$ ——(2)

(2) − (1): $156 = 13d$

$d = 12$ ✓ 1mark

Substitute $d = 12$ into Eqn(1):

$37 = a + 5 \times 12$

$a = -23$ ✓ 1mark

Question 18 (4 marks)

The table shows the future value of a $1 annuity at different interest rates over different numbers of time periods.

Future values of a $1 annuity

Time Period	*Interest rate*				
	1%	*2%*	*3%*	*4%*	*5%*
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

(a) What would be the future value of a $7000 per year annuity at 4% per annum for 5 years, with interest compounding yearly? **1**

Table factor : when $n = 5$ and $r = 4\%$

Factor $= 5.4163$

Future Value $= \$7000 \times 5.4163$

$= \$37914.10$ ✓ 1 mark

(b) What is the value of an annuity that would provide a future value of $1020285 after 6 years at 5% per annum compound interest? **1**

Table factor : when $n = 6$ and $r = 5\% \rightarrow$ factor $= 6.8019$

Let A = Annuity. $FV = A \times 6.8019$

$A = \frac{1020285}{6.8019}$

$= \$150000$ ✓ 1 mark

(c) An annuity of $4000 per six months is invested at 2% per annum, compounded every six months for 3 years. What will be the amount of interest earned? **2**

$4000 per 6 months ., $n = 3 \text{ years} \times 2 = 6$, $r = \frac{2\%}{2} = 1\%$ per six months

Table factor when $n = 6$ and $r = 1\%$: Factor 6.1520

$FV = 4000 \times 6.1520$

$= \$24608$ ✓ 1 mark

Interest $= \$24608 - (6 \times \$4000)$

$= \$608$. ✓ 1 mark

Question 19 (4 marks)

Given that $f(x) = \dfrac{2x}{e^{5x-2}}$,

(a) Show that $f'(x) = \dfrac{2(1-5x)}{e^{5x-2}}$. **2**

Let $u = 2x$ $\quad v = e^{5x-2}$

$u' = 2$ $\quad v' = 5e^{5x-2}$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

$$= \frac{2xe^{5x-2} - 2x \times 5e^{5x-2}}{(e^{5x-2})^2}$$

$$= \frac{\cancel{e^{5x-2}}[2-10x]}{(e^{5x-2})^{\cancel{2}}}$$

$$= \frac{2-10x}{e^{5x-2}} \quad \text{1 mark}$$

$$= \frac{2(1-5x)}{e^{5x-2}} \quad \text{1 mark} \quad \ldots \text{as required.}$$

(b) Find the equation of the normal at $x = \dfrac{2}{5}$. **2**

$$m_T = \frac{2(1-5x)}{e^{5x-2}}$$

At $x = \frac{2}{5}$: $m_T = \dfrac{2(1-5\times\frac{2}{5})}{e^{5\times\frac{2}{5}-2}}$

$$= \frac{-2}{1}$$

$$= -2$$

At $x = \frac{2}{5}$: $f(\frac{2}{5}) = \dfrac{2\times\frac{2}{5}}{e^{5\times\frac{2}{5}-2}} = \frac{4}{5}$ $\therefore$ $(\frac{2}{5}, \frac{4}{5})$

$m_N = -\frac{1}{-2} = \frac{1}{2}$ 1 mark

Eqn of Normal: $y - \frac{4}{5} = \frac{1}{2}(x - \frac{2}{5})$

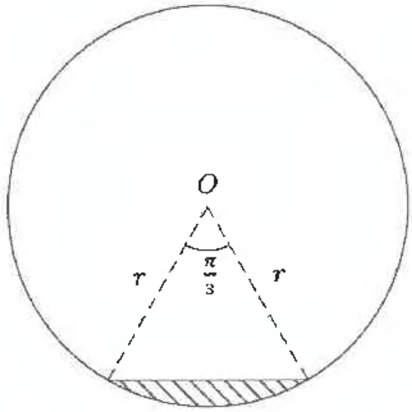
$$2y - \frac{8}{5} = x - \frac{2}{5}$$

$$10y - 8 = 5x - 2$$

$$5x - 10y + 6 = 0 \quad \text{1 mark}$$

Question 20 (2 marks)

The area of a minor segment of the circle pictured below is 2.5 m^2.



Find the radius of the circle to 2 decimal places. **2**

$A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2}r^2\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$

$= \frac{1}{2}r^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2}r^2\left(\frac{2\pi - 3\sqrt{3}}{6}\right)$

$A = r^2\left(\frac{2\pi - 3\sqrt{3}}{12}\right)$

But $A = 2.5\ m^2$

$\therefore r^2\left(\frac{2\pi - 3\sqrt{3}}{12}\right) = 2.5$ ✓ 1 mark.

$r^2 = \frac{2.5 \times 12}{(2\pi - 3\sqrt{3})}$

$r = \sqrt{\frac{30}{2\pi - 3\sqrt{3}}}$

$= 5.25338\ldots$

$= 5.25\ m$ (to 2d.p.) ✓ 1 mark.

Penalise for incorrect rounding.

Question 21 (6 marks)

Consider the probability density function f for a random variable X given by

$$f(x) = \begin{cases} \dfrac{m}{\sqrt{25+3x}} & ;0 \le x \le 8 \\ 0 & ;otherwise \end{cases}$$

where m is a constant.

(a) Show that $m = \dfrac{3}{4}$. **2**

Since $f(x)$ is a probability density function, PDF over the range must be 1.

$0 + \int_0^8 f(x)\,dx = 1$

$m \int_0^8 (25+3x)^{-\frac{1}{2}}\,dx = 1$ ✓ 1 mark

$m\left[2 \times (25+3x)^{\frac{1}{2}} \div 3\right]_0^8 = 1$

$m\left[\frac{2}{3}(7-5)\right] = 1$

$m = \frac{3}{4}$ ... as required. ✓ 1 mark

(b) Find the cumulative distribution function, $F(x)$. **1**

For $0 \le x \le 8$: $F(x) = \frac{3}{4}\int_0^x (25+3x)^{-\frac{1}{2}}\,dx$

$= \frac{3}{4}\left[\frac{2}{3}(25+3x)^{\frac{1}{2}}\right]_0^x$

$= \frac{3}{4}\left[\frac{2}{3}\sqrt{25+3x} - \frac{2}{3}\times 5\right]$

$= \frac{3}{4}\left[\frac{2}{3}\left(\sqrt{25+3x} - 5\right)\right]$

$= \frac{\sqrt{25+3x} - 5}{2}$ ✓ 1 mark

$$\therefore F(x) = \begin{cases} 0 & \text{for } x < 0 \\ \dfrac{\sqrt{25+3x}-5}{2} & \text{for } 0 \le x \le 8 \\ 1 & \text{for } x > 8 \end{cases}$$

(c) Hence find the interquartile range. $IQR = Q_3 - Q_1$ **3**

$F(x) = \frac{\sqrt{25+3x} - 5}{2}$

$\sqrt{25+3x} = 2F(x) + 5$

$x = \frac{[2 \times F(x) + 5]^2 - 25}{3}$

For Q_3: $F(x) = \frac{3}{4}$: $Q_3 = \frac{[2\times\frac{3}{4} + 5]^2 - 25}{3} = \frac{23}{4}$ ✓ 1 mark

For Q_1: $F(x) = \frac{1}{4}$: $Q_1 = \frac{[2\times\frac{1}{4} + 5]^2 - 25}{3} = \frac{7}{4}$ ✓ 1 mark

$\therefore IQR = \frac{23}{4} - \frac{7}{4}$

$= 4$ ✓ 1 mark

Question 22 (2 marks)

Thomas is playing two games. He is equally likely to win each game. The probability that he will win at least one of the games is 90%. What is the probability that Thomas will win both games? **2**

$P(\text{at least } 1W) = 1 - P(LL) = 0.9$

$\therefore P(LL) = 1 - 0.9$

$= 0.1$ ✓ 1 mark.

$P(L) = \sqrt{0.1} = 0.316$

$P(W) = 1 - 0.316$

$= 0.684$

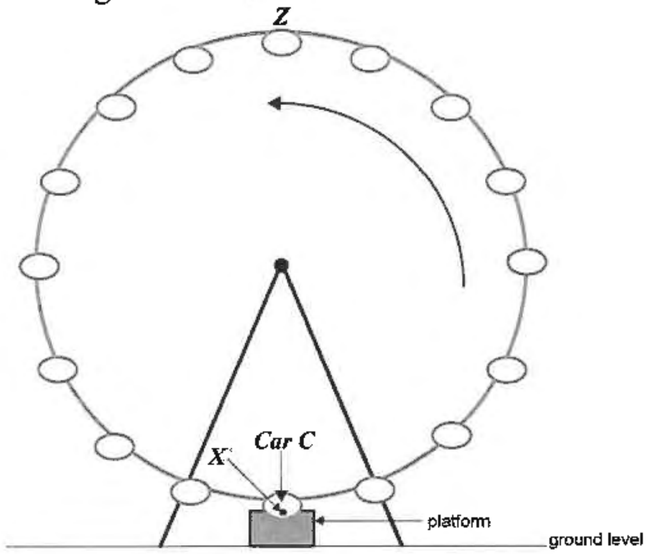
$P(WW) = 0.684 \times 0.684$

$P(WW) = 0.467856$.

$\therefore$ The probability that Thomas will win both games is 46.8%. ✓ 1 mark.

Question 23 (5 marks)

A Ferris wheel at a city rotates in an anticlockwise direction at a constant rate. Passengers enter the cars of the Ferris wheel from a platform which is above ground level. The Ferris wheel does not stop at any time and has 16 cars which is evenly spaced around the circular structure as shown in the diagram below.



A camera was attached to point X on the side of car C when point X was at its lowest point at 2:00pm.

The height, h in metres, of the point X above the ground level, at time t hours after 2:00pm is given by:

$$h(t) = 52 + 50\sin\left(\frac{(5t-1)\pi}{2}\right)$$

(a) What is the minimum height, in metres, of point X above the ground? **1**

Minimum height when $\sin\left(\frac{(5t-1)\pi}{2}\right) = -1$

$\therefore h_{min} = 52 + 50(-1) = 2\,m$ ✓ 1 mark

(b) At what time after 2:00pm does point X first return to its lowest point? **1**

Period $T = \frac{2\pi}{\frac{5\pi}{2}} = 2\pi \times \frac{2}{5\pi}$

$= \frac{4}{5}$

$= 0.8$ hours or 48 mins.

$\therefore$ The point X returns to its lowest point at 2:00pm + 48 min

$= 2:48$ pm ✓ 1 mark

Question 23 continues on the next page.

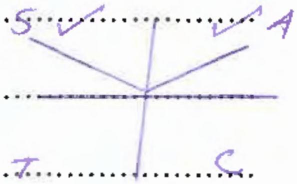
Question 23 (continued)

(c) Find the number of minutes during one rotation that point X will be at least 77 m above ground level. **3**

$$77 = 52 + 50 \sin\left(\frac{(5t-1)\pi}{2}\right)$$

$$\frac{25}{50} = \sin\left(\frac{(5t-1)\pi}{2}\right)$$

$$\sin\left(\frac{(5t-1)\pi}{2}\right) = \frac{1}{2}$$



$$\frac{(5t-1)\pi}{2} = \frac{\pi}{6}, \frac{5}{6}\pi$$ ✓

1 mark

1st time: $\frac{(5t-1)\pi}{2} = \frac{\pi}{6}$ and 2nd time $\frac{(5t-1)\pi}{2} = \frac{5\pi}{6}$

$5t - 1 = \frac{1}{3}$ $\quad$ $5t - 1 = \frac{5}{3}$

$5t = \frac{4}{3}$ $\quad$ $5t = \frac{8}{3}$

$t = \frac{4}{15}$ $\quad$ $t = \frac{8}{15}$ ✓ 1 mark for both t values correct.

Duration at least 77 m above ground level $= \frac{8}{15} - \frac{4}{15}$

$= \frac{4}{15}$

$= 16$ minutes.

✓ 1 mark.

Mathematics Advanced
Section II Answer Booklet 2

NAME: ______________________________

Booklet 2 – Attempt Questions 24 – 34 (45 marks)

Instructions

- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your response should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 24 (2 marks)

Evaluate $\int_0^2 \frac{3}{7x+2}\,dx$. **2**

$$\int_0^2 \frac{3}{7x+2}\,dx = \frac{3}{7}\int_0^2 \frac{7}{7x+2}\,dx$$

$$= \frac{3}{7}\left[\ln|7x+2|\right]_0^2$$ ✓ 1 mark

$$= \frac{3}{7}\left[\ln|7\times2+2| - \ln|7\times0+2|\right]$$

$$= \frac{3}{7}\left[\ln 16 - \ln 2\right]$$

$$= \frac{3}{7}\left[4\ln 2 - \ln 2\right]$$

$$= \frac{3}{7}\times 3\ln 2$$

$$= \frac{9}{7}\ln 2$$ ✓ 1 mark

OR $\frac{3}{7}\ln 8$

Question 25 (2 marks)

Find $\int \left(\sin 5x - \frac{3}{e^{2x}}\right)dx$. **2**

$$\int\left(\sin 5x - \frac{3}{e^{2x}}\right)dx$$

$$= \int\left[-\frac{1}{5}(-5\sin 5x) - 3e^{-2x}\right]dx$$

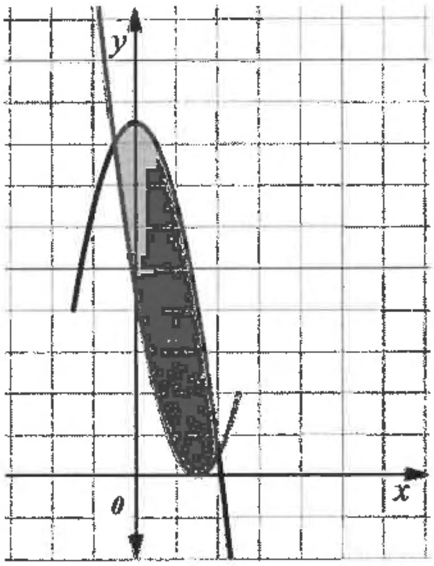
$$= \int\left[-\frac{1}{5}(-5\sin 5x) - \frac{3\times(-2)}{(-2)}e^{-2x}\right]dx$$

$$= -\frac{1}{5}\cos 5x + \frac{3}{2}e^{-2x} + C$$ ✓ 1 mark for partially correct integration.

✓ 2 marks for ALL correct.

Question 26 (5 marks)

The curves $y = (x-3)^2$ and $y = 17 - x^2$ intersect at two points, as shown in the diagram.



NOT DRAWN TO SCALE

(a) Solve simultaneously to find the x-coordinates of the points of intersection of the two curves. **2**

$y=(x-3)^2$ and $y=17-x^2$

To find points of intersection:

$(x-3)^2 = 17-x^2$

$x^2-6x+9 = 17-x^2$

$2x^2-6x-8=0$

$2(x^2-3x-4)=0$

$2(x-4)(x+1)=0$

$x=-1$ or $x=4$

✓ 1 mark ✓ 1 mark

(b) Hence find the area enclosed by the two curves. **3**

$\text{Area} = \int_{-1}^{4} (17-x^2)-(x-3)^2\,dx$ ✓ 1 mark

$= \int_{-1}^{4} (17-x^2-x^2+6x-9)\,dx$

$= \int_{-1}^{4} (-2x^2+6x+8)\,dx$

$= \left[-\frac{2}{3}x^3+3x^2+8x\right]_{-1}^{4}$ ✓ 1 mark

$= \left[-\frac{2}{3}(4)^3+3\times4^2+8\times4\right]-\left[-\frac{2}{3}(-1)^3+3(-1)^2+8(-1)\right]$

$= \frac{125}{3}$ ✓ 1 mark

Question 27 (5 marks)

The velocity of a particle moving along a straight line is given by $v = 6 - 4t$, where t is time in seconds. Initially the particle is 8 units to the right of the origin.

$= t=0$

(a) Show that the particle is initially moving to the right. **1**

$v = 6 - 4t$

When $t=0$, $v = 6 - 4\times 0$

$v = 6$

> 0 $\therefore$ the particle starts to move to the right. ✓ 1 mark

(b) Show that the displacement function is $x = 8 + 6t - 2t^2$. **2**

$x = \int (6-4t)\,dt$

$x = 6t - 2t^2 + C$ ✓ 1 mark (Must have "+C")

When $t=0$, $x=8$

$8 = 6\times 0 - 2\times 0^2 + C$

$C = 8$ ✓ 1 mark

$\therefore x = 8 + 6t - 2t^2$ ... as required.

(c) Find the distance the particle travelled in the first 3 seconds. **2**

$x = 8 + 6t - 2t^2$

When $t=0$, $x = 8 + 6\times 0 - 2\times 0^2$

$x = 8$

When $t=3$, $x = 8 + 6\times 3 - 2\times 3^2$

$x = 8$

Change in direction at $v=0$

$6 - 4t = 0$

$t = 3/2$

When $t=\frac{3}{2}$, $x = 8 + 6\times\frac{3}{2} - 2\times\left(\frac{3}{2}\right)^2 = \frac{25}{2}$ ✓ 1 mark

Total distance $= \left(x_{t=0} \text{ to } x_{t=\frac{25}{2}}\right) + \left(x_{t=\frac{25}{2}} \text{ to } x_{t=3}\right)$

$= \frac{9}{2} + \frac{9}{2}$

$= 9$ units ✓ 1 mark

Question 28 (3 marks)

A small meteor passes through a dust shower. It picks up particles and gains weight (kg) in time t (hours) according to the expression $W_t = 1.7e^{0.05t}$.

(a) What was the initial weight of the meteor? **1**

When $t=0$, $W_0 = 1.7e^{0.05\times 0}$

$W_0 = 1.7\text{kg}$ ✓ 1 mark

(b) How long will it take for the meteor to triple in weight? **2**

$W_t = 3\times 1.7\text{kg} = 5.1\text{kg}$

$5.1 = 1.7e^{0.05t}$ ✓ 1 mark

$e^{0.05t} = \frac{5.1}{1.7}$

$e^{0.05t} = 3$

$0.05t = \ln 3$

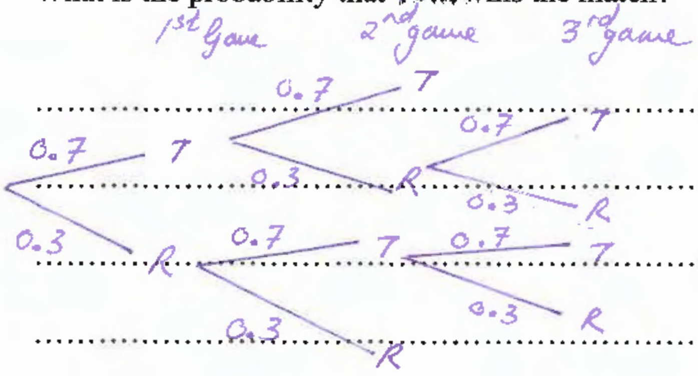
$t = \frac{\ln 3}{0.05}$

$t = 21.9722\ldots$ hours ✓ 1 mark

Question 29 (2 marks)

Tom and Ricky play against each other in a chess match. The champion will be the first person to win two games. Tom's probability of winning each game is 0.7 and Ricky's probability of winning each game is 0.3.

What is the probability that Tom wins the match? **2**



Sample Space

TT: 0.7×0.7

TRT: $0.7\times 0.3\times 0.7$

TRR: $0.7\times 0.3\times 0.3$

RTT: $0.3\times 0.7\times 0.7$

RTR: $0.3\times 0.7\times 0.3$

RR: 0.3×0.3

P(Tom is champion)

= P(Tom wins in 2 games) + P(Tom wins in 3 games)

✓ 1 mark

$= (0.7\times 0.7) + (0.7\times 0.3\times 0.7) + (0.3\times 0.7\times 0.7)$

$= 0.49 + 0.147 + 0.147$

$= 0.784$ ✓ 1 mark

Question 30 (3 marks)

Mike is a building contractor who is planning to build a brick wall that approximates the shape of a trapezoid as shown in the diagram below. The shorter base of the trapezoid needs to start with a row of 5 bricks, and each row must increase by 2 bricks on each side until all of the 495 bricks that he has are used. How many rows will Mike be able to lay with the 495 bricks? 3

This is an arithmetic sequence:

$5, 9, 13, \dots$

$a = 5,\ d = 4$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 5 + (n-1)4]$

$= \frac{n}{2}[10 + 4n - 4]$

$= \frac{n}{2}[4n + 6]$

But $S_n = 495$

$\therefore \frac{n}{2}[4n+6] = 495$ ✓ 1 mark

$2n^2 + 3n - 495 = 0$

$n = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-495)}}{2 \times 2}$

$= \frac{-3 \pm \sqrt{3969}}{4}$

$n = \frac{-3+63}{4}$ or $n = \frac{-3-64}{4}$

$n = 15$ or $n = -\frac{33}{2}$ ✓ 1 mark for both

Since $n = \frac{-33}{2}$ is not an integer, the number of rows that Mike will be able to lay is 15. ✓ 1 mark

Question 31 (7 marks)

Consider the curve given by $y = x^3 - 12x + 2$.

(a) Find the coordinates of the stationary points and determine their nature. **3**

$y = x^3 - 12x + 2$

$\frac{dy}{dx} = 3x^2 - 12$

To find stationary points, let $\frac{dy}{dx} = 0$: $3x^2 - 12 = 0$

$x^2 = 4$

$x = -2$ or $x = 2$

When $x = -2$, $y = (-2)^3 - 12 \times (-2) + 2 = 18$ $\therefore (-2, 18)$ 1 mark

When $x = 2$, $y = 2^3 - 12 \times 2 + 2 = -14$ $\therefore (2, -14)$

The stationary points are $(-2, 18)$ and $(2, -14)$.

Must state values for $\frac{dy}{dx}$ in table

Nature

x	-3	-2	0	2	3
dy/dx	15	0	-12	0	15
grad.	/	—	\	—	/

1 mark

$\therefore$ Stationary point $(-2, 18)$ is a maximum turning point and $(2, -14)$ is a minimum turning point. 1 mark

OR

$\frac{d^2y}{dx^2} = 6x$

When $x = -2$, $\frac{d^2y}{dx^2} = 6 \times (-2) = -12 < 0$ $\cap$

$\therefore$ stat. pt $(-2, 18)$ is a maximum.

When $x = 2$, $\frac{d^2y}{dx^2} = 6 \times 2 = 12 > 0$ $\cup$

$\therefore$ stat. pt. $(2, 14)$ is a minimum.

(b) Determine any point of inflection. **1**

Let $\frac{d^2y}{dx^2} = 0$, $6x = 0$

$x = 0$

When $x = 0$, $y = 0^3 - 12 \times 0 + 2 = 2$

$\therefore$ The point $(0, 2)$ is a possible point of inflection.

Concavity:

x	-1	0	1
$\frac{d^2y}{dx^2}$	-6	0	6
Concavity	$\cap$	•	$\cup$

1 mark

$\therefore$ $(0, 2)$ is a point of inflection.

Must state values for $\frac{d^2y}{dx^2}$ in table.

Question 31 continues on the next page

Question 31 (continued)

(c) Sketch the curve for the domain $-4 \le x \le 3$. Clearly label endpoints, turning points, point of inflection, and y-intercept. **2**

At $x = -4$, $y = x^3 - 12x + 2$

$= (-4)^3 - 12(-4) + 2$

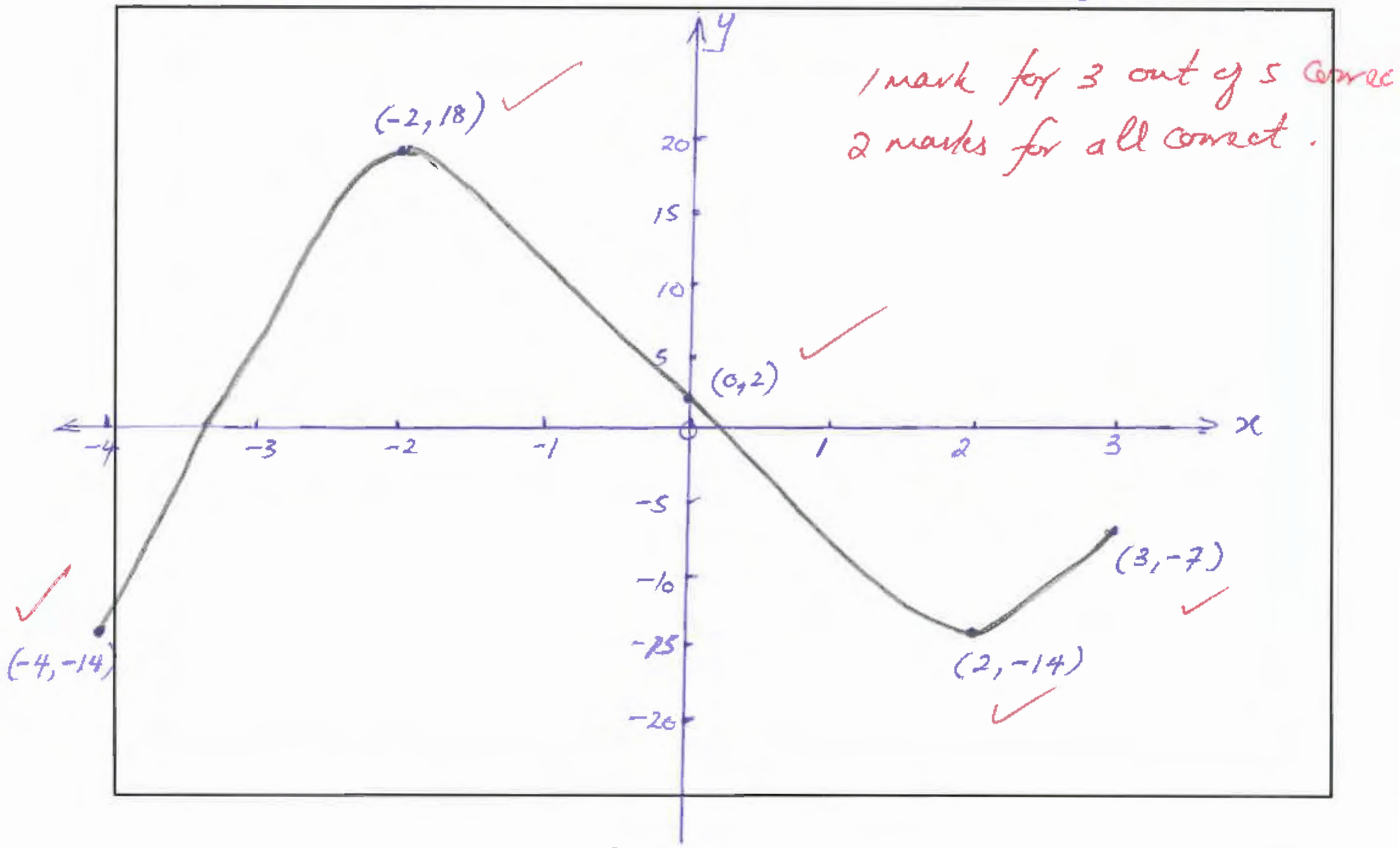
$= -14$ $\therefore$ End point $(-4, -14)$.

At $x = 3$, $y = 3^3 - 12 \times 3 + 2$

$= -7$ $\therefore$ End point $(3, -7)$

y-intercept: When $x = 0$, $y = 2$ $\therefore$ y-int $(0, 2)$

Turning points: $(-2, 18)$ and $(2, -14)$. Point of Inflection: $(0, \ldots$



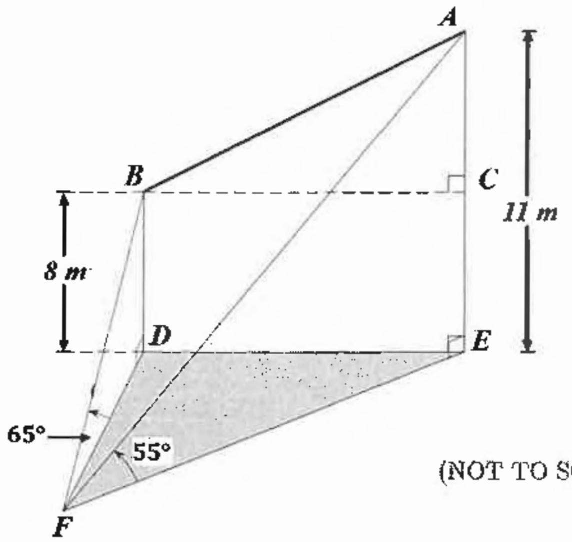
(d) What is the maximum value of $x^3 - 12x + 2$ in the domain $-4 \le x \le 3$. **1**

Maximum value is 18. 1 mark

Question 32 (5 marks)

Ravi plans to better support a section of roof truss *ABC* which sits on top of a wall *CBDE*, by installing two steel ropes *AF* and *BF*, as shown in the diagram below. The *CBDE* sits on the ground which is a level surface. The apex *A* of the roof is *11 m* above the ground and the other end of the roof truss (Point *B*) is *8 m* above the ground.

Ravi installs a steel rope from Point *B* to a peg at Point *F* which is located at ground level and another steel rope from Point *A* to Point *F*. The angle of elevation from Point *F* to Point *B* is 65° and angle of elevation from Point *F* to Point *A* is 55°.



(NOT TO SCALE)

(a) Show that $DF = 8\cot 65°$. **1**

In ΔBDF: $\tan 65° = \frac{8}{DF}$

$DF = \frac{8}{\tan 65°}$

$DF = 8\cot 65°$ --- As required ✓ 1 mark

(b) The area of ΔDFE is $12.34\ m^2$. Show that $\angle DFE = 59°12'$. **2**

In ΔAFE: $\tan 55° = \frac{11}{EF}$

$EF = 11\cot 55°$

In ΔDFE: $A = \frac{1}{2}(DF)(EF)\sin\angle DFE$

$12.340 = \frac{1}{2}(8\cot 65°)(11\cot 55°)\sin\angle DFE$ ✓ 1 mark

$12.340 = \frac{44\sin\angle DFE}{(\tan 65°)(\tan 55°)}$

$\sin\angle DFE = \frac{12.340\times\tan 65°\times\tan 55°}{44}$

$\angle DFE = \sin^{-1}\left[\frac{12.340\times\tan 65°\times\tan 55°}{44}\right]$

$\angle DFE = 59°12'$ (to the nearest minute).

✓ 1 mark

Question 32 continues next page.

Question 32 (continued)

(c) Hence find the slant length of the roof truss AB, correct to two decimal places. **2**

In $\triangle DFE$: $DE^2 = DF^2 + EF^2 - 2(DF)(EF)\cos\angle DFE$

$DE^2 = (8\cot 65°)^2 + (11\cot 55°)^2 - 2(8\cot 65°)(11\cot 55°)\cos 59°12'$

$DE = 6.619394114$

$\therefore BC = DE = 6.619394114$

$AC = 11 - 8 = 3\text{ m}$ ✓ 1 mark

In $\triangle BCA$: $AB^2 = BC^2 + AC^2$

$AB^2 = (6.619394114)^2 + 3^2$

$AB = 7.267\ldots$

$\therefore$ The slant height of the roof truss AB is 7.27 m (to 2 dp) ✓ 1 mark

Question 33 (5 marks)

$50000 is borrowed at 9% per annum reducible interest, calculated monthly. The loan is repaid in equal monthly instalments of $$M$ over 6 years. Interest is charged before repayments are made every month.

Let A_n be the amount owing after n monthly repayments.

(a) Show that $A_n = 50\,000\left(\frac{403}{400}\right)^n \times -400M\left(\frac{\left(\frac{403}{400}\right)^n - 1}{3}\right)$. **3**

$P = \$50000$, 9% p.a. $= \frac{9}{12} \times \frac{1}{100}$ per month $= \frac{3}{400}$ per month, 6 years $= 72$ months.

Let $R = 1 + \frac{3}{400} = \frac{403}{400}$

Amount owing after 1st month:

$A_1 = 50000 \times R - M$.

Amount owing after 2nd month:

$A_2 = (50000 \times R - M) \times R - M$

$= 50000R^2 - MR - M$

Amount owing after 3rd month:

$A_3 = (50000R^2 - MR - M)R - M$

$= 50000R^3 - MR^2 - MR - M$

$= 50000R^3 - M(R^2 + R + 1)$

1 mark for correct A_1, A_2, A_3.

$\vdots$

$A_n = 50000R^n - M(R^{n-1} + R^{n-2} + \cdots + R^2 + R + 1)$

$A_n = 50000R^n - M(\underbrace{1 + R + R^2 + \cdots R^{n-1}}_{\text{G.P. } a=1,\ r=R})$

$S_n = \frac{a(R^n - 1)}{R - 1}$

$S_n = \frac{1(R^n - 1)}{R - 1}$ ✓ 1 mark

$\therefore A_n = 50000R^n - M\left(\frac{R^n - 1}{R - 1}\right)$ where $R = \frac{403}{400}$

$A_n = 50\,000\left(\frac{403}{400}\right)^n - M\left(\frac{\left(\frac{403}{400}\right)^n - 1}{\frac{403}{400} - 1}\right)$

$A_n = 50\,000\left(\frac{403}{400}\right)^n - M\left(\frac{\left(\frac{403}{400}\right)^n - 1}{\frac{3}{400}}\right)$ ✓ 1 mark.

$A_n = 50\,000\left(\frac{403}{400}\right)^n - 400M\left(\frac{\left(\frac{403}{400}\right)^n - 1}{3}\right)$ .... as required.

Question 33 continues next page.

Question 33 (continued)

(b) Show that the monthly repayment is \$901.28. **1**

Loan is paid off when $A_n = 0$ or $A_{72} = 0$

$$50000\left(\frac{403}{400}\right)^n - 400M\left(\frac{\left(\frac{403}{400}\right)^n - 1}{3}\right) = 0$$

$$M = \frac{50000\left(\frac{403}{400}\right)^{72} \times \frac{3}{400}}{\left(\frac{403}{400}\right)^{72} - 1}$$

$M = \$901.28$ .... as required. ✓ 1 mark

(c) What is the balance owing after the 21st payment ? **1**

Balance owing after 21^{st} payment:

$$A_{21} = 50000\left(\frac{403}{400}\right)^{21} - 400 \times (901.28)\left(\frac{\left(\frac{403}{400}\right)^{21} - 1}{3}\right)$$

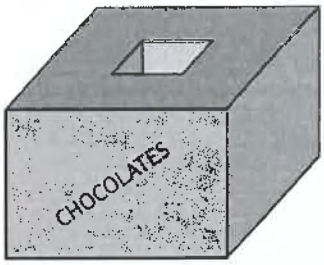
$A_{21} = \$38078.49$

$\therefore$ the balance owing after the 21^{st} payment is \$38078.49

✓ 1 mark

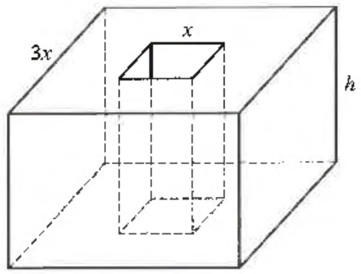
Question 34 (6 marks)

A manufacturer of chocolates is launching a new product in novelty shaped cardboard boxes.



The box is a rectangular prism with a different rectangular prism tunnel through it.

- The height of the box is h centimetres
- The top of the box is a square of side $3x$ centimetres
- The end of the tunnel is a square of side x centimetres
- The volume of the box is 3456 cm^3



(a) Show that $h = \dfrac{432}{x^2}$. **1**

$V = [3x \times 3x \times h] - [x \times x \times h]$

$V = 9x^2h - x^2h = 8x^2h$

But $V = 3456$ cm^3. (given)

$\therefore \; 8x^2h = 3456$. ✓ 1 mark.

$h = \dfrac{3456}{8x^2} = \dfrac{432}{x^2}$ - - - as required.

(b) Hence, show that the total surface area, $A\ cm^2$, of the box is given by: **2**

$$A = 16x^2 + \frac{6912}{x}$$

✓ 1 mark.

$SA_{Total} = 2[3x \times 3x - x \times x] + 4[3x \times h] + 4[x \times h]$

$= 16x^2 + 12xh + 4xh$

$= 16x^2 + 16xh$

$= 16x^2 + 16x \times \dfrac{432}{x^2}$ ✓ 1 mark.

$= 16x^2 + \dfrac{6912}{x}$ - - - - as required.

Question 34 continues next page.

Question 34 (continued)

(c) To minimise the cost of production, the surface area, A, of the box should be as small as possible. **3**

Find the minimum value of A.

$A_T = 16x^2 + \frac{6912}{x}$

$A_T = 16x^2 + 6912x^{-1}$

$\frac{dA_T}{dx} = 32x - \frac{6912}{x^2}$ ✓ 1 mark

Let $\frac{dA_T}{dx} = 0$: $32x - \frac{6912}{x^2} = 0$

$6912 = 32x^3$

$x^3 = 216$

$x = \sqrt[3]{216}$

$x = 6$ ✓ 1 mark.

When $x = 6$, $A_T = 16 \times 6^2 + \frac{6912}{6}$

$A_T = 1728$ ∴ Stationary point at $(6, 1728)$

Nature:

x	5	6	7
dA_T/dx	-116.48	0	82.939
Slope	\	—	/

1 mark (Must state values).

∴ The stationary point $(6, 1728)$ is a minimum value:

∴ the minimum value of A is $1728\ cm^2$ ✓

OR

$\frac{d^2A_T}{dx^2} = 32 + 6912 \times \frac{2}{x^3}$

$\frac{d^2A_T}{dx^2} = 32 + \frac{13824}{x^3}$

When $x = 6$, $\frac{d^2A_T}{dx^2} = 32 + \frac{13824}{6^3}$

$\frac{d^2A_T}{dx^2} = 96$

> 0 ∴ ∪ Minimum turning point

∴ The minimum value of $A = 1728\ cm^2$.

End of Exam