CARINGBAH HIGH SCHOOL

2025 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Advanced

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen.
- Calculators approved by NESA may be used.
- Reference sheet is provided.
- For questions in Section II, show relevant mathematical reasoning and/or calculations.

Section I – 10 marks (pages 3 - 6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section.
- Answer the questions on page 37.

Section II – 90 marks (pages 9 - 32)

- Attempt Questions 11–33
- Allow about 2 hours and 45 minutes for this section.

Student Number: ________________________ Class: __________________

Marker's Use Only								
Section I	**Section II**							**Total**
Q1-10	**11 - 15**	**16 - 19**	**20 - 23**	**24 - 26**	**27- 28**	**29 - 31**	**32-33**	
/10	/14	/14	/13	/15	/13	/12	/9	/100

Section I

10 marks
Attempt Question 1-10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10.

---

1 If $f(x) = 2x + 1$, what is the solution of the equation $f(f(x)) = 11$?

(A) $x = 2$

(B) $x = 3$

(C) $x = 4$

(D) $x = 5$

2 What is $\int (e^{2x} + e^{2x+1})\, dx$?

(A) $\dfrac{e^{2x}(3-2e)}{2} + c$

(B) $\dfrac{e^{2x}(3+2e)}{2} + c$

(C) $\dfrac{e^{2x}(1-e)}{2} + c$

(D) $\dfrac{e^{2x}(1+e)}{2} + c$

3 An arithmetic sequence has a common difference of -2.5 and a fifth term of 15.

What is the tenth term?

(A) -17.5

(B) -14.5

(C) 2.5

(D) 15

4 If A and B are two independent events such that $P(A) = 0.5$ and $P(B) = 0.4$, what is the value of $P(A \cup B)$?

(A) 0.6

(B) 0.7

(C) 0.8

(D) 0.9

5 The population of a type of bacteria is given by the equation $P = 500e^{0.3t}$, where P is the number of bacteria after t hours.

What is the hourly percentage change in the population?

(A) 0.3

(B) 0.35

(C) 30%

(D) 35%

6 What is the domain of the function $f(x) = \sqrt{x+3} + \ln(4-x)$?

(A) $(-3,4)$

(B) $[-3,4]$

(C) $[-3,4)$

(D) $(-3,4]$

7 The function $y = 2\sin x + 1$ is translated up m units and then vertically dilated by a factor of n from the x-axis, where m and n are positive numbers.

What is the range of the new curve?

(A) $[mn - 2n, mn + n]$

(B) $[mn - n, mn + 3n]$

(C) $[mn + n, mn + 3n]$

(D) $[mn + 2n, mn + n]$

8 What is the solution to the equation $x = \log_2 2\sqrt{2} + \log_2 \sqrt{2}$?

(A) $x = 2$

(B) $x = 3$

(C) $x = 4$

(D) $x = 5$

9 What is the derivative of $y = \ln\left(\dfrac{x-2}{4x+1}\right)$?

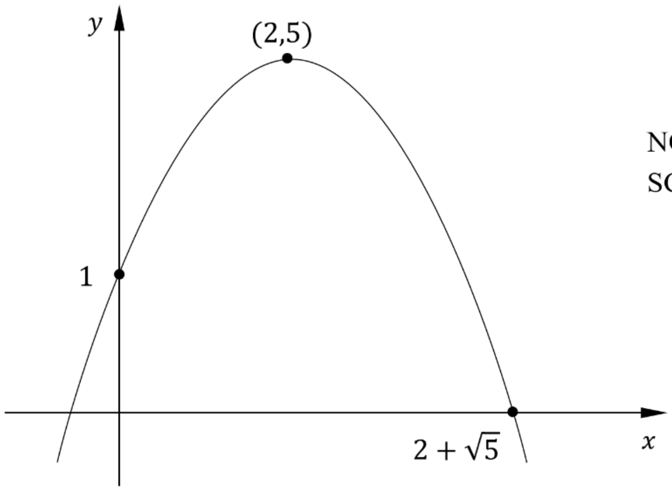
(A) $\dfrac{3x+3}{(x-2)(4x+1)}$

(B) $\dfrac{9}{(x-2)(4x+1)}$

(C) $\dfrac{3x-1}{(x-2)(4x+1)}$

(D) $\dfrac{4x+1}{x-2}$

10 Consider the diagram of the parabola $y = -x^2 + bx + c$ below.



NOT TO SCALE

What are the values of b and c?

(A) $b = 4$, $c = 1$

(B) $b = 2$, $c = 2 + \sqrt{5}$

(C) $b = -2$, $c = 2 + \sqrt{5}$

(D) $b = -4$, $c = 1$

End of Section I

2025 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Advanced
Section II Answer Booklet

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section.

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Marks

Question 11 (2 marks)

Solve $\sin x = -\frac{1}{2}$, where $0 \leq x \leq 2\pi$. Giving the answer in exact form in radians 2

Question 12 (3 marks)

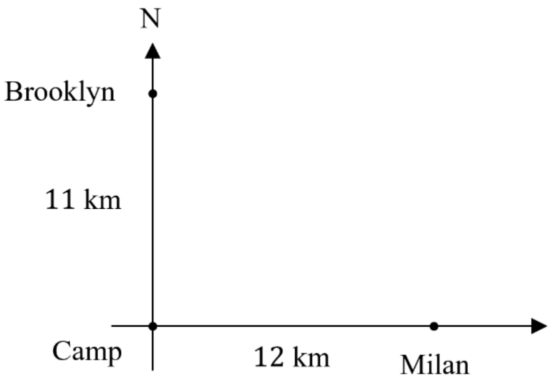
Use two applications of the Trapezoidal rule to find an approximate value, correct to 2 decimal places for, $\int_1^3 \ln(x^2 + 1)\, dx$. 3

Question 13 (2 marks)

Find $\int 10^{5x-1}\, dx$. 2

Question 14 (2 marks)

At 9 am, Brooklyn is 11 km due north of camp, while Milan is 12 km due east of camp. Their positions are shown on the diagram below.



NOT TO
SCALE

They both start walking directly towards camp at a speed of 4 kilometres per hour.

Find the distance between Brooklyn and Milan at 11 am. **2**

Question 15 (5 marks)

A group of medical researchers conducted an investigation to study the impact of different dosages of a new drug on the duration of relief from an allergic reaction.

An experiment was carried out using a random sample of seven patients. The data collected is recorded in the table.

Dosage (mL)	3	4	4	5	7	7	9
Duration of relief (hours)	5	9	12	9	18	22	22

a) Find the equation of the least-squares regression line. Give your answer in the form $y = Ax + B$, where A & B are correct to 2 decimal places. **2**

----

b) By finding Pearson's correlation coefficient, describe the strength and direction of the relationship between the dosage and the duration of relief. **2**

----

c) Using the equation found in part (a), estimate the hours of relief with a dosage of 4.3 mL. Give your answer correct to the nearest hour. **1**

Question 16 (3 marks)

Find the values of x for which the function $f(x) = e^{5x^3 - 10x}$ is increasing. **3**

Question 17 (2 marks)

Evaluate in simplest form $\int_0^{\frac{\pi}{2}} \sin 2x \, dx$. **2**

Question 18 (4 marks)

Differentiate the following with respects to x.

a) $\dfrac{3x + 2}{x + 4}$ **2**

b) $x(1 + 2x)^4$ **2**

Question 19 (5 marks)

Finn is working in the garden when he has an accident and puts a hole in the bottom of his rainwater tank. At the time of the accident, the tank contained 10 200 litres of water.

In the first hour after the accident, Finn noticed that the volume of water in the tank decreased by 200 litres.

In the second hour, he noticed that the volume of water in the tank decreased by a further 250 litres.

Finn is concerned that the leak is growing, but he is not sure what will happen next. He wonders if the amount of water leaking from the tank each hour might form an arithmetic sequence or geometric sequence.

a) If the amount of water that leaks out of the tank each hour forms an arithmetic sequence, how many hours will it take until the tank is empty? **3**

Question 19 continues of page 15

b) If the amount of water that leaks out of the tank each hour forms a geometric sequence, show that eight hours after the leak, over 60% of the water remains in the tank **2**

Question 20 can be found on page 16

Question 20 (4 marks)

a) Prove that $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$. **2**

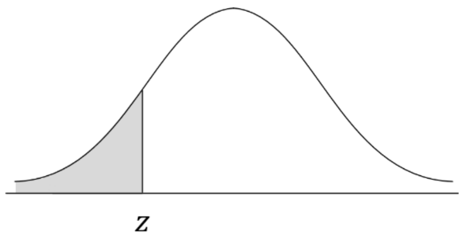
b) Hence, or otherwise, find the exact value of $\displaystyle\int_0^{\frac{\pi}{3}} \sec x + \tan x \, dx$. **2**

Question 21 (3 marks)

A random variable is normally distributed with a mean of 0 and a standard deviation of 1. The table gives the probability that this random variable lies below z for some negative values of z.

z	-1.0	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
Probability	0.1587	0.1841	0.2119	0.242	0.2743	0.3085	0.3446	0.3821	0.4207	0.4602	0.5000

The probability values given in the table are represented by the shaded area in the following diagram.



The time it takes for a worker to drive from home to work is normally distributed with a mean of 32 minutes and a standard deviation of 4 minutes.

a) What is the probability that on a particular day the drive takes between 30 and 32 minutes? **2**

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b) Explain why the probability that the drive takes between 30 and 34 minutes is twice your answer to part (a). **1**

Question 22 (4 marks)

A survey of all the Zoology students at the local university showed that $\frac{3}{4}$ of them had a pet cat, $\frac{3}{8}$ of them had a pet dog and $\frac{1}{4}$ of them had both a pet cat and a pet dog. 25 of the students had either a pet cat or a pet dog but not both types of pet.

a) Show that a total of 40 Zoology students where surveyed **2**

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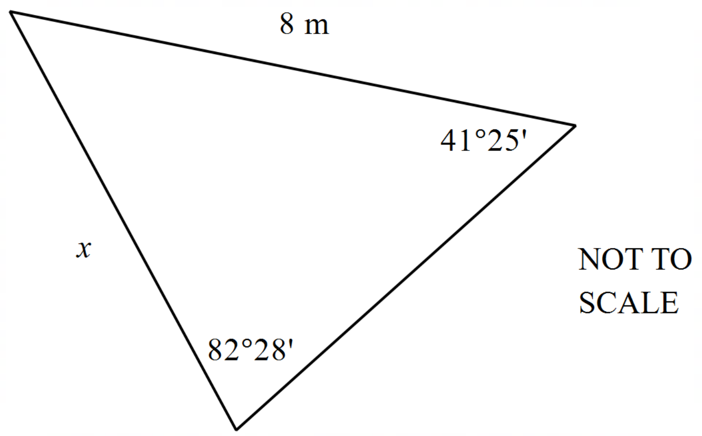
b) If two of the Zoology students are chosen at random, find in simplest fraction form the probability that neither of these students has a pet dog **2**

Question 23 (2 marks)

A triangle has one side that is 8 metres in length with the angle opposite being $82°28'$. A second angle is $41°25'$, with the side opposite this angle being x. **2**



Find the length of x, correct to the nearest metre.

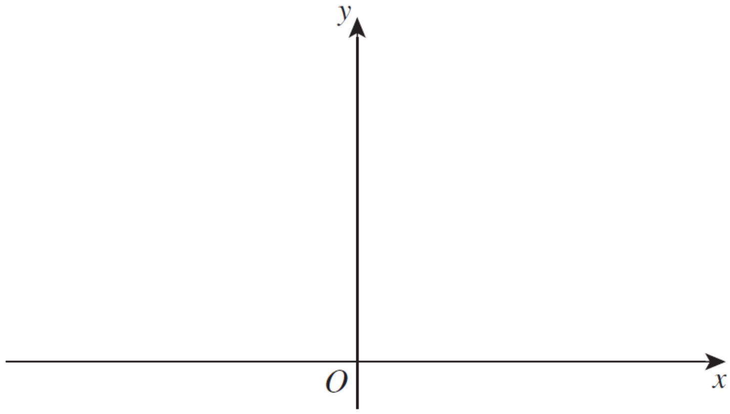
Question 24 can be found on page 20

Question 24 (7 marks)

The lifetime, X, in tens of hours, of a battery has a cumulative distribution function $F(x)$ that is given by

$$F(x)=\begin{cases} 0 & x<1 \\ \dfrac{25}{116}\left(x^2+3x-4\right) & 1<x<1.8 \\ 1 & x\geq 1.8 \end{cases}$$

a) Sketch the cumulative distribution function. **2**



b) An outdoor lantern requires three batteries, all of which must be working, to operate. Three new batteries are inserted into the lantern.

Find the probability that the lantern will still be working after 15 hours. Give your answer as a percentage, rounded to two decimal places. **3**

Question 24 continues on page 21

APPROXIMATELY HALFWAY – 56 marks out of 100 complete at this point.

c) Find the probability density function of the random variable X. **2**

Question 25 (4 marks)

The population of mosquitoes in a garden varies throughout the year according to

the formula $P = 60 + 50\cos\left[\frac{\pi}{6}t\right]$

where t is the time in months since the start of January.

a) Find the maximum and minimum mosquito population during the year. **2**

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b) Find the times in the year when $P = 85$ **2**

Question 26 (4 marks)

Point $P(-1,-2)$ lies on $f(x)$. Point P' is the image of point P on the function $g(x) = 2f(3x-1)$.

a) By clearly stating the transformations required to produce $g(x)$ from $f(x)$, show that the coordinates of point P' are $(0,-4)$. **2**

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b) If the equation of the tangent at point P on $f(x)$ is $y = x - 1$, find the equation of the tangent at point P' on $g(x)$. **2**

Question 27 (5 marks)

A particle moves along the x-axis, where left is in a positive direction. After t seconds its displacement, x metres, is given by the equation $x = (t^2 - 1)^2$.

a) Show that the acceleration of the particle is given by the equation **2**

$$\ddot{x} = 12t^2 - 4$$

----

b) The particle initially moves left, before turning around and moving to the right. **3**

Find the particle's acceleration when it returns to its initial position.

Question 28 (8 marks)

a) Differentiate $\frac{d}{dx}\cos(\ln x)$. **2**

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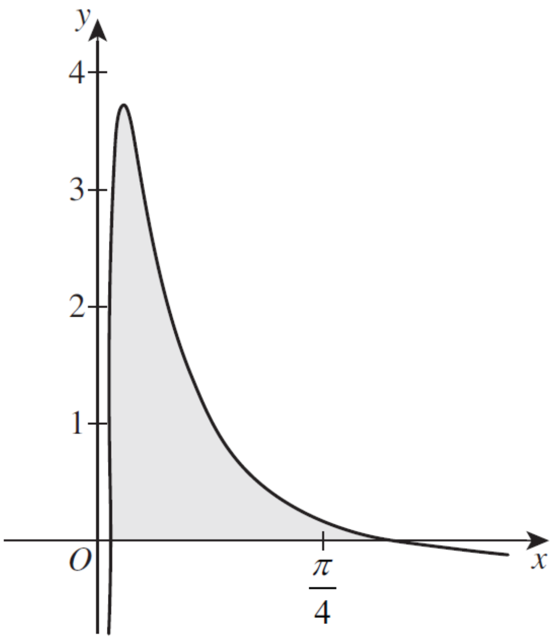
b) The graph of $y = \frac{-\sin(\ln x)}{kx}$ for $0 < x \le 1$ is shown below.



Show that one of the x-intercepts has a value of $e^{-\pi}$. **2**

Question 28 continues on page 25

c) The probability density function of a random variable, x, is given by

$$f(x)=\begin{cases}\dfrac{-\sin(\ln x)}{kx} & \text{for } e^{-\pi}\le x\le 1\\ 0 & \text{elsewhere}\end{cases}$$

i) Show that $k=2$. **2**

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ii) Find $P\left(x<\dfrac{\pi}{4}\right)$. Give your answer correct to three decimal places. **2**

End of Question 28

APPROXIMATELY THREE QUARTERS COMPLETE – 79 marks out of 100 complete

Question 29 (3 marks)

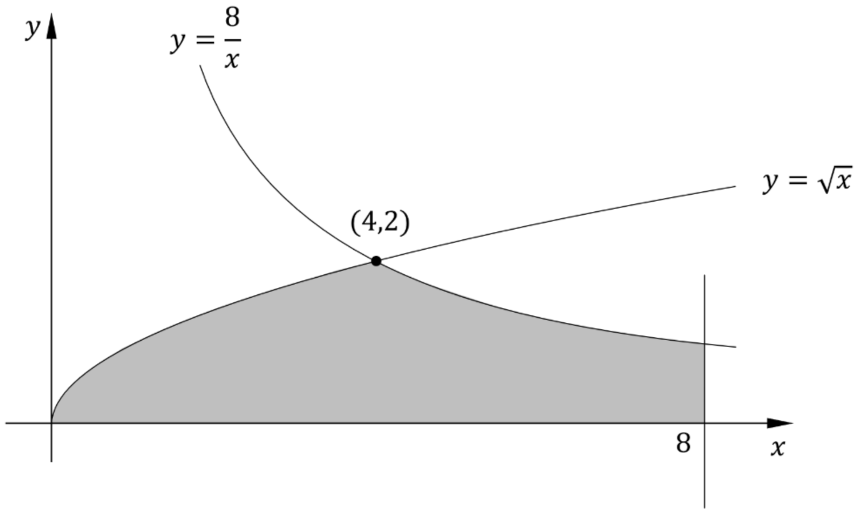
The parabola $y = x^2 - 8x + 17$ is dilated vertically by a factor of 2 and reflected about the y-axis. Find the equation of the transformed parabola in the form $y = a(x+b)^2 + c$, where a, b and c are integers. **3**

Question 30 can be found on page 27

Question 30 (3 marks)

The curve $y = \sqrt{x}$ intersects the curve $y = \dfrac{8}{x}$ at (4,2).

The region bounded by the curve $y = \sqrt{x}$, the curve $y = \dfrac{8}{x}$, the line $x = 8$ and the x-axis is shaded in the diagram.



Show that the exact area of the added region is $\dfrac{8(2 + 3\ln 2)}{3}$. **3**

Question 31 (6 marks)

Consider the function $y = \dfrac{x^2}{(x-1)^2}$

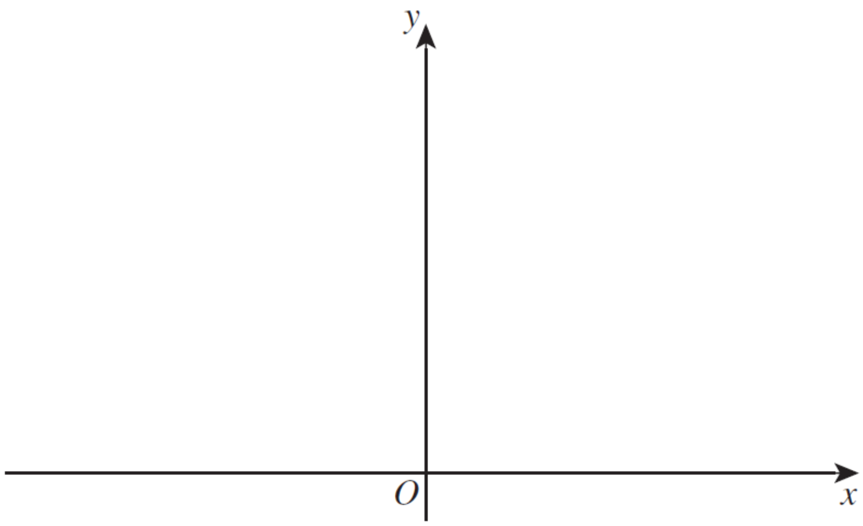
a) Find $\lim\limits_{x \to \infty} \dfrac{x^2}{(x-1)^2}$ **1**

b) Find the domain of the function **1**

c) Find the coordinates of any stationary point(s) of the function **2**

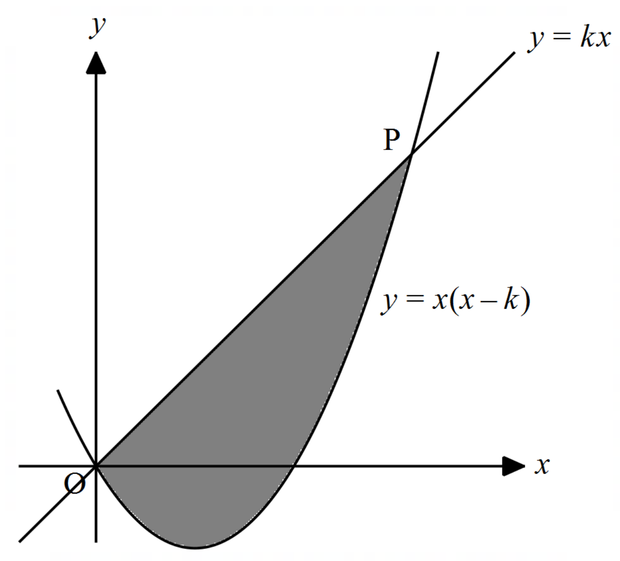
Question 31 will continue on page 29

d) Sketch the graph of $y = \dfrac{x^2}{(x-1)^2}$ on the set of axes below. Show all critical features, including the equations of any asymptotes. **2**



End of Question 31

Question 32 (4 marks)



The diagram shows the graphs of the parabola $y = x(x-k)$ and the line $y = kx$, where $k > 0$ is a constant, intersecting at the origin $O(0,0)$ and the point P.

a) Find, in terms of k, the x-coordinate of the point P. **1**

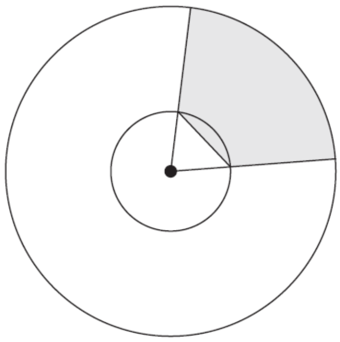
---

b) Find in simplest form, in terms of k, the area of the shaded region enclosed by the parabola and the line. **3**

Question 33 (5 marks)

The diagram shows two circles, one with radius $r\sqrt{r}$ and the other with radius r units.

The shaded segment has an angle of $\frac{\pi}{3}$ subtended at the centre of the circle.



a) Given that $r > 1$, show that the area of the shaded region is $\frac{\pi}{6}r^3 - \frac{\sqrt{3}}{4}r^2$ square units. **2**

Question 33 continues of page 32

b) Find the value of r at which the area of the shaded segment is a minimum. **3**

End of Examination!!! 😊

* Solutions

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2025 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Advanced

- Reading time – 10 minutes
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Section I	**Section II**							**Total**
Q1-10	**11 - 15**	**16 - 19**	**20 - 23**	**24 - 26**	**27- 28**	**29 - 31**	**32-33**	
/10	/14	/14	/13	/15	/13	/12	/9	/100

Section I

10 marks
Attempt Question 1-10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10.

---

1 If $f(x) = 2x + 1$, what is the solution of the equation $f(f(x)) = 11$?

(A) $x = 2$

(B) $x = 3$

(C) $x = 4$

(D) $x = 5$

$2(2x+1)+1 = 11$

$4x+3 = 11$

$4x = 8$

$x = 2$

2 What is $\int (e^{2x} + e^{2x+1})\, dx$? $= \frac{1}{2}e^{2x} + \frac{1}{2}e^{2x+1} + c$

(A) $\dfrac{e^{2x}(3-2e)}{2} + c$

(B) $\dfrac{e^{2x}(3+2e)}{2} + c$

(C) $\dfrac{e^{2x}(1-e)}{2} + c$

(D) $\dfrac{e^{2x}(1+e)}{2} + c$

$= \dfrac{e^{2x} + e^{2x+1}}{2} + c$

3 An arithmetic sequence has a common difference of -2.5 and a fifth term of 15.

What is the tenth term?

(A) -17.5

(B) -14.5

(C) 2.5

(D) 15

$T_n = a + d(n-1)$

$T_5 = a + 4(-2.5) = 15$

$a = 25$

$T_{10} = 25 + 9(-2.5)$

4 If A and B are two independent events such that $P(A) = 0.5$ and $P(B) = 0.4$, what is the value of $P(A \cup B)$?

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.4 - 0.2$

$P(A \cap B) = P(A)\,P(B)$

(A) 0.6

(B) 0.7

(C) 0.8

(D) 0.9

5 The population of a type of bacteria is given by the equation $P = 500e^{0.3t}$, where P is the number of bacteria after t hours.

What is the hourly percentage change in the population?

$e^{0.3} = 1.3498$

$135\% - 100\%$

(A) 0.3

(B) 0.35

(C) 30%

(D) 35%

6 What is the domain of the function $f(x) = \sqrt{x+3} + \ln(4-x)$?

$y = \sqrt{x+3}$ $\quad x+3 \geq 0$

$x \geq -3$

$y = \ln(4-x)$ $\quad 4-x > 0$

$x < 4$

(A) $(-3,4)$

(B) $[-3,4]$

(C) $[-3,4)$

(D) $(-3,4]$

7 The function $y = 2\sin x + 1$ is translated up m units and then vertically dilated by a factor of n from the x-axis, where m and n are positive numbers.
What is the range of the new curve?

$y \to y - m$: $y - m = 2\sin x + 1$

$y = 2\sin x + m + 1$

(A) $[mn - 2n, mn + n]$

(B) $[mn - n, mn + 3n]$

$y \to \frac{y}{n}$: $y = 2n\sin x + mn + n$

(C) $[mn + n, mn + 3n]$

Amp: $2n$

(D) $[mn + 2n, mn + n]$

Centre: $mn + n$

8 What is the solution to the equation $x = \log_2 2\sqrt{2} + \log_2 \sqrt{2}$?

(A) $x = 2$

$x = \log_2(2\sqrt{2} \times \sqrt{2})$

(B) $x = 3$

(C) $x = 4$

$= \log_2 4$

(D) $x = 5$

9 What is the derivative of $y = \ln\left(\frac{x-2}{4x+1}\right)$? $= \ln(x-2) - \ln(4x+1)$

(A) $\frac{3x+3}{(x-2)(4x+1)}$

$\frac{d}{dx} = \frac{1}{x-2} - \frac{4}{4x+1}$

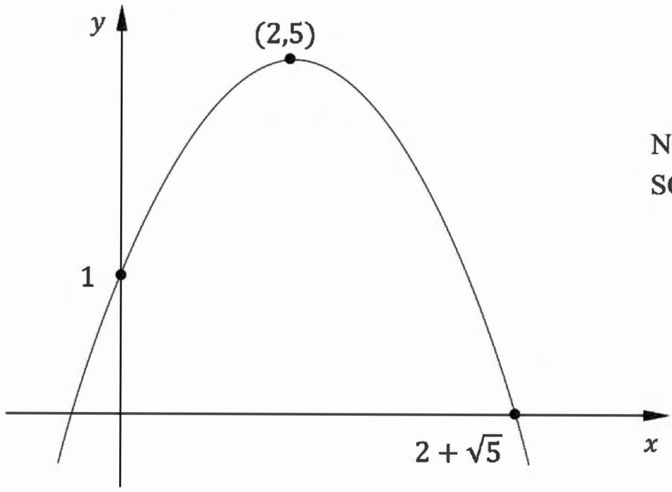
(B) $\frac{9}{(x-2)(4x+1)}$

(C) $\frac{3x-1}{(x-2)(4x+1)}$

$= \frac{4x+1-4(x-2)}{(x-2)(4x+1)}$

(D) $\frac{4x+1}{x-2}$

10 Consider the diagram of the parabola $y = -x^2 + bx + c$ below.



NOT TO SCALE

What are the values of b and c?

(A) $b = 4$, $c = 1$

(B) $b = 2$, $c = 2 + \sqrt{5}$

(C) $b = -2$, $c = 2 + \sqrt{5}$

(D) $b = -4$, $c = 1$

y-int (0,1) $c = 1$

Aos = $2 = \dfrac{-b}{-2}$

$b = 4$

End of Section I

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Marks

Question 11 (2 marks)

Solve $\sin x = -\frac{1}{2}$, where $0 \leq x \leq 2\pi$. Giving the answer in exact form in radians 2

$x = \frac{-\pi}{6}$ 1

$= \frac{11\pi}{6}, \frac{7\pi}{6}$ 1

Question 12 (3 marks)

Use two applications of the Trapezoidal rule to find an approximate value, correct to 2 decimal places for, $\int_1^3 \ln(x^2+1)\,dx$. 3

1 $\frac{1}{2}\left[\ln(1^2+1) + \ln(3^2+1) + 2\ln(2^2+1)\right]$ 1

$= 3.11$ 1

OR

$\frac{1}{2}\left[\ln(1^2+1) + \ln(2^2+1)\right] + \frac{3-2}{2}\left[\ln(2^2+1) + \ln(3^2+1)\right]$

Question 13 (2 marks)

Find $\int 10^{5x-1}\,dx$. 2

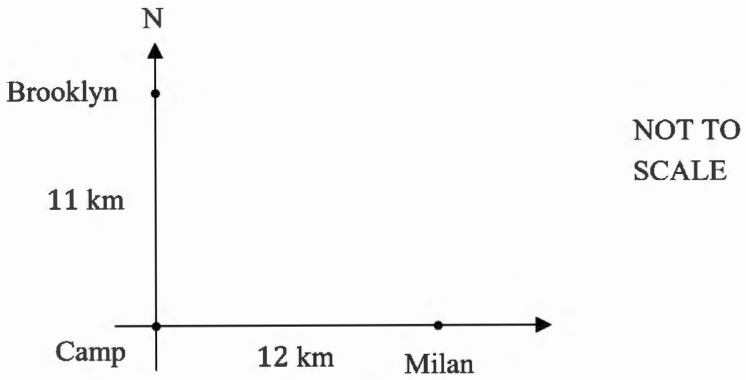
$= \frac{10^{5x-1}}{5\ln 10} + C$

1 MARK FOR:

- $10^{5x-1} + C$
- $\frac{10^{5x-1}}{5} + C$
- $\frac{10^{5x-1}}{\ln 10} + C$

Question 14 (2 marks)

At 9 am, Brooklyn is 11 km due north of camp, while Milan is 12 km due east of camp. Their positions are shown on the diagram below.



They both start walking directly towards camp at a speed of 4 kilometres per hour.

Find the distance between Brooklyn and Milan at 11 am. **2**

At 11am Brooklyn is $11-2\times4 = 3$ km away

Milan is $12-2\times4 = 4$ km away

$d = \sqrt{3^2+4^2}$ 1

$= 5$ km 1

Question 15 (5 marks)

A group of medical researchers conducted an investigation to study the impact of different dosages of a new drug on the duration of relief from an allergic reaction.

An experiment was carried out using a random sample of seven patients. The data collected is recorded in the table.

Dosage (mL)	3	4	4	5	7	7	9
Duration of relief (hours)	5	9	12	9	18	22	22

a) Find the equation of the least-squares regression line. Give your answer in the form $y = Ax + B$, where A & B are correct to 2 decimal places. **2**

$A = -2.54$ $B = 2.94$ 1

$y = 2.94x - 2.54$ 1

b) By finding Pearson's correlation coefficient, describe the strength and direction of the relationship between the dosage and the duration of relief. **2**

$r = 0.93$ 1

As r is positive and close to 1 the relationship is strong positive 1

c) Using the equation found in part (a), estimate the hours of relief with a dosage of 4.3 mL. Give your answer correct to the nearest hour. **1**

$y = 2.94 \times 4.3 - 2.54$

$= 10$ hours 1

Question 16 (3 marks)

Find the values of x for which the function $f(x) = e^{5x^3 - 10x}$ is increasing. **3**

Increasing at $f'(x) > 0$

$f'(x) = (15x^2 - 10)\, e^{5x^3 - 10x} > 0$ 1

$= 5(3x^2 - 2)\, e^{5x^3 - 10x} > 0$

$e^{5x^3 - 10x} > 0$ for all x

$3x^2 - 2 > 0$ 1

$x = \pm\sqrt{\frac{2}{3}}$

$\therefore \left(-\infty, -\sqrt{\frac{2}{3}}\right) \cup \left(\sqrt{\frac{2}{3}}, \infty\right)$ 1

Question 17 (2 marks)

Evaluate in simplest form $\int_0^{\frac{\pi}{2}} \sin 2x \, dx$. **2**

$\left[\frac{-1}{2}\cos 2x\right]_0^{\pi/2}$ 1

$= -\frac{1}{2}\cos\pi - -\frac{1}{2}\cos 0$

$= 1$ 1

Question 18 (4 marks)

Differentiate the following with respects to x.

a) $\dfrac{3x + 2}{x + 4}$ **2**

$$\frac{d}{dx} = \frac{3(x+4) - (3x+2)}{(x+4)^2} \quad 1 \qquad u = 3x+2 \quad v = x+4$$

$$u' = 3 \quad v' = 1$$

$$= \frac{3x + 12 - 3x - 2}{(x+4)^2}$$

$$= \frac{10}{(x+4)^2} \quad 1$$

b) $x(1 + 2x)^4$ **2**

$$\frac{d}{dx} = 8x(1+2x)^3 + (1+2x)^4 \quad 1 \qquad u = x \quad v = (1+2x)^4$$

$$u' = 1 \quad v' = 8(1+2x)^4$$

$$= (1+2x)^3(8x + 1 + 2x)$$

$$= (10x+1)(1+2x)^3 \quad 1$$

Question 19 (5 marks)

Finn is working in the garden when he has an accident and puts a hole in the bottom of his rainwater tank. At the time of the accident, the tank contained 10 200 litres of water.

In the first hour after the accident, Finn noticed that the volume of water in the tank decreased by 200 litres.

In the second hour, he noticed that the volume of water in the tank decreased by a further 250 litres.

Finn is concerned that the leak is growing, but he is not sure what will happen next. He wonders if the amount of water leaking from the tank each hour might form an arithmetic sequence or geometric sequence.

a) If the amount of water that leaks out of the tank each hour forms an arithmetic sequence, how many hours will it take until the tank is empty? **3**

$a = 200 \qquad d = 50 \qquad S_n = 10\ 200$

$10\ 200 = \frac{n}{2}\left[2(200) + (n-1) \times 50\right]$ 1

$20\ 400 = n\left[400 + 50n - 50\right]$

$20\ 400 = 350n + 50n^2$

$n^2 + 7n - 408 = 0$ 1

$$n = \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times (-408)}}{2}$$

$n = -24$ or 17

$\therefore$ Tank empty after 17 hours 1

Question 19 continues of page 15

b) If the amount of water that leaks out of the tank each hour forms a geometric sequence, show that eight hours after the leak, over 60% of the water remains in the tank 2

$a = 200 \quad r = \frac{250}{200} = 1.25 \quad n = 8$

$$S_8 = \frac{200(1.25^8 - 1)}{1.25 - 1} \quad 1$$

$= 3968.37$ L has leaked out

$10\,200 - 3968.37 = 6231.63$ L

$$\frac{6231.63}{10\,200} \times 100 = 61.09\% \quad 1$$

$\therefore$ After 8 hours more than 60% of water remain

Question 20 can be found on page 16

Question 20 (4 marks)

a) Prove that $\sec x + \tan x = \dfrac{\cos x}{1-\sin x}$. **2**

$$\text{LHS: } \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$= \frac{1 + \sin x}{\cos x}$$

$$1 \quad = \frac{(1+\sin x)(1-\sin x)}{\cos x\,(1-\sin x)}$$

$$= \frac{1-\sin^2 x}{\cos x\,(1-\sin x)}$$

$$1 \quad = \frac{\cos^2 x}{\cos x\,(1-\sin x)} = \frac{\cos x}{1-\sin x}$$

b) Hence, or otherwise, find the exact value of $\int_0^{\frac{\pi}{3}} \sec x + \tan x \, dx$. **2**

$$\int_0^{\frac{\pi}{3}} \frac{\cos x}{1-\sin x}\, dx = -\left[\ln|1-\sin x|\right]_0^{\pi/3} \quad 1$$

$$= -\ln\left|1-\sin \pi/3\right| + \ln|1-\sin 0|$$

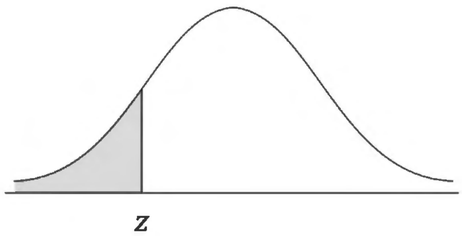
$$= -\ln \frac{2-\sqrt{3}}{2} \quad \text{or} \quad \ln \frac{2}{2-\sqrt{3}} \quad 1$$

Question 21 (3 marks)

A random variable is normally distributed with a mean of 0 and a standard deviation of 1. The table gives the probability that this random variable lies below z for some negative values of z.

z	−1.0	−0.9	−0.8	−0.7	−0.6	−0.5	−0.4	−0.3	−0.2	−0.1	0.0
Probability	0.1587	0.1841	0.2119	0.242	0.2743	0.3085	0.3446	0.3821	0.4207	0.4602	0.5000

The probability values given in the table are represented by the shaded area in the following diagram.



The time it takes for a worker to drive from home to work is normally distributed with a mean of 32 minutes and a standard deviation of 4 minutes.

a) What is the probability that on a particular day the drive takes between 30 and 32 minutes? **2**

$Z_{30} = -0.5$ $\mu = 32$ $\sigma = 4$ 1

$P(30 < t < 32) = P(z < 0) - P(z < -0.5)$

$= 0.5 - 0.3085$

$= 0.1915$ 1

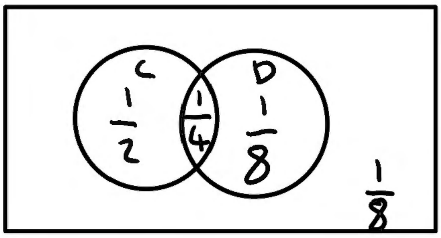
$= 19.5\%$

b) Explain why the probability that the drive takes between 30 and 34 minutes is twice your answer to part (a). **1**

Symmetry: $P(30 < x < 34) = 2P(30 < x < 32)$

Question 22 (4 marks)

A survey of all the Zoology students at the local university showed that $\frac{3}{4}$ of them had a pet cat, $\frac{3}{8}$ of them had a pet dog and $\frac{1}{4}$ of them had both a pet cat and a pet dog. 25 of the students had either a pet cat or a pet dog but not both types of pet.



a) Show that a total of 40 Zoology students where surveyed **2**

$\left(\frac{1}{2} + \frac{1}{8}\right)n = \frac{5}{8}n$ 1

$n = 25 \times 8/5 = 40$ students 1

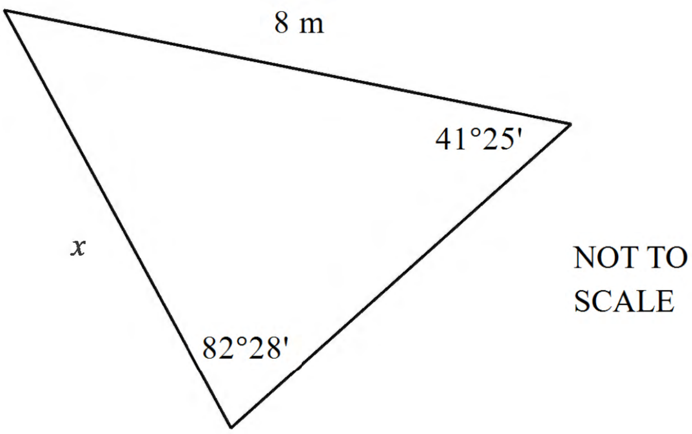
b) If two of the Zoology students are chosen at random, find in simplest fraction form the probability that neither of these students has a pet dog **2**

$5/8 \times 40 = 25$ do not have pet dog 1

$P(\text{neither has pet dog}) = \frac{25}{40} \times \frac{24}{39} = \frac{5}{13}$ 1

Question 23 (2 marks)

A triangle has one side that is 8 metres in length with the angle opposite being $82°28'$. A second angle is $41°25'$, with the side opposite this angle being x. **2**



Find the length of x, correct to the nearest metre.

$$\frac{x}{\sin 41° 25'} = \frac{8}{\sin 82° 28'}$$ 1 Attempt sine rule

$x = 5$ 1

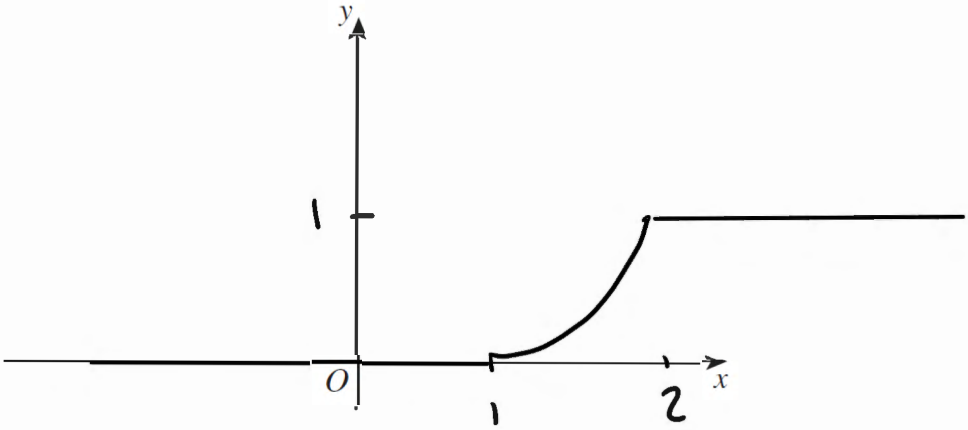
Question 24 can be found on page 20

Question 24 (7 marks)

The lifetime, X, in tens of hours, of a battery has a cumulative distribution function $F(x)$ that is given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{25}{116}(x^2 + 3x - 4) & 1 < x < 1.8 \\ 1 & x \geq 1.8 \end{cases}$$

a) Sketch the cumulative distribution function. **2**



b) An outdoor lantern requires three batteries, all of which must be working, to operate. Three new batteries are inserted into the lantern.

Find the probability that the lantern will still be working after 15 hours. Give your answer as a percentage, rounded to two decimal places. **3**

$P(X > 1.5) = 1 - P(X \leq 1.5) = 1 - P(1.5)$ ①

$1 - \frac{25}{116}(1.5^2 + 3 \times 1.5 - 4) = 189/464$ ①

$\left(\frac{189}{464}\right)^3 = 6.76\%$ ①

Question 24 continues on page 21

APPROXIMATELY HALFWAY – 56 marks out of 100 complete at this point.

c) Find the probability density function of the random variable X. **2**

$f(x) = \frac{d}{dx} F(x) = \frac{25}{116}\left(x^2 + 3x - 4\right)$

$= \frac{25}{116}(2x+3)$ 1

$\therefore f(x) = \begin{cases} \frac{25}{116}(2x+3) & 1 < x < 1.8 \\ 0 & \text{elsewhere} \end{cases}$ 1

Question 25 (4 marks)

The population of mosquitoes in a garden varies throughout the year according to

the formula $P = 60 + 50\cos\left[\frac{\pi}{6}t\right]$

where t is the time in months since the start of January.

a) Find the maximum and minimum mosquito population during the year. **2**

Amp: 50 Centre: 60 1

min: $60 - 50 = 10$ 1

Max: $60 + 50 = 110$

b) Find the times in the year when $P = 85$ **2**

$60 + 50\cos\left(\frac{\pi}{6}t\right) = 85$

$50\cos\left(\frac{\pi}{6}t\right) = 25$

$\cos\left(\frac{\pi}{6}t\right) = \frac{1}{2}$ 1

$\frac{\pi}{6}t = \frac{\pi}{3}$ or $\frac{5\pi}{3}$ $t = 2$ or 10 1

Question 26 (4 marks)

Point $P(-1,-2)$ lies on $f(x)$. Point P' is the image of point P on the function $g(x) = 2f(3x-1)$.

a) By clearly stating the transformations required to produce $g(x)$ from $f(x)$, show that the coordinates of point P' are $(0,-4)$. **2**

$2f(x)$: vertical dilation: SF = 2. New point $(-1,-4)$ 1

$2f(x-1)$: Translation right 1. New point $(0,-4)$

$2f(3x-1)$: horizontal dilation. SF = 1/3 new point $(0,-4)$ 1

b) If the equation of the tangent at point P on *f(x)* is $y = x-1$, find the equation of the tangent at point P' on *g(x)*. **2**

$g'(x) = 2f'(3x-1) \times 3$

$= 6f'(3x-1)$

$g'(0) = 6f'(-1)$ 1

Given tangent at point P is $y = x-1$

$f'(-1) = 1$, $g'(0) = 6f'(-1) = 6 \times -1 \times -1 = 6$

Equation of tangent at $(0,-4)$ is:

$y-(-4) = 6x$ 1

$y = 6x-4$

Question 27 (5 marks)

A particle moves along the x-axis, where left is in a positive direction. After t seconds its displacement, x metres, is given by the equation $x = (t^2 - 1)^2$.

a) Show that the acceleration of the particle is given by the equation **2**

$$\ddot{x} = 12t^2 - 4$$

$x = (t^2 - 1)^2$

$x = t^4 - 2t^2 + 1$ 1

$\dot{x} = 4t^3 - 4t$

$\ddot{x} = 12t^2 - 4$ 1

b) The particle initially moves left, before turning around and moving to the right. **3**

Find the particle's acceleration when it returns to its initial position.

When $t = 0$, $x = 1$

let $x = 1$, $1 = (t^2 - 1)^2$ 1

$t^2 - 1 = \pm 1$

$t^2 = 0, 2$

$t = 0$ or $\sqrt{2}$ $(t \geq 0)$ 1

Let $t = \sqrt{2}$

$\ddot{x} = 12(\sqrt{2})^2 - 4 = 20$ m/s^2 1

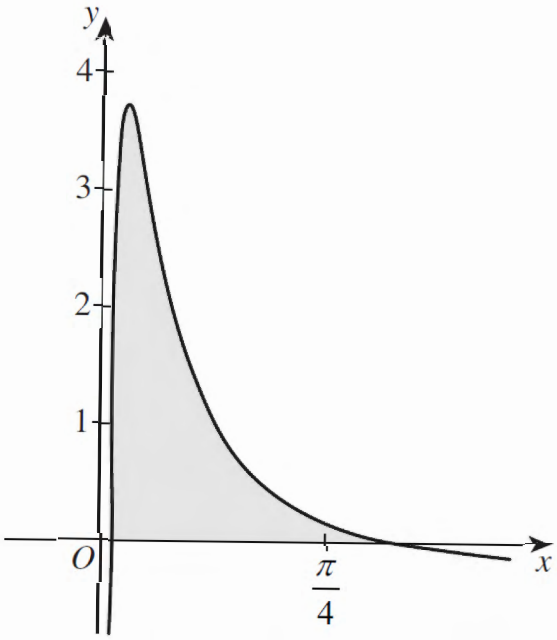
Question 28 (8 marks)

a) Differentiate $\frac{d}{dx}\cos(\ln x)$. **2**

$= -\sin(\ln x) \times \frac{1}{x}$ 1

$= \frac{-\sin \ln x}{x}$ 1

b) The graph of $y = \frac{-\sin(\ln x)}{kx}$ for $0 < x \leq 1$ is shown below.



Show that one of the x-intercepts has a value of $e^{-\pi}$. **2**

$\frac{-\sin(\ln x)}{x} = 0$

$\sin(\ln x) = 0$ 1

$\ln x = -\pi, 0$

$x = e^{-\pi}, 1$ 1

Question 28 continues on page 25

c) The probability density function of a random variable, x, is given by

$$f(x) = \begin{cases} \dfrac{-\sin(\ln x)}{kx} & \text{for } e^{-\pi} \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

i) Show that $k = 2$. **2**

$$\int_{e^{-\pi}}^{1} \frac{-\sin(\ln x)}{kx}\,dx = 1$$

$$\frac{1}{k}\int_{e^{-\pi}}^{1} \frac{-\sin(\ln x)}{x}\,dx = 1$$

$$\frac{1}{k}\Big[\cos(\ln x)\Big]_{e^{-\pi}}^{1} = 1 \quad 1$$

$$k = \cos(\ln 1) - \cos(\ln e^{-\pi})$$

$$= 1 - (-1) = 2 \quad 1$$

ii) Find $P\left(x < \frac{\pi}{4}\right)$. Give your answer correct to three decimal places. **2**

$$\int_{e^{-\pi}}^{\pi/4} \frac{-\sin(\ln x)}{2x}\,dx$$

$$= \frac{1}{2}\Big[\cos(\ln x)\Big]_{e^{-\pi}}^{\pi/4} \quad 1$$

$$= \frac{1}{2}\left(\cos\left(\ln \frac{\pi}{4}\right) - \cos\left(\ln e^{-\pi}\right)\right)$$

$$= 0.985 \quad 1$$

End of Question 28

APPROXIMATELY THREE QUARTERS COMPLETE – 79 marks out of 100 complete

Question 29 (3 marks)

The parabola $y = x^2 - 8x + 17$ is dilated vertically by a factor of 2 and reflected about the y-axis.

Find the equation of the transformed parabola in the form $y = a(x+b)^2 + c$, where a, b and c are integers. **3**

Vertically dilated by a factor of 2:

$\frac{y}{2} = x^2 - 8x + 17$

$= 2x^2 - 16x + 34$ 1

Reflected about y-axis:

$y = 2(-x)^2 - 16(-x) + 34$

$= 2x^2 + 16x + 34$ 1

Completing the square

$y = 2(x^2 + 8x + 17)$

$= 2((x+4)^2 + 1)$

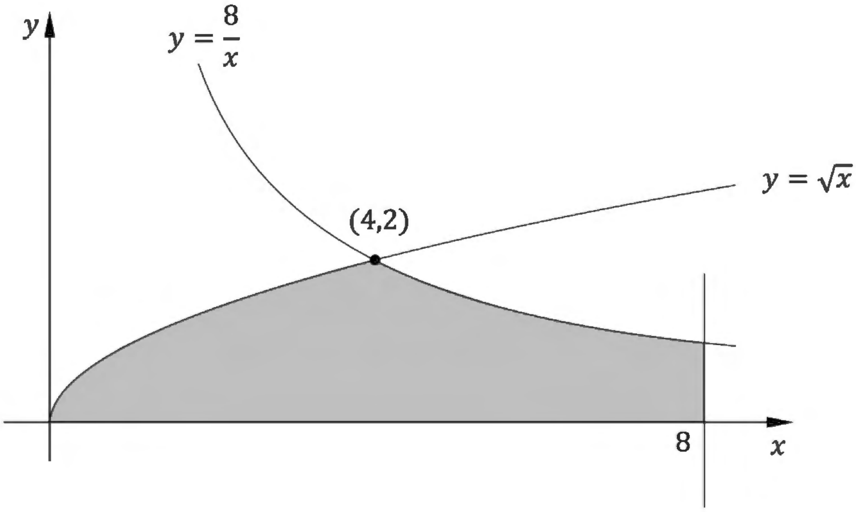
$= 2(x+4)^2 + 2$ 1

Question 30 can be found on page 27

Question 30 (3 marks)

The curve $y=\sqrt{x}$ intersects the curve $y=\frac{8}{x}$ at $(4,2)$.

The region bounded by the curve $y=\sqrt{x}$, the curve $y=\frac{8}{x}$, the line $x=8$ and the x-axis is shaded in the diagram.



Show that the exact area of the added region is $\frac{8(2+3\ln 2)}{3}$. **3**

$$A=\int_0^4 x^{1/2}\,dx+\int_4^8 \frac{8}{x}\,dx \quad 1$$

$$=\left[\frac{2}{3}x^{3/2}\right]_0^4+8\left[\ln x\right]_4^8$$

$$=\frac{2}{3}(8-0)+8(\ln 8-\ln 4) \quad 1$$

$$=\frac{16}{3}+8\ln 2$$

$$=\frac{16+24\ln 2}{3}=\frac{8(2+3\ln 2)}{3}$$

1

Question 31 (6 marks)

Consider the function $y = \dfrac{x^2}{(x-1)^2}$

a) Find $\lim\limits_{x \to \infty} \dfrac{x^2}{(x-1)^2}$ **1**

$$\lim_{x\to\infty} \frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2} - \frac{2x}{x^2} + \frac{1}{x^2}} = \lim_{x\to\infty} \frac{1}{1 - \frac{2x}{x^2} + \frac{1}{x^2}}$$

$$= \frac{1}{1-0+0} = 1$$

b) Find the domain of the function **1**

All real x, $x \neq 1$

c) Find the coordinates of any stationary point(s) of the function **2**

Stat point $y' = 0$

$$y' = \frac{2x(x-1)^2 - 2x^2(x-1)}{(x-1)^4}$$ 1

$$= \frac{2x(x-1)(x-1-x)}{(x-1)^4}$$

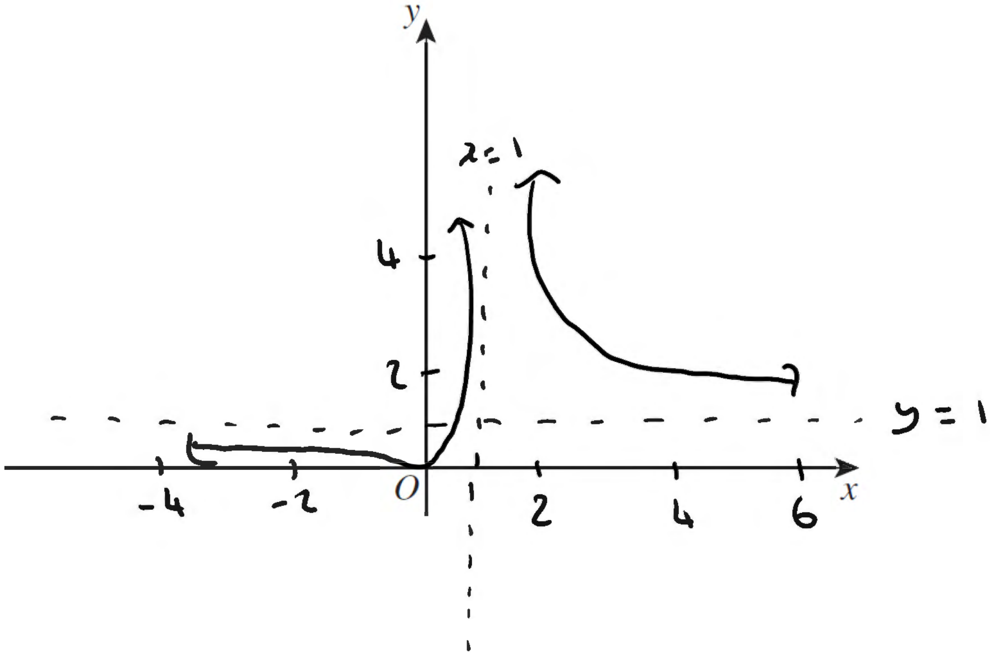
$$= \frac{-2x}{(x-1)^3} = 0$$

x	-0.1	0	0.1
y'	-0.15	0	0.27
	$\searrow$	$-$	$\nearrow$

when $x = 0$, $y = 0$ $\therefore (0,0)$ = stat point 1

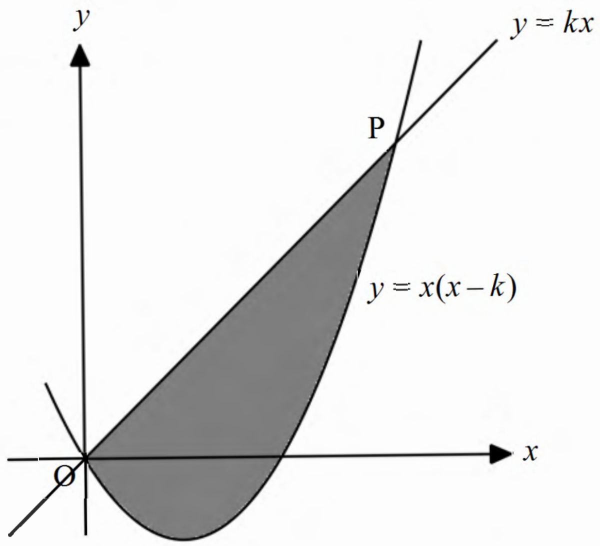
Question 31 will continue on page 29

d) Sketch the graph of $y = \dfrac{x^2}{(x-1)^2}$ on the set of axes below. Show all critical features, including the equations of any asymptotes. **2**



End of Question 31

Question 32 (4 marks)



The diagram shows the graphs of the parabola $y = x(x-k)$ and the line $y = kx$, where $k > 0$ is a constant, intersecting at the origin $O(0,0)$ and the point P.

a) Find, in terms of k, the x-coordinate of the point P. **1**

At P, $x(x-k) = kx$

$x(x-2k) = 0$ $\therefore$ x-coordinate of $2k$

b) Find in simplest form, in terms of k, the area of the shaded region enclosed by the parabola and the line. **3**

$A = \int_0^{2k} kx - x(x-k)\ dx$ 1

$= \int_0^{2k} (2kx - x^2)\ dx$

$= \left[kx^2 - \frac{1}{3}x^3\right]_0^{2k}$ 1

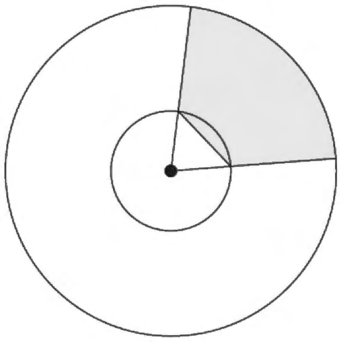
$= 4k^3 - \frac{1}{3}8k^3$

$A = \frac{4}{3}k^3$ units^2 1

Question 33 (5 marks)

The diagram shows two circles, one with radius $r\sqrt{r}$ and the other with radius r units.

The shaded segment has an angle of $\frac{\pi}{3}$ subtended at the centre of the circle.



a) Given that $r > 1$, show that the area of the shaded region is $\frac{\pi}{6}r^3 - \frac{\sqrt{3}}{4}r^2$ square units. **2**

$A = \frac{1}{2}r^2\theta - \frac{1}{2}ab\sin C$

$= \frac{1}{2}(r\sqrt{r})^2 \times \frac{\pi}{3} - \frac{1}{2} \times r \times r \times \sin\frac{\pi}{3}$ 1

$= \frac{r^3}{2} \times \frac{\pi}{3} - \frac{1}{2} \times r^2 \times \frac{\sqrt{3}}{2}$

$= \frac{\pi}{6}r^3 - \frac{\sqrt{3}}{4}r^2$ 1

Question 33 continues of page 32

b) Find the value of r at which the area of the shaded segment is a minimum. **3**

Min $A' = 0$

$A' = \frac{1}{2} r(\pi r - \sqrt{3}) = 0$ 1

$r = 0$ or $\frac{\sqrt{3}}{\pi}$ $(r > 0)$ 1

$A'' = \pi r - \frac{\sqrt{3}}{2}$

When $r = \sqrt{3}/\pi$, $A'' = \pi\left(\frac{\sqrt{3}}{\pi}\right) - \frac{\sqrt{3}}{2} > 0$ 1

$A'' > 0$ $\therefore$ min

End of Examination!!! 😊