NESA No | | | | | | | | | |

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2022

YEAR 12

AP4

MATHEMATICS ADVANCED

Time allowed – 3 hours plus 10 minutes reading time

General Instructions	• Attempt all questions • Write your student NESA number on the question paper • Write using black pen • NESA approved calculators may be used • The NESA reference sheet has been provided • Answer the questions in the spaces provided • For questions in Section II, show relevant mathematical reasoning and/or calculations
Total marks: 100	**Section I – 10 marks (pages 3 – 7)** • Attempt Questions 1-10 • Allow about 15 minutes for this section **Section II – 90 marks (pages 8 – 29)** • Attempt Questions 11-31 • Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 Let $f\ (x) = \frac{1}{x}, x \neq 0$ and $g\ (x) = x + 2$. Find the domain of $f(g(x))$.

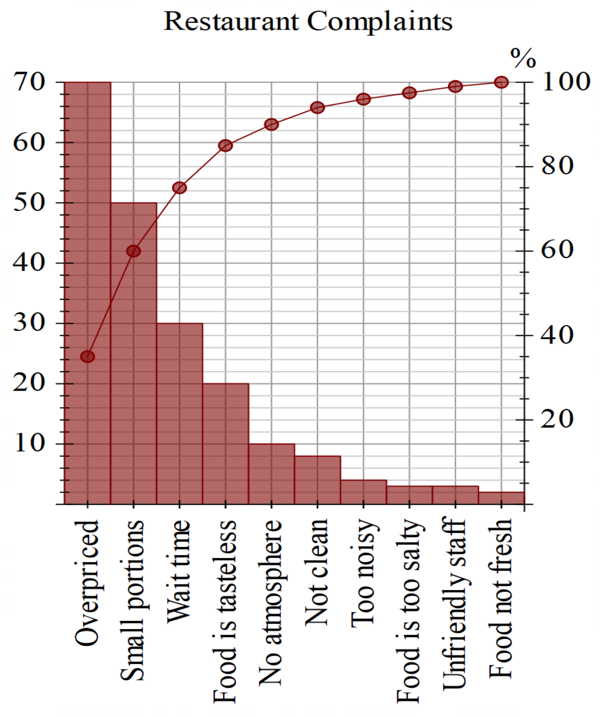
(A) All real x

(B) $x\ \neq -2$

(C) $x\ \neq 0$

(D) $x = -2$

2 A restaurant collected data related to the reasons given by customers for complaints. The Pareto chart shows the data collected.



What percentage of customers gave the reason 'Wait time'?

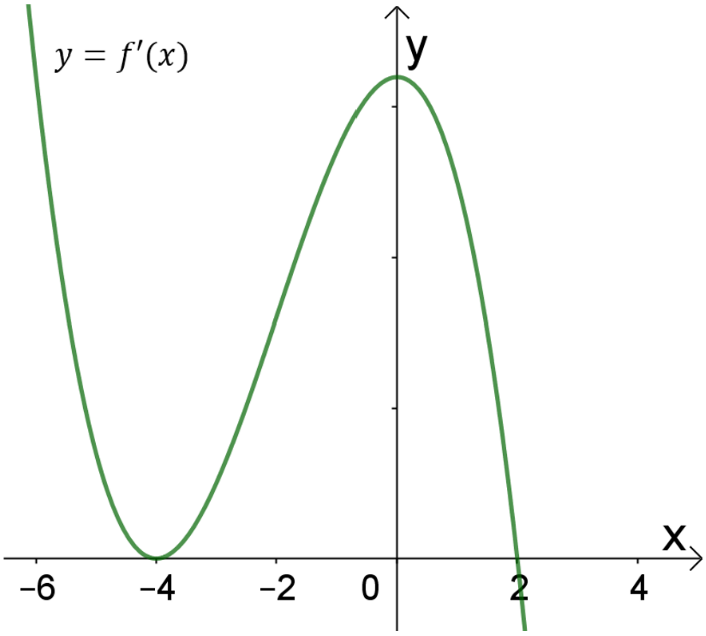
(A) 15%

(B) 30%

(C) 60%

(D) 75%

3 The diagram below shows a sketch of the gradient function of the curve $y = f(x)$.



Which of the following is a true statement?

The curve $y = f(x)$ has

(A) a minimum turning point at $x = -4$

(B) a horizontal point of inflection at $x = 2$

(C) a horizontal point of inflection at $x = -4$

(D) a minimum turning point at $x = 2$

4 The point $P(8, -3)$ lies on the graph of $f(x)$. Transformations map the graph of $f(x)$ to the graph of g(x), where $g(x) = -2f(x + 7) + 5$. The same transformation maps the point *P* to the point *P'*. What are the coordinates of the point *P'*?

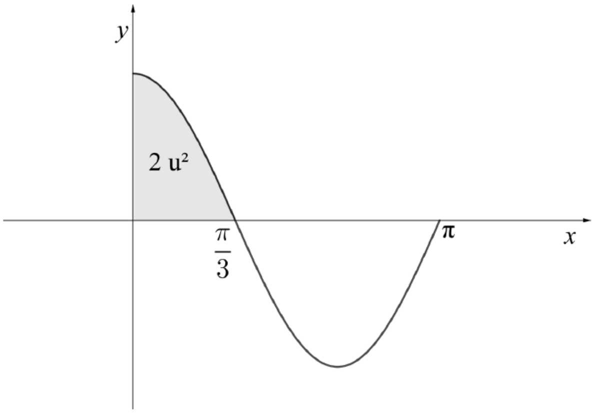
(A) $(1, 11)$

(B) $(1, -1)$

(C) $(15, 11)$

(D) $(15, -1)$

5 The diagram below shows the graph of $f(x) = a\cos bx$.



The area of the shaded region is equal to 2 square units. What is the value of $\int_0^{\pi} f(x)\ dx$?

(A) -2

(B) -4

(C) 2

(D) 6

6 What are the values of x for which $|5 - 3x| \geq 11$?

(A) $x \leq -2$ or $x \leq \dfrac{16}{3}$

(B) $x \leq -2$ and $x \geq \dfrac{16}{3}$

(C) $x \leq -2$ and $x \leq \dfrac{16}{3}$

(D) $x \leq -2$ or $x \geq \dfrac{16}{3}$

7 Initially, a water tank holds 800 litres of water. Water drains from the bottom of the tank at a rate of $\frac{dV}{dt}$ litres per minute where $\frac{dV}{dt} = -100t$ after t minutes. How long does it take the tank to empty?

(A) 2 minutes

(B) 4 minutes

(C) 6 minutes

(D) 8 minutes

8 What is the equation of the axis of symmetry of the quadratic function
$y = -k\ (x + b)^2 + c$?

(A) $x = -b$

(B) $x = b$

(C) $x = 2a$

(D) $x = 2ak$

9 How many solutions are there to the equation $2 - x = 4\ \cos x$? $(0\ \leq x \leq 2\pi)$

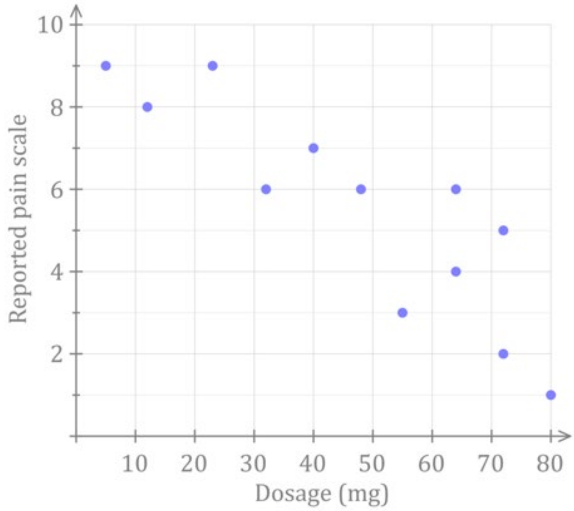
(A) 0

(B) 1

(C) 2

(D) 3

10 A scatterplot of pain (as reported by patients) compared to the dosage (in mg) of a drug is shown below.



Which of the following describes the correlation between the pain and the dosage?

(A) A weak negative correlation.

(B) A strong positive correlation.

(C) A strong negative correlation.

(D) A weak positive correlation.

End of Section I

Section II

90 marks
Attempt Questions 11 – 31
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet.
  If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks) **Marks**

Solve $x + \dfrac{x-4}{3} = 2.$ **2**

Question 12 (4 marks) **Marks**

A continuous random variable, T, represents the time taken in days to show symptoms after contracting a virus. It has the following probability density function.

$$f(x) = \begin{cases} \dfrac{k}{2t-1} & for\ 1 \le t \le 14 \\ 0 & for\ 0 \le t < 1\ \ or\ t > 14 \end{cases}$$

(a) Show that $k = \frac{2}{\ln 27}$. **2**

(b) After how many days will a person have a 75% chance of showing symptoms after they have contracted the virus? **2**

Question 13 (3 marks) **Marks**

(a) Show that $\frac{\sec\theta - \sec\theta cos^4\theta}{1+cos^2\theta} = \sin\theta\tan\theta$ **2**

(b) What are the possible values of a if $1 + 2a + 4a^2 + 8a^3 + \cdots$ is to have a limiting sum? **1**

Question 14 (5 marks)

(a) Find the equation of the normal to the curve $y = x^2 + 4x - 7$ at the point (1,–2) **2**

(b) The curve $y = f(x)$ has a gradient function $f'(x) = 3x^2 - k$, where k is a constant.

(i) Find the value of k if the curve has a stationary point at $(-1,3)$. **1**

(ii) Hence, find the equation of the curve $y = f(x)$. **2**

Question 15 (5 marks)

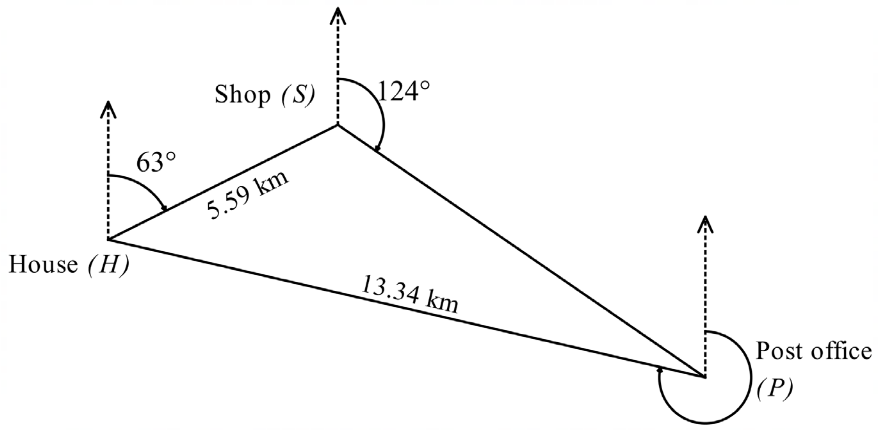
A curve is defined by $f(x) = \frac{1}{4}x^4 - 2x^2 + 1$. **5**

Sketch the graph of the curve for the domain $[-3, 3]$ showing all key features including, classifying the nature of stationary points, and finding any points of inflection. Also state where the curve is concave down. You **do not** need to find the x-intercepts

Space is provided on the next page to draw the graph.

Question 16 (3 marks)

Robert walks 5.59 km from his house to the shop on a bearing of $063°$. He then turns on a bearing of $124°$ to walk to the post office, where he is 13.34 km from home.

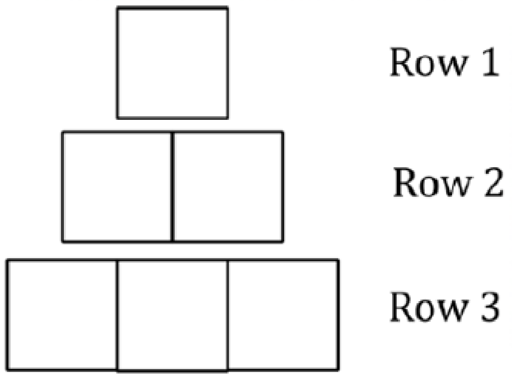


(a) Show that $\angle HSP = 119°$. **1**

(b) Find the bearing of Robert's house from the post office. Give your answer correct to the nearest degree. **2**

Question 17 (6 marks)

James has some sticks that are all of the same length. He arranges them in squares and has made the following 3 rows of patterns.



He notices that 4 sticks are required to make the single square in the first row, 7 sticks in the second row and in the third row he needs 10 sticks to make 3 squares.

(a) Find an expression, in terms of n, for the number of sticks required to make a similar arrangement of n squares in the n^{th} row. **1**

(b) James continues to make squares following the same pattern. He makes 4 squares in the fourth row and so on until he has completed 10 rows. Find the total number of sticks James uses in making these 10 rows. **2**

(c) James started with 1750 sticks. Given that James continues the pattern to complete k rows but does not have sufficient sticks to complete the $(k+1)^{\text{th}}$ row. Show that k satisfies $(3k-100)(\ k+35) < 0$. **2**

(d) What is the value of k? **1**

Question 18 (5 marks) **Marks**

(a) Find the value of k such that **3**

$$\int_{-2}^{0} \frac{x^2}{x^3-2}\, dx = \ln k$$

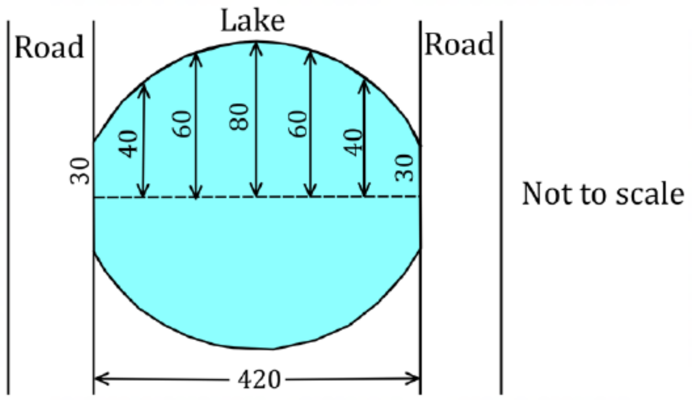
(b) **2**

The sign for 'Pizza π' restaurant is in the shape of a circle with a sector cut out of it. The sector has an angle $\frac{\pi}{3}$. If the area of the sign is 30π, what is the radius of the circle?

Question 19 (3 marks) **Marks**

(a) For what values of k does the quadratic equation $5x^2 - 2x + (8k - 15) = 0$ have real roots? **1**

(b) A symmetrical lake has two roads, 420 metres apart, forming two of its sides. **2**



Equally spaced measurements of the lake, in metres are shown on the above diagram. Use the trapezoidal rule to estimate the area of the lake.

Question 20 (4 marks) **Marks**

Differentiate with respect to x

(a) $e^{x \sin 2x}$ **2**

(b) $\dfrac{x^2}{\ln(x)}$ **2**

Question 21 (5 marks) **Marks**

A particle is moving in a straight line. Initially, it is travelling to the left at 1 cm/min. Its acceleration as a function of time(t) is given by $a = \pi \ \cos \pi t + \pi \ \sin \pi t$ for $0 \leq t \leq 2$ where time and displacement are measured in minutes and cm respectively.

(a) Find when the particle changes its direction for the first time. **2**

(b) Find the exact total distance travelled in the first half a minute. **3**

Question 22 (4 marks) **Marks**

Integrate the following:

(a) $\int x^4\,(x^5-3)^2 dx =$ **2**

(b) $\int (\sqrt{x}-\frac{2}{x}+3e^{-x}+1\,)dx =$ **2**

Question 23 (3 marks) **Marks**

(a) Show that the derivative of $\ln\left(\frac{3+x}{3-x}\right)$ is $\frac{6}{9-x^2}$. **2**

(b) Hence or otherwise find $\int \frac{1}{9-x^2}\,dx$. **1**

Question 24 (3 marks)

Given that $\int_0^a 5\ \sin 3x\ dx = \frac{10}{3}, 0\ \leq a\ <\ \pi$, calculate the value of a. **3**

Question 25 (3 marks)

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Solve $2\cos\left[2(x-\frac{\pi}{6})\right] = 1$ in the domain $[0, 2\pi]$. **3**

Question 26 (5 marks) **Marks**

The probability distribution of a discrete random variable X is shown.

x	-1	0	1	2
$\mathrm{P}(X = x)$	t^2	t^2	$\frac{t}{4}$	$\frac{4t+1}{8}$

(a) What is the value of t? **3**

(b) Find the expected value of the probability distribution. **2**

Question 27 (4 marks) **Marks**

(a) The probability that a person chosen at random has green eyes is 0.04
Two people are chosen at random.

(i) What is the probability that at least ONE person has green eyes? **2**

____

____

____

(ii) What is the smallest number of people that can be chosen at random so that the probability that at least ONE has green eyes is greater than 0.4? **2**

____

____

____

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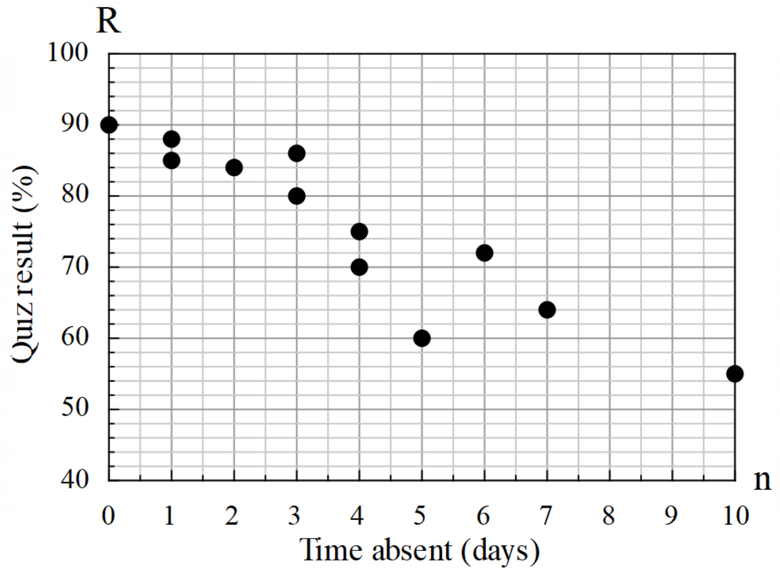
Question 28 (5 marks) **Marks**

A teacher was investigating the impact of absences on their students' grades in a quiz.
The diagram shows the dataset used in the investigation.



(a) Find the correlation coefficient, correct to two decimal places, and describe the correlation. **2**

____

____

____

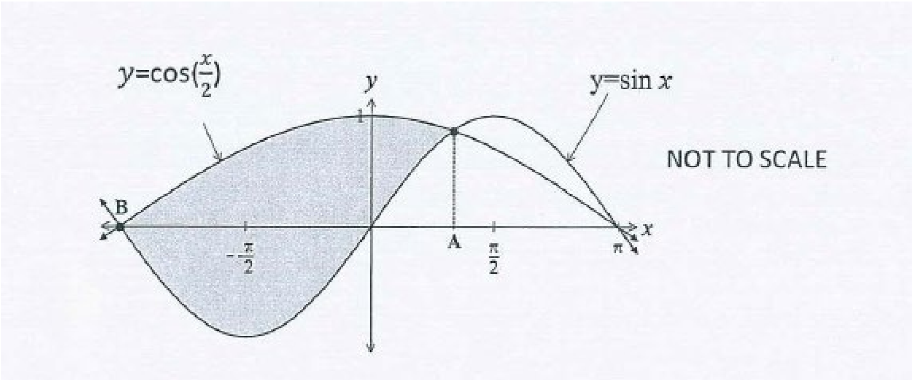
(b) Find the equation of the least-squares regression line in terms of results, R, and number of days absent, n. Give values correct to 2 decimal places. **1**

(c) Comment on the validity of the equation of the least-squares regression line for this context. **1**

(d) The result of a student was not recorded when they were absent for 8 days. Using the equation from part (b), predict the result for a student absent for 8 days, correct to the nearest percentage. **1**

Question 29 (5 marks) **Marks**

A section of the graphs of $y = \sin x$ and $y = \cos\left(\frac{x}{2}\right)$ is shown.



(a) The curves meet at A and B. Show by substitution, that the x values of point A and B are $\frac{\pi}{3}$ and $-\pi$ respectively. **2**

(b) Hence or otherwise, find the exact area of the shaded region. **3**

Question 30 (6 marks) **Marks**

In a laboratory, a food product is heated in an oven at 210° C in an effort to kill harmful bacteria. The number of bacteria recorded when the food product is first placed in the oven is 1800. After ten minutes the number of bacteria recorded is 550. It is found that N, the number of bacteria remaining t minutes after being in the oven is given by $N = A\,e^{-kt}$ where A and k are constants.

(a) Find the value of A. **1**

(b) Calculate the value of k correct to 4 decimal places. **2**

(c) If an acceptable number of bacteria present is 100 or less, For how long should the food product to be in the oven? Express your answer to the nearest minute. **3**

Question 31 (7 marks)

After a week of heavy rain, the local dam starts to fill until at 10am on Sunday the dam overflows into the river. At time t (hours after the dam begins spilling), the height (H) of the river changes at the rate of $\left[1 - \frac{t}{20}\right]$ metres per hour. Initially the height of the river is 5 metres.

(a) Show that the height of the river is given by the formula $H = -\frac{t^2}{40} + t + 5$ **2**

(b) Find the maximum height of the river during this flood. **2**

(c) A bridge over this river will be flooded and closed once the height of the river reaches 12.5 metres. At what time(s) and day(s) will the bridge be closed and reopened? **3**

Questions 11-31 are worth 90 marks in total

End of Paper

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

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If you use this space, clearly indicate which question you are answering.

Year 12 Mathematics Advanced AP4 2022, Section I – Answer Sheet

NESA No.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct

A ● B ⊗ C ○ D ○

---

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

NESA No | | | | | | | | |

Solutions

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2022

YEAR 12

AP4

MATHEMATICS ADVANCED

Time allowed – 3 hours plus 10 minutes reading time

General Instructions	• Attempt all questions • Write your student NESA number on the question paper • Write using black pen • NESA approved calculators may be used • The NESA reference sheet has been provided • Answer the questions in the spaces provided • For questions in Section II, show relevant mathematical reasoning and/or calculations
Total marks: 100	**Section I – 10 marks (pages 3 – 7)** • Attempt Questions 1-10 • Allow about 15 minutes for this section **Section II – 90 marks (pages 8 – 29)** • Attempt Questions 11-31 • Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

---

1 Let $f\ (x) = \frac{1}{x}, x \neq 0$ and $g\ (x) = x + 2$. Find the domain of $f(g(x))$.

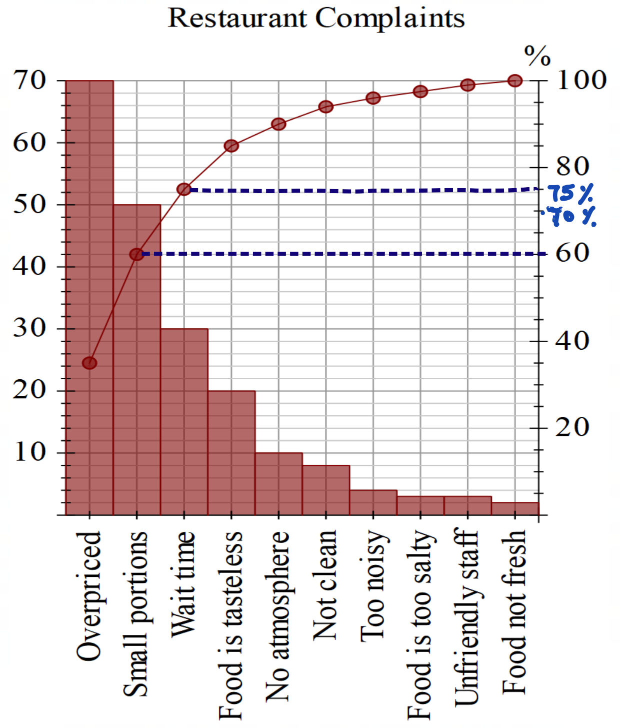
(A) All real x

(B) $x \neq -2$

(C) $x \neq 0$

(D) $x = -2$

2 A restaurant collected data related to the reasons given by customers for complaints. The Pareto chart shows the data collected.



What percentage of customers gave the reason 'Wait time'?

(A) 15%

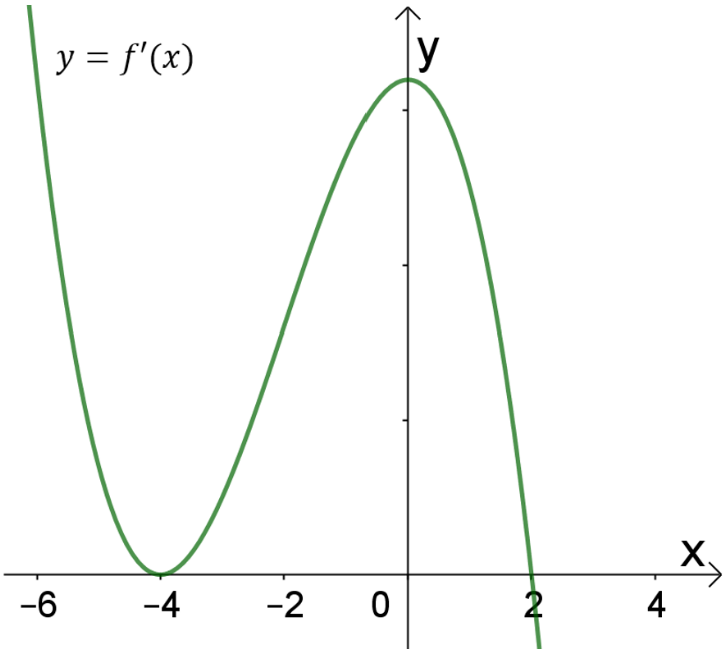
(B) 30%

(C) 60%

(D) 75%

75% - 60% = 15%

3 The diagram below shows a sketch of the gradient function of the curve $y = f(x)$.



Which of the following is a true statement?

The curve $y = f(x)$ has

(A) a minimum turning point at $x = -4$

(B) a horizontal point of inflection at $x = 2$

(C) a horizontal point of inflection at $x = -4$

(D) a minimum turning point at $x = 2$

4 The point $P(8, -3)$ lies on the graph of $f(x)$. Transformations map the graph of $f(x)$ to the graph of g(x), where $g(x) = -2f(x + 7) + 5$. The same transformation maps the point *P* to the point *P'*. What are the coordinates of the point *P'*?

(A) $(1, 11)$

(B) $(1, -1)$

(C) $(15, 11)$

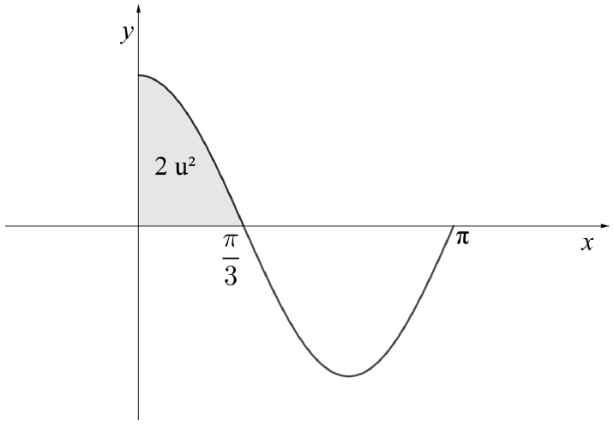
(D) $(15, -1)$

Vertical dilation $(8, 6)$

Vertical translation $(8, 11)$

Horizontal translation $(1, 11)$

5 The diagram below shows the graph of $f(x) = a\cos bx$.



The area of the shaded region is equal to 2 square units. What is the value of $\int_0^{\pi} f(x)\ dx$?

(A) -2

(B) -4

(C) 2

(D) 6

6 What are the values of x for which $|5 - 3x| \geq 11$?

(A) $x \leq -2$ or $x \leq \frac{16}{3}$

(B) $x \leq -2$ and $x \geq \frac{16}{3}$

(C) $x \leq -2$ and $x \leq \frac{16}{3}$

(D) $x \leq -2$ or $x \geq \frac{16}{3}$

$5 - 3x \leq -11$ or $5 - 3x \geq 11$

$-3x \leq -16$ or $-3x \geq 6$

$x \geq \frac{16}{3}$ or $x \leq -2$

7 Initially, a water tank holds 800 litres of water. Water drains from the bottom of the tank at a rate of $\frac{dV}{dt}$ litres per minute where $\frac{dV}{dt} = -100t$ after t minutes. How long does it take the tank to empty?

(A) 2 minutes

(B) 4 minutes

(C) 6 minutes

(D) 8 minutes

$V = -\frac{100t^2}{2} + C$

$800 = -50(0)^2 + C$

$C = 800$

$\therefore V = -50t^2 + 800$

$0 = -50t^2 + 800$

$t^2 = 16 \quad \therefore t = 4$ minutes

8 What is the equation of the axis of symmetry of the quadratic function $y = -k\,(x + b)^2 + c$?

(A) $x = -b$

(B) $x = b$

(C) $x = 2a$

(D) $x = 2ak$

$x = -b$

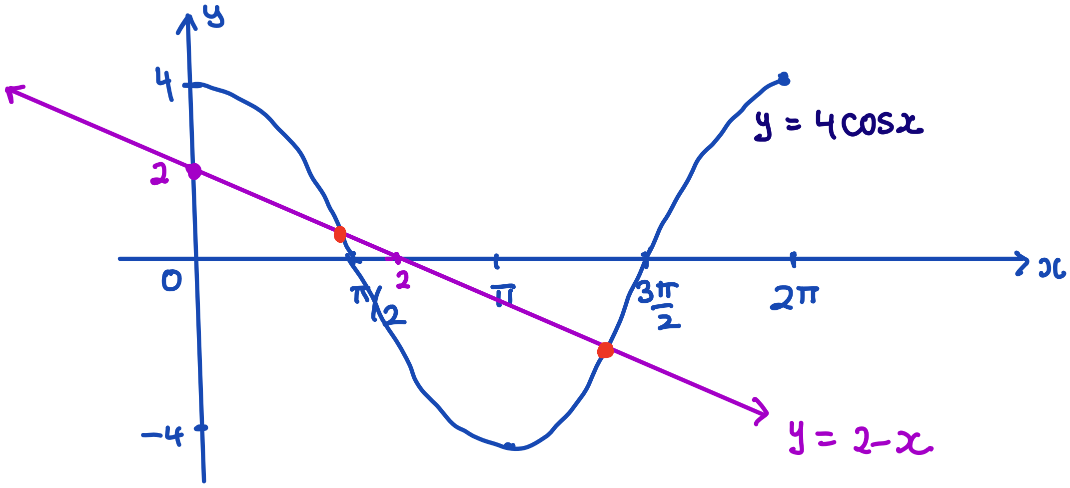
9 How many solutions are there to the equation $2 - x = 4\cos x$? $(0 \le x \le 2\pi)$

(A) 0

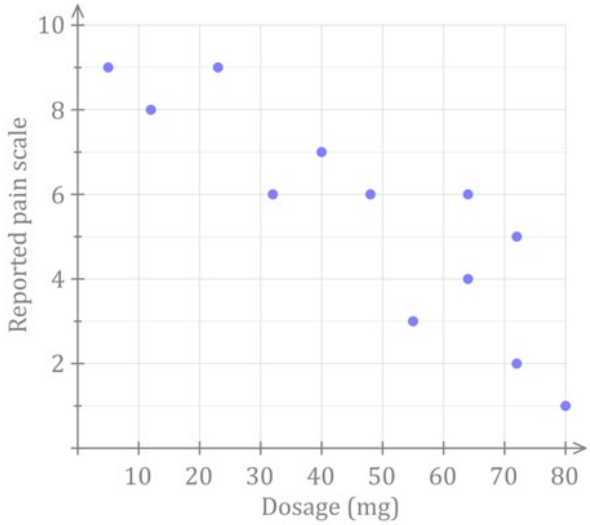
(B) 1

(C) 2

(D) 3



10 A scatterplot of pain (as reported by patients) compared to the dosage (in mg) of a drug is shown below.



Which of the following describes the correlation between the pain and the dosage?

(A) A weak negative correlation.

(B) A strong positive correlation.

(C) A strong negative correlation.

(D) A weak positive correlation.

End of Section I

Section II

90 marks
Attempt Questions 11 – 31
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks) **Marks**

Solve $x + \frac{x - 4}{3} = 2$. **2**

$3x + x - 4 = 6$

$4x - 4 = 6$

$4x = 10$

$x = \frac{5}{2}$

Question 12 (4 marks) **Marks**

A continuous random variable, T, represents the time taken in days to show symptoms after contracting a virus. It has the following probability density function.

$$f(x) = \begin{cases} \dfrac{k}{2t-1} & for\ 1 \le t \le 14 \\ 0 & for\ 0 \le t < 1\ \ or\ t > 14 \end{cases}$$

(a) Show that $k = \frac{2}{\ln 27}$. **2**

For a PDF $\int_1^{14} f(t)\,dt = 1$ i.e. $\int_1^{14} \frac{k}{2t-1}\,dt = 1$

$\left[\frac{k}{2}\ln|2t-1|\right]_1^{14} = 1$

$\frac{k}{2}\left[\ln 27 - \ln 1\right] = 1$

$\frac{k}{2}\ln 27 = 1$

$k = \frac{2}{\ln 27}$

(b) After how many days will a person have a 75% chance of showing symptoms after they have contracted the virus? **2**

CDF $= F(t) = \int_1^{t} f(t)\,dt = 0.75$

$\frac{2}{\ln 27} \cdot \frac{1}{2}\left[\ln|2t-1| - \ln|1|\right] = 0.75$

$\ln|2t-1| = 0.75 \times \ln 27$

$2t-1 = e^{0.75\ln 27}$

$t = \frac{e^{0.75\times\ln 27}+1}{2} \approx 6.42$ days.

Question 13 (3 marks) **Marks**

(a) Show that $\frac{\sec\theta - \sec\theta cos^4\theta}{1+cos^2\theta} = \sin\theta\tan\theta$ **2**

LHS. $\frac{\sec\theta - \sec\theta\cos^4\theta}{1+\cos^2\theta}$

$= \frac{\sec\theta(1-\cos^4\theta)}{1+\cos^2\theta}$

$= \frac{\sec\theta(1^2-(\cos^2\theta)^2)}{1+\cos^2\theta}$

$= \frac{\sec\theta\,\cancel{(1+\cos^2\theta)}(1-\cos^2\theta)}{\cancel{1+\cos^2\theta}}$

$= \sec\theta\sin^2\theta$

$= \frac{1}{\cos\theta}\times\sin\theta\times\sin\theta$

$= \frac{\sin\theta}{\cos\theta}\times\sin\theta$

$= \sin\theta\tan\theta = \text{RHS}$ proved.

(b) What are the possible values of a if $1 + 2a + 4a^2 + 8a^3 + \cdots$ is to have a limiting sum? **1**

$S_\infty = \frac{a}{1-r}$

$|2a| < 1$

$-1 < 2a < 1$

$-\frac{1}{2} < a < \frac{1}{2}$

Question 14 (5 marks)

(a) Find the equation of the normal to the curve $y = x^2 + 4x - 7$ at the point $(1,-2)$ **2**

$\frac{dy}{dx} = 2x + 4$ $\quad m_{\text{tangent}}$ at $x = 1 = 2 \times 1 + 4$

$= 6$

$\therefore m_{\text{normal}} = -\frac{1}{6}$

$\therefore$ Eq. of normal using $m = -\frac{1}{6}$, pt$(1,-2)$

$y + 2 = -\frac{1}{6}(x - 1)$

$y = -\frac{1}{6}x + \frac{1}{6} - 2$ $\quad$ or $\quad x + 6y + 11 = 0$

$y = -\frac{1}{6}x - \frac{11}{6}$

(b) The curve $y = f(x)$ has a gradient function $f'(x) = 3x^2 - k$, where k is a constant.

(i) Find the value of k if the curve has a stationary point at $(-1,3)$. **1**

Stationary pts occur when $f'(x) = 0$

$3x^2 - k = 0$

$3(-1)^2 - k = 0$

$3 - k = 0$

$k = 3$

(ii) Hence, find the equation of the curve $y = f(x)$. **2**

$f(x) = \int f'(x)\,dx$

$= \int (3x^2 - 3)\,dx$

$= x^3 - 3x + C$

When $x = -1$, $y = 3$ $\quad 3 = (-1)^3 - 3(-1) + C$

$3 = -1 + 3 + C$

$C = 1$ $\quad \boxed{\therefore f(x) = x^3 - 3x + 1}$

Question 15 (5 marks)

A curve is defined by $f(x) = \frac{1}{4}x^4 - 2x^2 + 1$. **5**

Sketch the graph of the curve for the domain $[-3, 3]$ showing all key features including, classifying the nature of stationary points, and finding any points of inflection. Also state where the curve is concave down. You **do not** need to find the x-intercepts

Space is provided on the next page to draw the graph.

- y-intercept $f(0) = 1$ $(0, 1)$
- $f'(x) = x^3 - 4x = 0$

$= x(x^2 - 4) = 0$

$x = 0$ $x = \pm 2$

$y = 1$,

$(0, 1)$ $(2, -3)$, $(-2, -3)$

$f''(x) = 3x^2 - 4$.

$f''(0) = -4 <$ ∩

∴ $(0, 1)$ is a local maximum t.p.

$f''(2) = 12 - 4 = 8 > 0$ ∪ ∴ $(2, -3)$ and $(-2, -3)$ are

$f''(-2) = 8 > 0$ ∪ local minimum turning points.

Possible inflection pt:

$f''(x) = 0$

$3x^2 - 4 = 0$

$x^2 = \frac{4}{3}$

$x = \pm\sqrt{\frac{4}{3}} = \pm \frac{2}{\sqrt{3}} \approx 1.15$

x	1.1	$\frac{2}{\sqrt{3}}$	1.2
$f''(x)$	$-0.37 < 0$ ∩		$0.32 > 0$ ∪

since concavity changes

∴ $\left(\frac{2}{\sqrt{3}}, -\frac{11}{9}\right)$ is a point of inflection

x	-1.2	$-\frac{2}{\sqrt{3}}$	-1.1
f''	$0.32 > 0$ ∪		$-0.37 < 0$ ∩

since concavity changes ∴ $\left(-\frac{2}{\sqrt{3}}, -\frac{11}{9}\right)$ is a point of inflection

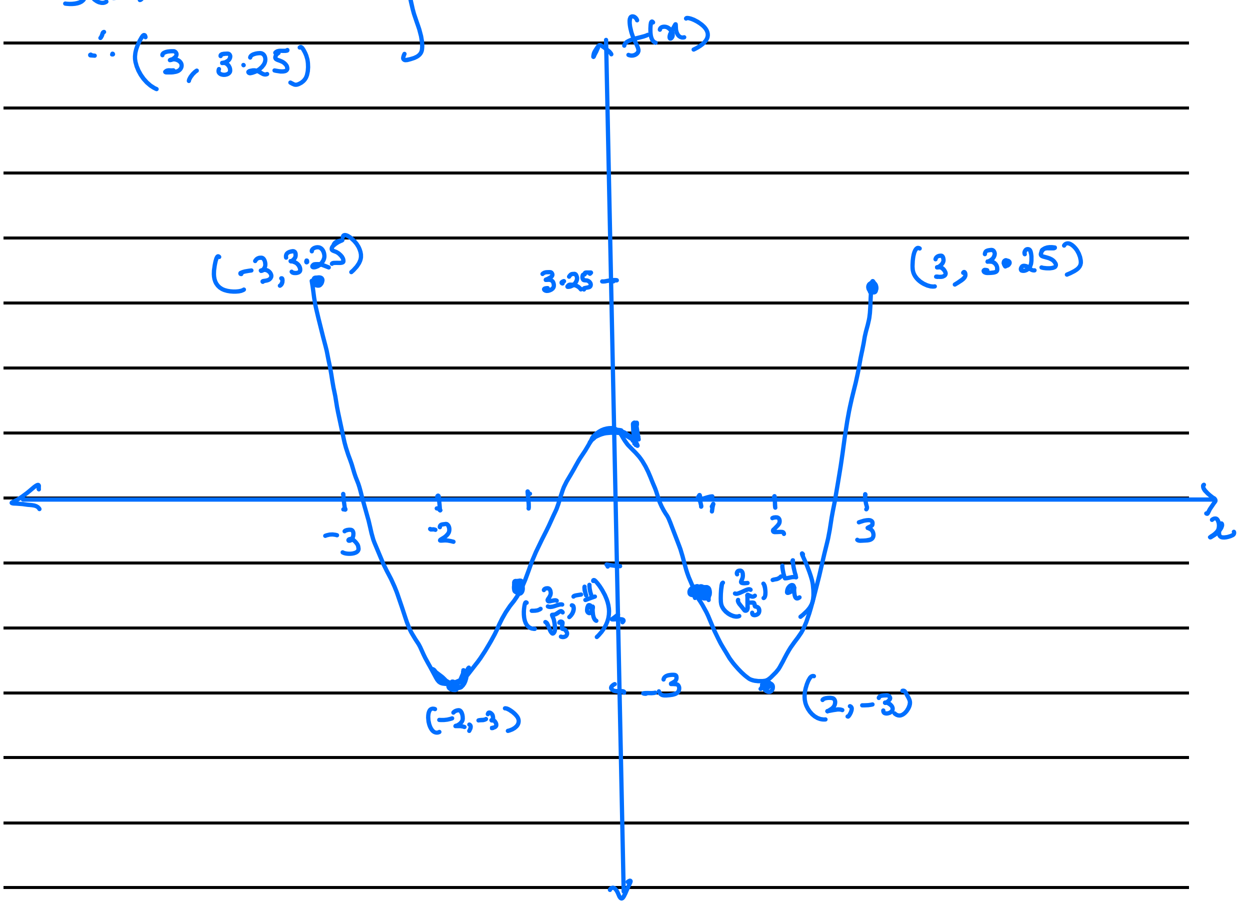
$f(-3) = 3.25$

$\therefore (-3, 3.25)$

$f(3) = 3.25$

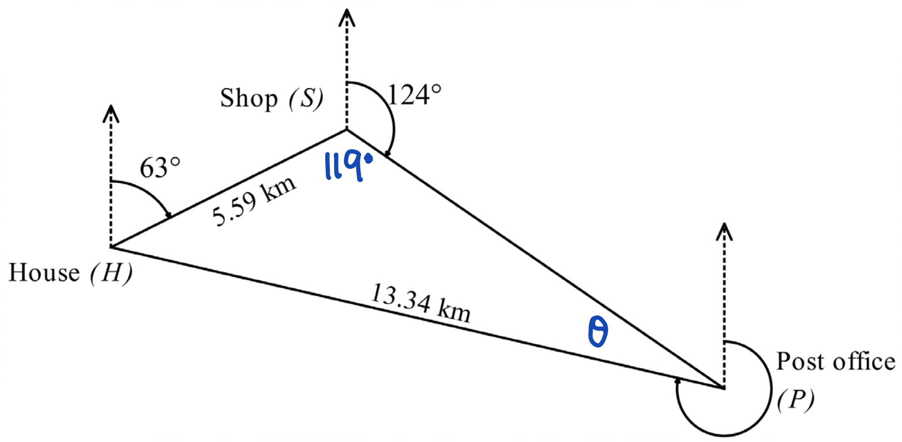
$\therefore (3, 3.25)$

End points



Question 16 (3 marks)

Robert walks 5.59 km from his house to the shop on a bearing of 063°. He then turns on a bearing of 124° to walk to the post office, where he is 13.34 km from home.



(a) Show that $\angle HSP = 119°$. **1**

$\angle HSP = 360° - 124° - \underbrace{(180° - 63°)}$

co-interior angle sum with parallel lines is 180°

$= 119°$

(b) Find the bearing of Robert's house from the post office. Give your answer correct to the nearest degree. **2**

Using Sine Rule

$$\frac{\sin\theta}{5.59} = \frac{\sin 119°}{13.34}$$

$$\theta = \sin^{-1}\left(\frac{\sin 119°}{13.34} \times 5.59\right)$$

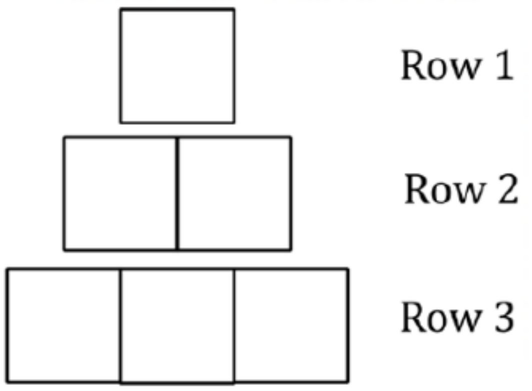
$$\approx 21° \text{ (nearest degree)}$$

Bearing of house from post-office $= 360° - 21° - (180° - 124°)$

$= 283°\,T$

Question 17 (6 marks)

James has some sticks that are all of the same length. He arranges them in squares and has made the following 3 rows of patterns.



He notices that 4 sticks are required to make the single square in the first row, 7 sticks in the second row and in the third row he needs 10 sticks to make 3 squares.

(a) Find an expression, in terms of n, for the number of sticks required to make a similar arrangement of n squares in the n^{th} row. **1**

$T_n = 4 + (n-1)\times 3$

$= 4 + 3n - 3$

$= 1 + 3n$

(b) James continues to make squares following the same pattern. He makes 4 squares in the fourth row and so on until he has completed 10 rows. Find the total number of sticks James uses in making these 10 rows. **2**

$S_{10} = \frac{10}{2}\left[2\times 4 + (10-1)\times 3\right]$

$= 5\left[8 + 27\right]$

$= 175$ sticks.

(c) James started with 1750 sticks. Given that James continues the pattern to complete k rows but does not have sufficient sticks to complete the $(k+1)^{\text{th}}$ row. Show that k satisfies $(3k-100)(k+35)<0$. **2**

$$S_k = \frac{k}{2}\left[2\times 4 + (k-1)\times 3\right] < 1750$$

$$k[8 + 3k - 3] < 3500$$

$$k[5 + 3k] < 3500$$

$$3k^2 + 5k - 3500 < 0 \qquad P = -10500$$

$$3k^2 + 105k - 100k - 3500 < 0 \qquad S = 5$$

$$3k(k+35) - 100(k+35) < 0 \qquad \text{Two numbers.}$$

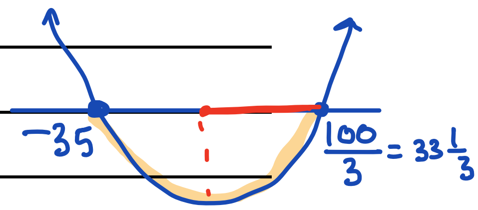
$$(3k-100)(k+35) < 0$$

(d) What is the value of k? **1**

$$(3k-100)(k+35) < 0$$

$$k \geq 0$$

$k = 33$ complete rows.



Question 18 (5 marks) **Marks**

(a) Find the value of k such that 3

$$\int_{-2}^{0} \frac{x^2}{x^3 - 2}\, dx = \ln k$$

$$\frac{1}{3}\int_{-2}^{0} \frac{3x^2}{x^3-2}\, dx = \ln k$$

$$\frac{1}{3}\Big[\ln|x^3-2|\Big]_{-2}^{0} = \ln k$$

$$\ln|-2| - \ln|-8-2| = 3\ln k$$

$$\ln\left|\frac{-2}{-10}\right| = \ln k^3$$

$$\frac{|-2|}{|-10|} = k^3$$

$$k^3 = \frac{1}{5}$$

$$k = \left(\frac{1}{5}\right)^{\frac{1}{3}}$$

$$= \frac{1}{\sqrt[3]{5}}$$

(b) **2**

The sign for 'Pizza π' restaurant is in the shape of a circle with a sector cut out of it. The sector has an angle $\frac{\pi}{3}$. If the area of the sign is 30π, what is the radius of the circle?

$2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\frac{1}{2} r^2 \times \frac{5\pi}{3} = 30\pi$

$r^2 = \frac{30\pi \times 6}{5\pi}$

$r^2 = 36$

$r = 6$ units since $r > 0$

Question 19 (3 marks) **Marks**

(a) For what values of k does the quadratic equation $5x^2 - 2x + (8k - 15) = 0$ have real roots? **1**

$a = 5,\ b = -2,\ c = 8k - 15$

$\Delta \geq 0$

$b^2 - 4ac \geq 0$

$(-2)^2 - 4 \times 5 \times (8k - 15) \geq 0$

$4 - 20 \times 8k + 20 \times 15 \geq 0$

$4 - 160k + 300 \geq 0$

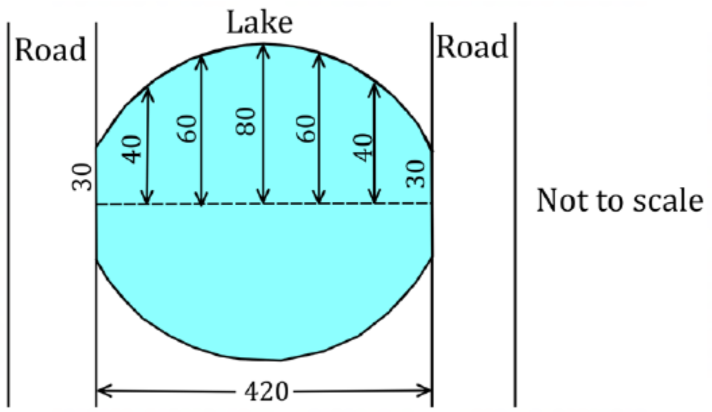
$304 - 160k \geq 0$

$-160k \geq -304$

$k \leq \frac{304}{160}$

$k \leq \frac{19}{10}$

(b) A symmetrical lake has two roads, 420 metres apart, forming two of its sides. **2**



Equally spaced measurements of the lake, in metres are shown on the above diagram. Use the trapezoidal rule to estimate the area of the lake.

$$\text{Area} \approx 2 \times \left[\frac{1}{2} \times \frac{420}{6}\left[30 + 30 + 2(40 + 60 + 80 + 60 + 40)\right]\right]$$

$$= 43400 \text{ square metres.}$$

Question 20 (4 marks) **Marks**

Differentiate with respect to x

(a) $e^{x \sin 2x}$ **2**

$$\frac{d(e^{x\sin 2x})}{dx} = e^{x\sin 2x} \times \frac{d(x\sin 2x)}{dx}$$

$$= e^{x\sin 2x}\left[\sin 2x + 2x\cos 2x\right.$$

(b) $\dfrac{x^2}{\ln(x)}$ **2**

$$\frac{d}{dx}\left(\frac{x^2}{\ln x}\right) = \frac{2x\ln x - x}{[\ln x]^2}$$

$u = x^2$

$u' = 2x$

$v = \ln x$

$v' = \frac{1}{x}$

Question 21 (5 marks) **Marks**

A particle is moving in a straight line. Initially, it is travelling to the left at 1 cm/min. Its acceleration as a function of time(t) is given by $a = \pi \cos \pi t + \pi \sin \pi t$ for $0 \leq t \leq 2$ where time and displacement are measured in minutes and cm respectively.

(a) Find when the particle changes its direction for the first time. **2**

Particle changes its direction when $v=0$

$$v = \int_{-1}^{v} \frac{dv}{dt}\,dt = \int_0^t (\pi \cos \pi t + \pi \sin \pi t)\,dt \quad \text{OR}$$

$$v + 1 = \Big[\sin \pi t - \cos \pi t\Big]_0^t \qquad v = \sin \pi t - \cos \pi t + c$$

$t=0 \Rightarrow -1 = -1 + c$

$c = 0$

$$v + 1 = \sin \pi t - \cos \pi t - \sin 0 + \cos 0$$

$$v + 1 = \sin \pi t - \cos \pi t + 1$$

$$v = \sin \pi t - \cos \pi t$$

$v = 0 \Rightarrow$

$$\sin \pi t - \cos \pi t = 0$$

$$\sin \pi t = \cos \pi t$$

$$\tan \pi t = 1 \Rightarrow \pi t = \frac{\pi}{4}, \frac{5\pi}{4}$$

$t = \frac{1}{4}, \frac{5}{4}$ minutes

First time $t = \frac{1}{4}$ min. $= 15$ secs.

(b) Find the exact total distance travelled in the first half a minute. **3**

$$x = \int \frac{dx}{dt}\,dt = \int_0 (\sin \pi t - \cos \pi t)\,dt$$

$$x = -\frac{\cos \pi t}{\pi} - \frac{\sin \pi t}{\pi} + C$$

$x = 0,\ t = 0$

$$0 = -\frac{1}{\pi} - 0 + C \quad \therefore C = \frac{1}{\pi}$$

$$\therefore x = -\frac{1}{\pi}\left[\cos \pi t + \sin \pi t - 1\right]$$

$\left|-\frac{1}{\pi}(\sqrt{2}-1)\right|$

$t = 1/4$ $\quad$ $t = 0$

$t = 1/2$

When $t = \frac{1}{4}$

$$x = -\frac{1}{\pi}\left(\cos \frac{\pi}{4} + \sin \frac{\pi}{4} - 1\right)$$

$$= -\frac{1}{\pi}\left(\sqrt{2} - 1\right)$$

When $t = \frac{1}{2}$

$x = -\frac{1}{\pi}(0 + 1 - 1)$

$= 0$

Distance travelled for $0 \leq t \leq \frac{1}{2}$ is $2 \times \frac{1}{\pi}(\sqrt{2}-1)$ cm.

Question 22 (4 marks) **Marks**

Integrate the following:

(a) $\int x^4\,(x^5-3)^2 dx =$ **2**

OR $\int x^4\,(x^{10}-6x^5+9)\,dx$

$\frac{1}{5}\int 5x^4(x^5-3)^2 dx$

$= \int (x^{14}-6x^9+9x^4)\,dx$

$\frac{1}{5}\,\frac{(x^5-3)^3}{3}+C$

$= \frac{x^{15}}{15}-\frac{3}{5}x^{10}+\frac{9}{5}x^5+C$

$= \frac{1}{15}(x^5-3)^3+C$

(b) $\int (\sqrt{x}-\frac{2}{x}+3e^{-x}+1\,)dx =$ **2**

$\int (x^{1/2}-2x^{-1}+3e^{-x}+1)\,dx$

$= \frac{2}{3}x^{3/2}-2\ln|x|-3e^{-x}+x+C$

Question 23 (3 marks) **Marks**

(a) Show that the derivative of $\ln\left(\frac{3+x}{3-x}\right)$ is $\frac{6}{9-x^2}$. **2**

let $y = \ln\left(\frac{3+x}{3-x}\right)$

$= \ln(3+x)-\ln(3-x)$

$\frac{dy}{dx} = \frac{1}{3+x}+\frac{1}{3-x}$

$= \frac{3-x+3+x}{(3+x)(3-x)}$

$= \frac{6}{(9-x^2)}$

(b) Hence or otherwise find $\int \frac{1}{9-x^2}\,dx$. **1**

$\frac{d}{dx}\left(\ln\left(\frac{3+x}{3-x}\right)\right) = \frac{6}{9-x^2}$

Integrate both sides

$\int \frac{6}{9-x^2}\,dx = \ln\left|\frac{3+x}{3-x}\right|+C$

$\therefore \int \frac{1}{9-x^2}\,dx = \frac{1}{6}\ln\left|\frac{3+x}{3-x}\right|+C$

Question 24 (3 marks)

Given that $\int_0^a 5\ \sin 3x\ dx = \frac{10}{3}, 0\ \leq a\ <\ \pi$, calculate the value of a. **3**

$$\int_0^a 5 \sin 3x \, dx = \frac{10}{3}$$

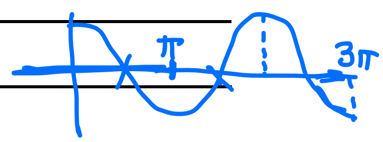
$$\left[-\frac{5\cos 3x}{3}\right]_0^a = \frac{10}{3}$$

$$-\frac{5}{3}\left[\cos 3a - \cos 0\right] = \frac{10}{3}$$

$$\cos 3a - 1 = \frac{\cancel{10}^{2}}{\cancel{3}} \times -\frac{\cancel{3}}{\cancel{5}}$$

$$\cos 3a = -2 + 1$$

$$\cos 3a = -1 \qquad 0 \leq 3a < 3\pi$$



$$3a = \pi$$

$$a = \frac{\pi}{3}$$

Question 25 (3 marks)

.

Solve $2\ \cos\left[2(x - \frac{\pi}{6})\right] = 1$ in the domain $[0, 2\pi]$. **3**

$$\cos\left[2\left(x - \frac{\pi}{6}\right)\right] = \frac{1}{2} \qquad \text{Reference angle} = \frac{\pi}{3}$$

$0 \leq x \leq 2\pi$

$-\frac{\pi}{6} \leq x - \frac{\pi}{6} \leq \frac{11\pi}{6}$

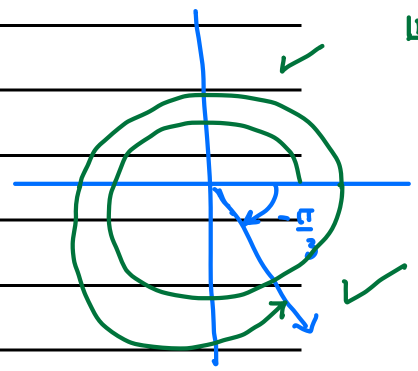
$-\frac{\pi}{3} \leq 2\left(x - \frac{\pi}{6}\right) \leq \frac{11\pi}{3}$

$$2\left(x - \frac{\pi}{6}\right) = -\frac{\pi}{3}, \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, 2\pi + \frac{\pi}{3}, 4\pi - \frac{\pi}{3}$$

$$x - \frac{\pi}{6} = -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$\frac{11\pi}{3} = 3\frac{2}{3}\pi$

$$x = 0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi$$



Question 26 (5 marks) **Marks**

The probability distribution of a discrete random variable X is shown.

x	-1	0	1	2
$P(X = x)$	t^2	t^2	$\frac{t}{4}$	$\frac{4t+1}{8}$

(a) What is the value of t? **3**

$\sum P(X=x) = 1$

$t^2 + t^2 + \frac{t}{4} + \frac{4t+1}{8} = 1$

$2t^2 + \frac{6t+1}{8} = 1$

$16t^2 + 6t - 7 = 0$

$t = \frac{-6 + \sqrt{36+448}}{32}$ as $0 \leq P(X=x) \leq 1$

$= \frac{-6+22}{32}$

$= \frac{1}{2}$

(b) Find the expected value of the probability distribution. **2**

$E(X) = -1 \times \left(\frac{1}{2}\right)^2 + 0\left(\frac{1}{2}\right)^2 + 1 \times \frac{1}{8} + 2 \times \frac{3}{8}$

$= -\frac{1}{4} + 0 + \frac{1}{8} + \frac{3}{4} = \frac{5}{8}$

Question 27 (4 marks) **Marks**

(a) The probability that a person chosen at random has green eyes is 0.04
Two people are chosen at random.

(i) What is the probability that at least ONE person has green eyes? **2**

P(at least one person)

$= 1 - P(\text{none})$

$= 1 - 0.96 \times 0.96 = 0.0784$

(ii) What is the smallest number of people that can be chosen at random so that the probability that at least ONE has green eyes is greater than 0.4? **2**

$1 - (0.96)^n > 0.4$

$-(0.96)^n > -0.6$

$(0.96)^n < 0.6$

$n \ln 0.96 < 0.6$

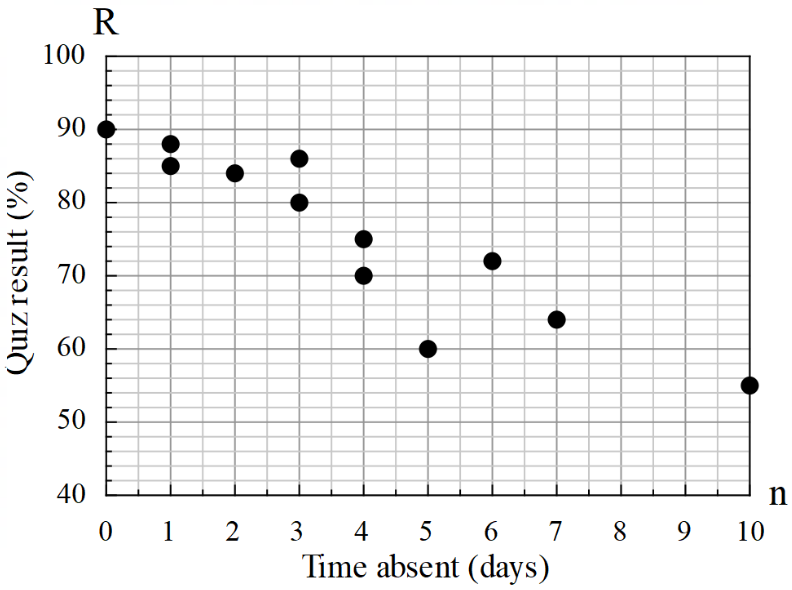
$n > \dfrac{\ln 0.6}{\ln 0.96}$

$n > 12.51\ldots$

$\therefore n = 13$ At least 13 people

Question 28 (5 marks) **Marks**

A teacher was investigating the impact of absences on their students' grades in a quiz.
The diagram shows the dataset used in the investigation.



n	R
0	90
1	85
1	88
2	84
3	80
3	86
4	70
4	75
5	60
6	72
7	64
10	55

(a) Find the correlation coefficient, correct to two decimal places, and describe the correlation. **2**

$r = -0.916569\ldots$

Strong negative correlation.

(b) Find the equation of the least-squares regression line in terms of results, R, and number of days absent, n. Give values correct to 2 decimal places. **1**

$A = 90.09 \quad B = -3.74$

$\therefore R = 90.09 - 3.74n.$

(c) Comment on the validity of the equation of the least-squares regression line for this context. **1**

– As absences increase

R is decreases

but R cannot be negative.

$R = 90.09 - 3.74n$

when $n = 25$ $R = 90.09 - 3.74 \times 25 = -3.41$

After $n=10$, it is extrapolation.

(d) The result of a student was not recorded when they were absent for 8 days. Using the equation from part (b), predict the result for a student absent for 8 days, correct to the nearest percentage. **1**

$R = 90.09 - 3.74n$

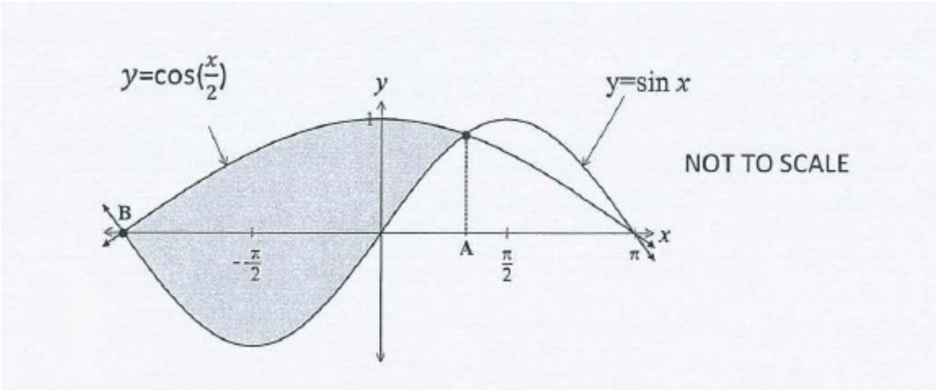
$= 90.09 - 3.74 \times 8$

$= 60.17\%$

$\approx 60\%$

Question 29 (5 marks) **Marks**

A section of the graphs of $y = \sin x$ and $y = \cos\left(\frac{x}{2}\right)$ is shown.



(a) The curves meet at A and B. Show by substitution, that the x values of point A and B are $\frac{\pi}{3}$ and $-\pi$ respectively. **2**

When $x = \frac{\pi}{3}$ $\quad y = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$y = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\therefore$ Two curves meet at $x = \frac{\pi}{3}$ which is A.

When $x = -\pi$ $\quad y = \sin(-\pi) = -\sin\pi = 0$

$y = \cos\left(-\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$

$\therefore$ Two curves meet at $x = -\pi$

(b) Hence or otherwise, find the exact area of the shaded region. **3**

$$\text{Area} = \int_{-\pi}^{\pi/3} \left(\cos\left(\frac{x}{2}\right) - \sin x\right) dx$$

$$= \left[\frac{\sin x/2}{1/2} + \cos x\right]_{-\pi}^{\pi/3}$$

$$= 2\sin\frac{\pi}{6} + \cos\frac{\pi}{3} - 2\sin\left(-\frac{\pi}{2}\right) - \cos(-\pi)$$

$$= 2\times\frac{1}{2} + \frac{1}{2} + 2 - (-1)$$

$$= 1 + \frac{1}{2} + 2 + 1$$

$= 4\frac{1}{2}$ square units.

Question 30 (6 marks) **Marks**

In a laboratory, a food product is heated in an oven at 210° C in an effort to kill harmful bacteria. The number of bacteria recorded when the food product is first placed in the oven is 1800. After ten minutes the number of bacteria recorded is 550. It is found that N, the number of bacteria remaining t minutes after being in the oven is given by $N = A\,e^{-kt}$ where A and k are constants.

(a) Find the value of A. **1**

when $t=0$

$1800 = A\,e^{-k\times 0}$

$A = 1800$

(b) Calculate the value of k correct to 4 decimal places. **2**

$550 = 1800 \times e^{-k\times 10}$

$e^{-10k} = \frac{550}{180}$

$-10k = \ln\left(\frac{55}{180}\right)$

$k = -\frac{1}{10}\ln\left(\frac{55}{180}\right)$

≈ 0.1186

(c) If an acceptable number of bacteria present is 100 or less, For how long should the food product to be in the oven? Express your answer to the nearest minute. **3**

$1800\,e^{-0.1186\cdot\times t} \leq 100$

$e^{-0.1186t} \leq \frac{1}{18}$

$-0.1186t \leq \ln\left(\frac{1}{18}\right)$

$t \geq \frac{-\ln 18}{-0.1186}$

≈ 24.3708

$t = 25$ minutes.

Question 31 (7 marks)

After a week of heavy rain, the local dam starts to fill until at 10am on Sunday the dam overflows into the river. At time t (hours after the dam begins spilling), the height (H) of the river changes at the rate of $\left[1 - \frac{t}{20}\right]$ metres per hour. Initially the height of the river is 5 metres.

(a) Show that the height of the river is given by the formula $H = -\frac{t^2}{40} + t + 5$ **2**

$$\frac{dH}{dt} = \left(1 - \frac{t}{20}\right)$$

$$\int_5^H dH = \int_0^t \left(1 - t/20\right) dt$$

$$H - 5 = \left[t - \frac{t^2}{40}\right]_0^t$$

$$H - 5 = t - \frac{t^2}{40}$$

$$H = -\frac{t^2}{40} + t + 5$$

or $H = t - \frac{t^2}{40} + c$

$5 = 0 - 0 + c$

$\therefore H = t - \frac{t^2}{40} + 5$

(b) Find the maximum height of the river during this flood. **2**

$-\frac{t^2}{40} + t + 5$ is $\cap$

$\therefore$ Vertex$_t$ $= \frac{-1}{2 \times -\frac{1}{40}}$

$= 20$

When $t = 20$

$H = -\frac{20^2}{40} + 20 + 5$

$= 15$ metres

OR use first or second derivative.*

$\frac{dH}{dt} = -\frac{t}{20} + 1$

$\frac{d^2H}{dt^2} = -\frac{1}{20} < 0 \therefore \cap$

i.e. $\frac{dH}{dt} = 0$ gives maximum t.p.

$-\frac{t}{20} + 1 = 0$

$-t = -20$

$t = 20$

find $H(20) = 15$ metres.

(c) A bridge over this river will be flooded and closed once the height of the river reaches 12.5 metres. At what time(s) and day(s) will the bridge be closed and reopened? **3**

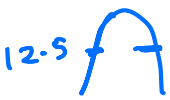
$$12.5 = -\frac{t^2}{40} + t + 5$$

$$t^2 - 40t - 200 + 500 = 0$$

$$t^2 - 40t + 300 = 0$$

$$t = \frac{40 \pm \sqrt{1600 - 1200}}{2}$$

$$= \frac{40 \pm 20}{2} = 10, 30$$



10 am Sunday + 10 = 8 pm Sunday Bridge closes.

10 am Sunday + 24 + 6 = 10 am Monday + 6

= 4 pm Monday

Bridge opens.

Questions 11-31 are worth 90 marks in total

End of Paper