Student Number:

Teacher: Blair / Karagiannis (Circle)

Danebank
An Anglican School for Girls

Year 12 Advanced Mathematics
Task 4 – Trial HSC Examination
Wednesday 3rd August 2022

General Instructions	
Write using black or blue pen Graphs can be drawn in pencil No liquid paper or correction tape to be used.	
Reading Time	10 minutes
Working time	3 hours

Structure of the Paper		
Section I	/10 Marks	Allow about 15 minutes for this section
Section II	/ 90 Marks	Allow about 2 hours 45 minutes for this section
Total	**/100 Marks**	**Weighting: 30%**

Special Instructions
NESA approved Calculator may be used. A separate reference sheet and Multiple Choice answer sheet is provided. Attempt all questions. This examination must not be removed from the Examination Room. In Question 11-33 show relevant mathematical reasoning and/or calculations. Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Extra writing space is provided at the back of the examination paper on pages 32 - 35. If you use this space, clearly indicate which question you are answering.

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SECTION I Multiple Choice 10 marks

Attempt Questions 1-10.

Allow about 15 minutes for this section.

Use the multiple choice answer sheet for questions 1 - 10.

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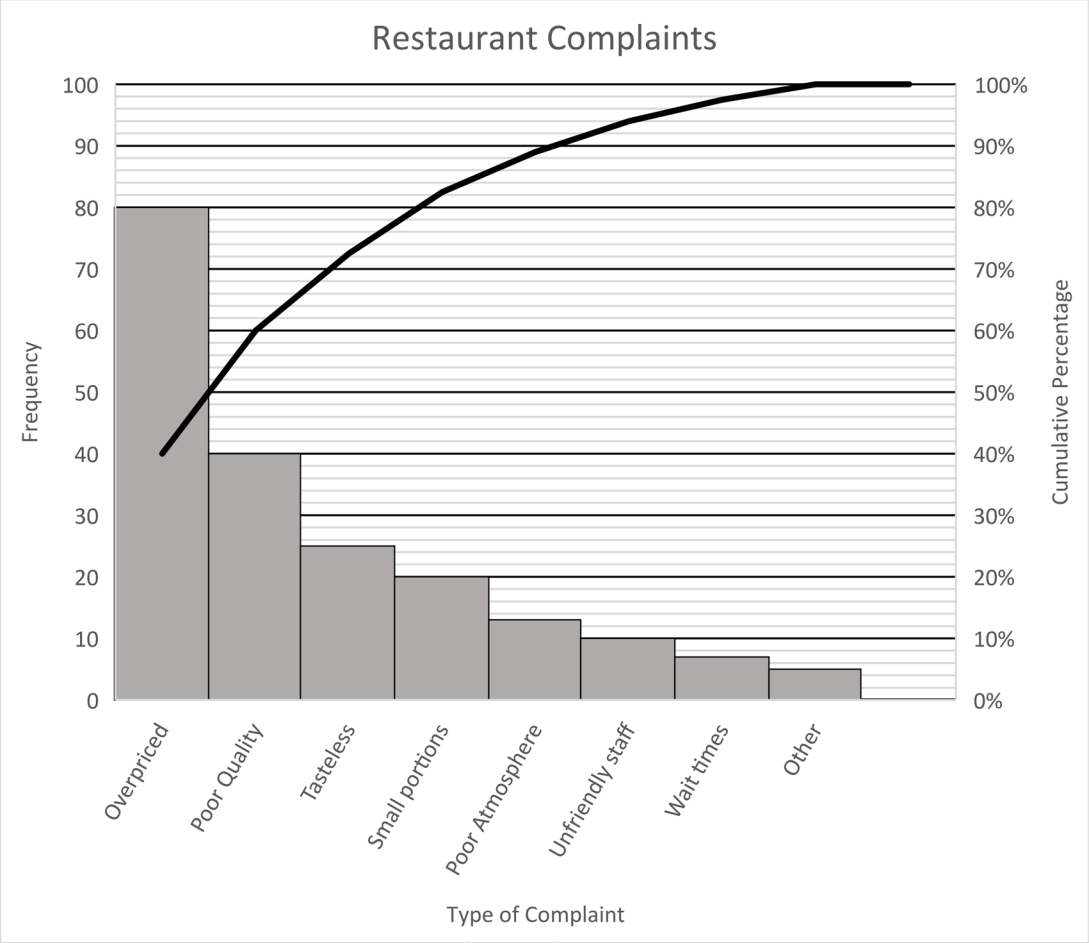
1. The Pareto chart below shows the frequency of types of complaints received by a restaurant.



What percentage of complaints were for poor quality?

A. 20% B. 40%

C. 60% D. 80%

2. The first three terms of an arithmetic series are 2, 7 and 12.
What is the 15th term of this series?

A. 72 B. 77

C. 555 D. 595

3. The mean of a set of five numbers is 11. A sixth number is added to the set and the mean decreases to 10.
What is the sixth number?

A. 1 B. 3

C. 5 D. 7

4. The stem and leaf plot shows the number of shots needed to complete a game of pool on two tables. Twenty games were played on each table.

Table A							Table B					
			7	6	4	**0**	6	8				
		8	4	4	1	**1**	2	6	7	7		
8	8	7	7	5	3	**2**	0	2	3	4	4	
		6	5	5	4	**3**	1	3	5			
				2	1	**4**	5	5	6	8	9	
						5						
					9	**6**	9					

Is the result of 69 shots an outlier for either of the tables?

A. No, it isn't for either table. B. Yes, for Table A only

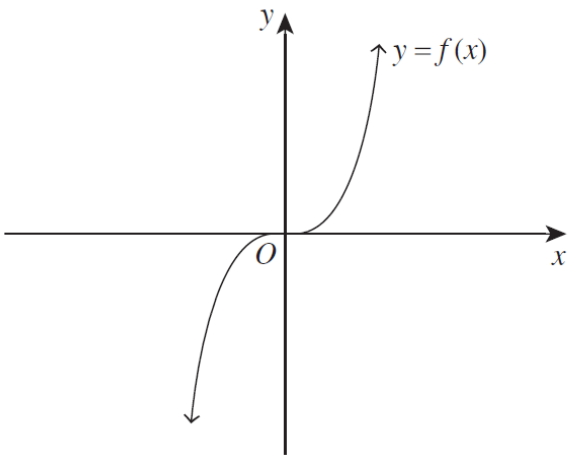
C. Yes, for Table B only D. Yes, for both tables

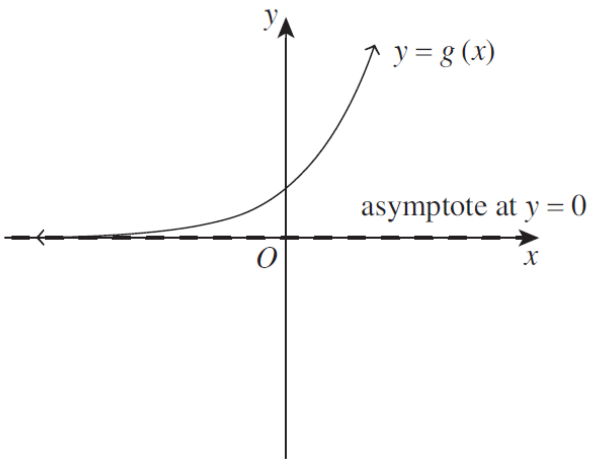
5. What is the exact value of $\sin 135° + \tan 300°$?

A. $\frac{\sqrt{2}+2\sqrt{3}}{2}$ B. $\frac{\sqrt{2}-2\sqrt{3}}{2}$

C. $\frac{\sqrt{5}}{2}$ D. $\frac{-\sqrt{5}}{2}$

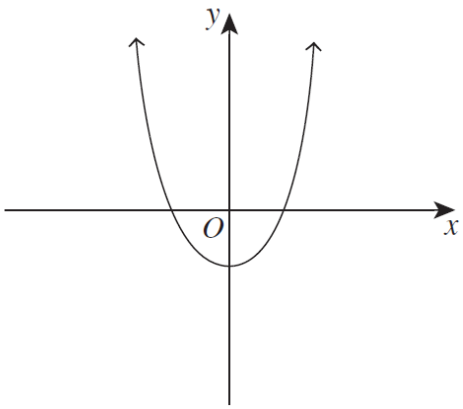
6. The graphs of $f(x)$ and $g(x)$ are shown.



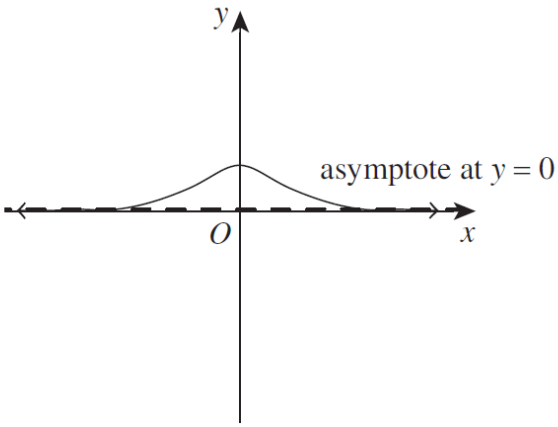


Which of the following best represents the graph of $y = f(g(-x))$?

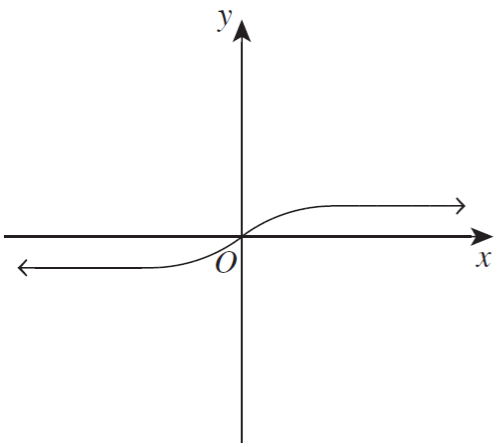
A.



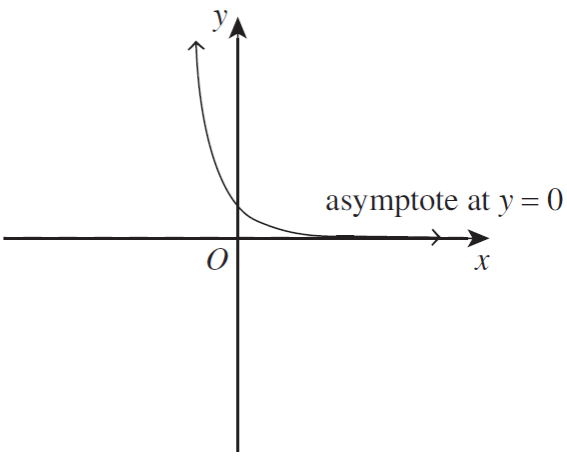
B.



C.



D.



7. The graph of $y = g(x)$ has been transformed by translating it vertically by 3 units and then dilating it horizontally by a factor of 2. Which of the following is the equation of the transformed graph?

A. $y = 2g(x) + 3$

B. $y = 2g(x + 3)$

C. $y = g(2x) + 3$

D. $y = g\left(\frac{x}{2}\right) + 3$

8. What is $\int 6x^2 (4x^3 - 5)^3 \, dx$?

A. $\frac{(4x^3 - 5)^4}{8} + C$

B. $12x (4x^3 - 5)^4 + C$

C. $2x^3(4x^3 - 5)^4 + C$

D. $\frac{2x^3(4x^3-5)^4}{8} + C$

9. Which is the complete solution set to the equation $\sin^2\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$,

for the domain $0 \leq x \leq 2\pi$?

A. $x = \frac{\pi}{12}, \frac{13\pi}{12}$

B. $x = 0, \frac{\pi}{2}, , \pi, \frac{3\pi}{2}$

C. $x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$

D. $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{9\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{15\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{21\pi}{12}, \frac{23\pi}{12}$

10. It is known that $f(x)$ is an odd function and $g(x)$ is an even function.
Given that $f(2) = 2$ and $g(2) = -2$, what is the value of $f\big(g(-2)\big) + g\big(f(-2)\big)$?

A. –4

B. –2

C. 0

D. 4

END OF SECTION 1

SECTION II 90 marks

Attempt Questions 11-33

Allow about 2 hours and 45 minutes for this section

Show all necessary working in the space provided. If extra space is required, use the lined pages at the end of this section, carefully labelling the additional work with the appropriate question number and part.

Question 11 (2 marks)

Solve $|5-3x| \geq 11$. **2**

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Question 12 (3 marks)

Find the equation of the normal to the curve $y = x^2 + 3x$ at $x = 1$. **3**

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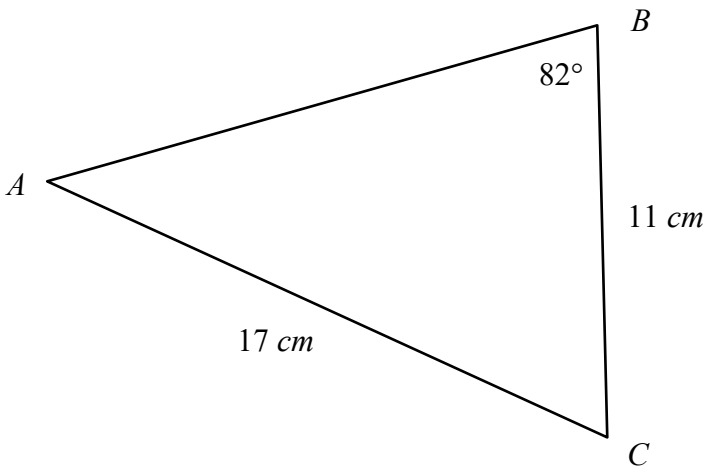
Question 13 (4 marks)

Triangle ABC is shown below



(a) Using the sine rule, show that $\angle A \approx 40°$. **2**

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(b) Hence, calculate the area of triangle ABC, correct to the nearest cm^2. **2**

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Question 14 (2 marks)

Evaluate $\sum_{r=2}^{10} 3^r$ **2**

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Question 15 (2 marks)

Express $\frac{2x}{x^2-4} - \frac{x+1}{x^2-x-2}$ as a single algebraic fraction, in simplest form. **2**

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Question 16 (4 marks)

Differentiate with respect to x :

(a) $y = e^{2x}\sin(3x)$ **2**

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(b) $y = \dfrac{\cos x}{6 - x}$ **2**

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Question 17 (3 marks)

The circle $x^2 - 2x + y^2 + 8y + 8 = 0$ is reflected in the x-axis. Find the coordinates of the centre and the radius. **3**

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Question 18 (5 marks)

(a) Find $\int \frac{1}{2x+3}\,dx$ **2**

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(b) Given that $\int_0^3 \frac{1}{2x+3}dx = \ln\sqrt{P}$, find the value of P. **3**

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Question 19 (4 marks)

Sketch the graph of the curve $y = x^3 - 2x^2 + 1$, labelling the stationary points, any points of inflection and the y-intercept. Do NOT determine the x intercepts of the curve. **4**

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Question 20 on the next page

Question 20 (4 marks)

The following table shows a probability distribution for the random variable X, where $E(X) = \frac{6}{5}$.

x	0	1	2	3
$P(X=x)$	p	$\frac{1}{2}$	$\frac{3}{10}$	q

(a) Find the values of p and q. **3**

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(b) Find Var(X) **1**

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Question 21 (5 marks)

Let $f(x)$ be a continuous function, such that $\int_1^5 3f(x)\,dx = 12$.

(a) Show that $\int_5^1 f(x)\,dx = -4$. **2**

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(b) Find the value of $\int_1^2 (x + f(x))\,dx + \int_2^5 (x + f(x))\,dx$. **3**

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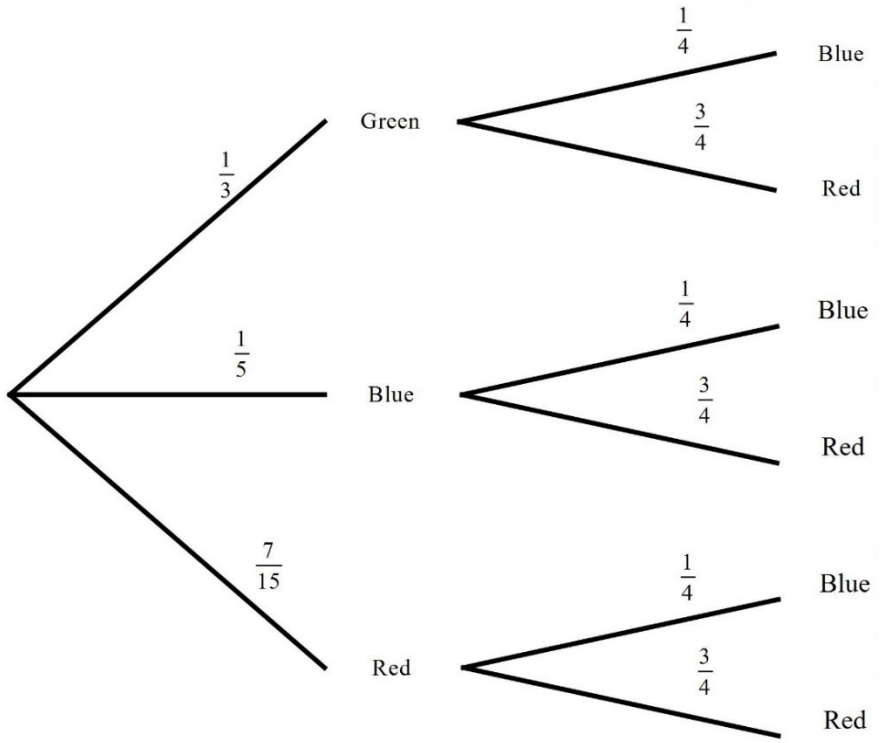
Question 22 (3 marks)

An artist has two boxes of paints beside his easel. The first box contains 15 tubes which contain green, red or blue paint and the second contains four tubes which have only red or blue paint.

The artist chooses one tube of paint from each box.

The tree diagram below shows the probabilities of him choosing a given colour.



(a) What is the probability that the tubes he chooses are red and blue? **1**

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(b) What is the probability that the tubes he chooses are not the same colour? **2**

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Question 23 (3 marks)

Find $f'(4)$ given $f(x) = \sqrt{x}\left(3x - \frac{2}{x\sqrt{x}}\right)$ **3**

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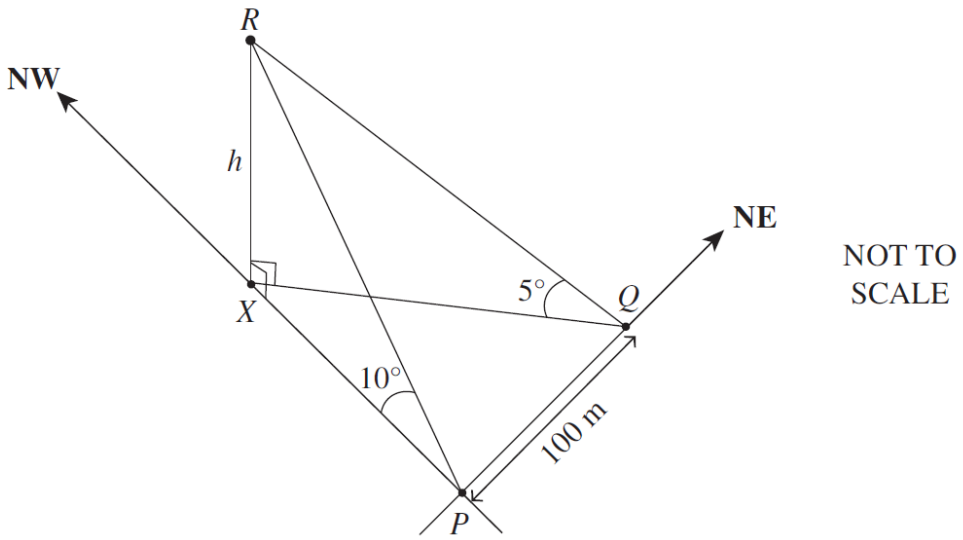
Question 24 on the next page

Question 24 (5 marks)

A hiker starts from point P and walks 100 metres along a straight path to the north-east, arriving at point Q. A peak is located at point R, which is directly above point X. From point P, the angle of elevation to peak R is 10°. At point Q, the angle of elevation to the peak is 5°.



(a) Show that $XP = h\cot 10°$ and find a similar expression for XQ. **2**

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Question 24 continues on page 19

(b) Hence, find the value of h. Give your answer correct to the nearest metre. **3**

Question 25 (3 marks)

Prove that $\dfrac{\sin A}{1+\cos A}+\dfrac{1+\cos A}{\sin A}=2\text{cosec}\,A$ **3**

Question 26 (4 marks)

(a) Find the area bounded by the curve $y = \frac{1}{x}$, the x-axis and the lines $x = 2$ and $x = 6$ by using the trapezoidal rule with seven function values. Answer correct to three decimal places. **2**

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(b) Calculate the same area by evaluating $\int_2^6 \frac{1}{x} dx$. Answer correct to 3 decimal places. **1**

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(c) Explain the difference between your answers in parts (a) and (b). **1**

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Question 27 (5 marks)

A scientist grows the number of bacteria according to the equation $N = N_0 e^{0.15t}$ where t is measured in days and N_0 is a constant.

(a) When $t = 3$ the number of bacteria was estimated at 1.5×10^8. Evaluate N_0. **2**

Answer in scientific notation correct to two significant figures.

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(b) The number of bacteria doubles every x days. Find x. **3**

Answer correct to one decimal place.

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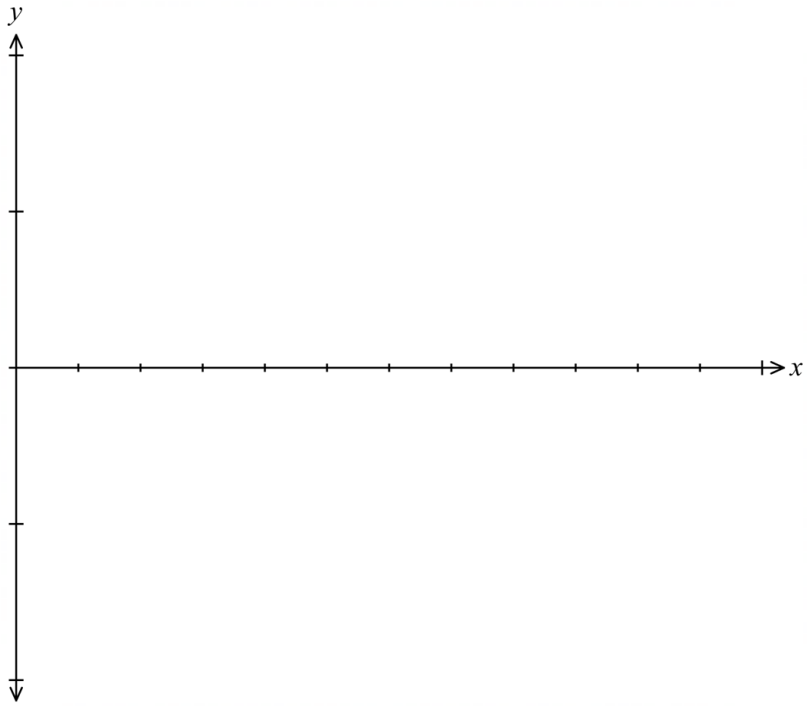
Question 28 (6 marks)

(a) Sketch the graphs of $y = \sin x$ and $y = \sqrt{3}\cos x$ over the domain $0 \le x \le 2\pi$. **2**



(b) The graphs intersect at points A and B. What are the coordinates of A and B ? **2**

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(c) Find the area enclosed by $y = \sin x$ and $y = \sqrt{3}\cos x$ between A and B. **2**

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Question 29 (7 marks)

A closed cylinder has a height h cm and a radius r cm. The volume of the cylinder is 450 cm^3.

(a) Find an expression for h in terms of r. **1**

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(b) Show that the surface area SA cm^2 of the cylinder is given by: **2**

$$SA = 2\pi r^2 + \frac{900}{r}$$

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Question 29 continues on page 25

(c) If the surface area of cylinder is to be minimised, find the radius of the cylinder. **4**

Answer correct to one decimal place.

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Question 30 (4 marks)

The population of a seal colony varies according to the equation $P = 100 \sin\left(\frac{\pi t}{6}\right) + 300$ where t is in months.

(a) Find the maximum and minimum populations of the colony. **2**

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(b) Find the number of months it takes for the population to vary from the maximum population to the minimum population. **2**

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Question 31 (3 marks)

Harrison received a lump sum from his superannuation fund when he retired. He deposited some of this into a new bank account, earning 2% per annum compound interest.

The table below shows present value interest factors for an annuity of \$1 for various interest rates (r) and numbers of periods (N).

Number of interest periods (N)	Interest rate per period as a decimal r			
	0.01	0.015	0.02	0.025
5	4.8534	4.7826	4.7135	4.6458
10	9.4713	9.2222	8.9826	8.7521
15	13.8651	13.3432	12.8493	12.3814
20	18.0456	17.1686	16.3514	15.5892

(a) For the first 5 years following retirement, Harrison was able to withdraw \$36 000 at the end of each year to pay his remaining mortgage and living expenses. **1**

Show that the present value of a \$36 000 annuity for 5 years is \$169 686.

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(b) Harrison wants to reduce his withdrawal to \$20 000 per year for the next 10 years, starting 5 years after his retirement. **2**

If the amount he deposited when he opened the account was \$340 000, would he be able to make these planned withdrawals over the ten years? (Justify your answer with calculations)

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Question 32 (4 marks)

A particle is initially at the origin and moves along a straight line with its displacement given by

$$x = \frac{20\ln(1+t)}{(1+t)^2}$$

(a) Show that the velocity of the particle is given by **2**

$$v = \frac{20(1-2\ln(1+t))}{(1+t)^3}$$

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Question 32 continues on page 29

(b) Describe what happens to the displacement and velocity eventually. **2**

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Question 33 on the next page

Question 33 (5 marks)

Ryan opens a bank account. At the start of each month he deposits \$$M$ into the bank account. At the end of each month, after interest is added into the bank account, the bank withdraws \$1 950 from the bank account as a loan repayment. Let A_n be the amount in the bank account after the nth withdrawal. The bank account pays interest of 3.6% per annum compounded monthly.

(a) Show that after the second withdrawal the amount in the bank account is given by: **2**

$A_2 = M[(1.003)^2 + (1.003)] - 1\,950(1.003 + 1)$

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(b) What is the amount Ryan needs to deposit into the bank account each month, \$$M$, so that \$70 000 remains after the last withdrawal of the fifth year ? **3**
Answer correct to the nearest dollar.

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END OF EXAMINATION

Section II Extra Writing Space

If you use this space, clearly indicate which questions you are answering.

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Student Number:

Student Name:_____

SECTION 1 Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample

2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

A ⊗ B ⊗ *correct* C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word correct and drawing an arrow as follows: correct

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1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

Teacher: Blair / Karagiannis (Circle)

Student Number:

SOLUTIONS

Danebank
An Anglican School for Girls

Year 12 Advanced Mathematics
Task 4 – Trial HSC Examination
Wednesday 3rd August 2022

General Instructions	
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Working time	3 hours

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Total	**/100 Marks**	**Weighting: 30%**

Special Instructions
NESA approved Calculator may be used. A separate reference sheet and Multiple Choice answer sheet is provided. Attempt all questions. This examination must not be removed from the Examination Room. In Question 11-33 show relevant mathematical reasoning and/or calculations. Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Extra writing space is provided at the back of the examination paper on pages 32 - 35. If you use this space, clearly indicate which question you are answering.

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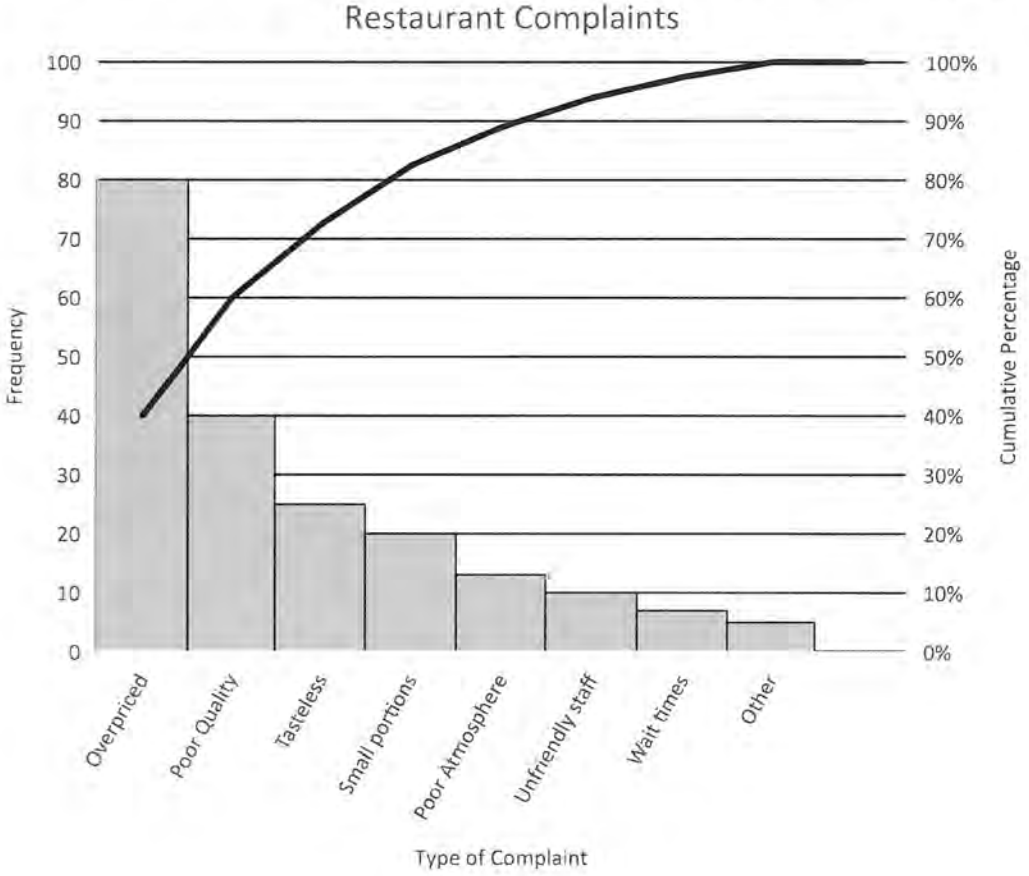
SECTION I — Multiple Choice — 10 marks

Attempt Questions 1-10.

Allow about 15 minutes for this section.

Use the multiple choice answer sheet for questions 1 - 10.

1. The Pareto chart below shows the frequency of types of complaints received by a restaurant.



What percentage of complaints were for poor quality?

A. 20%
B. 40%
C. 60%
D. 80%

2. The first three terms of an arithmetic series are 2, 7 and 12. What is the 15th term of this series?

$2, 7, 12, \ldots$

$a = 2, d = 5, n = 15$

$T_{15} = 2 + 14 \times 5$

$= 72$

A. 72
B. 77
C. 555
D. 595

3. The mean of a set of five numbers is 11. A sixth number is added to the set and the mean decreases to 10.
What is the sixth number?

$\frac{55+x}{6} = 10$

$x = 60 - 55$

A. 1

B. 3

C. 5

D. 7

4. The stem and leaf plot shows the number of shots needed to complete a game of pool on two tables. Twenty games were played on each table.

Table A						Table B					
		7	6	4	**0**	6	8				
	8	4	4	1	**1**	2	6	7	7		
8	8	7	7	5	3	**2**	0	2	3	4	4
	6	5	5	4	**3**	1	3	5			
			2	1	**4**	5	5	6	8	9	
					5						
				9	**6**	9					

$Q_1 = 14$

$Q_2 = 27$

$Q_3 = 35$

$IQR = 21$

$35 + 1.5 \times 21 = 66.5$

$Q_1 = 17$

$Q_2 = 24$

$Q_3 = 45$

$IQR = 28$

$45 + 1.5 \times 28 = 87$

Is the result of 69 shots an outlier for either of the tables?

A. No, it isn't for either table.

B. Yes, for Table A only

C. Yes, for Table B only

D. Yes, for both tables

5. What is the exact value of $\sin 135° + \tan 300°$?

$\frac{1}{\sqrt{2}} - \sqrt{3} = \frac{\sqrt{2} - 2\sqrt{3}}{2}$

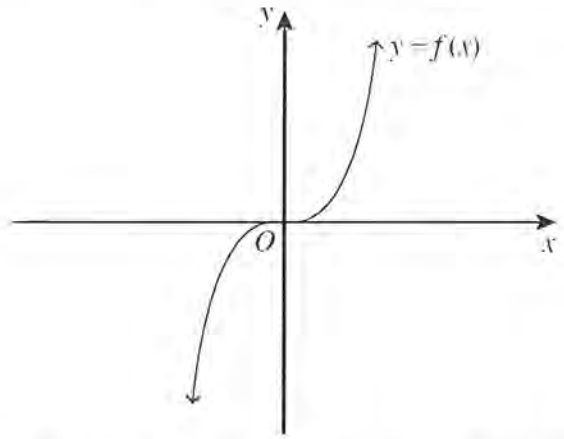
A. $\frac{\sqrt{2}+2\sqrt{3}}{2}$

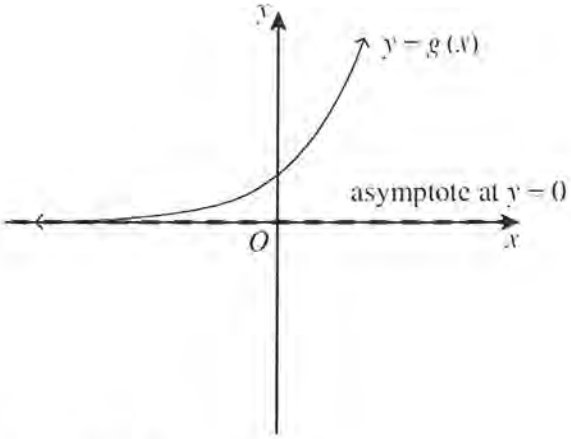
B. $\frac{\sqrt{2}-2\sqrt{3}}{2}$

C. $\frac{\sqrt{5}}{2}$

D. $\frac{-\sqrt{5}}{2}$

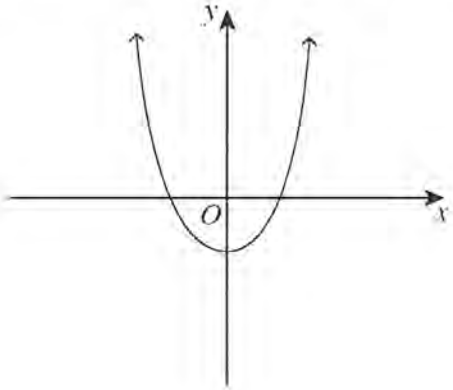
6. The graphs of $f(x)$ and $g(x)$ are shown.



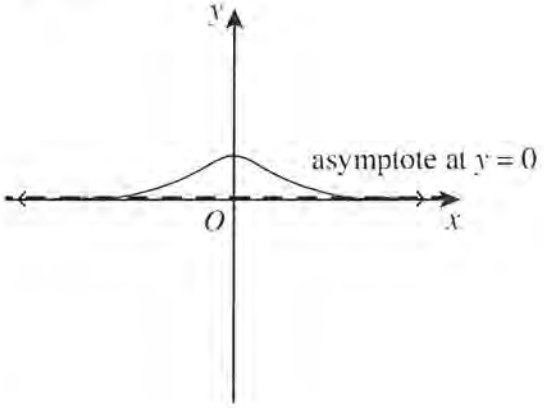


Which of the following best represents the graph of $y = f(g(-x))$?

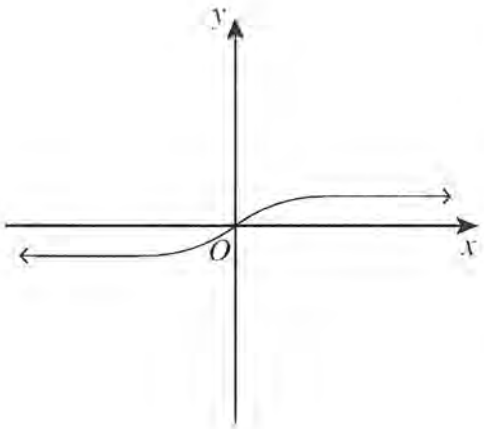
A.



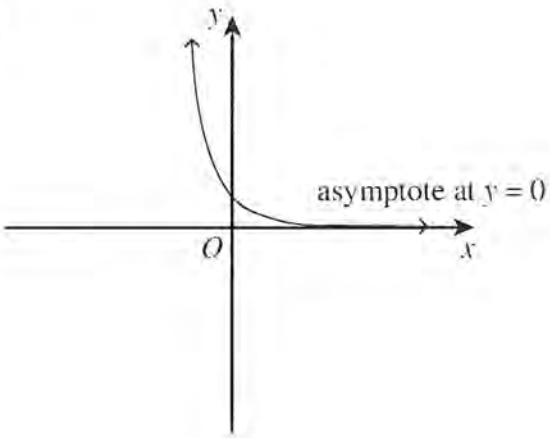
B.



C.



D.



7. The graph of $y = g(x)$ has been transformed by translating it vertically by 3 units and then dilating it horizontally by a factor of 2. Which of the following is the equation of the transformed graph?

A. $y = 2g(x) + 3$

B. $y = 2g(x + 3)$

C. $y = g(2x) + 3$

D. $y = g\left(\frac{x}{2}\right) + 3$

8. What is $\int 6x^2 (4x^3 - 5)^3 \, dx$?

$\frac{1}{2}\int 12x^2(4x^3-5)^3 dx = \frac{1}{2} \times \frac{1}{4}(4x^3-5)^4 + C$

A. $\frac{(4x^3 - 5)^4}{8} + C$

B. $12x (4x^3 - 5)^4 + C$

C. $2x^3(4x^3 - 5)^4 + C$

D. $\frac{2x^3(4x^3-5)^4}{8} + C$

9. Which is the complete solution set to the equation $\sin^2\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$, for the domain $0 \leq x \leq 2\pi$?

$\frac{\pi}{6} \leq x + \frac{\pi}{6} \leq 2\pi + \frac{\pi}{6}$

$\sin\left(x + \frac{\pi}{6}\right) = \pm\frac{1}{\sqrt{2}}$

$x + \frac{\pi}{6} = \frac{\pi}{4}, \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$= \frac{\pi}{4} - \frac{\pi}{6}, 3\pi - \frac{\pi}{6}, \frac{5\pi}{4} - \frac{\pi}{6}, \frac{7\pi}{4} - \frac{\pi}{6}$

A. $x = \frac{\pi}{12}, \frac{13\pi}{12}$

B. $x = 0, \frac{\pi}{2}, , \pi, \frac{3\pi}{2}$

C. $x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$

D. $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{9\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{15\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{21\pi}{12}, \frac{23\pi}{12}$

10. It is known that $f(x)$ is an odd function and $g(x)$ is an even function.
Given that $f(2) = 2$ and $g(2) = -2$, what is the value of $f(g(-2)) + g(f(-2))$?

$f(-2) = -2 \quad g(-2) = -2$

$f(-2) + g(-2) = -2 - 2$

A. -4

B. -2

C. 0

D. 4

END OF SECTION 1

SECTION II

90 marks

Attempt Questions 11-33

Allow about 2 hours and 45 minutes for this section

Show all necessary working in the space provided. If extra space is required, use the lined pages at the end of this section, carefully labelling the additional work with the appropriate question number and part.

Question 11 (2 marks)

Solve $|5-3x| \geq 11$. **2**

$-11 \geq 5-3x \geq 11$

$-16 \geq -3x \geq 6$

$\frac{16}{3} \leq x \leq -2$

$\therefore x \leq -2, \; x \geq \frac{16}{3}$

Question 12 (3 marks)

Find the equation of the normal to the curve $y = x^2 + 3x$ at $x = 1$. **3**

$y' = 2x+3$ When $x=1$, $y=4$

When $x=1$, $y'=5$

$m_1 m_2 = -1$ when lines are perpendicular

$m_1 = 5$

$\therefore m_2 = -\frac{1}{5}$

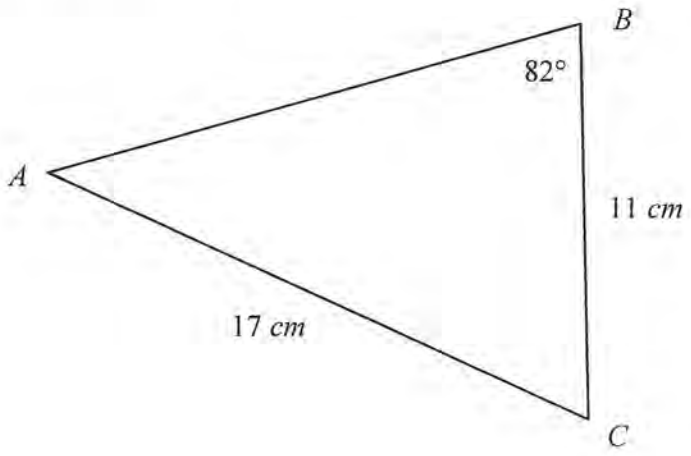
Equation of normal is

$y-4 = -\frac{1}{5}(x-1)$

$y = -\frac{1}{5}x + \frac{21}{5}$

Question 13 (4 marks)

Triangle ABC is shown below



(a) Using the sine rule, show that $\angle A \approx 40°$. **2**

$\frac{\sin A}{11} = \frac{\sin 82°}{17}$

$\therefore \angle A \doteq 39°51'$

$\therefore \angle A \doteq 40°$

(b) Hence, calculate the area of triangle ABC, correct to the nearest cm^2. **2**

$A = \frac{1}{2}(11)(17)\sin C$

$\angle C \doteq 58°9'$

$= \frac{1}{2}(11)(17)\sin 58°9'$

$A \doteq 79\ cm^2$

Question 14 (2 marks)

Evaluate $\sum_{r=2}^{10} 3^r$ 2

$10-2+1=9$

$3^2+3^3+3^4+\ldots+3^{10}$

$a=9,\ r=3,\ n=9$

$S_9 = \frac{9(3^9-1)}{3-1}$

$= 88569$

Question 15 (2 marks)

Express $\frac{2x}{x^2-4} - \frac{x+1}{x^2-x-2}$ as a single algebraic fraction, in simplest form. 2

$= \frac{2x}{(x-2)(x+2)} - \frac{x+1}{(x-2)(x+1)}$

$= \frac{2x-(x+2)}{(x-2)(x+2)}$

$= \frac{x-2}{(x-2)(x+2)}$

$= \frac{1}{x+2}$

Question 16 (4 marks)

Differentiate with respect to x :

(a) $y = e^{2x}\sin(3x)$ **2**

$u = e^{2x}$ $\quad v = \sin 3x$

$u' = 2e^{2x}$ $\quad v' = 3\cos 3x$

$y' = 2e^{2x}\sin 3x + 3e^{2x}\cos 3x$

$= e^{2x}(2\sin 3x + 3\cos 3x)$

(b) $y = \dfrac{\cos x}{6-x}$ **2**

$u = \cos x$ $\quad v = 6-x$

$u' = -\sin x$ $\quad v' = -1$

$y' = -\dfrac{(6-x)\sin x + \cos x}{(6-x)^2}$ OR

$= \dfrac{\cos x - (6-x)\sin x}{(6-x)^2}$

Question 17 (3 marks)

The circle $x^2 - 2x + y^2 + 8y + 8 = 0$ is reflected in the x-axis. Find the coordinates of the centre and the radius. **3**

$x^2 - 2x + 1 + y^2 + 8y + 16 = -8 + 1 + 16$

$(x-1)^2 + (y+4)^2 = 9$

Centre $(1, -4)$, radius is 3 units

When it is reflected in the x-axis

Centre $(1, 4)$, radius is 3 units

Question 18 (5 marks)

(a) Find $\int \frac{1}{2x+3}\,dx$ **2**

$\frac{1}{2}\int \frac{2}{2x+3}\,dx = \frac{1}{2}\ln|2x+3| + C$

(b) Given that $\int_0^3 \frac{1}{2x+3}\,dx = \ln\sqrt{P}$, find the value of P. **3**

$\frac{1}{2}\left[\ln|2x+3|\right]_0^3 = \frac{1}{2}(\ln 9 - \ln 3)$

$= \frac{1}{2}\left(\ln\frac{9}{3}\right)$

$= \frac{1}{2}\ln 3$

$= \ln\sqrt{3}$

$\therefore P = 3$

Question 19 (4 marks)

Sketch the graph of the curve $y = x^3 - 2x^2 + 1$, labelling the stationary points, any points of inflection and the y-intercept. Do NOT determine the x intercepts of the curve. **4**

$y' = 3x^2 - 4x$

$y'' = 6x - 4$

Stationary pts occur when $y' = 0$

$3x^2 - 4x = 0$

$x(3x - 4) = 0$

$\therefore x = 0, \quad x = \frac{4}{3}$

When $x = 0$, $y = 1$

$x = \frac{4}{3}, \quad y = \frac{-5}{27}$

When $x = 0$

$y'' = -4$

$< 0 \cap$

$\therefore (0, 1)$ is a max turning pt

When $x = \frac{4}{3}$

$y'' = 6\left(\frac{4}{3}\right) - 4$

$= 4$

$> 0 \cup$

$\therefore \left(\frac{4}{3}, \frac{-5}{27}\right)$ is a min turning pt

Possible pts of inflection when $y'' = 0$

$6x - 4 = 0$

$6x = 4$

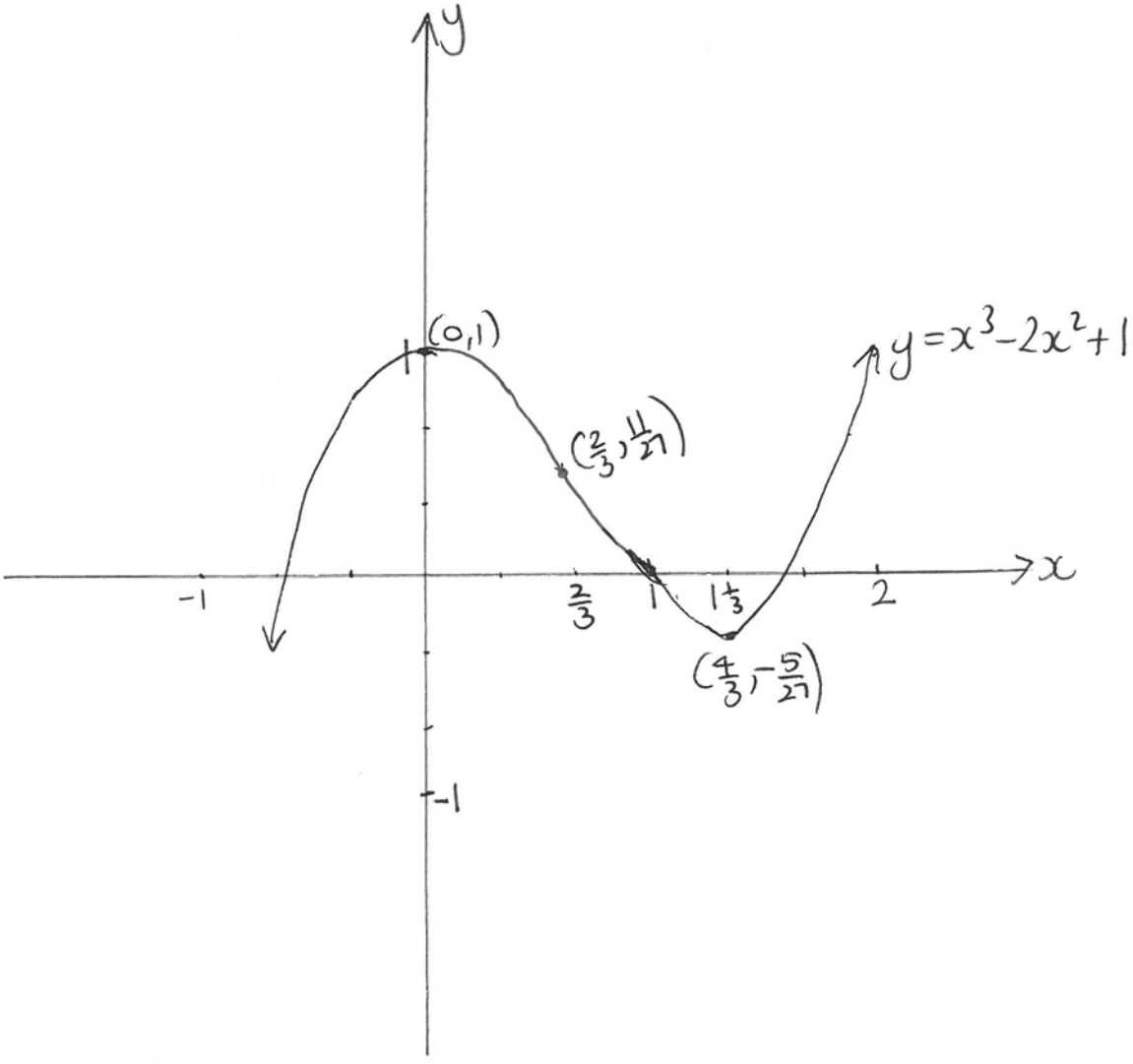
$x = \frac{2}{3}$

When $x = \frac{2}{3}$, $y = \frac{11}{27}$

x	$\frac{1}{3}$	$\frac{2}{3}$	1
y''	-2	0	2

∴ change in concavity

∴ $(\frac{2}{3}, \frac{11}{27})$ is a point of inflection since concavity changes



Question 20 on the next page

Question 20 (4 marks)

The following table shows a probability distribution for the random variable X, where $E(X)=\frac{6}{5}$.

x	0	1	2	3	Sum
$P(X=x)$	p	$\frac{1}{2}$	$\frac{3}{10}$	q	1
$x\,p(x)$	0	$\frac{1}{2}$	$\frac{6}{10}$	$3q$	$\frac{6}{5}$ $E(X)$
$x^2\,p(x)$	0	$\frac{1}{2}$	$\frac{12}{10}$	$9q$	2

(a) Find the values of p and q. 3

$0\times p+\frac{1}{2}\times 1+2\times\frac{3}{10}+3\times q=\frac{6}{5}$

$3q=\frac{1}{10}$

$\therefore q=\frac{1}{30}$

$p+\frac{1}{2}+\frac{3}{10}+\frac{1}{30}=1$

$\therefore p=\frac{1}{6}$

(b) Find Var(X) 1

$Var(x)=2-\left(\frac{6}{5}\right)^2$

$=\frac{14}{25}$

Question 21 (5 marks)

Let $f(x)$ be a continuous function, such that $\int_1^5 3f(x)\,dx = 12$.

(a) Show that $\int_5^1 f(x)\,dx = -4$. **2**

$$\int_5^1 f(x)\,dx = -\int_1^5 f(x)\,dx$$

$$= -\frac{1}{3}\int_1^5 3f(x)\,dx$$

$$= -\frac{1}{3} \times 12$$

$$\therefore \int_5^1 f(x)\,dx = -4$$

(b) Find the value of $\int_1^2 (x + f(x))\,dx + \int_2^5 (x + f(x))\,dx$. **3**

$$\int_1^5 (x + f(x))\,dx = \int_1^5 x\,dx + \int_1^5 f(x)\,dx$$

$$= \left[\frac{x^2}{2}\right]_1^5 + 4$$

$$= \frac{25}{2} - \frac{1}{2} + 4$$

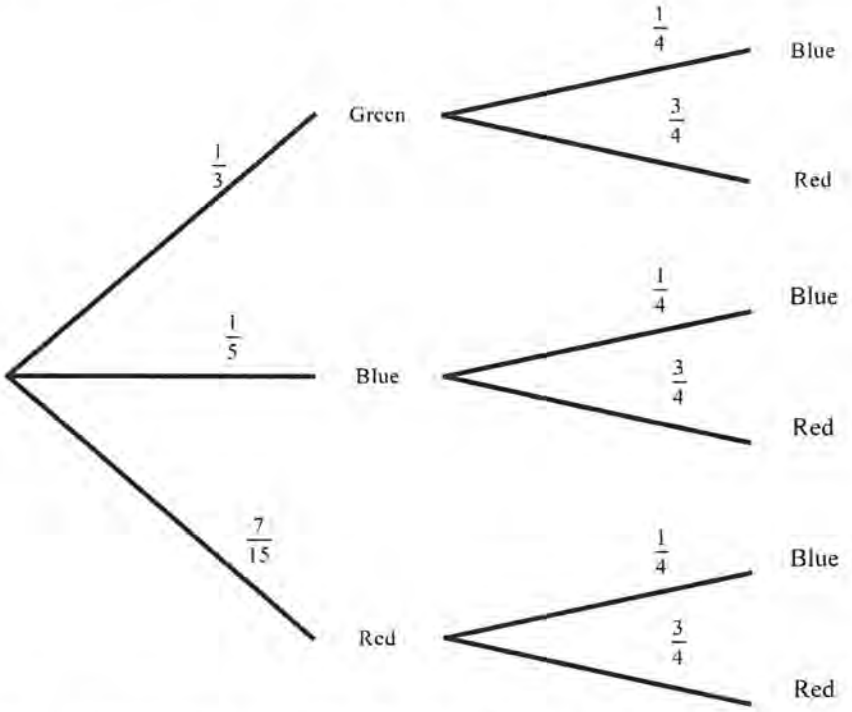
$$= 16$$

Question 22 (3 marks)

An artist has two boxes of paints beside his easel. The first box contains 15 tubes which contain green, red or blue paint and the second contains four tubes which have only red or blue paint.

The artist chooses one tube of paint from each box.

The tree diagram below shows the probabilities of him choosing a given colour.



(a) What is the probability that the tubes he chooses are red and blue? **1**

$P(R \text{ and } B) = \frac{1}{5} \times \frac{3}{4} + \frac{7}{15} \times \frac{1}{4}$

$= \frac{3}{20} + \frac{7}{60}$

$= \frac{4}{15}$

(b) What is the probability that the tubes he chooses are not the same colour? **2**

$P(\text{not same}) = 1 - P(\text{same})$

$= 1 - \left(\frac{1}{5} \times \frac{1}{4} + \frac{7}{15} \times \frac{3}{4}\right)$

$= 1 - \frac{2}{5}$

$= \frac{3}{5}$

Question 23 (3 marks)

Find $f'(4)$ given $f(x) = \sqrt{x}\left(3x - \frac{2}{x\sqrt{x}}\right)$ **3**

$f(x) = 3x^{\frac{3}{2}} - 2x^{-1}$

$f'(x) = 3 \times \frac{3}{2} x^{\frac{1}{2}} + 2x^{-2}$

$= \frac{9}{2}\sqrt{x} + \frac{2}{x^2}$

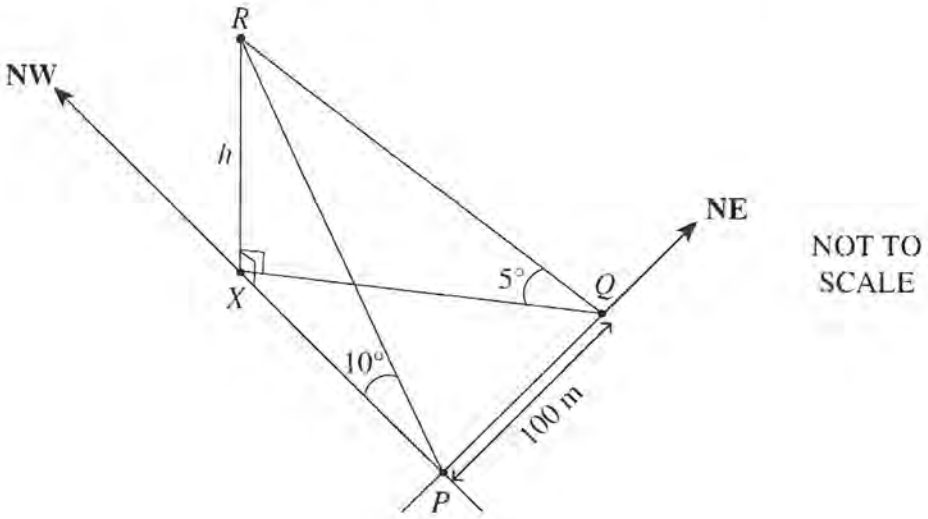
$f'(4) = \frac{9}{2} \times 2 + \frac{2}{16}$

$= 9.125$

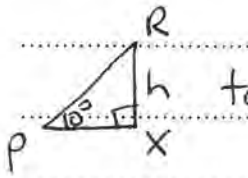
Question 24 on the next page

Question 24 (5 marks)

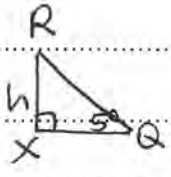
A hiker starts from point P and walks 100 metres along a straight path to the north-east, arriving at point Q. A peak is located at point R, which is directly above point X. From point P, the angle of elevation to peak R is 10°. At point Q, the angle of elevation to the peak is 5°.



(a) Show that $XP = h\cot 10°$ and find a similar expression for XQ. **2**



$\tan 10° = \frac{h}{XP}$



$\tan 5° = \frac{h}{XQ}$

$XP = \frac{h}{\tan 10°}$

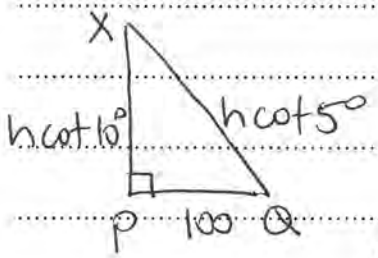
$XQ = \frac{h}{\tan 5°}$

$\therefore XP = h\cot 10°$

$\therefore XQ = h\cot 5°$

Question 24 continues on page 19

(b) Hence, find the value of h. Give your answer correct to the nearest metre. **3**



$(h\cot 5°)^2 = 100^2 + (h\cot 10°)^2$

$10\,000 = h^2\cot^2 5° - h^2\cot^2 10°$

$= h^2(\cot^2 5° - \cot^2 10°)$

$h^2 = \dfrac{10\,000}{(\cot^2 5° - \cot^2 10°)}$

$= 101.5407198$

$h \doteq 10.0767$

$\therefore h \doteq 10\text{m}$

Question 25 (3 marks)

Prove that $\frac{\sin A}{1+\cos A}+\frac{1+\cos A}{\sin A}=2\text{cosec } A$ **3**

$$LHS = \frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A}$$

$$= \frac{\sin^2 A + (1+\cos A)^2}{\sin A(1+\cos A)}$$

$$= \frac{\sin^2 A + 1 + 2\cos A + \cos^2 A}{\sin A(1+\cos A)}$$

$$= \frac{1 + 1 + 2\cos A}{\sin A(1+\cos A)} \quad \text{since } \sin^2 A + \cos^2 A = 1$$

$$= \frac{2 + 2\cos A}{\sin A(1+\cos A)}$$

$$= \frac{2(1+\cos A)}{\sin A(1+\cos A)}$$

$$= \frac{2}{\sin A}$$

$$= 2\,\text{cosec } A$$

$$= RHS$$

Question 26 (4 marks)

(a) Find the area bounded by the curve $y = \frac{1}{x}$, the x-axis and the lines $x = 2$ and $x = 6$ by using the trapezoidal rule with seven function values. Answer correct to three decimal places. **2**

x	2	$\frac{8}{3}$	$\frac{10}{3}$	$\frac{12}{3}$	$\frac{14}{3}$	$\frac{16}{3}$	6
y	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{10}$	$\frac{3}{12}$	$\frac{3}{14}$	$\frac{3}{16}$	$\frac{1}{6}$
		$\times 2$	$\times 2$	$\times 2$	$\times 2$	$\times 2$	

$h = \frac{6-2}{6}$

$h = \frac{2}{3}$

$\text{Total} = \frac{1}{2} + \frac{1}{6} + 2\left(\frac{3}{8} + \frac{3}{10} + \frac{3}{12} + \frac{3}{14} + \frac{3}{16}\right)$

$= \frac{2789}{840}$

$A \doteq \frac{1}{2} \times \frac{2}{3} \times \frac{2789}{840}$

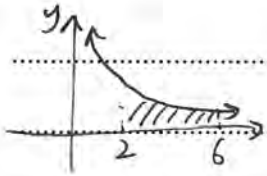
$\therefore A \doteq 1.107 \text{ units}^2$

(b) Calculate the same area by evaluating $\int_2^6 \frac{1}{x} dx$. Answer correct to 3 decimal places. **1**

$[\ln x]_2^6 = \ln 6 - \ln 2$

$\therefore A \doteq 1.099 \text{ units}^2$

(c) Explain the difference between your answers in parts (a) and (b). **1**



The curve of $y = \frac{1}{x}$ is concave up between $x = 2$ and $x = 6$, so the trapezoidal rule will overestimate the area.

Question 27 (5 marks)

A scientist grows the number of bacteria according to the equation $N = N_0 e^{0.15t}$ where t is measured in days and N_0 is a constant.

(a) When $t = 3$ the number of bacteria was estimated at 1.5×10^8. Evaluate N_0. 2

Answer in scientific notation correct to two significant figures.

$N = N_0 e^{0.15t}$

$1.5 \times 10^8 = N_0 e^{0.15 \times 3}$

$N_0 = \dfrac{1.5 \times 10^8}{e^{0.15 \times 3}}$

$= 95644222.74$

$\therefore N_0 \doteq 9.6 \times 10^7$

(b) The number of bacteria doubles every x days. Find x. 3

Answer correct to one decimal place.

$N = 2N_0$

$2N_0 = N_0 e^{0.15x}$

$2 = e^{0.15x}$

$x = \dfrac{\ln 2}{0.15}$

$x \doteq 4.6$ days

OR

when $t = 3 + x$ $N = 2 \times 1.5 \times 10^8$

since when $t = 3$ $N = 1.5 \times 10^8$

$3 \times 10^8 = 9.6 \times 10^7 e^{0.15(3+x)}$

$e^{0.15(3+x)} = \dfrac{3 \times 10^8}{9.6 \times 10^7}$

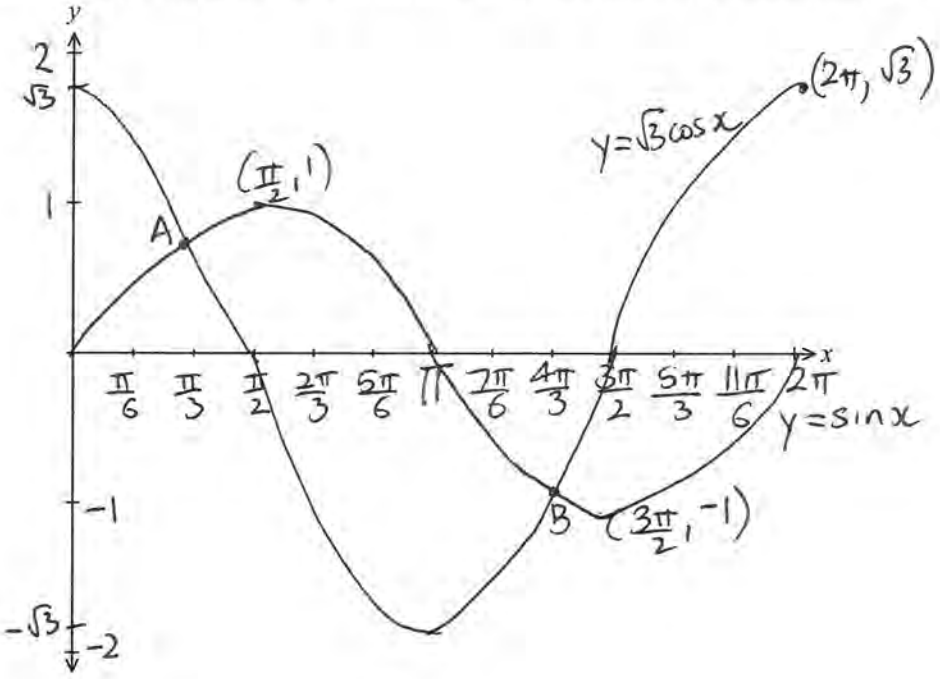
$0.15(3+x) = \log_e\left(\dfrac{3 \times 10^8}{9.6 \times 10^7}\right)$

$3 + x = \dfrac{\log_e\left(\dfrac{3 \times 10^8}{9.6 \times 10^7}\right)}{0.15}$

$x \doteq 4.6$

Question 28 (6 marks)

(a) Sketch the graphs of $y = \sin x$ and $y = \sqrt{3}\cos x$ over the domain $0 \le x \le 2\pi$. **2**



(b) The graphs intersect at points A and B. What are the coordinates of A and B? **2**

$\sin x = \sqrt{3}\cos x$

$\tan x = \sqrt{3}$

$x = \frac{\pi}{3}, \frac{4\pi}{3}$

When $x = \frac{\pi}{3}$, $y = \frac{\sqrt{3}}{2}$ $\therefore A\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$, $B\left(\frac{4\pi}{3}, -\frac{\sqrt{3}}{2}\right)$

When $x = \frac{4\pi}{3}$, $y = -\frac{\sqrt{3}}{2}$

(c) Find the area enclosed by $y = \sin x$ and $y = \sqrt{3}\cos x$ between A and B. **2**

$$A = \int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} (\sin x - \sqrt{3}\cos x)\, dx$$

$$= \left[-\cos x - \sqrt{3}\sin x\right]_{\frac{\pi}{3}}^{\frac{4\pi}{3}}$$

$$= -\cos\frac{4\pi}{3} - \sqrt{3}\sin\frac{4\pi}{3} - \left(-\cos\frac{\pi}{3} - \sqrt{3}\sin\frac{\pi}{3}\right)$$

$$= \frac{1}{2} + \sqrt{3} \times \frac{\sqrt{3}}{2} + \frac{1}{2} + \sqrt{3} \times \frac{\sqrt{3}}{2}$$

$\therefore A = 4$ units^2

Question 29 (7 marks)

A closed cylinder has a height h cm and a radius r cm. The volume of the cylinder is 450 cm^3.

(a) Find an expression for h in terms of r. **1**

$V = \pi r^2 h$

$\therefore h = \frac{450}{\pi r^2}$

(b) Show that the surface area SA cm^2 of the cylinder is given by: **2**

$SA = 2\pi r^2 + \frac{900}{r}$

$SA = 2\pi r^2 + 2\pi rh$

$= 2\pi r^2 + 2\pi r\left(\frac{450}{\pi r^2}\right)$

$= 2\pi r^2 + \frac{900\pi r}{\pi r^2}$

$\therefore SA = 2\pi r^2 + \frac{900}{r}$, as required

Question 29 continues on page 25

(c) If the surface area of cylinder is to be minimised, find the radius of the cylinder. **4**

Answer correct to one decimal place.

$SA = 2\pi r^2 + 900r^{-1}$

$\frac{dSA}{dr} = 4\pi r - 900r^{-2}$

$\frac{d^2SA}{dr^2} = 4\pi + 1800r^{-3}$

When $\frac{dSA}{dr} = 0$

$4\pi r - \frac{900}{r^2} = 0$

$\frac{900}{r^2} = 4\pi r$

$4\pi r^3 = 900$

$r^3 = \frac{900}{4\pi}$

$r = \sqrt[3]{\frac{225}{\pi}}$

When $r = \sqrt[3]{\frac{225}{\pi}}$, $\frac{d^2SA}{dr^2} = 4\pi + \frac{1800}{\frac{225}{\pi}}$

$\doteq 37.699$

$> 0 \quad \cup$

$\therefore$ the SA is minimised when $r \doteq 4.2$ cm

Question 30 (4 marks)

The population of a seal colony varies according to the equation $P = 100\sin\left(\frac{\pi t}{6}\right) + 300$ where t is in months.

(a) Find the maximum and minimum populations of the colony. **2**

Minimum = 300 − 100

= 200

Maximum = 300 + 100

= 400

(b) Find the number of months it takes for the population to vary from the maximum population to the minimum population. **2**

Period = $\frac{2\pi}{\frac{\pi}{6}}$

= 12

∴ it takes 6 months for the popn to vary from max to min

Question 31 (3 marks)

Harrison received a lump sum from his superannuation fund when he retired. He deposited some of this into a new bank account, earning 2% per annum compound interest.

The table below shows present value interest factors for an annuity of $1 for various interest rates (r) and numbers of periods (N).

Number of interest periods (N)	Interest rate per period as a decimal r			
	0.01	0.015	0.02	0.025
5	4.8534	4.7826	4.7135	4.6458
10	9.4713	9.2222	8.9826	8.7521
15	13.8651	13.3432	12.8493	12.3814
20	18.0456	17.1686	16.3514	15.5892

(a) For the first 5 years following retirement, Harrison was able to withdraw $36 000 at the end of each year to pay his remaining mortgage and living expenses. **1**

Show that the present value of a $36 000 annuity for 5 years is $169 686.

$PV = 36000 \times 4.7135$

$= \$169\,686$, as required

(b) Harrison wants to reduce his withdrawal to $20 000 per year for the next 10 years, starting 5 years after his retirement. **2**

If the amount he deposited when he opened the account was $340 000, would he be able to make these planned withdrawals over the ten years? (Justify your answer with calculations)

$PV = 20000 \times 12.8493$ (15 yrs)

$= 256986$

$PV = 20000 \times 4.7135$ (5 yrs)

$= 94270$

$PV = 256986 - 94270$

$= 162716$ (10 yrs)

$\therefore \$162716 + \$169686 = \$332402$

$\therefore \$340000$ would be enough

Question 32 (4 marks)

A particle is initially at the origin and moves along a straight line with its displacement given by

$$x = \frac{20\ln(1+t)}{(1+t)^2} \quad \frac{u}{w}$$

(a) Show that the velocity of the particle is given by **2**

$$v = \frac{20(1-2\ln(1+t))}{(1+t)^3}$$

$u = 20\ln(1+t)$ $\quad w = (1+t)^2$

$u' = \frac{20}{1+t}$ $\quad w' = 2(1+t)$

$$v = \frac{(1+t)^2 \times \frac{20}{1+t} - 2(1+t)(20\ln(1+t))}{(1+t)^4}$$

$$v = \frac{20(1+t) - 40(1+t)\ln(1+t)}{(1+t)^4}$$

$$= \frac{(1+t)[20 - 40\ln(1+t)]}{(1+t)^4}$$

$$= \frac{20(1 - 2\ln(1+t))}{(1+t)^3}, \text{ as required}$$

Question 32 continues on page 29

(b) Describe what happens to the displacement and velocity eventually. **2**

As $t \to \infty$, $v \to 0^-$

As $t \to \infty$, $x \to 0^+$

As time goes on the particle approaches the origin and slows down

Question 33 on the next page

Question 33 (5 marks)

Ryan opens a bank account. At the start of each month he deposits $\$M$ into the bank account. At the end of each month, after interest is added into the bank account, the bank withdraws \$1 950 from the bank account as a loan repayment. Let A_n be the amount in the bank account after the nth withdrawal. The bank account pays interest of 3.6% per annum compounded monthly.

(a) Show that after the second withdrawal the amount in the bank account is given by: **2**

$A_2 = M[(1.003)^2 + (1.003)] - 1\,950(1.003 + 1)$

$R = \frac{3.6\%}{12} = 0.003$

$A_1 = M(1.003) - 1950$

$A_2 = A_1(1.003) + M(1.003) - 1950$

$= [M(1.003) - 1950](1.003) + M(1.003) - 1950$

$= M(1.003)^2 - 1950(1.003) + M(1.003) - 1950$

$= M(1.003^2 + 1.003) - 1950(1.003 + 1)$

as required

(b) What is the amount Ryan needs to deposit into the bank account each month, $\$M$, so that \$70 000 remains after the last withdrawal of the fifth year ? **3**
Answer correct to the nearest dollar.

$A_3 = A_2(1.003) + M(1.003) - 1950$

$= [M(1.003^2 + 1.003) - 1950(1 + 1.003)](1.003) + M(1.003) - 1950$

$= M(1.003^3 + 1.003^2) - 1950(1.003 + 1.003^2) + M(1.003) - 1950$

$= M(1.003^3 + 1.003^2 + 1.003) - 1950(1 + 1.003 + 1.003^2)$

Following pattern, After 60 months

$A_{60} = M(1.003^{60} + 1.003^{59} + 1.003^{58} + \ldots + 1.003) - 1950(1 + 1.003 + 1.003^2 + \ldots + 1.003^{59})$

This is a GP in brackets
$a = 1.003, r = 1.003, n = 60$

This is a GP in brackets
$a = 1, r = 1.003, n = 60$

$70 000 remains after 60 months

$$70000 = M \times \frac{1.003(1.003^{60}-1)}{1.003-1} - \frac{1950(1.003^{60}-1)}{1.003-1}$$

$$\frac{M \times 1.003(1.003^{60}-1)}{0.003} = 70000 + \frac{1950(1.003^{60}-1)}{0.003}$$

$$M = \frac{0.003\left(70000 + \frac{1950(1.003^{60}-1)}{0.003}\right)}{1.003(1.003^{60}-1)}$$

$M \doteq \$3007.54$

∴ Ryan needs to deposit approx $3008 each month.

END OF EXAMINATION