Fort Street High School

NESA Number:

Name:

2025 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- Marks may be deducted for careless or badly arranged work.

Total marks : 90

Section I – 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt Questions 11 – 32
- Allow about 2 hours and 45 minutes for this section
- Write your student number on each answer booklet

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.
Question 1.

What is the solution to the equation $3x^2 + 5x - 1 = 0$?

A. $x = \dfrac{-5 \pm \sqrt{37}}{6}$

B. $x = \dfrac{-5 \pm \sqrt{13}}{6}$

C. $x = \dfrac{5 \pm \sqrt{37}}{6}$

D. $x = \dfrac{5 \pm \sqrt{13}}{6}$

Question 2.

Express $4x\sqrt{11}$ in the form of $\sqrt{a}$.

A. $\sqrt{44x^2}$

B. $\sqrt{44x}$

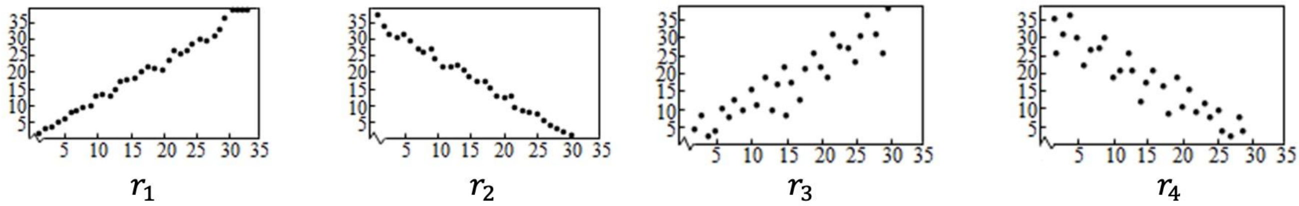
C. $\sqrt{176x^2}$

D. $\sqrt{176x}$

Question 3.

A statistics department conducted an analysis on four sets of data, producing the scatter plots shown.

Which of the following comparisons is correct based on the correlation coefficients?



A. $r_4 < r_2 < 0 < r_1 < r_3$

B. $r_2 < r_4 < 0 < r_1 < r_3$

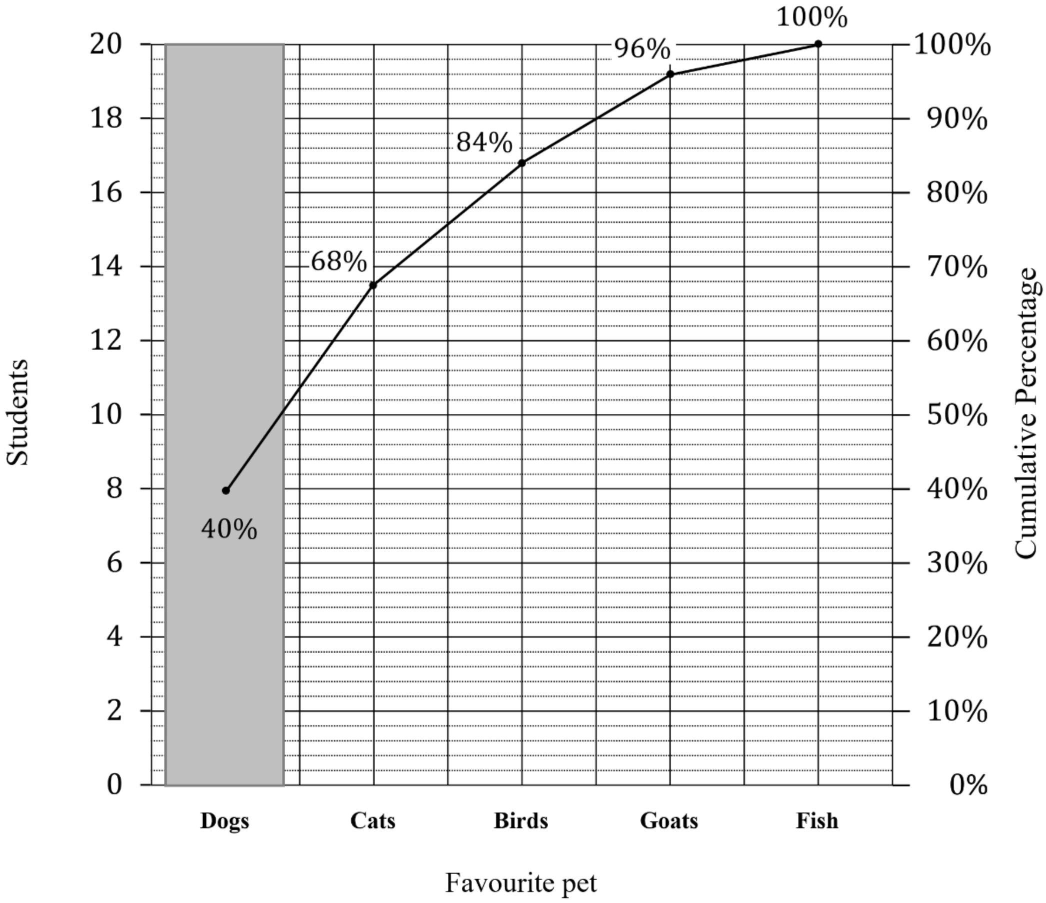
C. $r_2 < r_4 < 0 < r_3 < r_1$

D. $r_4 < r_2 < 0 < r_3 < r_1$

Question 4.

A teacher asked fifty students to identify their favourite type of pet.

A partially completed Pareto chart shows some of the data collected.



How many students chose cats as their favourite pet?

A. 12 students

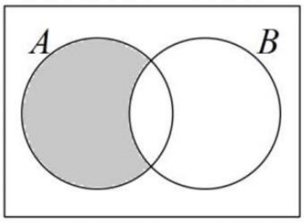
B. 14 students

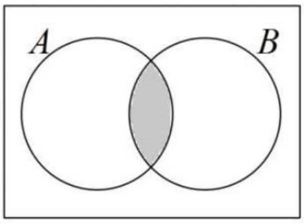
C. 28 students

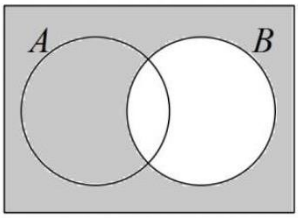
D. 34 students

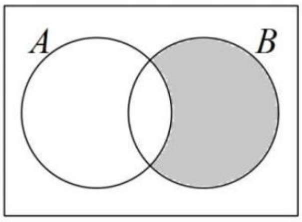
Question 5.

Which of the Venn diagrams best represent $A \cap \overline{B}$?

A. 

B. 

C. 

D. 

Question 6.

What is the domain of $f(x) = \log_{10}\left[(x-3)(x-5)\right]$?

A. $\{3 \le x \le 5\}$

B. $\{3 < x < 5\}$

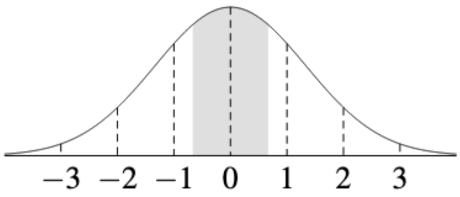
C. $\{x \le 3 \text{ or } x \ge 5\}$

D. $\{x < 3 \text{ or } x > 5\}$

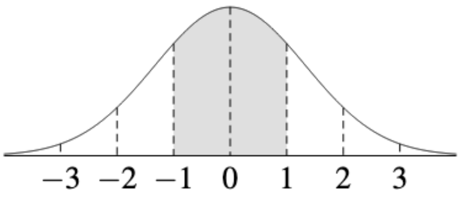
Question 7.

Which of the following diagrams best represents a standardised normally distributed dataset with the interquartile range shaded?

A.



B.



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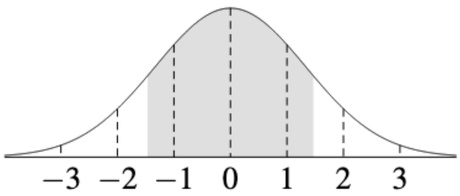
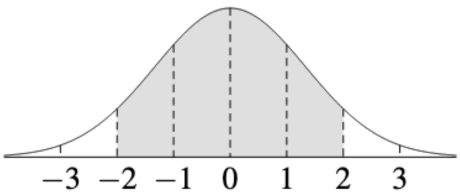


D.



Question 8.

The area enclosed by the function $y=x(x-1)(2-x)$ and the x-axis can be described as:

A. $-\int_0^2 x(x-1)(2-x)dx$

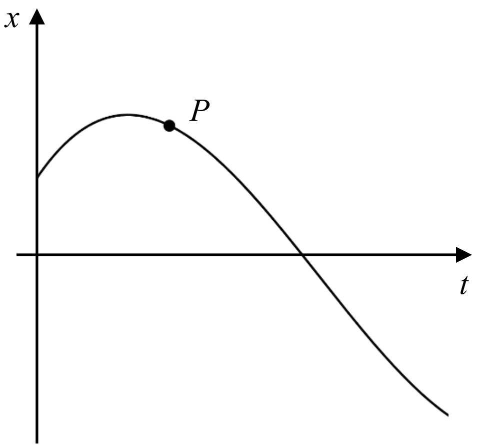
B. $\int_0^1 x(x-1)(2-x)\mathrm{d}x-\int_1^2 x(x-1)(2-x)\mathrm{d}x$

C. $\int_0^2 x(x-1)(2-x)\mathrm{d}x$

D. $-\int_0^1 x(x-1)(2-x)\mathrm{d}x+\int_1^2 x(x-1)(2-x)\mathrm{d}x$

Question 9.

The displacement of a car from a reference point along a straight road that runs east-west is x kilometres. The car is initially 100 metres east of the reference point. The graph below shows the displacement of the car at time t seconds.



When the car is at point P, which statement reflects its motion?

A. The car is moving east, and its speed is increasing.

B. The car is moving west, and its speed is increasing.

C. The car is moving east, and its speed is decreasing.

D. The car is moving west, and its speed is decreasing.

Question 10.

Which of the following function has a period of $\frac{\pi}{2}$ and is increasing in the domain $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$?

A. $f(x) = \sin|x|$

B. $f(x) = |\sin(2x)|$

C. $f(x) = \cos|x|$

D. $f(x) = |\cos(2x)|$

Mathematics Advanced
Trial HSC Examination 2025
Section II

Part A

NESA NUMBER

Attempt Questions 11–16 (17 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the ned of this part. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

a) Find an expression, in simplest form, for the general term of the arithmetic series: **1**

$$257 + 246 + 235 + \ldots$$

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b) Find the 125th term of the series above. **1**

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Question 12 (3 marks)

A discrete random variable X has the probability distribution table shown below.

$X = x$	5	10	15	20
$P(x)$	0.1	0.25	n	0.35

a) Find the value of n. **1**

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b) Find the expected value $E(X)$. **2**

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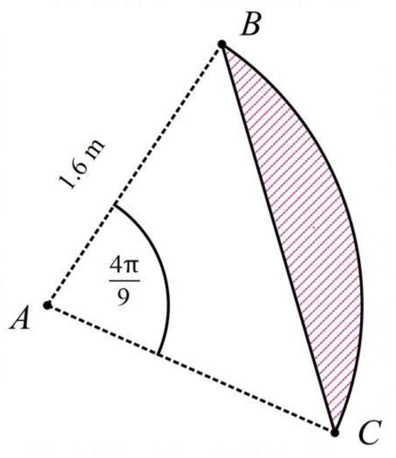
Question 13 (3 marks)

The shaded segment shown is formed by an arc BC of a circle with radius 1.6m.

The arc subtends an angle of $\frac{4\pi}{9}$ radians at its centre, A.



Find the area of the shaded segment, correct to 3 significant figures. **3**

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Question 14 (2 marks)

Given $\log_5 2 = a$ and $\log_5 3 = b$, express $\log_5\left(\frac{12}{\sqrt{2}}\right)$ in terms of a and b. **2**

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Question 15 (4 marks)

Differentiate with respect to x.

a) $y = (3x^2 - 7)^6$. **2**

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b) $y = \dfrac{2x-1}{x+8}$. **2**

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Question 16 (3 marks)

Find the equation of the tangent to the curve $y = x^3 + \ln x + 5$ at the point $(1, 6)$. **3**

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Mathematics Advanced
Trial HSC Examination 2025
Section II

Part B

NESA NUMBER

Attempt Questions 17–21(18 marks)

Instructions

- Write your student number at the top of this page.
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Please turn over

Question 17 (2 marks)

If $\sin x = \dfrac{\sqrt{5}}{5}$, $0 < x < \dfrac{\pi}{2}$, find the value of $\sin^4 x - \cos^4 x$. **2**

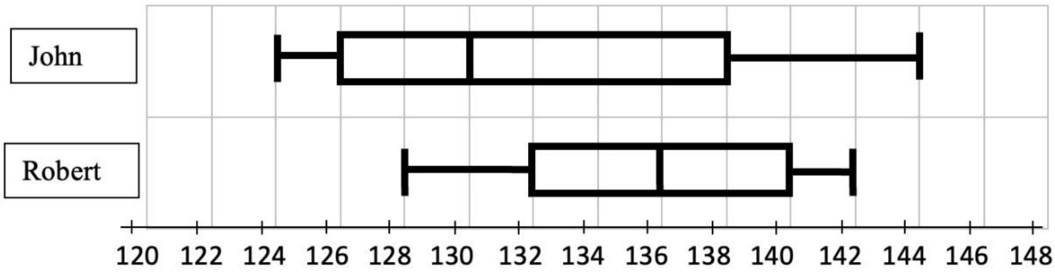
Question 18 (3 marks)

Solve for x where $0 \leq x \leq 2\pi$, $\operatorname{cosec}\left(x + \dfrac{\pi}{3}\right) = \sqrt{2}$ **3**

Question 19 (3 marks)

John and Robert each played equal number of computer games.

Their scores in these games are analysed to create the box plots as shown below.



Robert claims that his scores were better than John's scores. Is he correct? **3**

Support your answer by referring to the box plots' skewness of the distributions, measures of the central tendencies and the measures of the spreads.

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Question 20 (3 marks)

Show that: $\sec^2 A \equiv \dfrac{\operatorname{cosec} A}{\operatorname{cosec} A - \sin A}$ **3**

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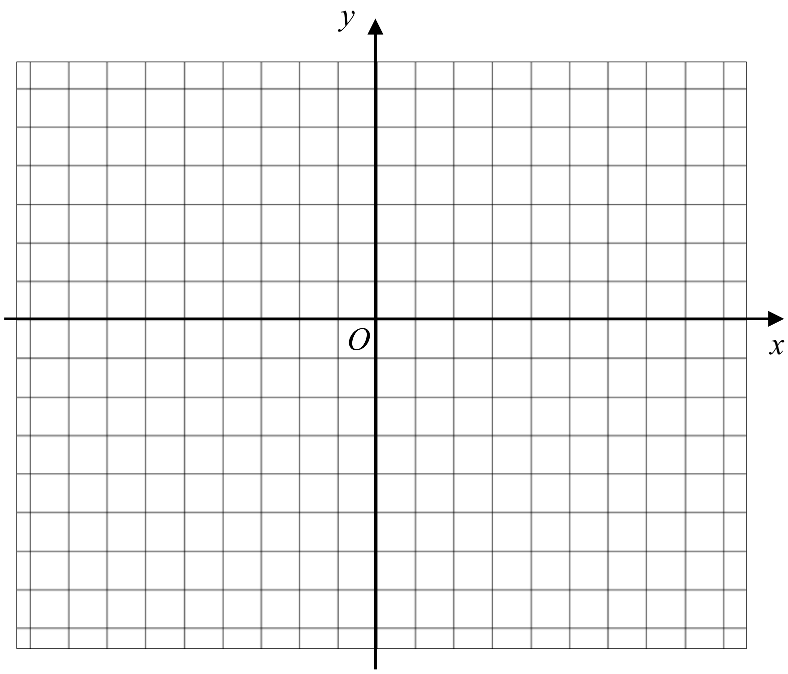
Question 21 (7 marks)

Given $f(x) = x - 2$ and $g(x) = (x+2)(x^2-4)$.

a) Find all points of intersection between $f(x)$ and $g(x)$. **3**

b) Sketch the graphs of $f(x)$ and $g(x)$ on the same axis, showing all important features (Do not find any stationary points). **3**



c) Hence, or otherwise, solve the inequality $x - 2 < (x+2)(x^2-4)$. **1**

Mathematics Advanced
Trial HSC Examination **2025**
Section II

Part C

NESA NUMBER

Attempt Questions 22–26 (18 marks)

Instructions

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Please turn over

Question 22 (2 marks)

A company produces resistors with resistance being normally distributed ($\mu = 50\Omega$, $\sigma = 2\Omega$).

Resistors below 48Ω or exceeds 52Ω are defective.

a) Find the percentage of defective resistors. **1**

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b) In a batch of 1000 resistors, how many are expected to be non-defective? **1**

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Question 23 (3 marks)

a) Show that **2**

$$\frac{d}{dx}\left(e^{x}\tan x\right)=e^{x}\left(\tan^{2}x+\tan x+1\right)$$

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b) Hence evaluate **1**

$$\int_{0}^{\frac{\pi}{3}} e^{x}\left(\tan^{2}x+\tan x+1\right)dx$$

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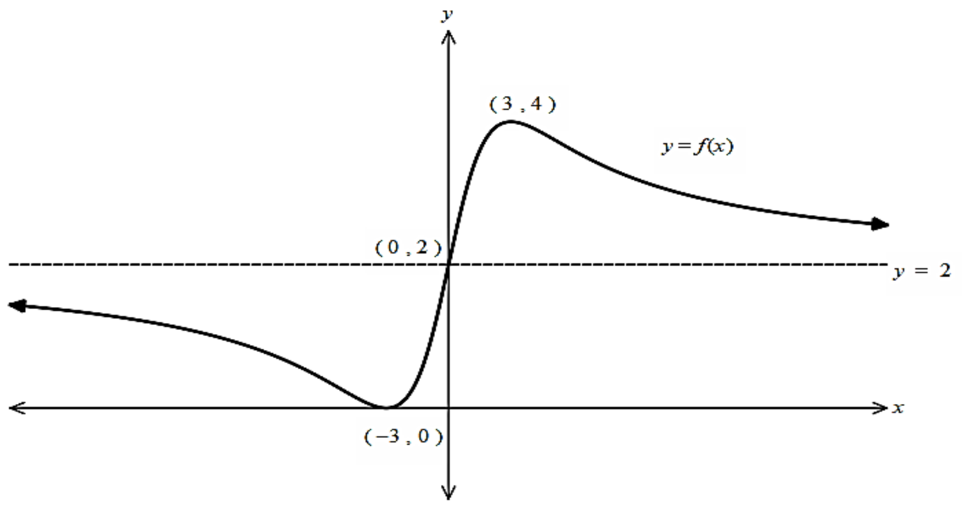
Question 24 (3 marks)

The figure below shows the graph of a curve with equation $y = f(x)$.

The curve meets the x-axis at $(-3, 0)$ and the y-axis at $(0, 2)$.

Its maximum point is at $(3, 4)$ and a minimum at $(-3, 0)$.

The line $y = 2$ is a horizontal asymptote to the curve.



a) What is the range of $\frac{1}{2}f(x)$? **1**

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b) Sketch the graph of $g(x) = f(3x) - 4$, labelling all key points and asymptotes. **2**

Question 25 (4 marks)

Events A and B are such that $P(A) = \frac{3k}{4}$, $P(B) = k$ and $P(\overline{A} \cap B) = k^2$ where $0 < k < 1$.

a) Find the expression of $P(A \cap B)$ in terms of k. **1**

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b) Given that the two events are independent, find the value of k. **3**

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Question 26 (6 marks)

The amount of time for Opstra Telecom. to respond to a customer query follows the probability density function:

$$f(x) = \begin{cases} \dfrac{k}{(5x-1)^2}, & x \geq 1 \\ 0 & , \ 0 < x < 1 \end{cases} \qquad \text{where } x \text{ is time in hours.}$$

a) Show that $k = 20$. **2**

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b) Calculate the median response time. **2**

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c) What is the probability that the company spends more than 2 hours but less than 3 hours to respond to the customer query? **2**

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Extra working space

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Mathematics Advanced
Trial HSC Examination 2025
Section II

Part D

NESA NUMBER

Attempt Questions 27– 29 (16 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
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- Extra writing space is provided at the ned of this part. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 27 (5 marks)

The velocity of a particle is given by $\dot{x}(t) = 8t^3e^{t^4}$, where $\dot{x}(t)$ is velocity in metres per second and t is time in seconds. Initially, the particle is at the origin.

a) Determine the times(s) the particle is at rest. **1**

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b) Show that the displacement function is $x(t) = 2e^{t^4} - 2$. **2**

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c) At what time does the particle reach a displacement of 100 metres? (Give your answer to 2 d.p.) **2**

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Question 28 (6 marks)

The mass of oranges at Fruit King farm are normally distributed with mean μ and standard deviation σ.

It is known that 25.78% of these oranges have mass more than 190 g and 34.46% of them have mass more than 180 g.

Using the table of z scores below, answer the following questions.

z	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75
Probability	0.6368	0.6554	0.6736	0.6915	0.7088	0.7257	0.7422	0.7580	0.7734

a) Find the mean μ and standard deviation σ. **3**

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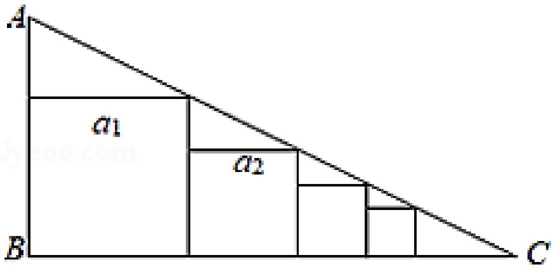
b) Two oranges from Fruit King farm are chosen at random.
Given that the mass of both oranges is greater than 180 grams, what is the probability that both of them have a mass less than 194 g. Give your answer as a percentage to 2 decimal places. **3**

Question 29 (5 marks)

As shown in the diagram, a sequence of squares is inscribed within a right-angled triangle ABC, where $AB = a$, $BC = 2a$ and $\angle ABC = 90°$. The squares are arranged such that each square touches the hypotenuse AC and the side length is labeled as a_1, a_2 ... a_n.



a) By considering similar triangles, show that $a_1 = \frac{2}{3}a$. **1**

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b) Show that the areas of the squares form a geometric sequence. State the common ratio. **2**

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c) Find, in terms of a, the total sum of the areas of all the squares. **2**

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Mathematics Advanced
Trial HSC Examination **2025**
Section II

Part E

NESA NUMBER

Attempt Questions 30– 32 (21 marks)

Instructions

- Write your student number at the top of this page.
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Please turn over

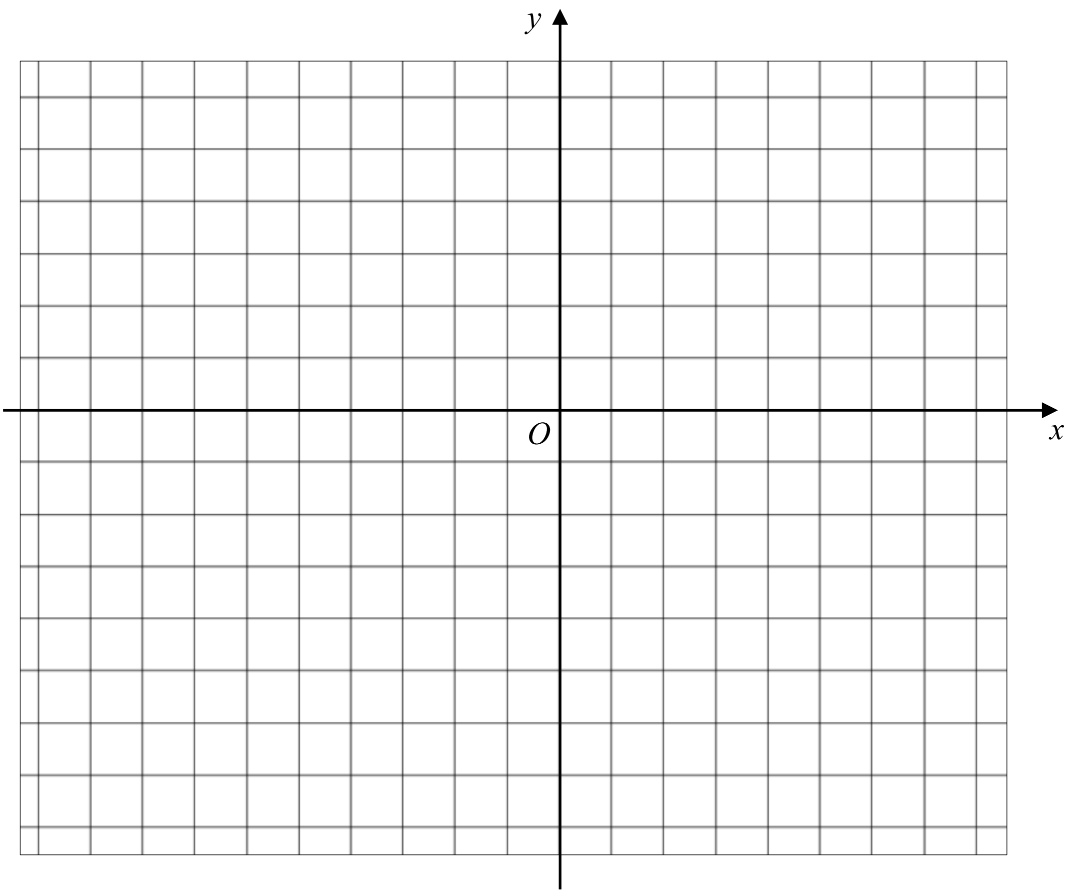
Question 30 (8 marks)

Let $f(x) = (x+1)e^x$.

a) Show that $f''(x) = (x+3)e^x$. **2**

b) Find any stationary points of $f(x)$, determine their nature, and find any points of inflection. **3**

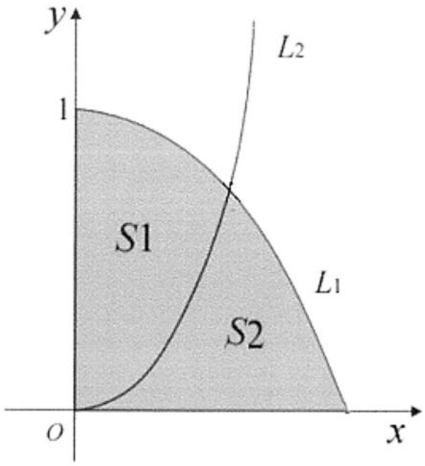
c) Sketch the curve $f(x)$, labelling any stationary points, points of inflection and intercepts with the x-axis and the y-axis. **3**



Question 31 (6 marks)

Consider the two curves $L_1: \ y = -x^2 + 1$ for $0 \le x \le 1$ and $L_2: \ y = ax^2$ for $x \ge 0$.

The diagram below shows the area enclosed by these two curves, and the x- and y-axis.



a) Show that the two curves intersect at the point $\left(\sqrt{\dfrac{1}{1+a}}, \dfrac{a}{1+a}\right)$. **2**

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b) Find the value of a such that the areas of the two regions are equal. **4**

Question 32 (7 marks)

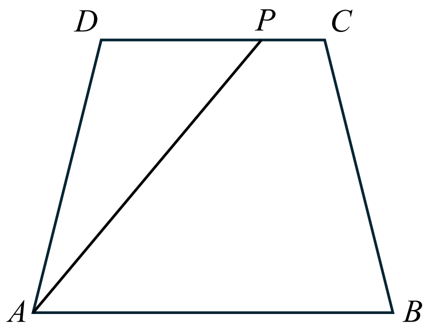
The diagram below shows four scenic points located at the vertices of the trapezium *ABCD*.

It is given that $AD = BC$, the distance between points *A* and *B* is 4 kilometres, the distance between points *C* and *D* is 2 km, and $\angle DAB = \angle CBA = 60°$.

A new lookout point, P, is to be constructed on segment *CD* (excluding endpoints), with direct walking tracks built from P to A.

The construction cost is:

- \$100,000 per km for track *AP*
- \$80,000 per km for track *PC*



NOT TO SCALE

a) Let $\angle BAP = \theta$, show that $AP = \dfrac{\sqrt{3}}{\sin\theta}$. **2**

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b) Find a similar expression for *DP* in terms of θ. **1**

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c) Find the value of $\cos\theta$ that minimises the total cost of constructing the two tracks. **4**

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End of Paper

NESA Number:

Fort Street High School

Name: Solution. :)

2025 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

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- Allow about 2 hours and 45 minutes for this section
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10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.
Question 1.

A

What is the solution to the equation $3x^2 + 5x - 1 = 0$?

A. $x = \dfrac{-5 \pm \sqrt{37}}{6}$

B. $x = \dfrac{-5 \pm \sqrt{13}}{6}$

C. $x = \dfrac{5 \pm \sqrt{37}}{6}$

D. $x = \dfrac{5 \pm \sqrt{13}}{6}$

C

Question 2.

Express $4x\sqrt{11}$ in the form of $\sqrt{a}$.

$\sqrt{(4x)^2 \cdot 11}$

A. $\sqrt{44x^2}$

B. $\sqrt{44x}$

C. $\sqrt{176x^2}$

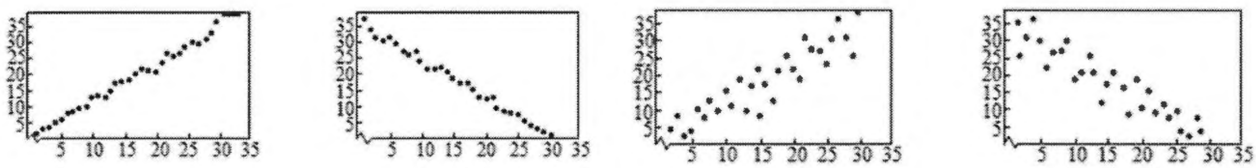
D. $\sqrt{176x}$

C

Question 3.

A statistics department conducted an analysis on four sets of data, producing the scatter plots shown.

Which of the following comparisons is correct based on the correlation coefficients?



r_1 r_2 r_3 r_4

A. $r_4 < r_2 < 0 < r_1 < r_3$

B. $r_2 < r_4 < 0 < r_1 < r_3$

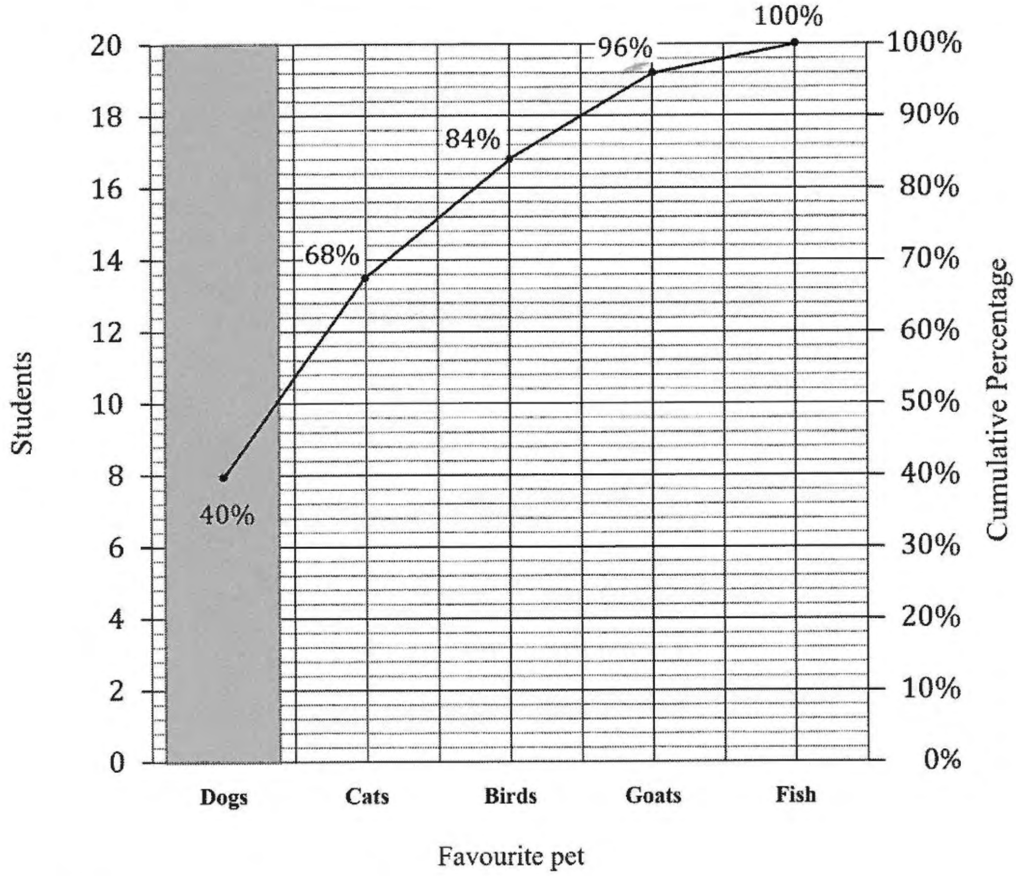
C. $r_2 < r_4 < 0 < r_3 < r_1$

D. $r_4 < r_2 < 0 < r_3 < r_1$

Question 4.

A teacher asked fifty students to identify their favourite type of pet.

A partially completed Pareto chart shows some of the data collected.



B How many students chose cats as their favourite pet?

A. 12 students

B. 14 students

C. 28 students

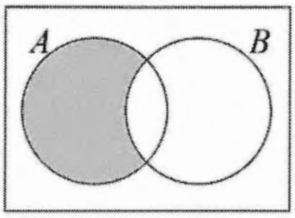
D. 34 students

$(68\% - 40\%) \cdot 50$
$= 14$

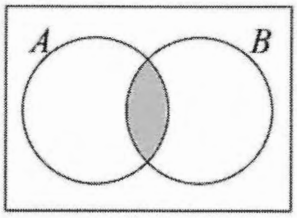
Question 5.

A Which of the Venn diagrams best represent $A \cap \overline{B}$?

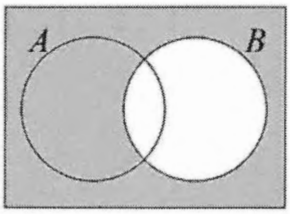
A.



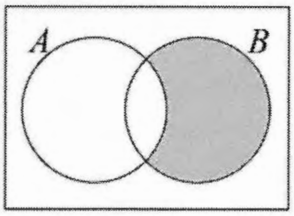
B.



C.



D.



Question 6.

D What is the domain of $f(x) = \log_{10}[(x-3)(x-5)]$?

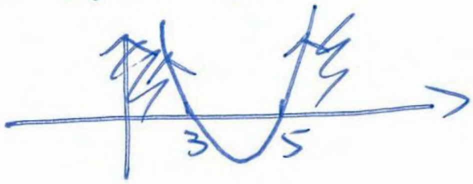
A. $\{3 \le x \le 5\}$

B. $\{3 < x < 5\}$

C. $\{x \le 3 \text{ or } x \ge 5\}$

D. $\{x < 3 \text{ or } x > 5\}$

$(x-3)(x-5) > 0$

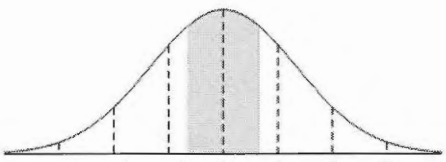


A

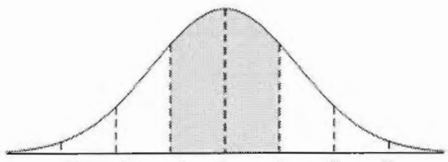
Question 7.

Which of the following diagrams best represents a standardised normally distributed dataset with the interquartile range shaded? IQR => 50%.

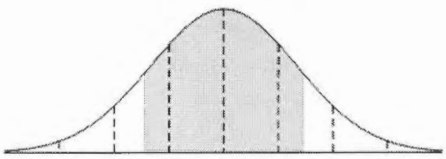
A.



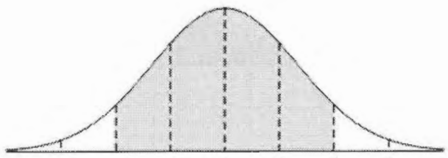
B.



C.



D.



D

Question 8.

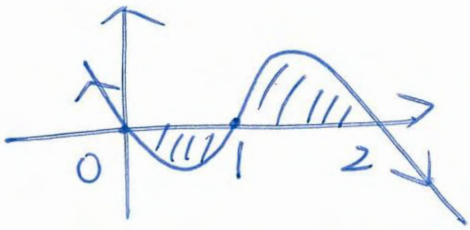
The area enclosed by the function $y = x(x-1)(2-x)$ and the x-axis can be described as:

A. $-\int_0^2 x(x-1)(2-x)dx$

B. $\int_0^1 x(x-1)(2-x)\mathrm{d}x - \int_1^2 x(x-1)(2-x)\mathrm{d}x$

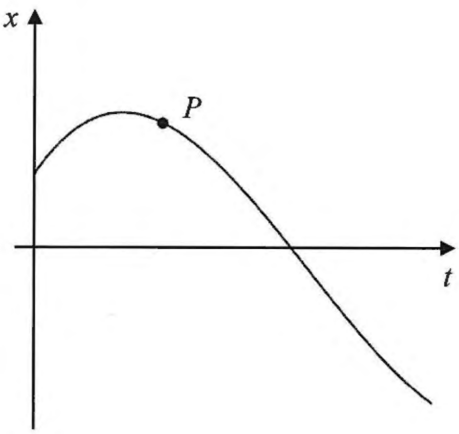
C. $\int_0^2 x(x-1)(2-x)\mathrm{d}x$

D. $-\int_0^1 x(x-1)(2-x)\mathrm{d}x + \int_1^2 x(x-1)(2-x)\mathrm{d}x$



Question 9.

The displacement of a car from a reference point along a straight road that runs east-west is x kilometres. The car is initially 100 metres east of the reference point. The graph below shows the displacement of the car at time t seconds.



B When the car is at point P, which statement reflects its motion?

A. The car is moving east, and its speed is increasing.

B. The car is moving west, and its speed is increasing.

C. The car is moving east, and its speed is decreasing.

D. The car is moving west, and its speed is decreasing.

D **Question 10.**

Which of the following function has a period of $\frac{\pi}{2}$ and is increasing in the domain $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$?

A. $f(x) = \sin|x|$

B. $f(x) = |\sin(2x)|$

C. $f(x) = \cos|x|$

D. $f(x) = |\cos(2x)|$

Mathematics Advanced
Trial HSC Examination 2025
Section I

Part A

NESA NUMBER

Attempt Questions 11–16 (17 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the ned of this part. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

a) Find an expression, in simplest form, for the general term of the arithmetic series: **1**

$257 + 246 + 235 + \ldots$

$a_1 = 257,\ d = 246 - 257$

$d = -11$

$T_n = a_1 + (n-1) \cdot d$

$= 257 + (n-1) \cdot (-11)$

$= 268 - 11n$ ✓

b) Find the 125th term of the series above. **1**

$T_{125} = 268 - 11(125)$

$= -1107$ ✓

Question 12 (3 marks)

A discrete random variable X has the probability distribution table shown below.

$X = x$	5	10	15	20
$P(x)$	0.1	0.25	n	0.35

a) Find the value of n. **1**

$0.1 + 0.25 + n + 0.35 = 1$

$n = 0.3$ ✓

b) Find the expected value $E(X)$. **2**

$E(X) = \Sigma xp$

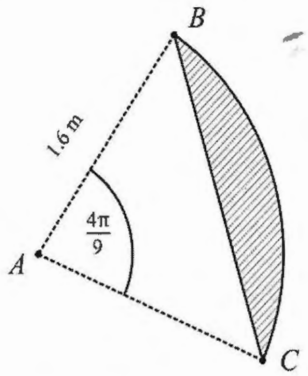
$= 5(0.1) + 10(0.25) + 15(0.3) + 20(0.35)$ ✓

$= 14.5$ ✓

Question 13 (3 marks)

The shaded segment shown is formed by an arc BC of a circle with radius 1.6m.

The arc subtends an angle of $\frac{4\pi}{9}$ radians at its centre, A.



Find the area of the shaded segment, correct to 3 significant figures. **3**

$A_{segment} = A_{sector} - A_{triangle}$.

$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ 1 mark Asector

$= \frac{1}{2}(1.6)^2\left(\frac{4\pi}{9} - \sin\frac{4\pi}{9}\right)$ 1 mark Atriangle.

$\approx 0.527\ m^2$ ✓ 1 mark final answer

Question 14 (2 marks)

Given $\log_5 2 = a$ and $\log_5 3 = b$, express $\log_5\left(\frac{12}{\sqrt{2}}\right)$ in terms of a and b. **2**

$\log_5\left(\frac{12}{\sqrt{2}}\right) = \log_5(6\sqrt{2})$

$= \log_5(2 \cdot 3 \cdot 2^{\frac{1}{2}})$ correctly

$= \log_5(2^{\frac{3}{2}} \cdot 3)$ apply log law

$= \log_5(2^{\frac{3}{2}}) + \log_5(3)$ to simplify ✓

$= \frac{3}{2}a + b$ ✓

Question 15 (4 marks)

Differentiate with respect to x.

a) $y = (3x^2 - 7)^6$. **2**

$y' = 6(3x^2-7)^5 \cdot (6x)$ ✓

$= 36x(3x^2-7)^5$ ✓

b) $y = \dfrac{2x-1}{x+8}$. **2**

$u = 2x-1, \ v = x+8$

$u' = 2, \ v' = 1$ ✓

$y' = \dfrac{2(x+8) - (2x-1)}{(x+8)^2}$

$= \dfrac{2x+16-2x+1}{(x+8)^2}$

$= \dfrac{17}{(x+8)^2}$ ✓

Question 16 (3 marks)

Find the equation of the tangent to the curve $y = x^3 + \ln x + 5$ at the point $(1, 6)$. **3**

$y = x^3 + \ln x + 5$

$y' = 3x^2 + \frac{1}{x}$ ✓

at $x = 1$, $m = 3(1)^2 + \frac{1}{1}$

$m = 4$ ✓

$y - y_1 = m(x - x_1)$

$y - 6 = 4(x - 1)$

$y = 4x + 2$ ✓

Mathematics Advanced
Trial HSC Examination 2025
Section I

Part B

NESA NUMBER

Attempt Questions 17–21(18 marks)

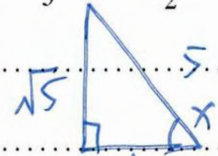
Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the ned of this part. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 17 (2 marks)

If $\sin x = \dfrac{\sqrt{5}}{5}$, $0 < x < \dfrac{\pi}{2}$, find the value of $\sin^4 x - \cos^4 x$. **2**



$\sqrt{25-5} = \sqrt{20}$

$\therefore \cos x = \dfrac{\sqrt{20}}{5} \quad \left(0 < x < \dfrac{\pi}{2}\right)$ ✓

$\sin^4 x - \cos^4 x = \left(\dfrac{\sqrt{5}}{5}\right)^4 - \left(\dfrac{\sqrt{20}}{5}\right)^4$

$= -\dfrac{3}{5}$ ✓

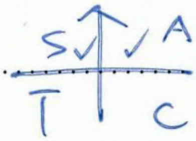
Question 18 (3 marks)

Solve for x where $0 \le x \le 2\pi$, $\quad \operatorname{cosec}\left(x + \dfrac{\pi}{3}\right) = \sqrt{2}$ **3**

$\dfrac{1}{\sin\left(x+\frac{\pi}{3}\right)} = \sqrt{2}$

$\sin\left(x+\frac{\pi}{3}\right) = \dfrac{1}{\sqrt{2}}$ ✓ $\quad \dfrac{\pi}{3} \le x + \dfrac{\pi}{3} \le \dfrac{7\pi}{3}$

$x + \dfrac{\pi}{3} = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{9\pi}{4}, \dfrac{11\pi}{4}$ ✓



$\dfrac{\pi}{4}$ invalid. $\quad \dfrac{11\pi}{4}$ invalid.

base angle $\dfrac{\pi}{4}$.

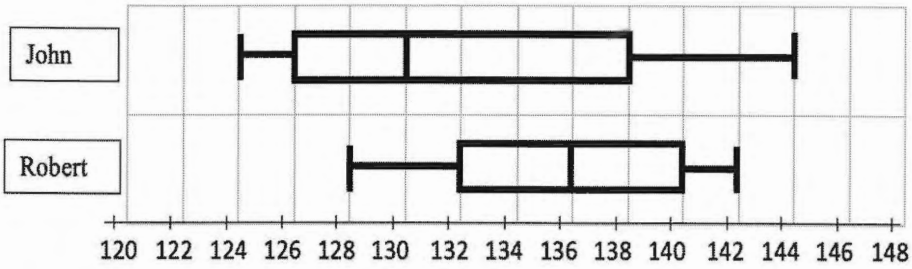
$\therefore x = \dfrac{3\pi}{4} - \dfrac{\pi}{3}$ or $x = \dfrac{9\pi}{4} - \dfrac{\pi}{3}$

$x = \dfrac{5\pi}{12}$ or $x = \dfrac{23\pi}{12}$ ✓

Question 19 (3 marks)

John and Robert each played equal number of computer games.

Their scores in these games are analysed to create the box plots as shown below.



Robert claims that his scores were better than John's scores. Is he correct? **3**

Support your answer by referring to the box plots' skewness of the distributions, measures of the central tendencies and the measures of the spreads.

skewness: John's scores are positively skewed ✓

Robert's scores are almost symmetrical.

centre & spread: John's median score is 130, IQR is 12, range is 20. ✓

Robert's median score is 136, IQR is 8, range is 14 ✓

Robert is correct because his scores are more consistent (smaller range (IQR), more than $\frac{3}{4}$ of his scores are above John's median (higher median).

Question 20 (3 marks)

Show that: $\sec^2 A \equiv \dfrac{\operatorname{cosec} A}{\operatorname{cosec} A - \sin A}$ **3**

$$RHS = \frac{\frac{1}{\sin A}}{\frac{1}{\sin A} - \sin A}$$

$$= \frac{\frac{1}{\sin A}}{\frac{1 - \sin^2 A}{\sin A}}$$

$$= \frac{1}{1 - \sin^2 A}$$

$$= \frac{1}{\cos^2 A}$$

$$= \sec^2 A$$

$$= LHS$$

Question 21 (7 marks)

Given $f(x) = x - 2$ and $g(x) = (x+2)(x^2-4)$.

a) Find all points of intersection between $f(x)$ and $g(x)$. **3**

PoI: $x-2 = (x+2)(x^2-4)$

$x-2 = (x+2)(x+2)(x-2)$

$(x+2)^2(x-2) - (x-2) = 0$

$(x+2)[(x+2)^2 - 1] = 0$ ✓ factorised form

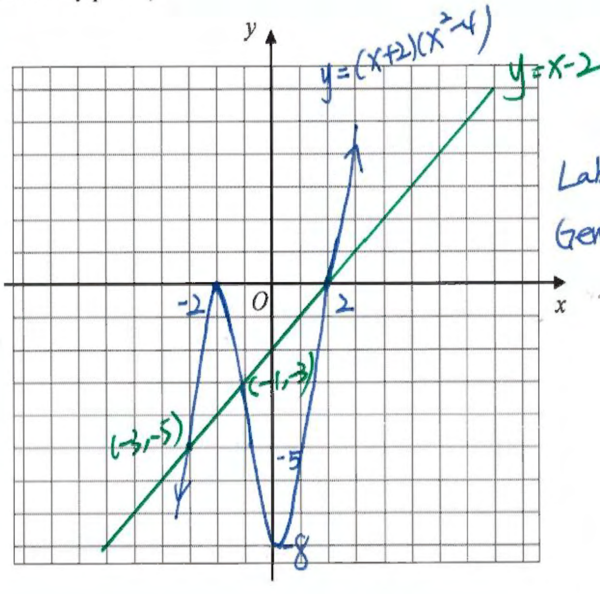
$x-2=0$ or $(x+2)^2 = 1$

$x=2$ or $x+2=1$ or $x+2=-1$

$x=2$ or $x=-1$ or $x=-3$

$\therefore$ PoI: $(2,0)$ ✓ ; $\underbrace{(-1,-3) ; (-3,-5)}_{✓}$

b) Sketch the graphs of $f(x)$ and $g(x)$ on the same axis, showing all important features (Do not find any stationary points). **3**



Label x & y int ✓
General shape ✓
PoI ✓

c) Hence, or otherwise, solve the inequality $x - 2 < (x+2)(x^2-4)$. **1**

$-3 < x < -1$; $x > 2$ ✓

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Section I

Part C

NESA NUMBER

Attempt Questions 22–26 (18 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the ned of this part. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 22 (2 marks)

A company produces resistors with resistance being normally distributed ($\mu = 50\Omega$, $\sigma = 2\Omega$).

Resistors below 48Ω or exceeds 52Ω are defective.

a) Find the percentage of defective resistors. **1**

$P(\text{defective}) = P(X < 48) + P(X > 52)$

$= 1 - 68\%$

$= 32\%$ ✓

b) In a batch of 1000 resistors, how many are expected to be non-defective? **1**

$1000(1 - 32\%)$

$= 680$ ✓ $\therefore$ Non defective $= 680$

Question 23 (3 marks)

a) Show that **2**

$$\frac{d}{dx}\left(e^x \tan x\right) = e^x\left(\tan^2 x + \tan x + 1\right)$$

Product rule: $u = e^x$, $v = \tan x$
$u' = e^x$, $v' = \sec^2 x$ ✓

$\frac{d}{dx}(e^x \tan x) = e^x \sec^2 x + e^x \tan x$

$= e^x (\sec^2 x + \tan x)$ ✓

$= e^x (\tan^2 x + 1 + \tan x)$ as required

since $\sec^2 x = \tan^2 x + 1$

b) Hence evaluate **1**

$$\int_0^{\frac{\pi}{3}} e^x\left(\tan^2 x + \tan x + 1\right) dx$$

from a), $I = \left[e^x \tan x\right]_0^{\frac{\pi}{3}}$

$= e^{\frac{\pi}{3}} \tan\left(\frac{\pi}{3}\right) - e^0 \tan(0)$

$= \sqrt{3}\, e^{\frac{\pi}{3}}$ ✓

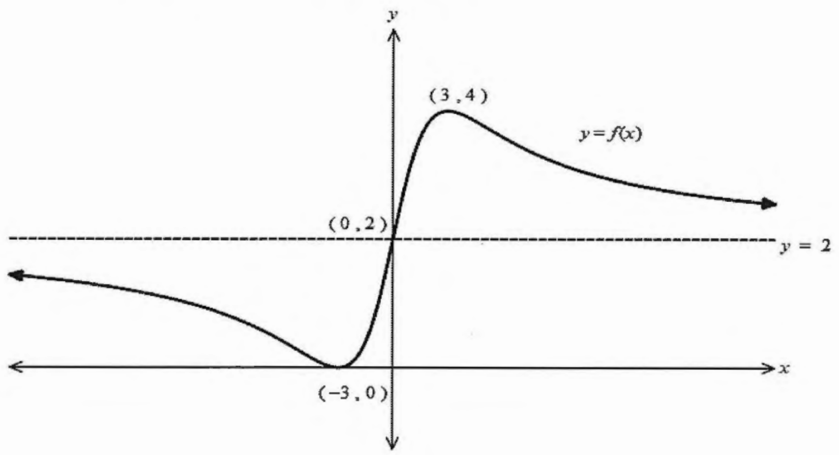
Question 24 (3 marks)

The figure below shows the graph of a curve with equation $y = f(x)$.

The curve meets the x-axis at $(-3, 0)$ and the y-axis at $(0, 2)$.

Its maximum point is at $(3, 4)$ and a minimum at $(-3, 0)$.

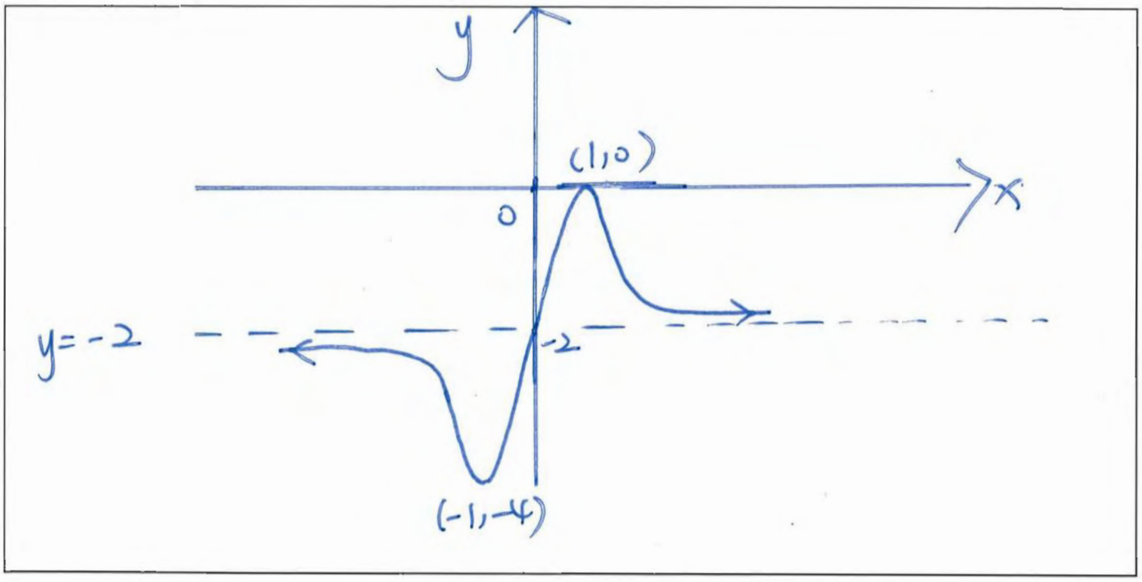
The line $y = 2$ is a horizontal asymptote to the curve.



a) What is the range of $\frac{1}{2}f(x)$? **1**

range $[0, 2]$ ✓

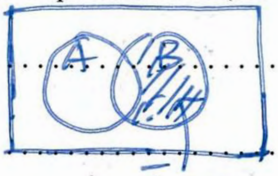
b) Sketch the graph of $g(x) = f(3x) - 4$, labelling all key points and asymptotes. ✓ ✓ **2**



Question 25 (4 marks)

Events A and B are such that $P(A) = \dfrac{3k}{4}$, $P(B) = k$ and $P(\overline{A} \cap B) = k^2$ where $0 < k < 1$.

a) Find the expression of $P(A \cap B)$ in terms of k. **1**



$\overline{A} \cap B$

$$P(A \cap B) = P(B) - P(\overline{A} \cap B)$$
$$= k - k^2 \checkmark$$

b) Given that the two events are independent, find the value of k. **3**

If two events are independent,

then $P(A \cap B) = P(A) \cdot P(B)$ $\checkmark$

$\therefore \quad k - k^2 = \dfrac{3k}{4} \cdot k$

$3k^2 = 4k - 4k^2$

$7k^2 - 4k = 0$

$k(7k - 4) = 0$

$k = 0$ or $k = \dfrac{4}{7}$ $\checkmark$

$k = 0$ is invalid since $0 < k < 1$

$\therefore k = \dfrac{4}{7}$ $\checkmark$

Question 26 (6 marks)

The amount of time for Opstra Telecom. to respond to a customer query follows the probability density function:

$$f(x)=\begin{cases}\dfrac{k}{(5x-1)^2}, & x\geq 1\\ 0 & , 0<x<1\end{cases}$$ where x is time in hours.

a) Show that $k=20$. **2**

$\int_1^{\infty} \frac{k}{(5x-1)^2}\,dx = 1$ ✓ set up the correct integral

$k\int_1^{\infty} (5x-1)^{-2}\,dx = 1$

$k\left[\frac{(5x-1)^{-1}}{-1(5)}\right]_1^{\infty} = 1$ ✓

$-\frac{k}{5}\left[0-\frac{1}{4}\right] = 1$

$\frac{k}{20} = 1$

$k = 20$

b) Calculate the median response time. **2**

Let the median be m:

$\int_1^{m} \frac{20}{(5x-1)^2}\,dx = \frac{1}{2}$ ✓

$-4\left[\frac{1}{5x-1}\right]_1^{m} = 0.5$

$-\frac{4}{5m-1}+1 = 0.5$

$\frac{4}{5m-1} = 0.5$

$m = \frac{9}{5}$ or 1.8 ✓

c) What is the probability that the company spends more than 2 hours but less than 3 hours to respond to the customer query? **2**

$$\int_2^3 \frac{20}{(5x-1)^2}\,dx$$

$$= 20\int_2^3 (5x-1)^{-2}\,dx$$

$$= 20\left[-\frac{1}{5(5x-1)}\right]_2^3$$ ✓ working out

$$= -\frac{20}{5}\left[\frac{1}{5x-1}\right]_2^3$$

$$= -4\cdot\left(\frac{1}{14}-\frac{1}{9}\right)$$

$$= \frac{10}{63}$$ ✓

Mathematics Advanced
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Section I

Part D

NESA NUMBER

Attempt Questions 27– 29 (16 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
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Please turn over

Question 27 (5 marks)

The velocity of a particle is given by $\dot{x}(t) = 8t^3e^{t^4}$, where $\dot{x}(t)$ is velocity in metres per second and t is time in seconds. Initially, the particle is at the origin.

a) Determine the times(s) the particle is at rest. **1**

At rest when $\dot{x} = 0$.

$8t^3 \cdot e^{t^4} = 0$ when $t = 0$ since $e^{t^4} > 0$ for all t ✓

b) Show that the displacement function is $x(t) = 2e^{t^4} - 2$. **2**

$x = \int 8t^3 e^{t^4} dt$

$= 2\int 4t^3 e^{t^4} dt$

$= 2e^{t^4} + C$ ✓

when $t = 0$, $x = 0$.

$\therefore 2e^0 + C = -2$ ✓

$C = -2$

$\therefore x(t) = 2e^{t^4} - 2$

c) At what time does the particle reach a displacement of 100 metres? (Give your answer to 2 d.p.) **2**

Let $x(t) = 100$

$\therefore 2e^{t^4} - 2 = 100$

$e^{t^4} = 51$

$t^4 = \ln(51)$ ✓

$t \approx 1.415$ ✓

Question 28 (6 marks)

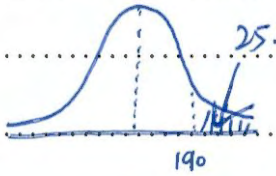
The mass of oranges at Fruit King farm are normally distributed with mean μ and standard deviation σ.

It is known that 25.78% of these oranges have mass more than 190 g and 34.46% of them have mass more than 180 g.

Using the table of z scores below, answer the following questions.

z	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75
Probability	0.6368	0.6554	0.6736	0.6915	0.7088	0.7257	0.7422	0.7580	0.7734

a) Find the mean μ and standard deviation σ. **3**



25.78% $\quad 100\% - 25.78\% = 74.22\%$

from the table, it corresponds to $z = 0.65$

$\therefore 0.65 = \dfrac{190-\mu}{\sigma}$

rearrange: $0.65\sigma = 190 - \mu$ — ①

Similarly, $100\% - 34.46\% = 65.54\%$, corresponds to $z = 0.4$

$\therefore 0.4 = \dfrac{180-\mu}{\sigma}$

rearrange: $0.4\sigma = 180 - \mu$ — ②

① − ②: $0.25\sigma = 10$

$\sigma = 40$

Sub $\sigma = 40$ into ②: $0.4(40) = 180 - \mu$

$\mu = 164$

$\therefore$ mean is 164g and standard deviation is 40.

1 mark for finding 74.22% and 65.54%

1 mark for set up two correct equations

1 mark for correct answer for μ and σ

b) Two oranges from Fruit King farm are chosen at random.
Given that the mass of both oranges is 180 grams, what is the probability that both of them have a mass less than 194 g. Give your answer as a percentage to 2 decimal places. **3**

Z-score for 194:

$Z = \frac{194-164}{40}$

$Z = 0.75$.

from the table, 77.34% of the oranges have mass less than 194g.

Let the mass of the orange be X.

$P(X<194 \mid X>180) = \frac{P(X<194g \cap X>180)}{P(X>180)}$

$= \frac{77.34\% - 65.54\%}{34.46\%}$

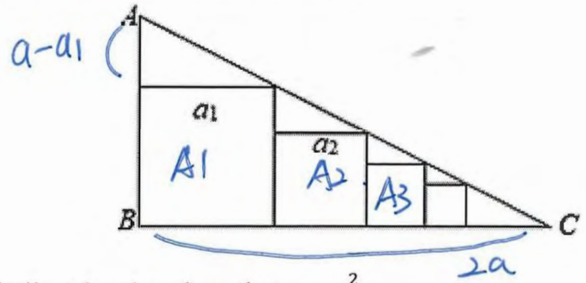
$= 34.24\%$

Hence, $P(\text{both}) = (34.24\%)^2$

$= 11.73\%$ (2 d.p)

Question 29 (5 marks)

As shown in the diagram, a sequence of squares is inscribed within a right-angled triangle ABC, where $AB = a$, $BC = 2a$ and $\angle ABC = 90°$. The squares are arranged such that each square touches the hypotenuse AC and the side length is labeled as $a_1, a_2 \ldots a_n$.



a) By considering similar triangles, show that $a_1 = \frac{2}{3}a$. **1**

$\frac{AB}{BC} = \frac{a}{2a} = \frac{1}{2}$.

By similar triangles : $\frac{a-a_1}{a_1} = \frac{1}{2}$

$a_1 = 2a - 2a_1 \Rightarrow 2a = 3a_1$.

$\therefore a_1 = \frac{2a}{3}$ as required.

b) Show that the areas of the squares form a geometric sequence. State the common ratio. **2**

Similarly, $a_2 = \frac{2}{3}a_1$; $a_3 = \frac{2}{3}a_2 \ldots$

$A_1 = (a_1)^2 = \frac{4}{9}a^2$ (use (a) result)

$A_2 = (a_2)^2 = (\frac{2}{3}a_1)^2 = \frac{4}{9} \cdot \frac{4}{9}a^2 = (\frac{4}{9})^2 a^2$

$A_3 = (a_3)^2 = (\frac{2}{3}a_2)^2 = \frac{4}{9} \cdot (\frac{4}{9})^2 a^2 = (\frac{4}{9})^3 a^2$

$\vdots$

$\therefore$ The areas of the squares form a geometric sequence with $r = \frac{4}{9}$

c) Find, in terms of a, the total sum of the areas of all the squares. **2**

$S_\infty = \frac{a}{1-r}$

first term: $A_1 = \frac{4}{9}a^2$; $r = \frac{4}{9}$

$\therefore S_\infty = \frac{\frac{4}{9}a^2}{1-\frac{4}{9}}$ ✓

$S_\infty = \frac{4}{5}a^2$ ✓

Mathematics Advanced
Trial HSC Examination 2025
Section I

Part E

NESA NUMBER

Attempt Questions 30– 32 (21 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the ned of this part. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 30 (8 marks)

Let $f(x) = (x+1)e^x$.

a) Show that $f''(x) = (x+3)e^x$. **2**

$f'(x) = e^x + (x+1)e^x$

$= e^x(x+2)$ ✓

$f''(x) = e^x + (x+2)e^x$ ✓

$= e^x(x+3)$ as required

b) Find any stationary points of $f(x)$, determine their nature, and find any points of inflection. **3**

Let $f'(x) = 0$ for stationary point

$e^x = 0$ or $x+2 = 0$.

No solution or $x = -2$. $\Rightarrow (-2, -e^{-2})$

$f''(-2) = e^{-2} > 0$ ✓ concave up.

$\therefore (-2, -e^{-2})$ is a local minimum t.p

point of inflection:

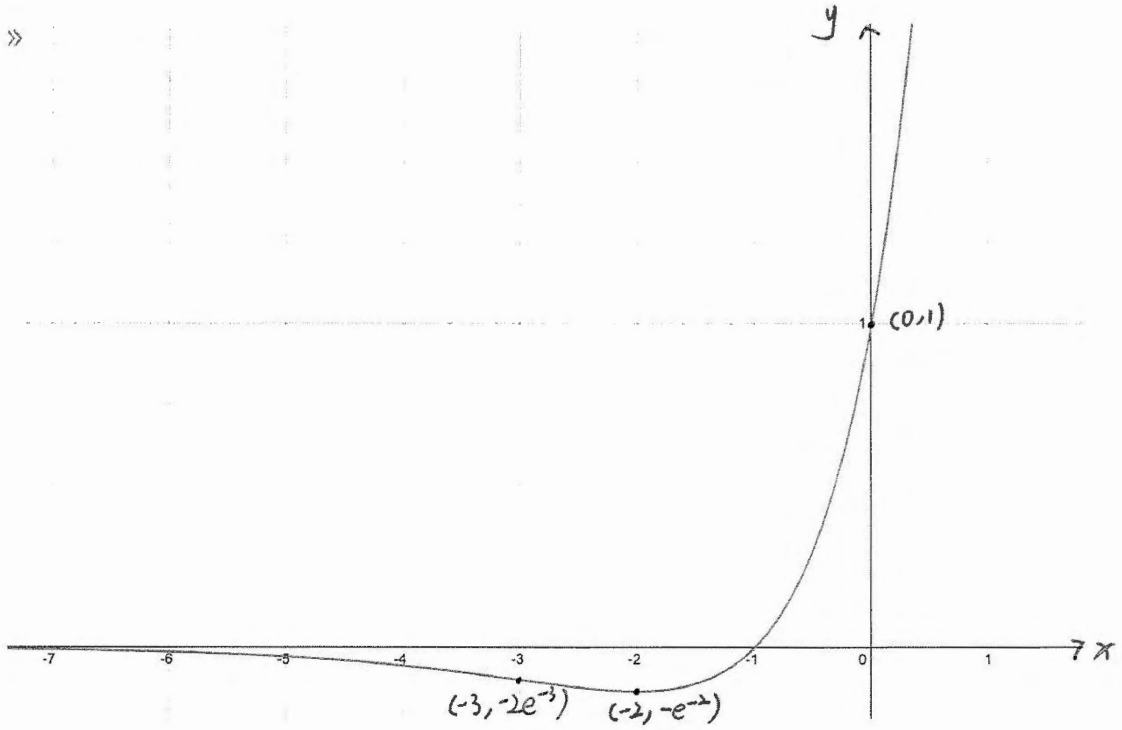
Let $f''(x) = 0$

$x = -3 \Rightarrow (-3, -2e^{-3})$

x	-3.1	-3	-2.9
$f''(x)$	0.0064	0	0.055

concavity changes $\therefore$ point of inflection at $(-3, -2e^{-3})$

c) Sketch the curve $f(x)$, labelling any stationary points, points of inflection and intercepts with the x-axis and the y-axis. **3**



asymptote at x-axis when $x \to -\infty$ ✓

Label all key points ✓

x&y int ✓

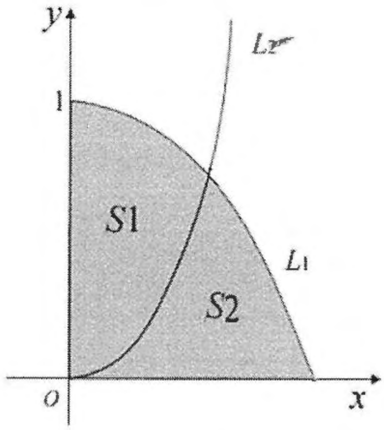
asymptotes: as $x \to \infty$, $y \to \infty$ y-int: (0, 1)

$x \to -\infty$, $y \to 0^-$ x-int: (-1, 0)

Question 31 (6 marks)

Consider the two curves $L_1: \ y=-x^2+1$ for $0 \leq x \leq 1$ and $L_2: \ y=ax^2$ for $x \geq 0$.

The diagram below shows the area enclosed by these two curves, and the x- and y-axis.



a) Show that the two curves intersect at the point $\left(\sqrt{\dfrac{1}{1+a}}, \dfrac{a}{1+a}\right)$. **2**

Equating the two equations:

$-x^2+1 = ax^2$

$(a+1)x^2 = 1$

$x^2 = \dfrac{1}{a+1}$

$x = \pm\sqrt{\dfrac{1}{a+1}}$ ✓

since $x \geq 0$ given:

$\therefore x = \sqrt{\dfrac{1}{a+1}}$

and $y = \left(\sqrt{\dfrac{1}{a+1}}\right)^2 = \dfrac{1}{a+1}$ as required. ✓

b) Find the value of a such that the areas of the two regions are equal. **4**

Method 1

$$S_1 + S_2 = \int_0^1 (1 - x^2)\,dx$$

$$= \left[x - \frac{x^3}{3}\right]_0^1$$

$$= \left(1 - \frac{1}{3}\right) - \left(0 - \frac{0}{3}\right)$$

$$= \frac{2}{3}$$

But $S_1 = S_2 \Rightarrow 2S_1 = \frac{2}{3}$

$$S_1 = S_2 = \frac{1}{3}$$

$$S_1 = \int_0^{\frac{1}{\sqrt{1+a}}} (-x^2 + 1 - ax^2)\,dx$$

$$= \int_0^{\frac{1}{\sqrt{1+a}}} (1 - (1+a)x^2)\,dx$$

$$= \left[x - \frac{1+a}{3}x^3\right]_0^{\frac{1}{\sqrt{1+a}}}$$

$$= \left(\frac{1}{\sqrt{1+a}} - \frac{(1+a)}{3}\cdot\frac{1}{(1+a)\sqrt{1+a}}\right) - \left(0 - \left(\frac{1+a}{3}\right)(0)\right)$$

$$= \frac{1}{\sqrt{1+a}}\left(1 - \frac{1+a}{3(1+a)}\right)$$

$$\Rightarrow \frac{1}{\sqrt{1+a}}\left(1 - \frac{1+a}{3(1+a)}\right) = \frac{1}{3}$$

$$2 + 2a = \frac{1}{3}\,3(1+a)\sqrt{1+a}$$

$$2(1+a) = (1+a)\sqrt{1+a}$$

$$4(1+a)^2 = (1+a)^2(1+a)$$

$$\Rightarrow (1+a)^2(1+a) - 4(1+a)^2 = 0$$

$$(1+a)^2(1+a-4) = 0$$

$(1+a)^2 = 0$ $\qquad a - 3 = 0$

$a = -1$ $\qquad a = 3$

$a > 0 \Rightarrow a = 3$

Method 2

$$S_2 = \int_0^1 (1 - x^2)\,dx - S_1$$

$$S_1 = S_2$$

$$\therefore S_1 = \int_0^1 (1 - x^2)\,dx - S_1$$

$$2S_1 = \int_0^1 (1 - x^2)\,dx$$

$$S_1 = \frac{1}{2}\int_0^1 (1 - x^2)\,dx$$

Similar to Method 1.

Method 3

$$S_1 = \int_0^{\frac{1}{\sqrt{1+a}}} (1 - x^2 - ax^2)\, dx$$

$$= \int_0^{\frac{1}{\sqrt{1+a}}} (1 - (1+a)x^2)\, dx$$

$$= \left[x - \frac{(1+a)}{3} x^3 \right]_0^{\frac{1}{\sqrt{1+a}}}$$

$$= \frac{1}{\sqrt{1+a}} - \frac{1+a}{3(1+a)\sqrt{1+a}}$$

$$= \frac{2(1+a)}{3(1+a)\sqrt{1+a}}$$

$$S_2 = \int_0^{\frac{1}{\sqrt{1+a}}} ax^2\, dx + \int_{\frac{1}{\sqrt{1+a}}}^{1} (1 - x^2)\, dx$$

$$= \left[\frac{a}{3} x^3 \right]_0^{\frac{1}{\sqrt{1+a}}} + \left[x - \frac{x^3}{3} \right]_{\frac{1}{\sqrt{1+a}}}^{1}$$

$$= \frac{a}{3(1+a)\sqrt{1+a}} + \left(1 - \frac{1}{3}\right) - \left(\frac{1}{\sqrt{1+a}} - \frac{1}{3(1+a)\sqrt{1+a}} \right)$$

$$= \frac{2}{3} - \frac{2+2a}{3(1+a)\sqrt{1+a}}$$

Equating S_1 & S_2:

$$\frac{2(1+a)}{3(1+a)\sqrt{1+a}} = \frac{2}{3} - \frac{2(1+a)}{3(1+a)\sqrt{1+a}}$$

$$\frac{2(1+a)}{(1+a)\sqrt{1+a}} = 1$$

$$2(1+a) = (1+a)\sqrt{1+a}$$

$$4(1+a)^2 = (1+a)^3$$

$$(1+a)^3 - 4(1+a)^2 = 0$$

$$(1+a)^2(1+a-4) = 0$$

$$(1+a)^2(a-3) = 0$$

$a = -1$ outside domain $\quad a = 3$

$\therefore a = 3$

Question 32 (7 marks)

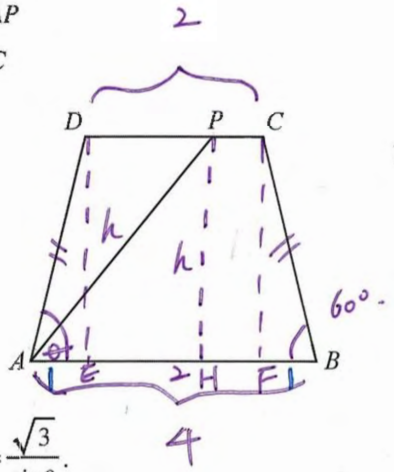
The diagram below shows four scenic points located at the vertices of the trapezium $ABCD$.

It is given that $AD = BC$, the distance between points A and B is 4 kilometres, the distance between points C and D is 2 km, and $\angle DAB = \angle CBA = 60°$.

A new lookout point, P, is to be constructed on segment CD (excluding endpoints), with direct walking tracks built from P to A.

The construction cost is:

- \$100,000 per km for track AP
- \$80,000 per km for track PC



NOT TO SCALE

a) Let $\angle BAP = \theta$, show that $AP = \dfrac{\sqrt{3}}{\sin\theta}$. **2**

Based on the given information as labelled, $\triangle ADE \equiv \triangle CBF$

$\therefore AE = BF = 1$ km

In $\triangle ADE$: $\tan 60° = \frac{h}{1} \Rightarrow h = \sqrt{3}$ km ✓

In $\triangle APH$: $\sin\theta = \frac{h}{AP}$

$AP = \frac{\sqrt{3}}{\sin\theta}$ as required. ✓

b) Find a similar expression for DP in terms of θ. **1**

In $\triangle APH$: $\tan\theta = \frac{h}{AH}$

$AH = \frac{\sqrt{3}}{\tan\theta}$

$DP = EH = AH - AE$

$= \frac{\sqrt{3}}{\tan\theta} - 1$

c) Find the value of $\cos\theta$ that minimises the total cost of constructing the two tracks. **4**

$AP = \frac{\sqrt{3}}{\sin\theta}$

$PC = CD - DP$

$= 2 - \left(\frac{\sqrt{3}}{\tan\theta} - 1\right)$

$= 3 - \frac{\sqrt{3}}{\tan\theta}$

Total cost $C = 100{,}000\, AP + 80{,}000\, PC$

$C = 100{,}000 \frac{\sqrt{3}}{\sin\theta} + 80{,}000 \left(3 - \frac{\sqrt{3}}{\tan\theta}\right)$

$C = 100{,}000\sqrt{3}(\sin\theta)^{-1} + 240{,}000 - 80{,}000\sqrt{3}(\tan\theta)^{-1}$

$C' = \frac{-100{,}000\sqrt{3} \cdot \cos\theta}{\sin^2\theta} + \frac{80{,}000\sqrt{3} \cdot \sec^2\theta}{\tan^2\theta}$

$= \frac{-100{,}000\sqrt{3} \cdot \cos\theta}{\sin^2\theta} + \frac{80{,}000\sqrt{3}}{\sin^2\theta}$

$\frac{\sec^2\theta}{\tan^2\theta} = \frac{1}{\cos^2\theta} \div \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \times \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

Let $C' = 0$.

$-100{,}000\sqrt{3}\cos\theta + 80{,}000\sqrt{3} = 0$.

$\cos\theta = \frac{80{,}000\sqrt{3}}{100{,}000\sqrt{3}}$

$\cos\theta = \frac{4}{5}$

$\theta \approx 36.9°$

θ	$36°$	$\cos^{-1}\left(\frac{4}{5}\right)$	$37°$
y'	-4520	0	652
	\	—	/

$\therefore$ minimum t.p when $\cos\theta = \frac{4}{5}$

End of Paper