



Girraween High School

2025

TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics Advanced

General Instructions

- Reading time: 10 minutes
- Working time: 3 hours
- Write using a black pen
- NESA approved calculators may be used
- Reference sheets are provided
- Answer multiple-choice questions by completely colouring in the appropriate circle on your multiple-choice answer sheet.
- Answer questions on the lines provided and show all relevant mathematical reasoning and/or calculations.

Total Marks: 100

Section 1 (Pages 1 – 5) **10 marks**

- Attempt Q1 - 10
- Allow about 15 minutes for this section

Section 2 (Pages 6 – 26) **90 marks**

- Attempt Q11 – Q33
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1– 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

Question 1

The amplitude and period of the curve $f(x) = 3 \sin (4x)$ are:

- A. amplitude = 3, period = 4
- B. amplitude = 4, period = 3
- C. amplitude = 3, period = $\frac{\pi}{4}$
- D. amplitude = 3, period = $\frac{\pi}{2}$

Question 2

What amount must be invested to obtain a future value of \$30 000 after 15 years at a rate of 12% p.a. compounded quarterly?

- A. \$182.93
- B. \$5091.99
- C. \$5480.89
- D. \$176 748.09

Examination continues next page...

Question 3

Which of the following represents a possible primitive function of $f'(x) = \frac{4}{2-x}$?

- A. $f(x) = -2 \log|2 - x|$
- B. $f(x) = -4 \log|2 - x|$
- C. $f(x) = 4 \log|2 + x|$
- D. $f(x) = 4 \log|2 - x|$

Question 4

What is the domain and range of the function $f(x) = \sqrt{4 - x^2}$?

- A. Domain: $[-2, 2]$ Range: $[-2, 2]$
- B. Domain: $[0, 2]$ Range: $[0, 4]$
- C. Domain: $[-4, 4]$ Range: $[0, 2]$
- D. Domain: $[-2, 2]$ Range: $[0, 2]$

Question 5

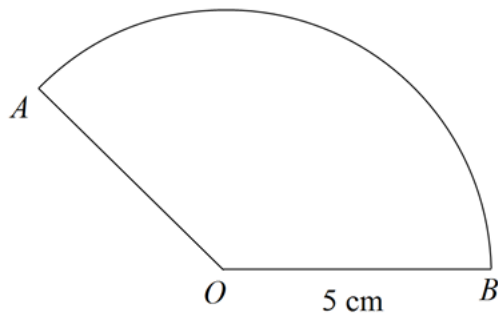
Differentiate: $y = 2(3x - 7)^4$

- A. $8(3x - 7)^3$
- B. $8(3x - 7)^4$
- C. $12(3x - 7)^3$
- D. $24(3x - 7)^3$

Examination continues next page...

Question 6

AOB is a sector of a circle with centre, O , and radius 5 cm. The sector has an area of $10\pi \text{ cm}^2$.



Not to scale

What is the arc length of the sector in exact form?

- A. $2\pi \text{ cm}$
- B. $4\pi \text{ cm}$
- C. $6\pi \text{ cm}$
- D. $10\pi \text{ cm}$

Question 7

The first term of an geometric series is 4 and the third term is 9. The common ratio is negative.

The 40th term of the series is given by:

- A. $T_{40} = 4 \times (-1.5)^{40}$
- B. $T_{40} = 4 + (40 - 1) \times (-2.5)$
- C. $T_{40} = 4 \times (-1.5)^{39}$
- D. $T_{40} = 4 \times (-2.5)^{39}$

Examination continues next page...

Question 8

A normally distributed random variable X has a mean of 90.

It is known that $P(X < 85) \approx 0.1587$.

Find $P(90 < X < 95)$.

- A. 0.1587
- B. 0.3174
- C. 0.3413
- D. 0.9207

Question 9

Given that $\sin \theta = -\frac{7}{13}$ and $\tan \theta > 0$, which of the following is equivalent to $\sec \theta$?

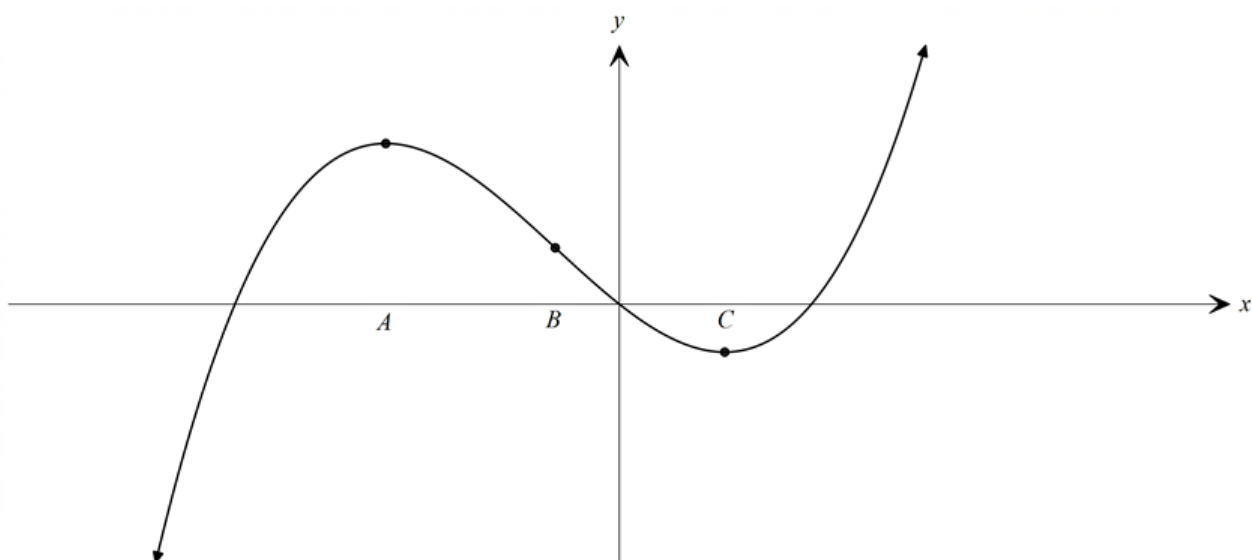
- A. $\frac{7}{\sqrt{120}}$
- B. $-\frac{13}{7}$
- C. $-\frac{13}{\sqrt{120}}$
- D. $-\frac{\sqrt{120}}{13}$

Examination continues next page...

Question 10

The graph of $y = f(x)$ is shown below:

At $x = A$ and $x = C$ there are stationary points, and at $x = B$ there is a point of inflection.



Which of the domains below show where $f'(x) < 0$ and $f''(x) > 0$?

- A. $x < A$
- B. $x > C$
- C. $A < x < B$
- D. $B < x < C$

END OF SECTION 1

Examination continues next page...

Section II

90 marks

Attempt Questions 11 – 33

Allow about 2 hours and 45 minutes for this section

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of the response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Differentiate: $\frac{5x^2 - 7}{x + 2}$

2

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Examination continues next page...

Question 12 (3 marks)

Solve the equation: $|2x - 3| = 9x$

3

Question 13 (3 marks)

A bag contains 5 red, 6 green and 3 white jellybeans.

Two jellybeans are chosen at random without replacement.

a) Draw a probability tree for the situation.

2

b) Find the probability of selecting at least 1 red jellybean.

1

Examination continues next page...

Question 14 (3 marks)

Find the coordinates of the point on the curve $y = x^2 - 2x - 3$, where the tangent is perpendicular to the line $x + 4y - 1 = 0$. **3**

Question 15 (2 marks)

Evaluate $\int_0^{\frac{\pi}{4}} \cos 2x \, dx$. **2**

Examination continues next page...

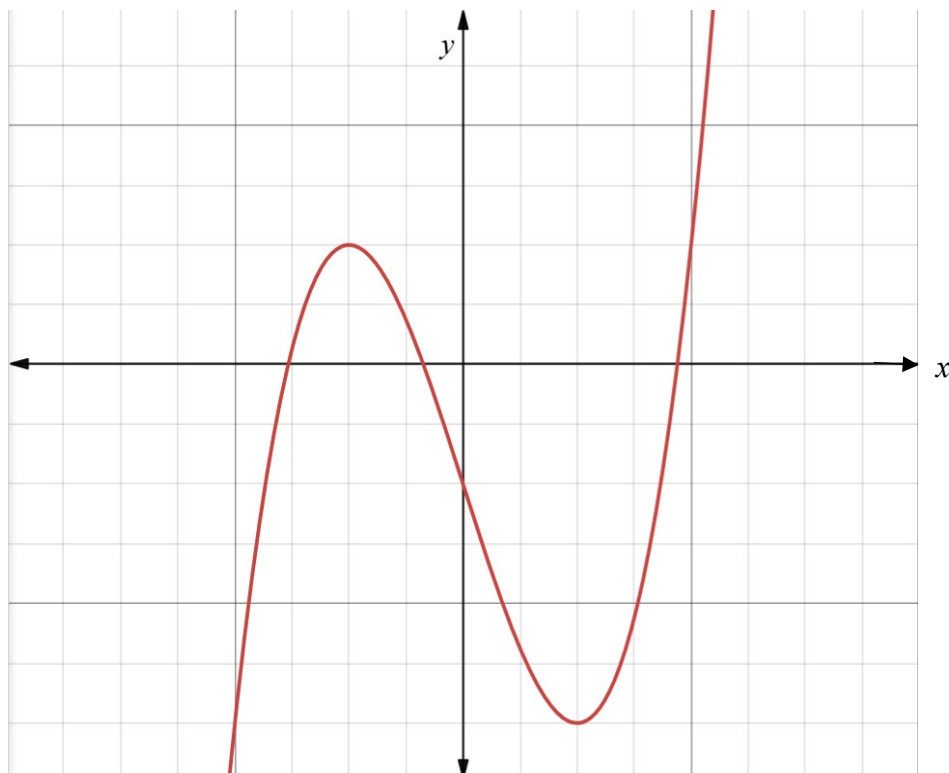
Question 16 (2 marks)

2

The graph shows the gradient function, $y = f'(x)$ for a particular curve.

On the same set of axes, sketch a possible primitive function, $y = f(x)$ of the curve.

Note: carefully line up any stationary points and points of inflection.

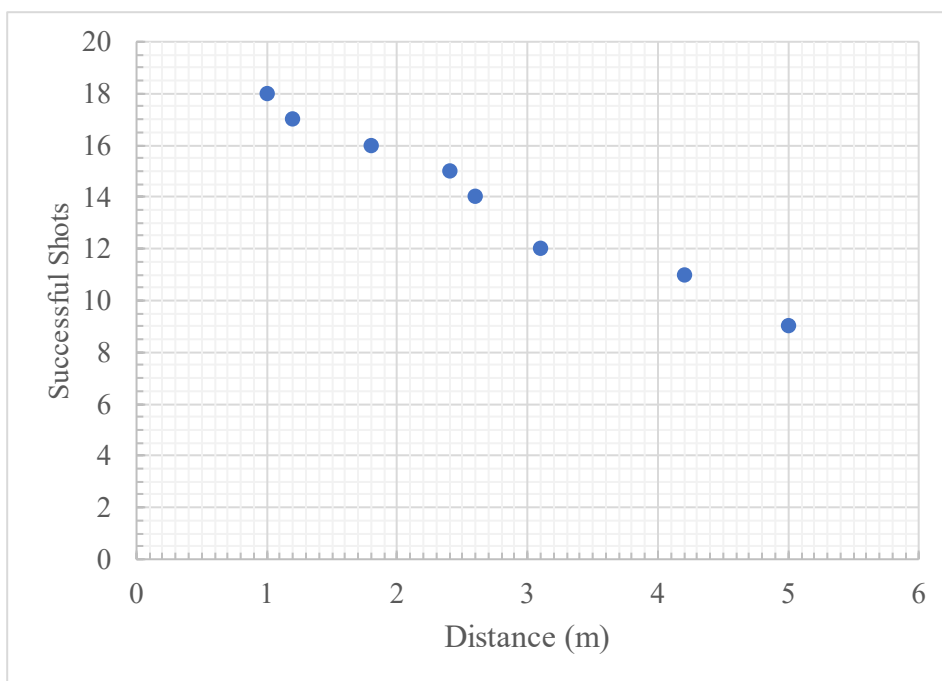


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Question 17 (3 marks)

A netball player took 20 shots from each of 8 different positions that were different distances from the goal. The results are shown in the table below.

Distance (m)	1.2	2.4	5.0	4.2	1.0	1.8	3.1	2.6
Successful Shots (s)	17	15	9	11	18	16	12	14



- a) By observing the scatterplot, describe the relationship between the variables in terms of strength, gradient and shape. 1
-
- b) Use your calculator to determine the equation of the least-squares regression line for this data, rounding any values to 2 decimal places. 1
-
- c) When extrapolating the data in the scatterplot, describe one limitation when it comes to predicting the number of successful shots. 1
-

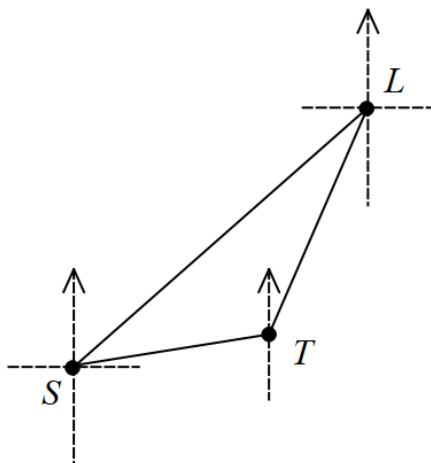
Examination continues next page...

Question 18 (3 marks)

A ship (S) observes a lighthouse (L) to be on a bearing of $N26^\circ E$ and a tower (T) to be on a bearing of $N70^\circ E$. The bearing of the lighthouse from the tower is $N15^\circ E$ and the distance from the lighthouse to the tower is 6.8 km.

- a) Complete the diagram showing the above information.

1



- b) Find the distance of the ship to the tower, correct to one decimal place.

2

Examination continues next page...

Question 19 (3 marks)

Find the equation of the new function if $f(x) = x^3$ is transformed in the following order:

3

1. Horizontal dilation by a factor of 3
2. Reflection across the y -axis
3. Translated to the left by 1 unit

Question 20 (4 marks)

A discrete random variable, X , has the probability distribution table below:

x	1	2	3
$P(X = x)$	0.7	m	0.2

- a) Find the value of m .

1

- b) Calculate the expected value $E(X)$.

1

- c) Calculate the standard deviation of the distribution, to 2 decimal places

2

Examination continues next page...

Question 21 (3 marks)

Cailey produces and sells tubs of slime. The weights of the tubs of slime are normally distributed. 84% of her tubs weigh more than 152 grams and 16% of her tubs weigh more than 200 grams.

- a) Find the mean, μ , and the standard deviation, σ , of the distribution.

2

- b) Find the percentage of tubs of slime that weigh between 152 grams and 224 grams.

1

Question 22 (3 marks)

Show that $\frac{\tan\theta \sec\theta}{1 + \tan^2\theta} = \sin\theta$

3

Examination continues next page...

Question 23 (4 marks)

A continuous probability distribution is given by $f(x) = \frac{3x^2}{279}$, defined in the domain $[4, 7]$.

a) Find the cumulative distribution function.

2

b) Find the median of the continuous probability distribution to 1 decimal place.

2

Examination continues next page...

Question 24 (10 marks)

Consider the function $y = (1 + x)e^{-2x}$.

- a) Find the coordinates of any stationary points and determine their nature.

3

- b) Find the point of inflection.

3

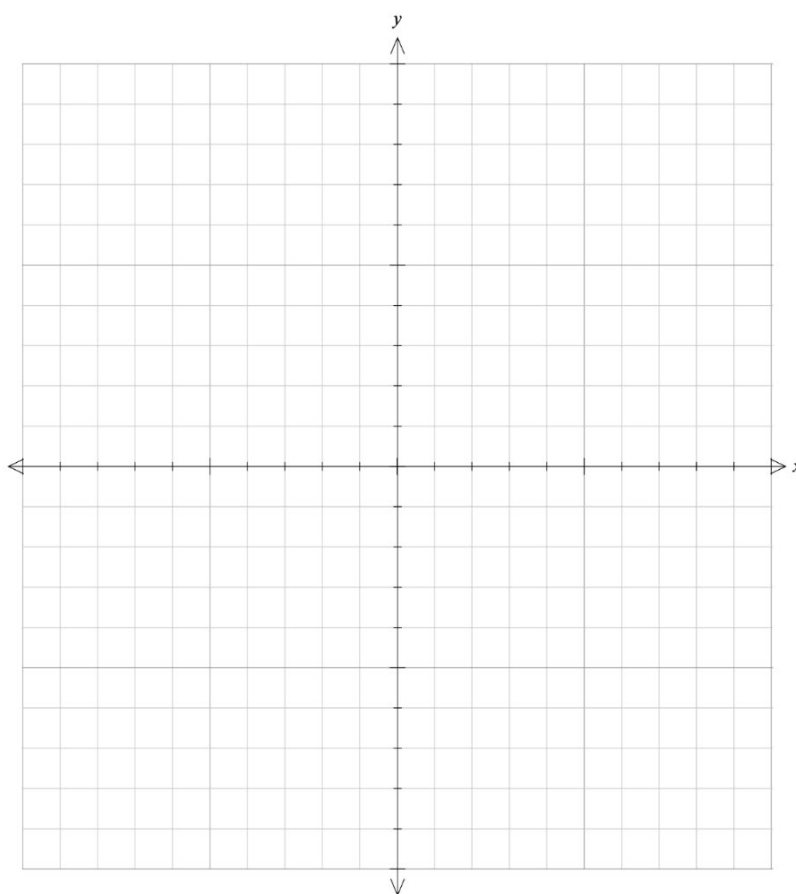
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c) Describe the behaviour of the curve as $x \rightarrow \pm \infty$.

2

d) Sketch the curve, showing all important features including intercepts.

2



Examination continues next page...

Question 25 (5 marks)

A particle moves with velocity given by $v = 12\sin\frac{\pi t}{2}$, where v and t are measured in cm/s and seconds respectively. The particle was initially at rest at $x = \frac{-24}{\pi}$.

- a) Find the equation for displacement, x , in terms of t .

2

- b) Find distance travelled by the particle in the first 3 seconds.

3

Examination continues next page...

4

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18

Question 27 (4 marks)

A company produces 40 tonnes of rice in the first year of production, 80 tonnes in the second year and 120 tonnes in its third year of production. The company continues to increase its rice production by this same amount each year thereafter.

- a) In which year of production did the amount of rice produced for that year equal 2000 tonnes? 2

- b) What was the total amount of rice produced in the first 20 years of production? 2

Question 28 (2 marks)

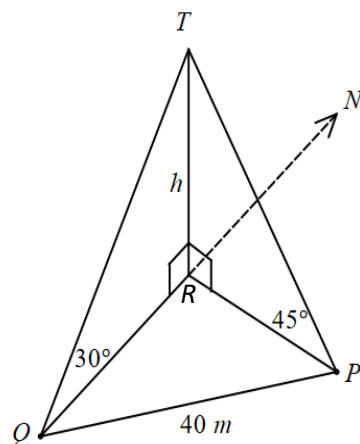
Solve the equation $\sqrt{3} + 2\cos\theta = 0$ in the domain $0 \leq \theta \leq 2\pi$ 2

Examination continues next page...

Question 29 (4 marks)

4

A vertical tower of height h metres stands on horizontal ground. From a point P on the ground due east of the tower the angle of elevation of the top of the tower is 45° . From a point Q on the ground due south of the tower the angle of elevation of the top of the tower is 30° . If the distance PQ is 40 metres, find the exact height of the tower.



Examination continues next page...

Question 30 (4 marks)

A population of insects varies according to the formula $P = 3000n^{\frac{t}{10}}$, where n is a positive constant and t is the time in days.

The initial population was 3000. After 20 days the population had grown to 12 000.

Find the value of t , correct to one decimal place, for which the instantaneous rate of increase is 6000 insects per day.

[illegible]

Examination continues next page...

Question 31 (5 marks)

a) Consider the geometric series: $1 - \tan^2\theta + \tan^4\theta - \dots$

i) When the limiting sum exists, find an expression for the limiting sum in simplest form. **3**

b) For what values of θ in the interval $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ does the limiting sum of the series exist? **2**

Examination continues next page...

Question 32 (8 marks)

Jackson inherits \$200 000. He invests it into an account earning interest at a rate of 6% p.a. compounding monthly. At the end of each month, immediately after the interest has been paid, Jackson withdraws \$2000.

- a) Find the total amount that is in the account immediately after the 3rd withdrawal. **3**

- b) How much interest was earned in the first three months? **2**

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- c) Calculate the amount of money in the account immediately after the 70th withdrawal.

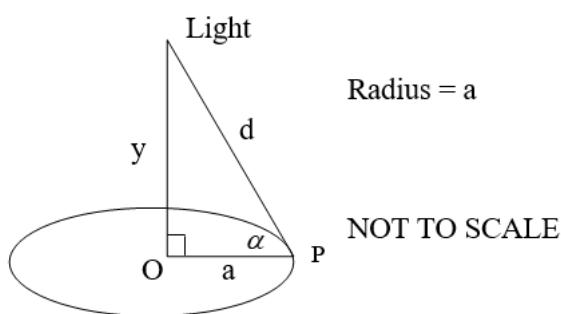
3

[illegible]

Examination continues next page...

Question 33 (6 marks)

A light is to be placed over the centre of a circle. The intensity (I) of the light varies as the sine of the angle (α) at which the rays strike the illuminated surface, divided by the square of the distance (d) from the light i.e. $I = \frac{k \sin \alpha}{d^2}$ where k is a positive constant.



a) Show that: $I = \frac{ky}{(y^2 + a^2)^{\frac{3}{2}}}$

3

Examination continues next page...

- b) Find the optimal height for a light to be placed over the centre of a circle in order to provide maximum illumination to the circumference. Give your final answer as an exact value. 3

[illegible]

END OF EXAMINATION



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Mathematics Advanced

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Section I

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Question 1

The amplitude and period of the curve $f(x) = 3 \sin(4x)$ are:

A. amplitude = 3, period = 4

B. amplitude = 4, period = 3

C. amplitude = 3, period = $\frac{\pi}{4}$

☒ D. amplitude = 3, period = $\frac{\pi}{2}$

$$T = \frac{2\pi}{4} \\ = \frac{\pi}{2}$$

Question 2

What amount must be invested to obtain a future value of \$30 000 after 15 years at a rate of 12% p.a. compounded quarterly?

A. \$182.93

☒ B. \$5091.99

C. \$5480.89

D. \$176 748.09

$$30000 = P(1.03)^{60} \\ \frac{30000}{(1.03)^{60}} = P$$

Examination continues next page...

Question 3

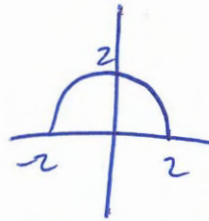
Which of the following represents a possible primitive function of $f'(x) = \frac{4}{2-x}$?

- A. $f(x) = -2 \log|2-x|$
- ☒ B. $f(x) = -4 \log|2-x|$
- C. $f(x) = 4 \log|2+x|$
- D. $f(x) = 4 \log|2-x|$

Question 4

What is the domain and range of the function $f(x) = \sqrt{4-x^2}$?

- A. Domain: $[-2, 2]$ Range: $[-2, 2]$
- B. Domain: $[0, 2]$ Range: $[0, 4]$
- C. Domain: $[-4, 4]$ Range: $[0, 2]$
- ☒ D. Domain: $[-2, 2]$ Range: $[0, 2]$

**Question 5**

Differentiate: $y = 2(3x-7)^4$

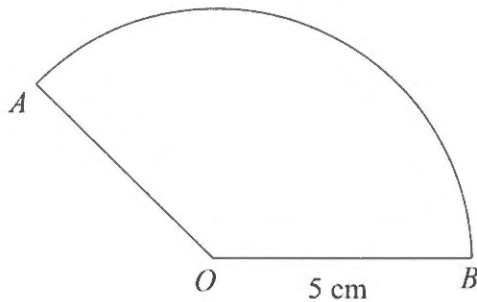
- A. $8(3x-7)^3$
- B. $8(3x-7)^4$
- C. $12(3x-7)^3$
- ☒ D. $24(3x-7)^3$

$$y' = 2 \times 4 \times 3(3x-7)^3$$

Examination continues next page...

Question 6

AOB is a sector of a circle with centre, O , and radius 5 cm. The sector has an area of $10\pi \text{ cm}^2$.



Not to scale

What is the arc length of the sector in exact form?

- A. $2\pi \text{ cm}$
- ☒ B. $4\pi \text{ cm}$
- C. $6\pi \text{ cm}$
- D. $10\pi \text{ cm}$

$$10\pi = \frac{1}{2} \times 5^2 \times \theta$$

$$\frac{20\pi}{25} = \theta$$

$$\frac{4\pi}{5} = \theta$$

$$L = r\theta$$

$$= \frac{4\pi}{5} \times 5$$

$$= 4\pi$$

Question 7

The first term of an geometric series is 4 and the third term is 9. The common ratio is negative.

The 40th term of the series is given by:

- A. $T_{40} = 4 \times (-1.5)^{40}$
- B. $T_{40} = 4 + (40 - 1) \times (-2.5)$
- ☒ C. $T_{40} = 4 \times (-1.5)^{39}$
- D. $T_{40} = 4 \times (-2.5)^{39}$

$$a = 4$$

$$4, x, 9$$

$$\therefore \frac{x}{4} = \frac{9}{x}$$

$$x^2 = 36$$

$$x = \pm 6$$

$$\therefore x = -6$$

$$\therefore r = \frac{-6}{4}$$

$$= -1.5$$

Examination continues next page...

Question 8

A normally distributed random variable X has a mean of 90.

It is known that $P(X < 85) \approx 0.1587$.

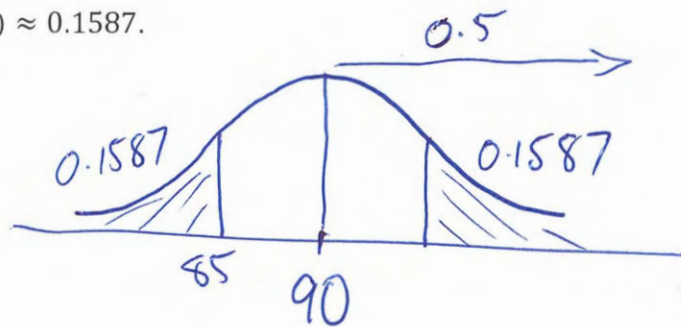
Find $P(90 < X < 95)$.

A. 0.1587

B. 0.3174

☒ C. 0.3413

D. 0.9207



$$P(90 < X < 95) = 0.5 - 0.1587 = 0.3413$$

Question 9

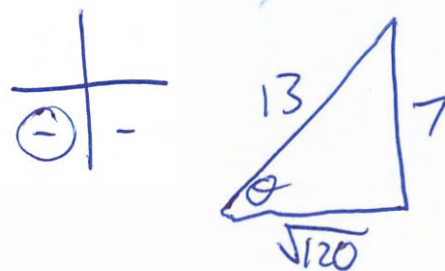
Given that $\sin \theta = -\frac{7}{13}$ and $\tan \theta > 0$, which of the following is equivalent to $\sec \theta$?

A. $\frac{7}{\sqrt{120}}$

B. $-\frac{13}{7}$

☒ C. $-\frac{13}{\sqrt{120}}$

D. $-\frac{\sqrt{120}}{13}$



$$\frac{1}{\cos \theta} = -\frac{1}{\frac{\sqrt{120}}{13}}$$

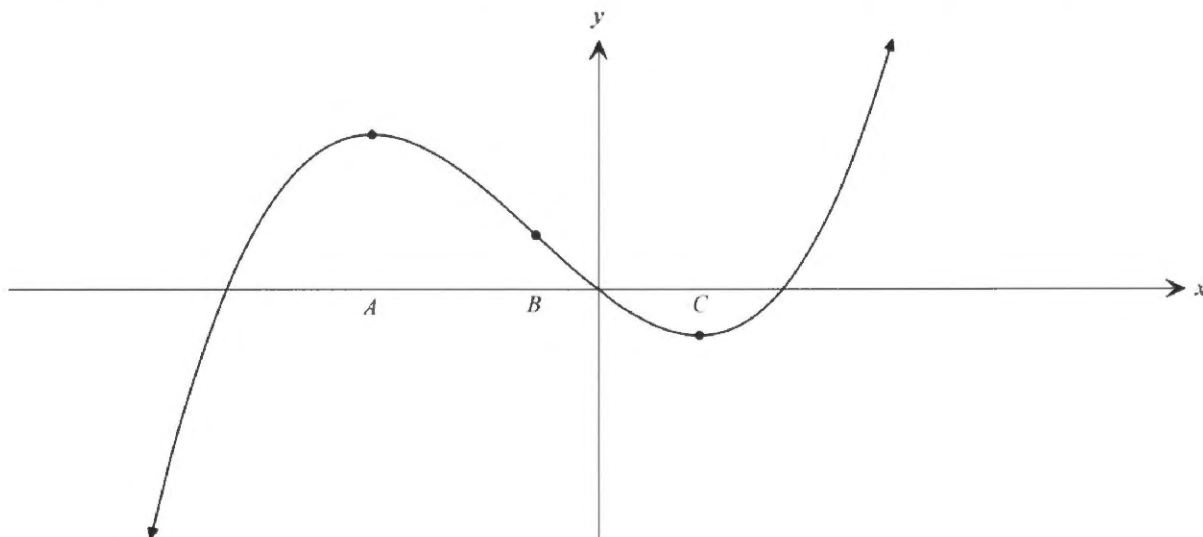
$$\sec \theta = -\frac{13}{\sqrt{120}}$$

Examination continues next page...

Question 10

The graph of $y = f(x)$ is shown below:

At $x = A$ and $x = C$ there are stationary points, and at $x = B$ there is a point of inflection.



Which of the domains below show where $f'(x) < 0$ and $f''(x) > 0$?

decreasing concave up

A. $x < A$

B. $x > C$

C. $A < x < B$

D. $B < x < C$

END OF SECTION 1

Examination continues next page...

Section II

90 marks

Attempt Questions 11 – 33

Allow about 2 hours and 45 minutes for this section

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of the response.
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Question 11 (2 marks)

Differentiate: $\frac{5x^2-7}{x+2} = y$

2

$$u = 5x^2 - 7$$

$$u' = 10x$$

$$v = x + 2$$

$$v' = 1$$

$$\frac{dy}{dx} = \frac{10x(x+2) - (5x^2-7)}{(x+2)^2}$$

$$= \frac{10x^2 + 20x - 5x^2 + 7}{(x+2)^2}$$

$$= \frac{5x^2 + 20x + 7}{(x+2)^2}$$

{ ② – correct sub into quotient rule
– correct simplification
① – correct sub into quotient rule

Examination continues next page...

Question 12 (3 marks)

Solve the equation: $|2x - 3| = 9x$

$$2x - 3 = 9x$$

$$-3 = 7x$$

$$-\frac{3}{7} = x$$

$$-2x + 3 = 9x$$

$$3 = 11x$$

$$\frac{3}{11} = x$$

Test Solutions:

$$|2 \times -\frac{3}{7} - 3| = 9 \times -\frac{3}{7}$$

$$\frac{27}{7} = -\frac{27}{7} \times$$

$\therefore$ not a solution

$$|2 \times \frac{3}{11} - 3| = 9 \times \frac{3}{11}$$

$$\frac{27}{11} = \frac{27}{11} \checkmark$$

$\therefore x = \frac{3}{11}$ is the only solution

③ - solves both equations & correctly tests solutions

② - solves two equations correctly

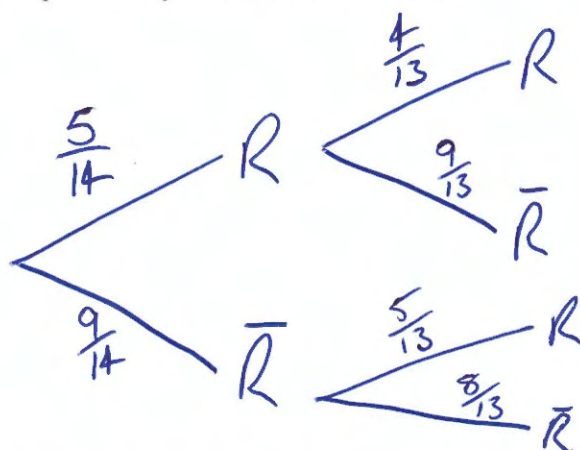
① - solves one equation correctly

Question 13 (3 marks)

A bag contains 5 red, 6 green and 3 white jellybeans.

Two jellybeans are chosen at random without replacement.

a) Draw a probability tree for the situation.



② - correctly writes probabilities on the correct tree

① - correct tree w/ one error

b) Find the probability of selecting at least 1 red jellybean.

$$P(\text{at least 1R}) = 1 - P(\bar{R}\bar{R})$$

$$= 1 - \frac{9}{14} \times \frac{8}{13}$$

$$= \frac{55}{91}$$

① - correct answer

Examination continues next page...

Question 14 (3 marks)

Find the coordinates of the point on the curve $y = x^2 - 2x - 3$, where the tangent is perpendicular to the line $x + 4y - 1 = 0$.

$$\begin{aligned}
 4y &= -x + 1 \\
 y &= -\frac{1}{4}x + \frac{1}{4} \\
 \therefore M &= -\frac{1}{4} \\
 \therefore M_{\perp} &= 4
 \end{aligned}
 \quad
 \begin{aligned}
 y' &= 2x - 2 \\
 4 &= 2x - 2 \\
 6 &= 2x \\
 \boxed{3} &= x \\
 y &= 3^2 - 2 \times 3 - 3 \\
 \boxed{y} &= \boxed{0} \\
 \therefore \text{Point } (3, 0)
 \end{aligned}$$

(3) - correct $M_{\perp}$
 - correct x
 - correct y
 (2) - two of the above correct
 (1) - one of the above correct

Question 15 (2 marks)

Evaluate $\int_0^{\frac{\pi}{4}} \cos 2x \, dx$.

2

$$\begin{aligned}
 &= \left[\frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{4}} \\
 &= \frac{1}{2} \sin \left(2 \cdot \frac{\pi}{4} \right) - \frac{1}{2} \sin 0 \\
 &= \frac{1}{2} \sin \frac{\pi}{2} - 0 \\
 &= \frac{1}{2} \times 1 \\
 &= \frac{1}{2}
 \end{aligned}$$

(2) - correctly integrates
 - correctly subs in & calculates answer.
 (1) - correct integral

Examination continues next page...

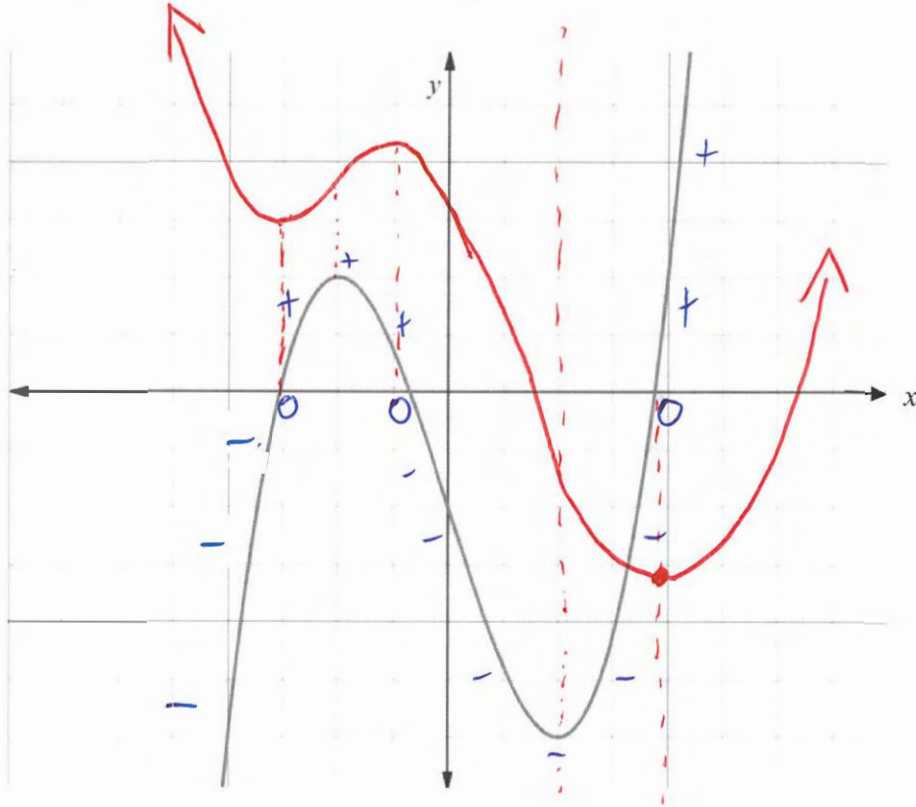
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The graph shows the gradient function, $y = f'(x)$ for a particular curve.

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On the same set of axes, sketch a possible primitive function, $y = f(x)$ of the curve.

Note: carefully line up any stationary points and points of inflection.



{ ② - correct shape
- correctly lines up all
st. points & points of inflection

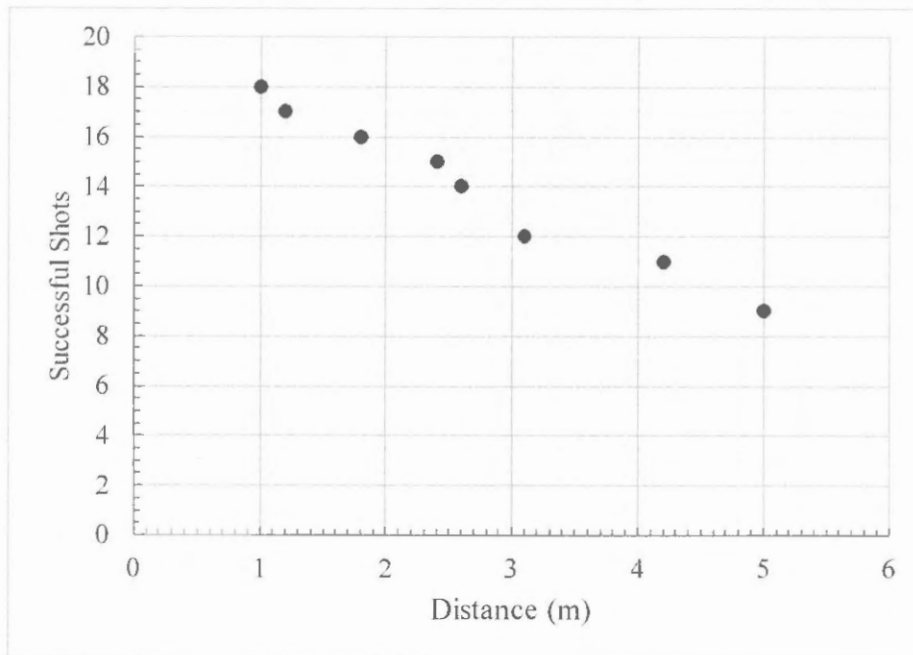
{ ① - correct shape

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Question 17 (3 marks)

A netball player took 20 shots from each of 8 different positions that were different distances from the goal. The results are shown in the table below.

Distance (m)	1.2	2.4	5.0	4.2	1.0	1.8	3.1	2.6
Successful Shots (s)	17	15	9	11	18	16	12	14



- a) By observing the scatterplot, describe the relationship between the variables in terms of strength, gradient and shape. 1

Strong, negative, linear ① - correct answer

- b) Use your calculator to determine the equation of the least-squares regression line for this data, rounding any values to 2 decimal places. 1

$a = 19.85$ $s = -2.20d + 19.85$
 $b = -2.20$ ① - correct equation

- c) When extrapolating the data in the scatterplot, describe one limitation when it comes to predicting the number of successful shots. 1

The successful shots will become negative at around 8-10 m

Examination continues next page...

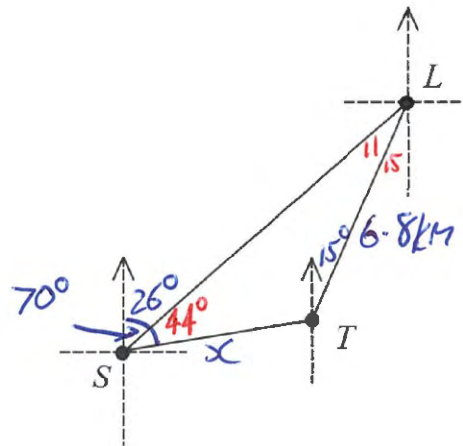
① - describes any legitimate limitation (teacher discretion)

Question 18 (3 marks)

A ship (S) observes a lighthouse (L) to be on a bearing of $N26^{\circ}E$ and a tower (T) to be on a bearing of $N70^{\circ}E$. The bearing of the lighthouse from the tower is $N15^{\circ}E$ and the distance from the lighthouse to the tower is 6.8 km.

- a) Complete the diagram showing the above information.

1



① - correctly shows bearings and length
 $26^{\circ}, 15^{\circ}, 70^{\circ}, 6.8\text{ km}$

- b) Find the distance of the ship to the tower, correct to one decimal place.

2

$$\frac{6.8}{\sin 44^{\circ}} = \frac{x}{\sin 11^{\circ}}$$

$$x = \frac{6.8 \times \sin 11^{\circ}}{\sin 44^{\circ}}$$

$$x = 1.867826$$

$$x \approx 1.9\text{ km}$$

} ② - correct solution
 ① - correctly subs into the sine rule

Examination continues next page...

Question 19 (3 marks)Find the equation of the new function if $f(x) = x^3$ is transformed in the following order:

3

1. Horizontal dilation by a factor of 3
2. Reflection across the y-axis
3. Translated to the left by 1 unit

$$1, f\left(\frac{x}{3}\right) = \left(\frac{x}{3}\right)^3$$

$$2, f\left(-\frac{x}{3}\right) = \left(-\frac{x}{3}\right)^3 = -\frac{x^3}{27}$$

$$3, f\left(-\frac{(x+1)}{3}\right) = -\frac{(x+1)^3}{27}$$

{ ③ - 1 mark for each transformation done correctly

Question 20 (4 marks)A discrete random variable, X , has the probability distribution table below:

x	1	2	3
$P(X = x)$	0.7	m	0.2

- a) Find the value of
- m
- .

1

$$M = 1 - 0.7 - 0.2 = 0.1$$

{ ① - correct answer

- b) Calculate the expected value
- $E(X)$
- .

1

$$E(X) = 1 \times 0.7 + 2 \times 0.1 + 3 \times 0.2 = 1.5 = \mu$$

{ ① - correct answer

- c) Calculate the standard deviation of the distribution, to 2 decimal places

2

$$\mu = 1.5 \quad \text{Var}(X) = E(X^2) - \mu^2$$

$$\mu^2 = 2.25 \quad = [1^2 \times 0.7 + 2^2 \times 0.1 + 3^2 \times 0.2] - 2.25$$

$$= 0.65$$

$$\therefore \sigma = \sqrt{0.65}$$

$$= 0.81 \text{ (to 2dp)}$$

Examination continues next page...

{ ② - correct variance
- correct s.d.

{ ① - correct variance
OR correct s.d. with
no working

Question 21 (3 marks)

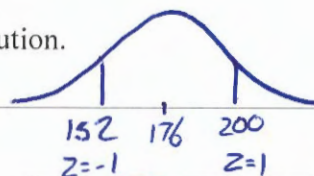
Cailey produces and sells tubs of slime. The weights of the tubs of slime are normally distributed. 84% of her tubs weigh more than 152 grams and 16% of her tubs weigh more than 200 grams.

- a) Find the mean, μ , and the standard deviation, σ , of the distribution.

2

$$\mu = \frac{152 + 200}{2}$$

$$\mu = 176 \quad \therefore \sigma = 24g$$



{ ② - both correct
① - one correct

- b) Find the percentage of tubs of slime that weigh between 152 grams and 224 grams.

1

$$47.5\% + 34\% = 81.5\%$$

{ ① - correct answer

Question 22 (3 marks)

Show that $\frac{\tan\theta \sec\theta}{1 + \tan^2\theta} = \sin\theta$

3

$$\text{LHS} = \left(\frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta} \right) \div \sec^2\theta$$

$$= \frac{\sin\theta}{\cos^2\theta} \div \frac{1}{\cos^2\theta}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{\cos^2\theta}{1}$$

$$= \sin\theta$$

$$= \text{RHS}$$

{ ③ - correctly changes numerator
- correctly changes denominator
- correctly shows = sinθ

{ ② - two of 16 above correct

{ ① - one of 16 above correct

Examination continues next page...

Question 23 (4 marks)

A continuous probability distribution is given by $f(x) = \frac{3x^2}{279}$, defined in the domain $[4, 7]$.

a) Find the cumulative distribution function.

2

$$CDF = \int_4^x \frac{3x^2}{279} dx$$

$$= \left[\frac{x^3}{279} \right]_4^x$$

$$= \frac{x^3}{279} - \frac{4^3}{279}$$

$$= \frac{x^3 - 64}{279}$$

② - correct integration
- correct function

① - correct integration
(or 1 error)

b) Find the median of the continuous probability distribution to 1 decimal place.

2

$$0.5 = \frac{x^3 - 64}{279}$$

$$139.5 = x^3 - 64$$

$$203.5 = x^3$$

$$x = \sqrt[3]{203.5}$$

$$x = 5.881951$$

$$x \approx 5.9 \text{ is the median}$$

② - correctly equates
to 0.5

- correctly calculates x

① - correctly equates to 0.5
- allow for 1 error

Examination continues next page...

Question 24 (10 marks)

Consider the function $y = (1+x)e^{-2x}$.

a) Find the coordinates of any stationary points and determine their nature.

3

$$\begin{array}{l|l}
 u = 1+x & y' = e^{-2x} + -2(1+x)e^{-2x} \\
 u' = 1 & = e^{-2x}(1-2(1+x)) \\
 v = e^{-2x} & = e^{-2x}(-1-2x) \\
 v' = -2e^{-2x} & \\
 \hline
 & \boxed{y' = -e^{-2x}(1+2x)}
 \end{array}$$

(3) - correct derivative
 - correct x & y
 - tests for max
 (2) - allow 1 error
 (1) - allow 2 errors

let $y' = 0$ $0 = -e^{-2x}(1+2x)$

$\therefore 0 = -e^{-2x}$ and $0 = 1+2x$

no solutions

$\boxed{-\frac{1}{2} = x}$

when $x = -\frac{1}{2}$ $y = (1-\frac{1}{2})e^{-2 \cdot -\frac{1}{2}}$

$\boxed{y = \frac{e}{2}}$

$\therefore (-\frac{1}{2}, \frac{e}{2})$ is a st. point

testing

x	-1	$-\frac{1}{2}$	0
y'	e^2	0	-1

$\therefore$ Maximum Turning Point at $(-\frac{1}{2}, \frac{e}{2})$

b) Find the point of inflection.

3

$$\begin{aligned}
 y'' &= -2e^{-2x} + (1+2x) \cdot 2e^{-2x} \\
 &= 2e^{-2x}(-1+1+2x) \\
 &= 4xe^{-2x}
 \end{aligned}$$

when $y'' = 0$ $0 = 4xe^{-2x}$



More writing space for this question over page

$$0 = 4x \text{ \& } 0 = e^{-2x}$$

$0 = x$
no solution

at $x=0$ $y=1$

$\therefore (0,1)$ is a possible P.O.I

x	$-\frac{1}{2}$	0	1
y''	$-e^{-2x}$	0	$4e^{-2}$

$\therefore$ concavity changes

so $(0,1)$ is a point of inflection

(3) - correct 2nd deriv.
- finds possible POI
- tests & concludes the P.O.I

(2) - 1 error or 2 of the above

(1) - 2 errors or 1 of the above correct

c) Describe the behaviour of the curve as $x \rightarrow \pm \infty$.

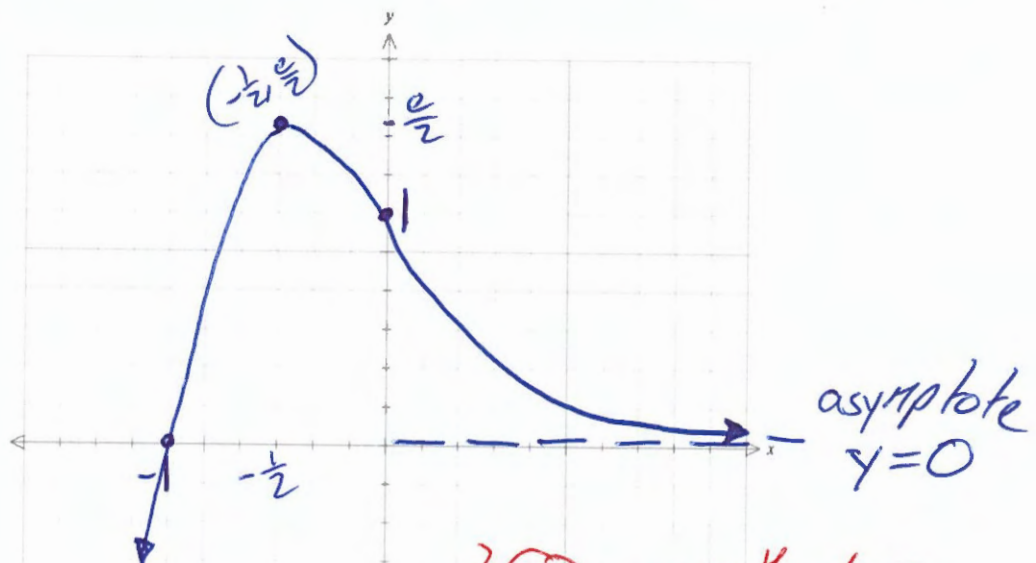
as $x \rightarrow \infty$, $e^{-2x} \rightarrow 0 \therefore y \rightarrow 0$

as $x \rightarrow -\infty$ $e^{-2x} \rightarrow \infty$ $\therefore y \rightarrow -\infty$
and $(1+x) \rightarrow -\infty$

(2) - correctly explains $y \rightarrow 0$ and $y \rightarrow -\infty$

(1) - 1 correct with explanation or both correct w/no explanation

d) Sketch the curve, showing all important features including intercepts.



(2) - correctly shows x,y intercepts, asymptote, st. point
- correct shape

(1) - missing one of the above

Examination continues next page...

Question 25 (5 marks)

A particle moves with velocity given by $v = 12\sin\frac{\pi t}{2}$, where v and t are measured in cm/s and seconds respectively. The particle was initially at rest at $x = \frac{-24}{\pi}$.

a) Find the equation for displacement, x , in terms of t .

2

$$\begin{aligned}
 x &= \int 12\sin\frac{\pi t}{2} dt \\
 x &= -12\cos\frac{\pi t}{2} + C \\
 x &= \frac{-24}{\pi}\cos\frac{\pi t}{2} + C \\
 \text{when } t=0 \quad x &= \frac{-24}{\pi} \\
 \frac{-24}{\pi} &= \frac{-24}{\pi}\cos\frac{\pi}{2} \times 0 + C \\
 0 &= C \\
 \therefore x &= \frac{-24}{\pi}\cos\frac{\pi t}{2}
 \end{aligned}$$

(2) - correctly integrates
 - correctly calculates C
 - correct solution
 (1) - one of the above correct

b) Find distance travelled by the particle in the first 3 seconds.

at rest $0 = 12\sin\frac{\pi t}{2}$
 $0 = \sin\frac{\pi t}{2}$
 $\therefore \frac{\pi t}{2} = 0, \pi, 2\pi, 3\pi, \dots$
 $t = 0, 2, 4, 6, \dots$

(3) - calculates t values
 - calculates x at $t=2$ & 3
 - calculates correct distance
 (2) - two of the above
 (1) - one of the above

at $t=2$ $x = \frac{-24}{\pi}\cos\pi = \frac{-24}{\pi} \times -1 = \frac{24}{\pi}$
 at $t=3$ $x = \frac{-24}{\pi}\cos\frac{3\pi}{2} = 0$

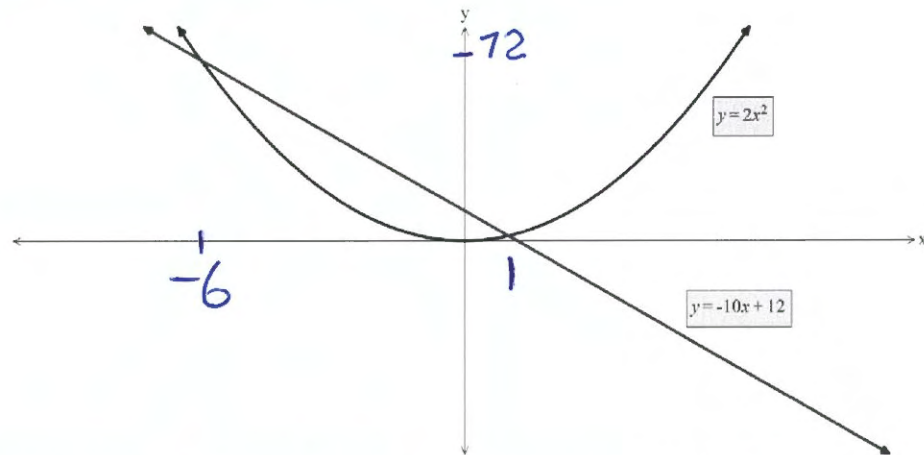
$\therefore \text{distance} = \frac{24}{\pi} \times 2 + \frac{24}{\pi}$
 $= \frac{72}{\pi} \text{ cm or } 22.9 \text{ cm (to 1 dp)}$

Examination continues next page...

Question 26 (4 marks)

Consider the curve $y = 2x^2$ and the line $y = -10x + 12$.

4



Find the area enclosed between the two functions. Give answer to 1 decimal place.

$$\begin{aligned} \text{P.O.I } 2x^2 &= -10x + 12 \\ 2x^2 + 10x - 12 &= 0 \\ x^2 + 5x - 6 &= 0 \\ (x-1)(x+6) &= 0 \\ \therefore x &= 1 \text{ and } -6 \end{aligned}$$

$$\begin{aligned} A &= \int_{-6}^1 (-10x + 12 - 2x^2) dx \\ &= \left[-5x^2 + 12x - \frac{2x^3}{3} \right]_{-6}^1 \end{aligned}$$

$$= \left(-5(1)^2 + 12(1) - \frac{2(1)^3}{3} \right) - \left(-5(-6)^2 + 12(-6) - \frac{2(-6)^3}{3} \right)$$

$$= \frac{19}{3} - 108$$

$$= \frac{343}{3}$$

$$= 114.3 \text{ u}^2$$

- ④ - correctly solves for x
 - correct expression/integral for the area
 - correctly integrates
 - correct answer.
- ③ - three of the above (or 1 error)
- ② - two of the above (or 2 errors)
- ① - one of the above.

Examination continues next page...

Question 27 (4 marks)

A company produces 40 tonnes of rice in the first year of production, 80 tonnes in the second year and 120 tonnes in its third year of production. The company continues to increase its rice production by this same amount each year thereafter.

- a) In which year of production did the amount of rice produced for that year

2

equal 2000 tonnes?

$$40 + 80 + 120 + \dots \quad AP: a=40 \quad d=40$$

$$T_n = a + (n-1)d$$

$$2000 = 40 + (n-1) \times 40$$

$$\frac{1960}{40} = n-1$$

$$49 = n-1$$

$$50 = n$$

$\therefore$ the 50th year

{ (2) - correct sub into formula
- correct n
(1) - 1' of the above

- b) What was the total amount of rice produced in the first 20 years of production?

2

$$S_{20} = \frac{20}{2} (2 \times 40 + (20-1) \times 40)$$

$$= 8400 \text{ tonnes}$$

{ (2) - correct sub into formula
(1) - 1 error

Question 28 (2 marks)

Solve the equation $\sqrt{3} + 2\cos\theta = 0$ in the domain $0 \leq \theta \leq 2\pi$

2

$$\begin{array}{l} 2\cos\theta = -\sqrt{3} \\ \cos\theta = -\frac{\sqrt{3}}{2} \\ \text{related angle: } \theta = \frac{\pi}{6} \text{ rad} \end{array}$$

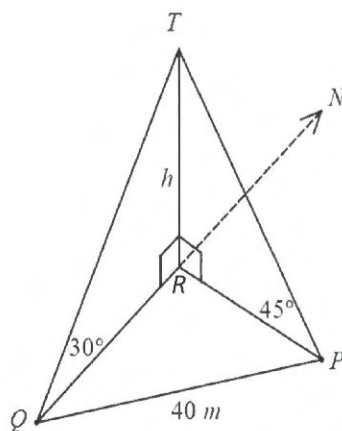
$$\begin{aligned} \therefore \theta &= \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6} \\ &= \frac{5\pi}{6}, \frac{7\pi}{6} \end{aligned}$$

{ (2) - correct solution
(1) - correct related angle
- 1 error

Examination continues next page...

Question 29 (4 marks)

A vertical tower of height h metres stands on horizontal ground. From a point P on the ground due east of the tower the angle of elevation of the top of the tower is 45° . From a point Q on the ground due south of the tower the angle of elevation of the top of the tower is 30° . If the distance PQ is 40 metres, find the exact height of the tower.



$$\tan 45 = \frac{h}{PR} \quad \tan 30 = \frac{h}{QR}$$

$$1 = \frac{h}{PR}$$

$$\boxed{PR = h}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{QR}$$

$$\boxed{QR = \sqrt{3}h}$$

Pythagoras' Theorem

$$40^2 = h^2 + (\sqrt{3}h)^2$$

$$1600 = h^2 + 3h^2$$

$$1600 = 4h^2$$

$$400 = h^2$$

$$\pm \sqrt{400} = h$$

$$\therefore h = \pm 20$$

$\therefore$ as $h > 0$

$$h = 20 \text{ m}$$

④ - correct PR
- correct QR
- correct sub into Pythagoras
- correct answer

③ - 3 of the above

② - 2 of the above

① - one of the above

Examination continues next page...

Question 30 (4 marks)

A population of insects varies according to the formula $P = 3000n^{\frac{t}{10}}$, where n is a positive constant and t is the time in days.

4

The initial population was 3000. After 20 days the population had grown to 12 000.

Find the value of t , correct to one decimal place, for which the instantaneous rate of increase is 6000 insects per day.

1, Find n $\hookrightarrow 12000 = 3000n^{\frac{20}{10}}$
 2, Find $\frac{dP}{dt}$ $4 = n^2$
 3, Find t $\pm 2 = n$

as $n > 0$

$$\boxed{n = 2}$$

2, $\frac{dP}{dt} = 3000 \times \ln 2 \times \frac{1}{10} \times 2^{\frac{t}{10}}$

$$\boxed{\frac{dP}{dt} = 300 \ln 2 \times 2^{\frac{t}{10}}}$$

3, When $\frac{dP}{dt} = 6000$

$$\boxed{6000 = 300 \ln 2 \times 2^{\frac{t}{10}}}$$

$$\frac{20}{\ln 2} = 2^{\frac{t}{10}}$$

$$\log_2 \left(\frac{20}{\ln 2} \right) = \frac{t}{10}$$

$$\log_2 20 - \log_2 (\ln 2) = \frac{t}{10}$$

$$10 \times \left(\frac{\ln 20}{\ln 2} - \frac{\ln (\ln 2)}{\ln 2} \right) = t$$

$$\therefore \boxed{t = 48.5 \text{ days}}$$

- ④ - finds n
- find $\frac{dP}{dt}$
- correctly subs 6000
 n to $\frac{dP}{dt}$
- correctly calculates t
- ③ - 3 of 16 above
 (or 1 error)
- ② - 2 of 16 above
 (or 2 errors)
- ① - one of 16 above

Examination continues next page...

Question 31 (5 marks)

a) Consider the geometric series: $1 - \tan^2\theta + \tan^4\theta - \dots$

i) When the limiting sum exists, find an expression for the limiting sum in simplest form.

3

$$a=1$$

$$r = -\tan^2\theta$$

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{1}{1+\tan^2\theta}$$

$$= \frac{1}{\sec^2\theta}$$

$$= \cos^2\theta$$

- ③ - correctly subs in S_{∞}
- changes $1+\tan^2\theta$
- changes $\frac{1}{\sec^2\theta}$
- ② - two of the above (one error)
- ① - one of the above

b) For what values of θ in the interval $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ does the limiting sum of the series exist?

2

For limiting sum to exist $|r| < 1$

$$\therefore -1 < r < 1$$

$$-1 < -\tan^2\theta < 1$$

$$1 > \tan^2\theta > -1$$

$$\therefore -1 < \tan^2\theta < 1$$

$$\text{Now } \tan\frac{\pi}{4} = 1$$

$$\text{and } \left(\tan\frac{\pi}{4}\right)^2 = 1 \quad \therefore -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

- ② - correctly states $-1 < \tan^2\theta < 1$
- correct answer
- ① - one of the above

Examination continues next page...

Question 32 (8 marks)

Jackson inherits \$200 000. He invests it into an account earning interest at a rate of 6% p.a. compounding monthly. At the end of each month, immediately after the interest has been paid, Jackson withdraws \$2000.

- a) Find the total amount that is in the account immediately after the 3rd withdrawal.

3

$$r = \frac{6\%}{12}$$
$$= 0.5\%$$

per month

$$A_1 = 200000(1 + 0.005)^1 - 2000$$
$$= 200000(1.005) - 2000$$

{ ③ - correct
A₁, A₂, A₃
② - 1 error
① - one correct
A₁

$$A_2 = A_1(1.005)^1 - 2000$$
$$= 200000(1.005)^2 - 2000(1.005) - 2000$$

$$A_3 = A_2(1.005)^1 - 2000$$
$$= 200000(1.005)^3 - 2000(1.005)^2 - 2000(1.005) - 2000$$
$$= 200000(1.005)^3 - 2000(1 + 1.005 + 1.005^2)$$
$$= \$196984.98$$

- b) How much interest was earned in the first three months?

2

$$I = A_3 - (P - M)$$
$$= 196984.98 - (200000 - (3 \times 2000))$$
$$= 196984.98 - 194000$$
$$= \$2984.98$$

{ ② - correct solution
① - correct (P-M)

This question continues over page...

c) Calculate the amount of money in the account immediately after the 70th withdrawal.

3

$$A_{70} = 200000(1.005)^{70} - 2000(1 + 1.005 + \dots + 1.005^{69})$$

$$= 200000(1.005)^{70} - 2000 \times S_{70}$$

$$= 200000(1.005)^{70} - 2000 \times \left(\frac{1 \times (1.005^{70} - 1)}{1.005 - 1} \right)$$

$$= 283566.1055 - 167132.211$$

$$= \$116433.89$$

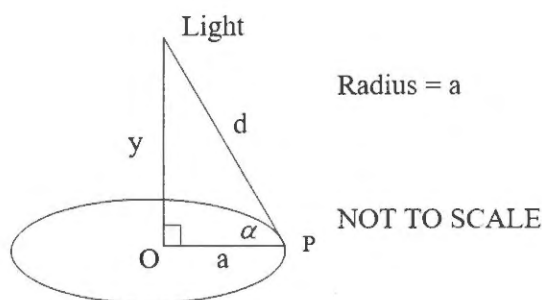
- ③ - correct A_{70}
- correct S_{70}
- correct answer
- ② - two of the above
- ① - one of the above

Examination continues next page...

Question 33 (6 marks)

A light is to be placed over the centre of a circle. The intensity (I) of the light varies as the sine of the angle (α) at which the rays strike the illuminated surface, divided by the square of the

distance (d) from the light i.e. $I = \frac{k \sin \alpha}{d^2}$ where k is a positive constant.



a) Show that: $I = \frac{ky}{(y^2 + a^2)^{\frac{3}{2}}}$

3

$$\boxed{\sin \alpha = \frac{y}{d}}$$

$$d^2 = y^2 + a^2$$

$$\boxed{d = (y^2 + a^2)^{\frac{1}{2}}} \text{ as } d > 0$$

$$I = \frac{k \sin \alpha}{d^2}$$

$$I = \frac{k \times \frac{y}{d}}{d^2}$$

$$I = \frac{k y}{d^3}$$

$$\boxed{I = \frac{k y}{(y^2 + a^2)^{\frac{3}{2}}}} \text{ as required}$$

(3) - correct $\sin \alpha$
- correct d
- correctly showing I

(2) - two of the above
(1 error)

(1) - one of the above

Examination continues next page...

- b) Find the optimal height for a light to be placed over the centre of a circle in order to provide maximum illumination to the circumference. Give your final answer as an exact value. 3

$$I = \frac{KY}{(y^2+a^2)^{\frac{3}{2}}} \quad u = KY \quad v = (y^2+a^2)^{\frac{3}{2}}$$

$$u' = K \quad v' = 3y(y^2+a^2)^{\frac{1}{2}}$$

$$\frac{dI}{dy} = \frac{K(y^2+a^2)^{\frac{3}{2}} - KY \cdot 3y(y^2+a^2)^{\frac{1}{2}}}{(y^2+a^2)^3}$$

$$= \frac{K(y^2+a^2)^{\frac{1}{2}}[(y^2+a^2) - 3y^2]}{(y^2+a^2)^3}$$

$$= \frac{K(y^2+a^2)^{\frac{1}{2}}(a^2-2y^2)}{(y^2+a^2)^3}$$

let $\frac{dI}{dy} = 0$ $\frac{dI}{dy} = \frac{K(a^2-2y^2)}{(y^2+a^2)^{\frac{5}{2}}}$

$$0 = K(a^2-2y^2)$$

$$0 = a^2-2y^2$$

$$2y^2 = a^2$$

$$y^2 = \frac{a^2}{2}$$

$$\boxed{y = \frac{a}{\sqrt{2}}} \text{ as } y > 0$$

test:

y	0	$\frac{a}{\sqrt{2}}$	a
$\frac{dI}{dy}$	$\frac{K}{a^3}$	0	$-\frac{Ka^2}{(2a^2)^{\frac{5}{2}}}$
	/	-	\

$\therefore$ Maximum value at $y = \frac{a}{\sqrt{2}}$ m

- ③ - correct derivative
- correctly finding y
- testing for max
- ② - two of the above (sl error)
- ① - one of the above

END OF EXAMINATION