
Student Number

2025 Glenwood High School
Year 12 – Trial HSC Examination

Mathematics Advanced

General Instructions

* Reading Time – 10 minutes
* Working time – 3 hours
* Write using black pen
* NESA approved calculators may be used
* A reference sheet is provided at the back of this paper
* For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2 – 7)

* Attempt Questions 1-10
* Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 30)

* Attempt Questions 11-33
* Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1. The stem and leaf plot below shows the number of lollies that fourteen students in the class have eaten this week.

0	0	0	2	5	
1	1	2	2	6	9
2	0	4	4	7	
3	3				

Two students were absent when this data was collected.

Once their data was added to the data of the rest of the class, the grouped frequency distribution table below was completed. The cumulative frequency for the $10 - 19$ class is marked with an $\boldsymbol{X}$.

Class	Frequency	Cumulative frequency
$0 - 9$		
$10 - 19$		**X**
$20 - 29$		
$30 - 39$		

Which of the following values for $\boldsymbol{X}$ could be true?

A. $\boldsymbol{X} = 9$

B. $\boldsymbol{X} = 12$

C. $\boldsymbol{X} = 14$

D. $\boldsymbol{X} = 16$

2. What is the gradient of the normal to the curve $f(x) = 3x^3 - 4x + 2$ at the point $(-1,3)$?

A. 5

B. -5

C. $-\frac{1}{5}$

D. $-\frac{1}{3}$

3. What is the solution of the equation $2a^{3x} = b$, where a and b are positive constants?

A. $x = \frac{1}{2}\log_a \frac{b}{3}$

B. $x = \frac{1}{2}\log_b \frac{a}{3}$

C. $x = \frac{1}{3}\log_a \frac{b}{2}$

D. $x = \frac{1}{3}\log_b \frac{a}{2}$

4. Savannah competed in four throwing events at the district athletics carnival.

The distances achieved in each event by the competitors were normally distributed.

	Savannah's result	Event mean	Event standard deviation
Shot put	15 m	13 m	2 m
Discus	37 m	43 m	6 m
Javelin	59 m	47 m	8 m
Hammer throw	33 m	48 m	10 m

Which was Savannah's weakest event compared with the other competitors?

A. Shot put

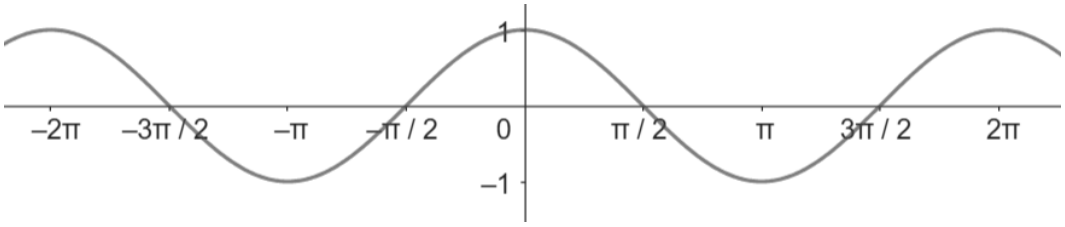
B. Discus

C. Javelin

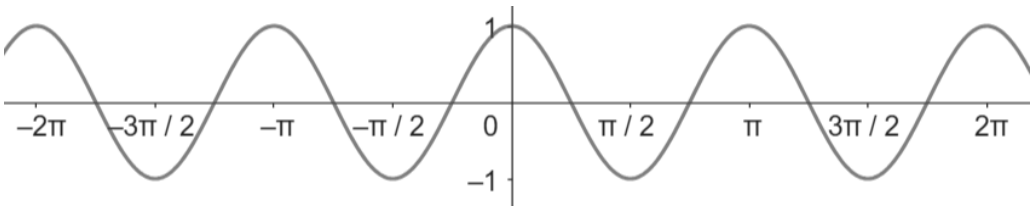
D. Hammer throw

5. Which diagram shows the graph of $y = \sin\left(\frac{\pi}{2} - 2x\right)$?

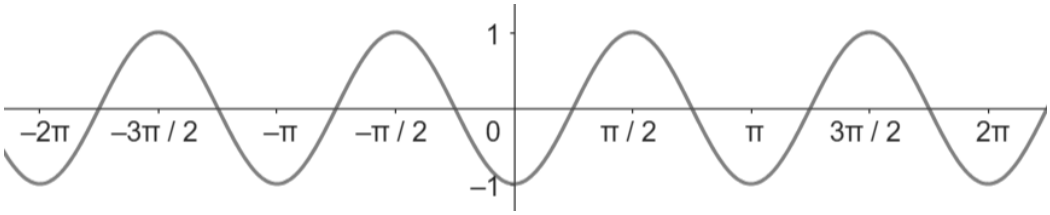
A.



B.



C.



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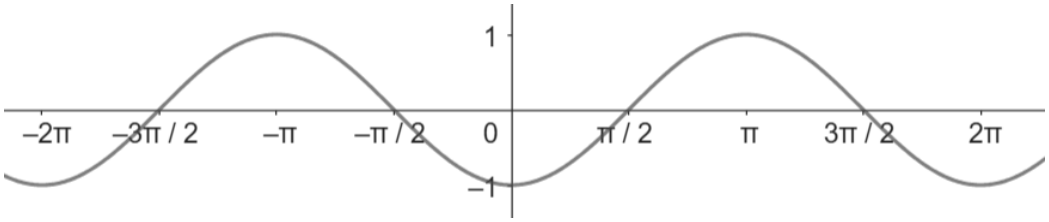


6. Two fair dice are thrown in a game. If the sum of the two numbers thrown is 7, the player gains $36. The player gains $6 on any other number rolled.

What is the expected gain from this game?

A. $11

B. $12

C. $30

D. $31

7. What is the range of the function $f(x) = \sqrt{3x^2 + 6x + 5}$?

A. $[0, \infty)$

B. $[\sqrt{2}, \infty]$

C. $[0, \sqrt{2}]$

D. $[0,1]$

8. What is

$$\int (e^{2x} + e^{2x+1})dx \, ?$$

A. $\frac{e^{2x}(3-2e)}{2} + c$

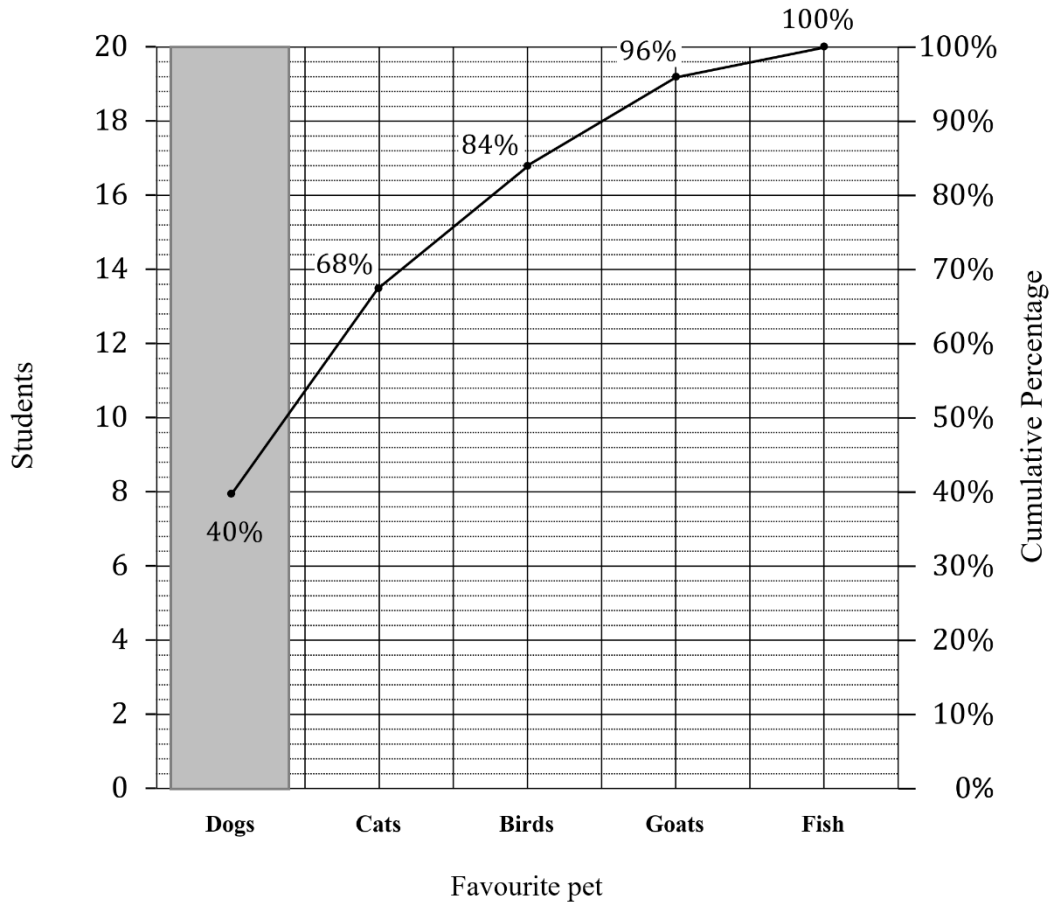
B. $\frac{e^{2x}(3+2e)}{2} + c$

C. $\frac{e^{2x}(1-e)}{2} + c$

D. $\frac{e^{2x}(1+e)}{2} + c$

9. A teacher asked fifty students to identify their favourite type of pet.

A partially completed Pareto chart shows some of the data collected.



How many students chose cats as their favourite pet?

A. 12 students

B. 14 students

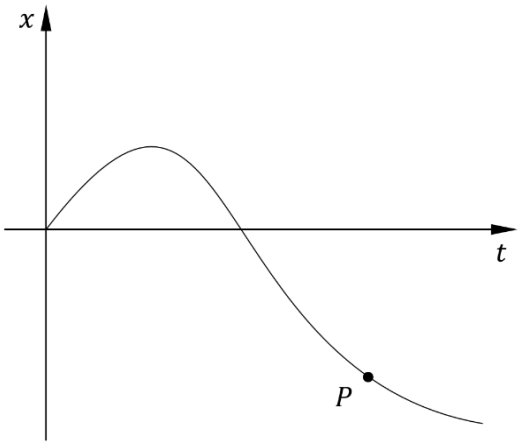
C. 28 students

D. 34 students

10. The displacement of a car to the east of its starting point along a straight road that runs east-west is x kilometres.

At time t hours the car's velocity is $\frac{dx}{dt}$ and its acceleration is $\frac{d^2x}{dt^2}$.

The graph of $x = f(t)$ is shown.



When the car is at point P, which statement reflects its motion?

A. The car is west of its starting point, it is moving east, and its speed is decreasing.

B. The car is west of its starting point, it is moving east, and its speed is increasing.

C. The car is west of its starting point, it is moving west, and its speed is decreasing.

D. The car is west of its starting point, it is moving west, and its speed is increasing.

END OF SECTION I

Student Number

2025 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11-33
Allow about 2 hours and 45 minutes for this section

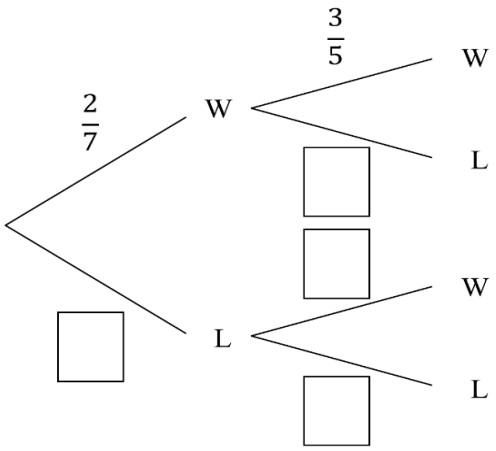
Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the booklet.
  If you use this space, clearly indicate which question you are answering.

Question 11 (3 marks)

When Cody plays Sofia at tennis, he wins the first set two out of every seven times they play.
If he wins the first set, he wins the second set three out of every five times, but if he loses the first set, he wins the second set one out of every five times.

(a) Complete the probability tree below and find the probability that Cody loses both sets. **2**



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(b) Suppose Cody and Sofia play 70 matches in a year. **1**

In how many of those matches would Cody expect to win at least one set?

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Question 12 (2 marks)

Solve: $|5 - x| = 4$ **2**

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Question 13 (5 marks)

A study is investigating the relationship between the number of hours students study and their scores on a statistics exam. The examination is marked in integers out of 100.

The data collected from seven students is shown in the table.

Hours studied (h)	2	4	3	5	7	8	1
Exam score (s)	47	50	32	75	83	94	48

(a) Calculate Pearson's coefficient to 3 d.p. and describe the correlation referring to strength and direction. **2**

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(b) Determine the equation of the regression line to 2 sig. fig. **2**

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(c) Use the equation of the regression line to predict the exam score of a student who studied for 6 hours. **1**

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Question 14 (3 marks)

Find all the values of θ, where $0° \leq \theta \leq 360°$, such that **3**

$$sin(2\theta - 30°) = -\frac{1}{2}$$

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Question 15 (2 marks)

A population of wasps, P, is to be modelled using the function $P = 100(1.3)^{-t}$, where t is the time in hours since a pesticide was applied. **2**

What is the percentage decrease in the wasp population over the first four hours since the pesticide was applied?

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Question 16 (3 marks)

Describe three transformations which, when applied in succession, change the graph of $y = 4\sin x$ to the graph with equation $y = \sin\left(\frac{x+\pi}{3}\right)$. **3**

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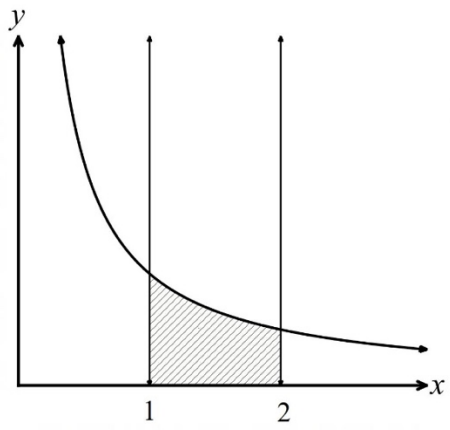
Question 17 (6 marks)

The curve $y = \frac{1}{x}$ and the lines $x = 1$ and $x = 2$ are shown on the graph below:



(a) By completing the table below and using the trapezoidal rule with six function values find an approximation for $\int_1^2 \frac{1}{x}\,dx$, correct to 2 significant figures. **3**

x	1	$\frac{6}{5}$	$\frac{7}{5}$	$\frac{8}{5}$	$\frac{9}{5}$	2
$f(x)$						

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(b) Find the exact value of $\int_1^2 \frac{1}{x}\,dx$ **2**

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(c) State whether your answer to part (a) is an under-estimate or over-estimate of the exact area and explain why this is the case. **1**

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Question 18 (3 marks)

Determine the global maximum and minimum values of y for the curve $y = e^2 + e^x - x\,e^2$ in the domain $0 \leq x \leq 5$. **3**

Give your answers as exact values.

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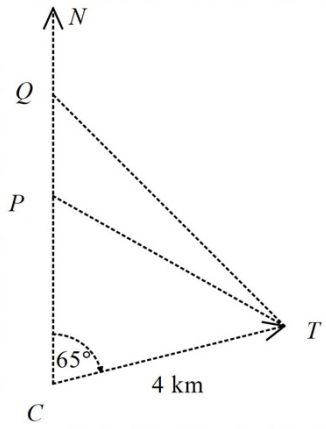
Question 19 (5 marks)

Two mountains, P and Q, lie directly north of a campground C. Tom walks 4 km from C on a bearing of $065°$ to a point T. From point T, the base of mountain P is on a bearing of $285°$ whilst the base of mountain Q is on a bearing of $315°$.

The diagram below shows the relative positions when viewed from above.



NOT TO SCALE

(a) Show that $\angle QTC = 70°$. **1**

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(b) Use the sine rule to find the distance CQ, correct to 1 decimal place. **2**

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(c) Hence or otherwise find the distance PQ, correct to 1 decimal place. **2**

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Question 20 (3 marks)

A semi-circle on the number plane is defined by the following equation:

$$y = \sqrt{-x^2 + 10x - 16}$$

(a) Find the centre and radius of the semi-circle. **2**

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(b) Sketch the curve on the axes below: **1**

Question 21 (3 marks)

The time it takes for a cupcake at a picnic to be eaten by ants (t minutes) varies inversely with the number of ants eating the cupcake (N ants). When there are 10 ants the cupcake is eaten in 120 minutes.

(a) The equation relating t and N is $t = \frac{k}{N}$. **1**

What is the value of k?

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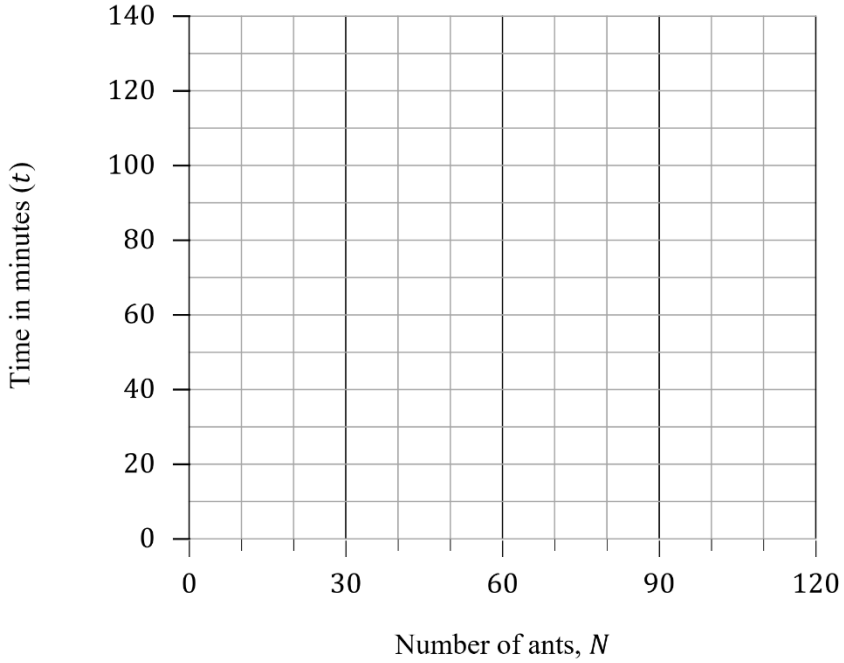
(b) By first completing this table of values, graph the curve representing the relationship between the time taken for the cupcake to be eaten and the number of ants eating the cupcake from $N = 10$ to $N = 120$. **2**

N	10	30	60	120
t	120			

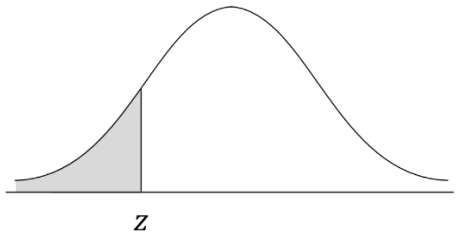


Question 22 (4 marks)

A random variable is normally distributed with a mean of 0 and a standard deviation of 1. The table gives the probability that this random variable lies less than z.

z	-1.0	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
Probability	0.1587	0.1841	0.2119	0.242	0.2743	0.3085	0.3446	0.3821	0.4207	0.4602	0.5000

The probability values given in the table are represented by the shaded area in the following diagram.



The time it takes for a worker to drive from home to work is normally distributed with a mean of 32 minutes and a standard deviation of 4 minutes.

(a) What is the probability that on a particular day the drive takes between 30 and 32 minutes? **2**

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(b) What is the probability that the drive takes 32.8 minutes or less? **2**

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Question 23 (4 marks)

(a) Show that **2**

$$\frac{d}{dx}(e^x \tan x) = e^x(\tan^2 x + \tan x + 1)$$

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(b) Find the exact value of **2**

$$\int_0^{\frac{\pi}{4}} 2e^x(\tan^2 x + \tan x + 1)dx$$

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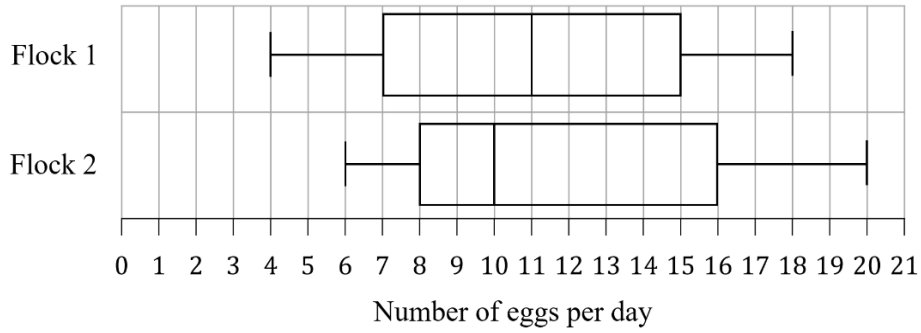
Questions 11- 23 are worth 46 marks in total

Question 24 (3 marks)

A farmer has two flocks of chickens. The number of eggs laid each day during the year was counted, with the data shown in a parallel box-plot. **3**



Compare the two datasets by examining the skewness of the distributions, and the measures of central tendency and spread.

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Question 25 (4 marks)

Emily's number of goals scored in football each game she has played this season has the following probability distribution.

x	0	1	2	3	4
$P(X = x)$	0.46	0.34	k	0.08	0.05

(a) Show that $k = 0.07$. **1**

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(b) Find $E(X)$, correct to 4 decimal place. **1**

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(c) Find Var(X). **2**

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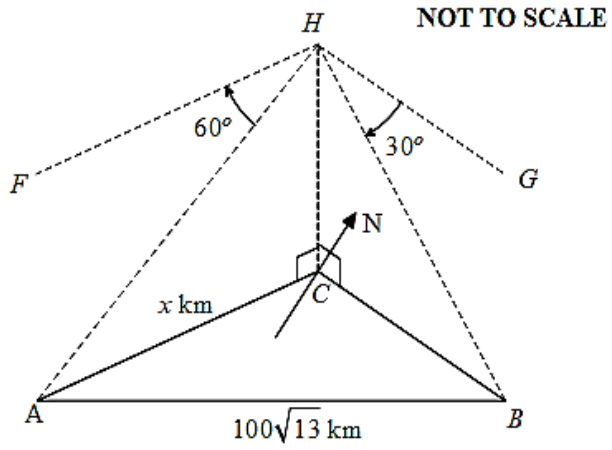
Question 26 (3 marks)

The figure below shows three buoys at sea at A, B and C. A helicopter is flying along FH and then HG, where F, G and H are three points of the same height vertically above A, B and C respectively. When the helicopter reaches H, the angles of depression of A and B from the helicopter are 60° and 30° respectively, while the true bearings of A and B from C are 250° and 130° respectively. The distance between A and B is $100\sqrt{13}\ km$.



Show that the distance between A and C is $100\,km$. **3**

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Question 27 (5 marks)

A continuous random variable X has the following cumulative distribution function $F(x)$, defined by

$$F(x) = \begin{cases} 0 & x < 0 \\ Bx^2\left(4 - x^2\right) & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

(a) Show that $B = \frac{1}{3}$. **1**

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(b) Find $P(X > 0.5)$. **1**

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(c) Find the mode of the function. **3**

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Question 28 (6 marks)

On the next page, sketch the curve $y = x^3(4 - x)$, showing x and y intercepts, any stationary points (classifying their nature) and stating if any inflection points exist and if so showing them. **6**

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Question 28 continues on page 25

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End of Question 28

Question 29 (5 marks)
The depth of water (in metres) in a lagoon in the coastal town of Schoolies is given by:

$$D = 8 + 2\cos\left(\frac{4\pi t}{25}\right) \text{ for } 0 \leq t \leq 25.$$

where t is the time in hours after noon.

(a) What is the depth of water at noon? **1**

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(b) At what time will the water next be at this depth? **2**

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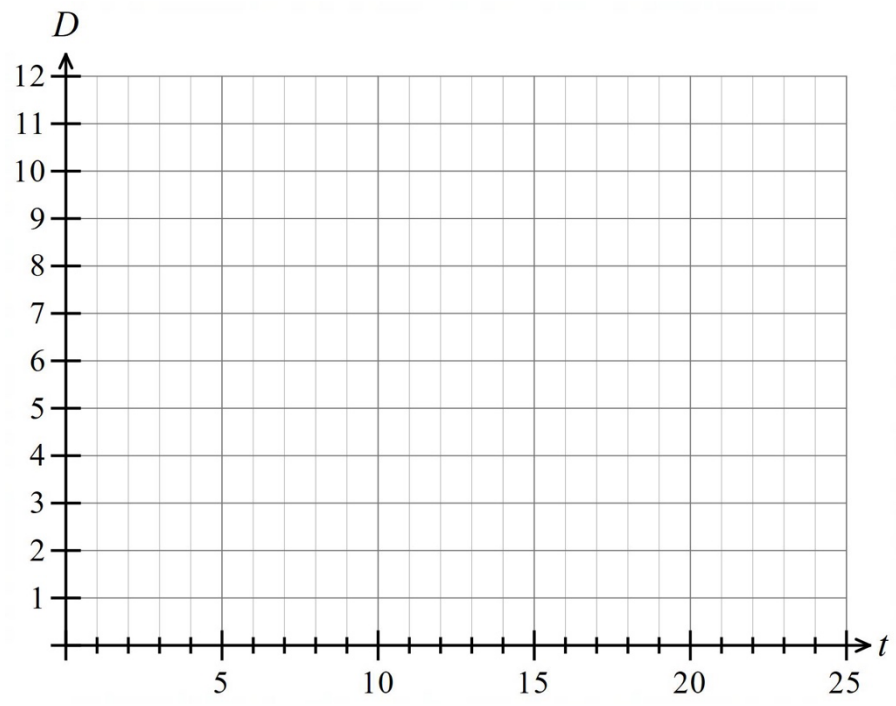
(c) Sketch the curve on the axes below, showing the maximum and minimum depths of the lagoon during the time period $0 \leq t \leq 25$. **2**



Question 30 (4 marks)

In a science experiment, a robot arm attempts to successfully pick up a small object. The probability that the robot arm successfully picks up the object on any single attempt is $\frac{1}{4}$.

(a) What is the probability that the robot arm successfully picks up the object on the third attempt only, after failing the first two attempts? **1**

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(b) What is the probability that the robot arm is unsuccessful in the first four attempts? **1**

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(c) Determine the minimum number of attempts required so that the probability of at least one successful pick-up exceeds 0.95. **2**

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Question 31 (6 marks)

A particle is moving along the x-axis such that at a time t seconds its displacement from the origin is x metres.

The acceleration of the particle is given by $a = \frac{8}{e^{2t}} + \frac{3}{e^{t}}$

Initially its velocity is -6 m/s and its displacement is 5 m.

(a) Find the equation for the displacement (x) of the particle in terms of t. **3**

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(b) At what time and where does the particle come to rest? **3**

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Question 32 (4 marks)

A municipal park needs a cylindrical water tank that holds 2 m^3 of water. The tank will have a flat top and bottom and the cost of the metal for all surfaces is the same per square metre.

Let the radius of the tank be r and its height be h.

Find the relationship between r and h that would minimise the surface area of the tank.

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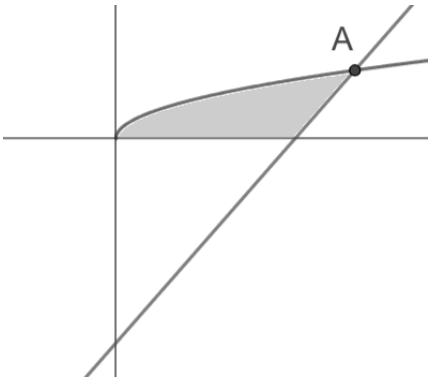
Question 33 (4 marks)

The graphs of $y = 2x - 3$ and $y = \sqrt{\frac{x}{2}}$ intersect at point A as shown in the diagram. **4**



Find the area of the shaded region.

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End of paper

Section I Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II Extra writing space

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Solutions

Student Number

2025 Glenwood High School
Year 12 – Trial HSC Examination

Mathematics Advanced

General Instructions

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Total marks: 100

Section I – 10 marks (pages 2 – 7)

* Attempt Questions 1-10
* Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 30)

* Attempt Questions 11-33
* Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1. The stem and leaf plot below shows the number of lollies that fourteen students in the class have eaten this week.

0	0 0 2 5
1	1 2 2 6 9
2	0 4 4 7
3	3

Two students were absent when this data was collected.

Once their data was added to the data of the rest of the class, the grouped frequency distribution table below was completed. The cumulative frequency for the $10 - 19$ class is marked with an $\boldsymbol{X}$.

Class	Frequency	Cumulative frequency
$0 - 9$		
$10 - 19$		**X**
$20 - 29$		
$30 - 39$		

Which of the following values for $\boldsymbol{X}$ could be true?

A. $\boldsymbol{X} = 9$

B. $\boldsymbol{X} = 12$

C. $\boldsymbol{X} = 14$

D. $\boldsymbol{X} = 16$

2. What is the gradient of the normal to the curve $f(x) = 3x^3 - 4x + 2$ at the point $(-1,3)$?

A. 5

B. -5

C. $-\frac{1}{5}$

D. $-\frac{1}{3}$

3. What is the solution of the equation $2a^{3x} = b$, where a and b are positive constants?

A. $x = \frac{1}{2}\log_a \frac{b}{3}$

B. $x = \frac{1}{2}\log_b \frac{a}{3}$

C. $x = \frac{1}{3}\log_a \frac{b}{2}$

D. $x = \frac{1}{3}\log_b \frac{a}{2}$

$a^{3x} = \frac{b}{2}$

$\log_a \frac{b}{2} = 3x$

$x = \frac{1}{3}\log_a \frac{b}{2}$

4. Savannah competed in four throwing events at the district athletics carnival.

The distances achieved in each event by the competitors were normally distributed.

	Savannah's result	Event mean	Event standard deviation
Shot put	15 m	13 m	2 m
Discus	37 m	43 m	6 m
Javelin	59 m	47 m	8 m
Hammer throw	33 m	48 m	10 m

Which was Savannah's weakest event compared with the other competitors?

A. Shot put

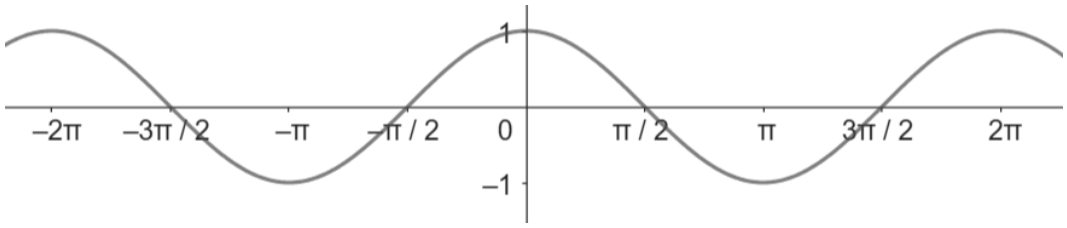
B. Discus

C. Javelin

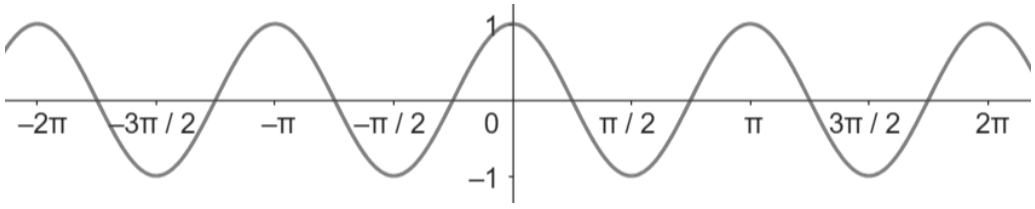
D. Hammer throw

5. Which diagram shows the graph of $y = \sin\left(\frac{\pi}{2} - 2x\right)$?

A.



B.



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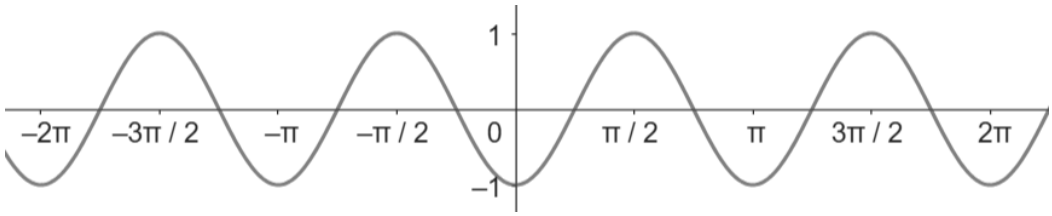

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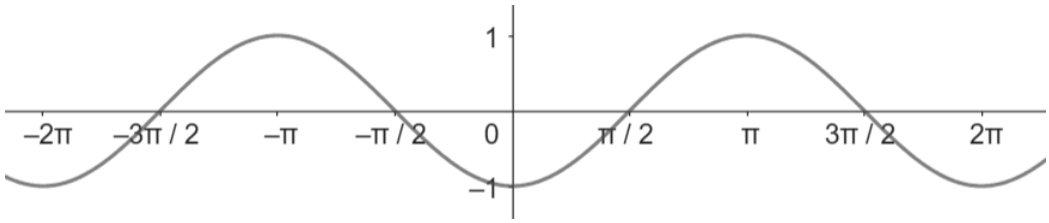


6. Two fair dice are thrown in a game. If the sum of the two numbers thrown is 7, the player gains $36. The player gains $6 on any other number rolled.

What is the expected gain from this game?

A. $11

B. $12

C. $30

D. $31

7. What is the range of the function $f(x) = \sqrt{3x^2 + 6x + 5}$?

A. $[0, \infty)$

B. $[\sqrt{2}, \infty]$

C. $[0, \sqrt{2}]$

D. $[0,1]$

8. What is

$$\int (e^{2x} + e^{2x+1})dx \, ?$$

A. $\frac{e^{2x}(3-2e)}{2} + c$

B. $\frac{e^{2x}(3+2e)}{2} + c$

C. $\frac{e^{2x}(1-e)}{2} + c$

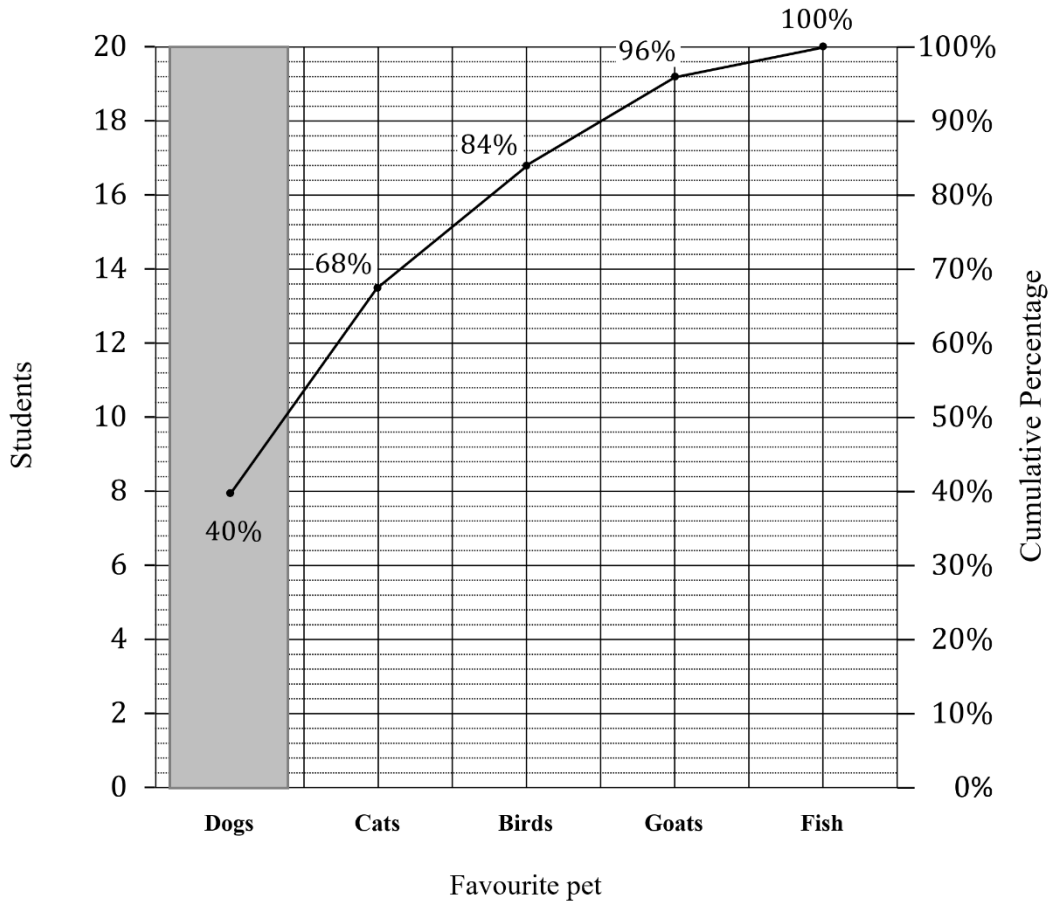
D. $\frac{e^{2x}(1+e)}{2} + c$

$$\frac{e^{2x}}{2} + \frac{e^{2x+1}}{2}$$

$$= \frac{e^{2x}(1+e)}{2}$$

9. A teacher asked fifty students to identify their favourite type of pet.

A partially completed Pareto chart shows some of the data collected.



How many students chose cats as their favourite pet?

68 − 40 = 28%

28% × 50

= 14

A. 12 students

B. 14 students

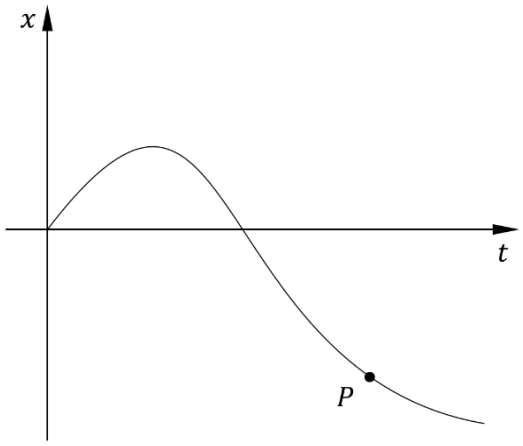
C. 28 students

D. 34 students

10. The displacement of a car to the east of its starting point along a straight road that runs east-west is x kilometres.

At time t hours the car's velocity is $\frac{dx}{dt}$ and its acceleration is $\frac{d^2x}{dt^2}$.

The graph of $x = f(t)$ is shown.



When the car is at point P, which statement reflects its motion?

A. The car is west of its starting point, it is moving east, and its speed is decreasing.

B. The car is west of its starting point, it is moving east, and its speed is increasing.

C. The car is west of its starting point, it is moving west, and its speed is decreasing.

D. The car is west of its starting point, it is moving west, and its speed is increasing.

END OF SECTION I

Student Number

2025 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11-33
Allow about 2 hours and 45 minutes for this section

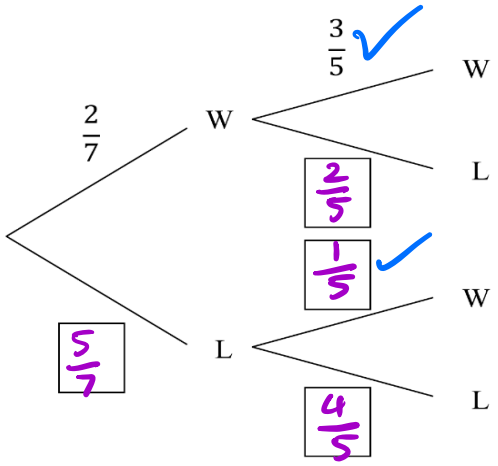
Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the booklet.
  If you use this space, clearly indicate which question you are answering.

Question 11 (3 marks)

When Cody plays Sofia at tennis, he wins the first set two out of every seven times they play.
If he wins the first set, he wins the second set three out of every five times, but if he loses the first set, he wins the second set one out of every five times.

(a) Complete the probability tree below and find the probability that Cody loses both sets. **2**



$P(LL) = \frac{5}{7} \times \frac{4}{5} = \frac{20}{35} = \frac{4}{7}$

(b) Suppose Cody and Sofia play 70 matches in a year. **1**

In how many of those matches would Cody expect to win at least one set?

$1 - \frac{20}{35} = \frac{15}{35}$

70 matches $= \frac{15}{35} \times 70 = 30$ matches.

Question 12 (2 marks)

Solve: $|5 - x| = 4$ **2**

$5 - x = 4$

$x = 1$

$-(5 - x) = 4$

$5 - x = -4 \quad x = 9$

$\therefore x = 1$ or 9

Question 13 (5 marks)

A study is investigating the relationship between the number of hours students study and their scores on a statistics exam. The examination is marked in integers out of 100.

The data collected from seven students is shown in the table.

Hours studied (h)	2	4	3	5	7	8	1
Exam score (s)	47	50	32	75	83	94	48

(a) Calculate Pearson's coefficient to 3 d.p. and describe the correlation referring to strength and direction. **2**

$r = 0.87999$

$= 0.880$ (3 dp)

Strength → strong positive correlation

(b) Determine the equation of the regression line to 2 sig. fig. **2**

$A = 27.9 \approx 28$

$B = 7.79 \approx 7.8$

∴ equation is

$S = 28 + 7.8h$

(c) Use the equation of the regression line to predict the exam score of a student who studied for 6 hours. **1**

$h = 6$ hours

$S = 28 + 7.8 \times 6$

$= 74.8$

≈ 75 (integer)

Question 14 (3 marks)

Find all the values of θ, where $0° \leq \theta \leq 360°$, such that **3**

$$sin(2\theta - 30°) = -\frac{1}{2}$$

$0° \leq 2\theta \leq 720°$ ✓

$-30° \leq 2\theta - 30 \leq 690°$ ✓

Base angle $= 30°$

$2\theta - 30° = -30°, 180+30, 360-30, 540+30, 720-30$

$= -30°, 210°, 330°, 570°, 690°$

$2\theta = 0°, 240°, 360°, 600°, 720°$

$\theta = 0°, 120°, 180°, 300°, 360°$

Question 15 (2 marks)

A population of wasps, P, is to be modelled using the function $P = 100(1.3)^{-t}$, where t is the time in hours since a pesticide was applied. **2**

What is the percentage decrease in the wasp population over the first four hours since the pesticide was applied?

$P_0 = 100$

$P_4 = 100(1.3)^{-4} = 35.01$

$\%\ \text{decrease} = \frac{100 - 35}{100} \times 100\%$

$= 65\%$

Question 16 (3 marks)

Describe three transformations which, when applied in succession, change the graph of $y = 4\sin x$ to the graph with equation $y = \sin\left(\frac{x+\pi}{3}\right)$. **3**

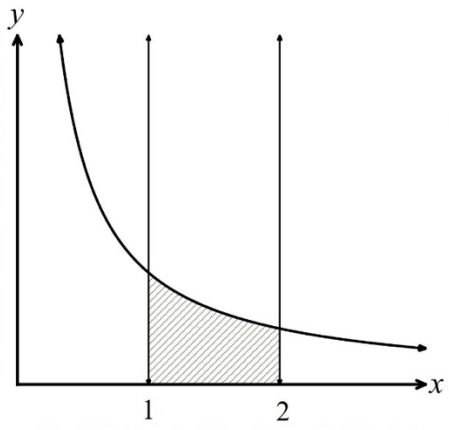
$4\sin x$ to $\sin x$ vertical dilation SF $\frac{1}{4}$

$\sin x$ to $\sin\left(\frac{x}{3}\right)$ Hoz. dilation SF 3

$\sin\left(\frac{x}{3}\right)$ to $\sin\left(\frac{x+\pi}{3}\right)$ Hoz. translation π units left.

Question 17 (6 marks)

The curve $y = \frac{1}{x}$ and the lines $x = 1$ and $x = 2$ are shown on the graph below:



(a) By completing the table below and using the trapezoidal rule with six function values find an approximation for $\int_1^2 \frac{1}{x}\,dx$, correct to 2 significant figures. **3**

x	1	$\frac{6}{5}$	$\frac{7}{5}$	$\frac{8}{5}$	$\frac{9}{5}$	2
$f(x)$	1	$\frac{5}{6}$	$5/7$	$5/8$	$5/9$	$1/2$

$h = \frac{1}{5}$

$\int_1^2 \frac{1}{x}\,dx = \frac{1/5}{2}\left(1 + \frac{1}{2} + 2\left(\frac{5}{6} + \frac{5}{7} + \frac{5}{8} + \frac{5}{9}\right)\right)$

$= 0.70$ (2 sig. fig.)

(b) Find the exact value of $\int_1^2 \frac{1}{x}\,dx$ **2**

$\int_1^2 \frac{1}{x}\,dx = [\ln x]_1^2$

$= \ln 2 - \ln 1$

$= \ln 2$

(c) State whether your answer to part (a) is an under-estimate or over-estimate of the exact area and explain why this is the case. **1**

Part a is over estimation since the curve is concave up.

Question 18 (3 marks)

Determine the global maximum and minimum values of y for the curve $y = e^2 + e^x - x\,e^2$ in the domain $0 \leq x \leq 5$. **3**

Give your answers as exact values.

$y = e^2 + e^x - xe^2$ $\quad u = x \quad v = e^2$

$y' = e^x - (0 + e^2)$ $\quad u' = 1 \quad v' = 0$

$= e^x - e^2$ $\quad uv' + vu'$

$0 + e^2$

when $y' = 0$

$e^x = e^2$

$\therefore x = 2$

$y(0) = e^2 + e^0 - 0 \times e^2$

$= e^2 + 1 \quad = 8.39$

$y(2) = e^2 + e^2 - 2e^2$

$= 0$

$y(5) = e^2 + e^5 - 5e^2$

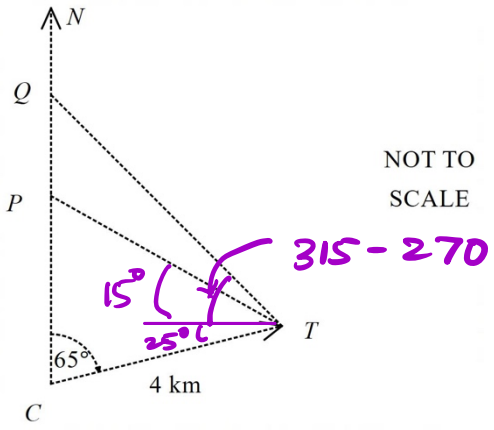
$= e^5 - 4e^2 = 118.86$

$\therefore$ global max of $e^5 - 4e^2$ global min of 0.

Question 19 (5 marks)

Two mountains, P and Q, lie directly north of a campground C. Tom walks 4 km from C on a bearing of 065° to a point T. From point T, the base of mountain P is on a bearing of 285° whilst the base of mountain Q is on a bearing of 315°.

The diagram below shows the relative positions when viewed from above.



(a) Show that $\angle QTC = 70°$. **1**

$\angle QTC = 25 + 315 - 270$

$= 70°$

(b) Use the sine rule to find the distance CQ, correct to 1 decimal place. **2**

$\frac{CQ}{\sin 70^\circ} = \frac{CT}{\sin(180 - 65 - 70)}$

$CQ = \frac{4 \times \sin 70}{\sin 45}$

$= 5.3 \text{ km (1dp)}$

(c) Hence or otherwise find the distance PQ, correct to 1 decimal place. **2**

$\frac{CP}{\sin 70} = \frac{4}{\sin(180 - 65 - 40)}$

$CP = \frac{4 \times \sin 70}{\sin 75^\circ} = 2.7 \text{ (1dp)}$

$\therefore PQ = 5.3 - 2.7$

$= 2.6 \text{ km}$

Question 20 (3 marks)

A semi-circle on the number plane is defined by the following equation:

$$y = \sqrt{-x^2 + 10x - 16}$$

(a) Find the centre and radius of the semi-circle. **2**

$y^2 = -x^2 + 10x - 16$

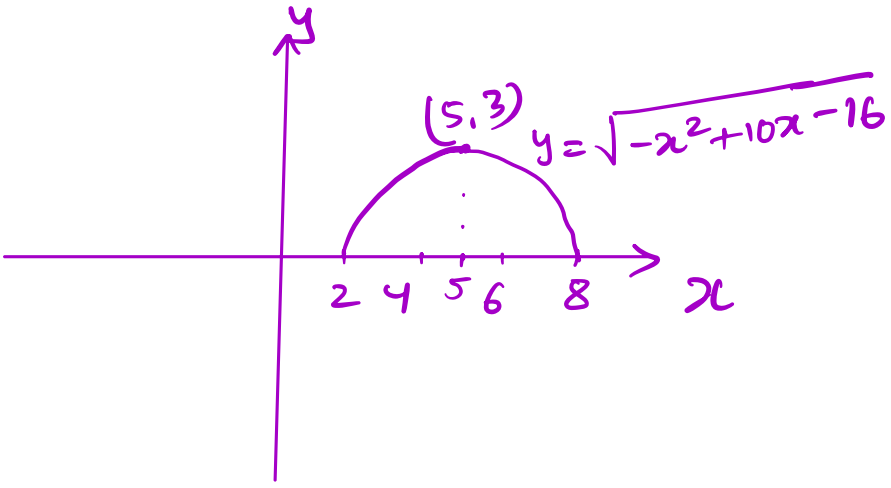
$x^2 - 10x + y^2 = -16$

$x^2 - 10x + 25 + y^2 = -16 + 25$

$(x-5)^2 + y^2 = 9$

$\therefore$ radius $= 3$ centre $(5, 0)$

(b) Sketch the curve on the axes below: **1**



Question 21 (3 marks)

The time it takes for a cupcake at a picnic to be eaten by ants (t minutes) varies inversely with the number of ants eating the cupcake (N ants). When there are 10 ants the cupcake is eaten in 120 minutes.

(a) The equation relating t and N is $t = \frac{k}{N}$. **1**

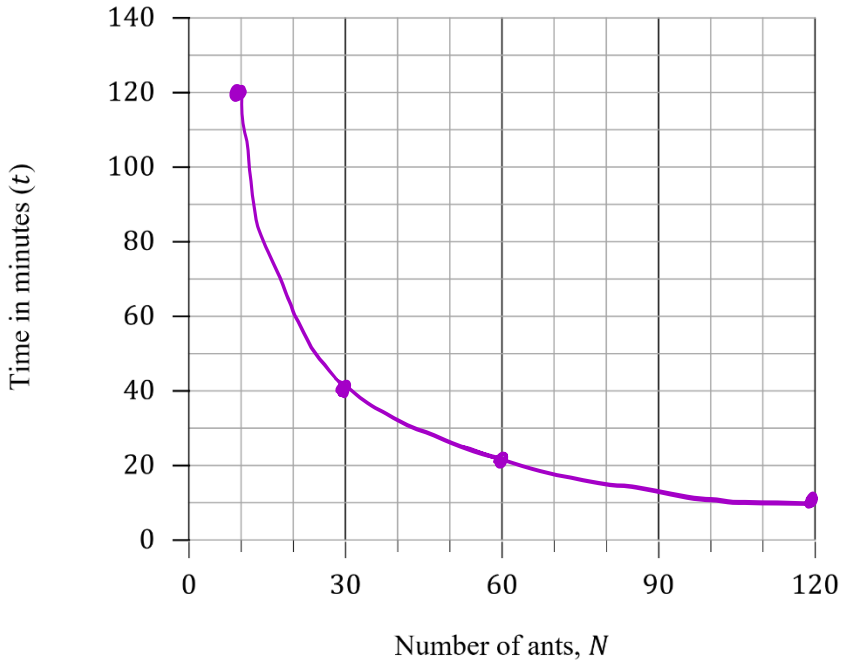
What is the value of k?

$120 = \frac{k}{10}$

$\therefore k = 1200$

(b) By first completing this table of values, graph the curve representing the relationship between the time taken for the cupcake to be eaten and the number of ants eating the cupcake from $N = 10$ to $N = 120$. **2**

N	10	30	60	120
t	120	40	20	10

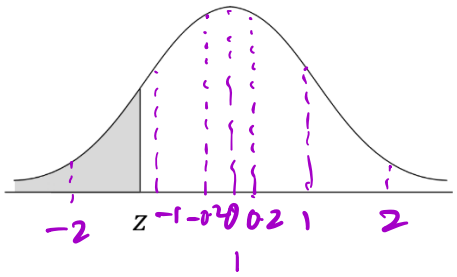


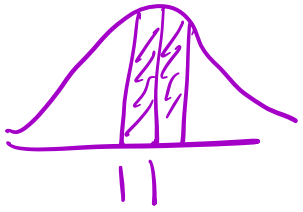
Question 22 (4 marks)

A random variable is normally distributed with a mean of 0 and a standard deviation of 1. The table gives the probability that this random variable lies less than z.

z	−1.0	−0.9	−0.8	−0.7	−0.6	−0.5	−0.4	−0.3	−0.2	−0.1	0.0
Probability	0.1587	0.1841	0.2119	0.242	0.2743	0.3085	0.3446	0.3821	0.4207	0.4602	0.5000

The probability values given in the table are represented by the shaded area in the following diagram.





The time it takes for a worker to drive from home to work is normally distributed with a mean of 32 minutes and a standard deviation of 4 minutes.

(a) What is the probability that on a particular day the drive takes between 30 and 32 minutes? **2**

$z = \frac{30-32}{4} = -0.5$

$z = \frac{32-32}{4} = 0$

$P(-0.5 \leq z \leq 0) = 0.5 - 0.3085 = 0.1915$

(b) What is the probability that the drive takes 32.8 minutes or less? **2**

$z = \frac{32.8-32}{4} = 0.2$

$P(-0.2 \leq z \leq 0) = 0.5 - 0.4207 = 0.0793$

$P(-0.2 \leq z \leq 0) = P(0 \leq z \leq 0.2) = 0.0793$

$P(x \leq 32.8) = 0.5 + 0.0793 = 0.5793$

Question 23 (4 marks)

(a) Show that **2**

$$\frac{d}{dx}(e^x \tan x) = e^x(\tan^2 x + \tan x + 1)$$

$u = e^x \qquad v = \tan x$

$u' = e^x \qquad v' = \sec^2 x$

$\frac{d}{dx}(e^x \tan x) = \tan x e^x + e^x \sec^2 x$

$= e^x(\sec^2 x + \tan x)$

$= e^x(1 + \tan^2 x + \tan x)$

as required.

(b) Find the exact value of **2**

$$\int_0^{\frac{\pi}{4}} 2e^x(\tan^2 x + \tan x + 1)dx$$

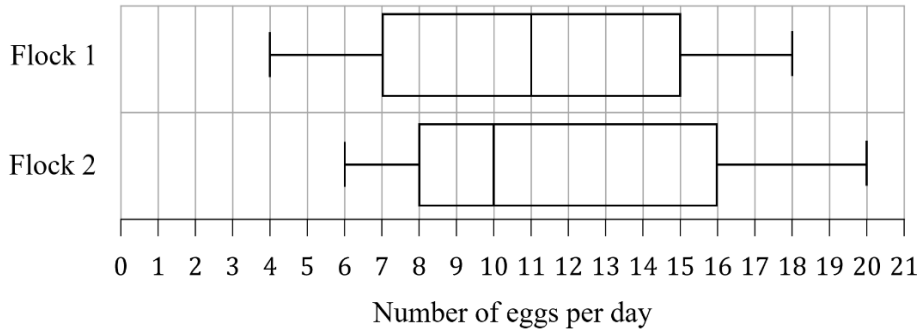
$= 2\left[e^x \tan x\right]_0^{\pi/4}$

$= 2\left[e^{\pi/4} \tan \pi/4 - e^0 \tan 0\right]$

$= 2e^{\pi/4}$

Question 24 (3 marks)

A farmer has two flocks of chickens. The number of eggs laid each day during the year was counted, with the data shown in a parallel box-plot. **3**



Compare the two datasets by examining the skewness of the distributions, and the measures of central tendency and spread.

	Flock 1	Flock 2
Skewness	Symmetrical	Positively skewed
Spread		
Range	18-4 = 14	20-6 = 14
IQR	15-7 = 8	16-8 = 8
Median	11	10

Spread is the same. However, Flock 1 seems to have more eggs indicated by median and skewness.

Question 25 (4 marks)

Emily's number of goals scored in football each game she has played this season has the following probability distribution.

x	0	1	2	3	4
$P(X = x)$	0.46	0.34	k	0.08	0.05

(a) Show that $k = 0.07$. **1**

$0.46 + 0.34 + K + 0.08 + 0.05 = 1$

$K = 1 - 0.93$

$= 0.07$

(b) Find $E(X)$, correct to 4 decimal place. **1**

$E(x) = 0 \times 0.46 + 1 \times 0.34 + 2 \times 0.07 + 3 \times 0.08 + 4 \times 0.05$

$= 0.92$

(c) Find Var(X). **2**

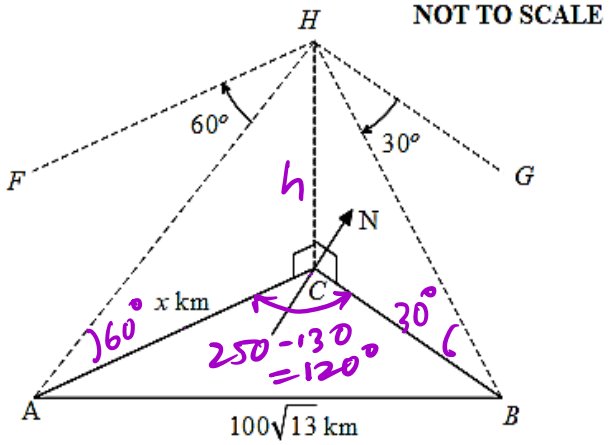
$Var(x) = E(x^2) - \mu^2$

$= 0^2 \times 0.46 + 1^2 \times 0.34 + 2^2 \times 0.07 + 3^2 \times 0.08 + 4^2 \times 0.05 - 0.92^2$

$= 1.2936$

Question 26 (3 marks)

The figure below shows three buoys at sea at A, B and C. A helicopter is flying along FH and then HG, where F, G and H are three points of the same height vertically above A, B and C respectively. When the helicopter reaches H, the angles of depression of A and B from the helicopter are 60° and 30° respectively, while the true bearings of A and B from C are 250° and 130° respectively. The distance between A and B is $100\sqrt{13}\ km$.



Show that the distance between A and C is $100\,km$. **3**

$\tan 60^\circ = \frac{h}{AC}$ $\tan 30^\circ = \frac{h}{BC}$

$AC = \frac{h}{\sqrt{3}}$ $BC = \sqrt{3}h$

In ΔABC, $AB^2 = AC^2 + BC^2 - 2AB\,AC\cos 120^\circ$

$(100\sqrt{13})^2 = \frac{h^2}{3} + 3h^2 - 2\times h^2 \times \cos 120^\circ$

$130000 = h^2\left(\frac{1}{3} + 3 - 2\times\left(-\frac{1}{2}\right)\right)$

$= h^2\left(\frac{13}{3}\right)$

$h^2 = \frac{130000 \times 3}{13}$

$h^2 = 30000$ $h = \sqrt{30000}$

$AC = \frac{h}{\sqrt{3}} = \frac{100\times\sqrt{3}}{\sqrt{3}} = 100\ km$

Hence, proved.

Question 27 (5 marks)

A continuous random variable X has the following cumulative distribution function $F(x)$, defined by

$$F(x)=\begin{cases} 0 & x<0 \\ Bx^2\left(4-x^2\right) & 0\le x\le 1 \\ 1 & x>1 \end{cases}$$

(a) Show that $B=\frac{1}{3}$. **1**

$F(1)=1$

$B\times 1^2(4-1^2)=1$

$B=\frac{1}{3}$

(b) Find $P(X>0.5)$. **1**

$F(0.5)=\frac{1}{3}\times 0.5^2(4-0.5^2)$

$=0.3125$

$P(x>0.5)=1-0.3125=0.6875$

(c) Find the mode of the function. **3**

$F(x)=\frac{1}{3}x^2(4-x^2)$

$=\frac{4}{3}x^2-\frac{x^4}{3}$

$PDF=F'(x)=f(x)$

$=\frac{8x}{3}-\frac{4x^3}{3}$

To find the mode $F''(x)=0$

$\frac{8}{3}-\frac{12x^2}{3}=0$

$8-12x^2=0$

$3x^2=2$

$x=\pm\sqrt{2/3}$

$\therefore$ mode $=\sqrt{2/3}$

$f(0)=0$

$f(\sqrt{\frac{2}{3}})=1.45$ (max)

$f(1)=\frac{4}{3}=1.33$

$\therefore x=\sqrt{\frac{2}{3}}$

Question 28 (6 marks)

On the next page, sketch the curve $y = x^3(4-x)$ showing x and y intercepts, any stationary points (classifying their nature) and stating if any inflection points exist and if so showing them. **6**

$y = x^3(4-x)$

$= 4x^3 - x^4$

$y' = 12x^2 - 4x^3$

$y'' = 24x - 12x^2$

For stationary points

$y' = 0$

$12x^2 - 4x^3 = 0$

$4x^2(3-x) = 0$

$x = 0$ or $x = 3$

$y = 0$ $y = 27$

∴ Stationary points at (0,0) and (3,27)

Concavity check at (0,0) at (3,27)

$y'' = 0$ $y'' = 24 \times 3 - 12 \times 3^2$

at $x = 1$, $y'' = -12(2+1) < 0$ $= -36 < 0$

$x = -1$, $y'' = 12(2-1) > 0$ Max. turning pt.

Concavity changes at (0,0)

For Pt. of inflection

$y'' = 0$

$24x - 12x^2 = 0$

$12x(2-x) = 0$

∴ $x = 0$ $x = 2$

$y = 0$ $y = 4 \times 2^3 - 2^4$

$= 16$

(0,0) is the horizontal pt. of inflection

(2,16) is the point of inflection.

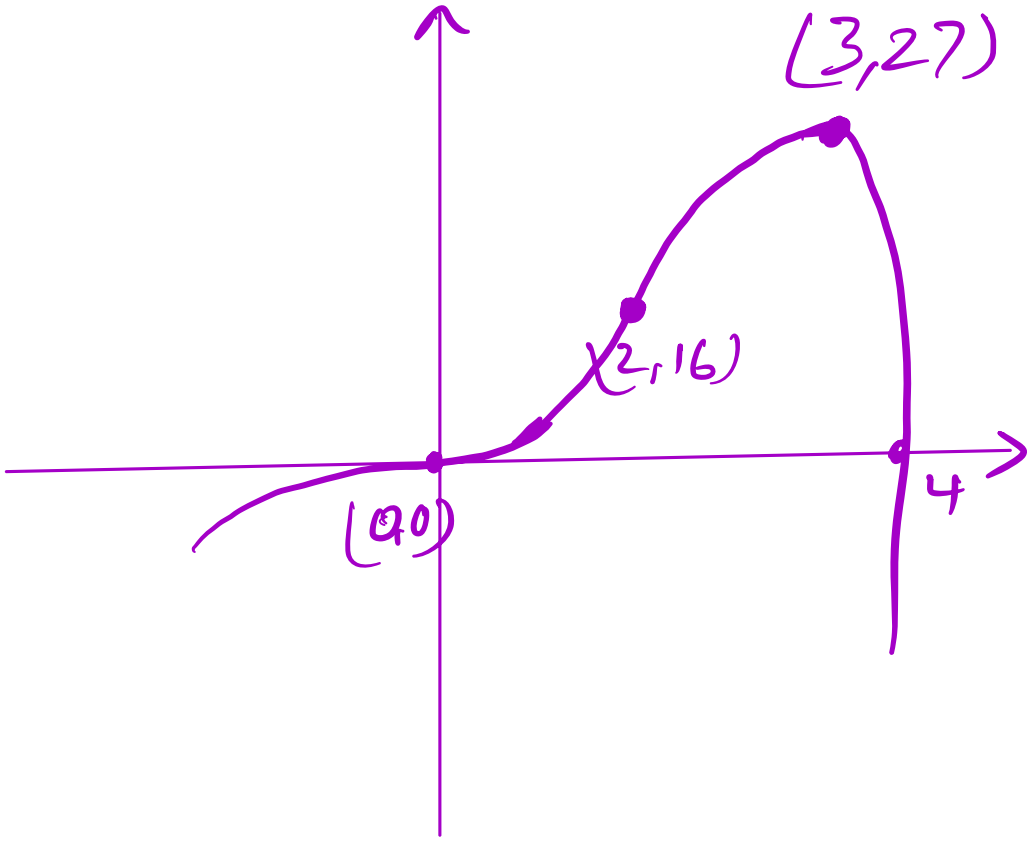
x-int: $y=0$

$x^3(4-x)=0$

$x=0, x=4$

y-int: $x=0$

$y=0$



Question 29 (5 marks)
The depth of water (in metres) in a lagoon in the coastal town of Schoolies is given by:

$$D = 8 + 2\cos\left(\frac{4\pi t}{25}\right) \text{ for } 0 \leq t \leq 25.$$

where t is the time in hours after noon.

(a) What is the depth of water at noon? **1**

$D(0) = 8 + 2\cos 0$

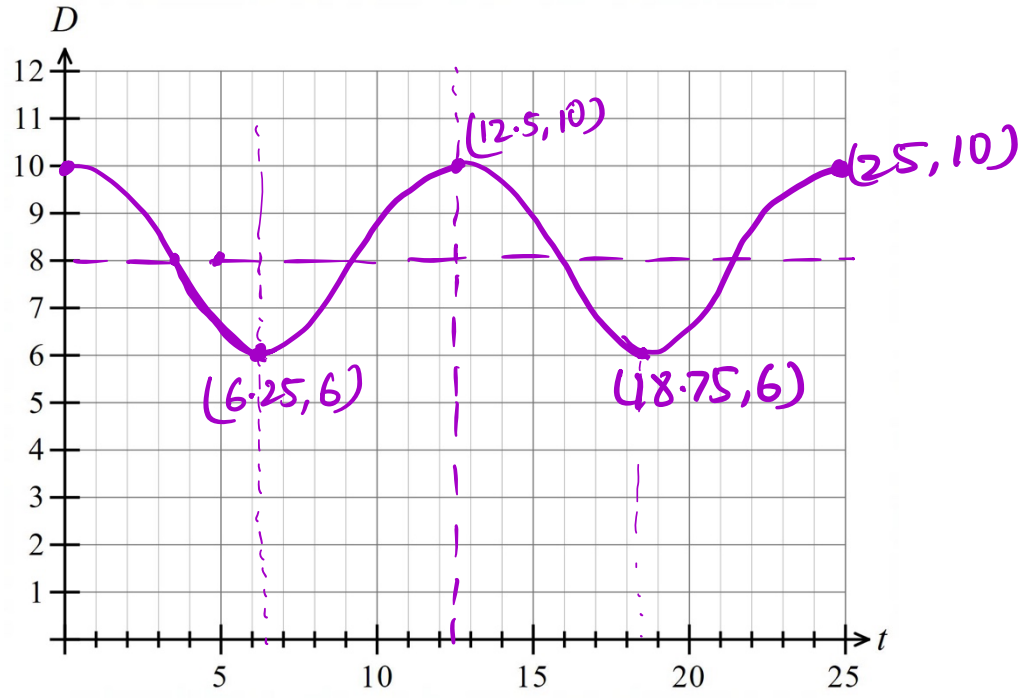
$= 10\text{m}$

(b) At what time will the water next be at this depth? **2**

$n = \frac{4\pi}{25}$ $T = \frac{2\pi}{4\pi/25} = 12.5$

$= 12:30\text{ am}$

(c) Sketch the curve on the axes below, showing the maximum and minimum depths of the lagoon during the time period $0 \leq t \leq 25$. **2**



Question 30 (4 marks)

In a science experiment, a robot arm attempts to successfully pick up a small object. The probability that the robot arm successfully picks up the object on any single attempt is $\frac{1}{4}$.

(a) What is the probability that the robot arm successfully picks up the object on the third attempt only, after failing the first two attempts? **1**

$\frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{9}{64}$

(b) What is the probability that the robot arm is unsuccessful in the first four attempts? **1**

$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{81}{256}$

(c) Determine the minimum number of attempts required so that the probability of at least one successful pick-up exceeds 0.95. **2**

$P(\text{at least 1 successful}) = 1 - P(\text{none successful})$

$= 1 - \left(\frac{3}{4}\right)^n > 0.95$

$\left(\frac{3}{4}\right)^n < 0.05$

Let $\left(\frac{3}{4}\right)^n = 0.05$

$n = \log_{0.75} 0.05$

$= \frac{\log 0.05}{\log 0.25} = 10.41$

$\left(\frac{3}{4}\right)^{10} = 0.056$

$\left(\frac{3}{4}\right)^{11} = 0.04$

$\therefore n = 11$

Question 31 (6 marks)

A particle is moving along the x-axis such that at a time t seconds its displacement from the origin is x metres.

The acceleration of the particle is given by $a = \frac{8}{e^{2t}} + \frac{3}{e^{t}}$

Initially its velocity is -6 m/s and its displacement is 5 m.

(a) Find the equation for the displacement (x) of the particle in terms of t. **3**

$a = 8e^{-2t} + 3e^{-t}$

$v = \int a = \frac{8e^{-2t}}{-4} + \frac{3e^{-t}}{-1} + C$

$= -4e^{-2t} - 3e^{-t} + C$

When $t = 0$, $v = -6$

$-6 = -4e^{0} - 3e^{0} + C$

$\therefore C = 1 \quad \therefore v = -4e^{-2t} - 3e^{-t} + 1$

$x = 2e^{-2t} + 3e^{-t} + t + C$

When $t = 0$, $x = 5$, $5 = 2 \times 1 + 3 \times 1 + 0 + C$

$\therefore C = 0$

$\therefore x = 2e^{-2t} + 3e^{-t} + t$

(b) At what time and where does the particle come to rest? **3**

$v = 0 \quad -4e^{-2t} - 3e^{-t} + 1 = 0$

$4e^{-2t} + 3e^{-t} - 1 = 0$

Let $u = e^{-t}$

$4u^2 + 3u - 1 = 0$

$4u^2 + 4u - u - 1 = 0$

$4u(u+1) - 1(u+1) = 0$

$(4u - 1)(u + 1) = 0$

$\therefore u = \frac{1}{4} \quad u = -1$ (no solution)

$e^{-t} = \frac{1}{4}$

$-t = \ln(\frac{1}{4})$

When $t = \ln 4$

where $x(\ln 4) = 2e^{-2\ln 4} + 3e^{-\ln 4} + \ln 4$

$= \frac{7}{8} + \ln 4$ m

Question 32 (4 marks)

A municipal park needs a cylindrical water tank that holds 2 m^3 of water. The tank will have a flat top and bottom and the cost of the metal for all surfaces is the same per square metre.

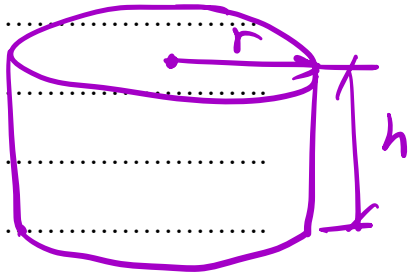
Let the radius of the tank be r and its height be h.

Find the relationship between r and h that would minimise the surface area of the tank. **4**

$V = 2\,m^3$

$\pi r^2 h = 2$

$h = \frac{2}{\pi r^2}$



$SA = 2\pi rh + 2\pi r^2$

$= 2\pi \times r \times \frac{2}{\pi r^2} + 2\pi r^2$

$= \frac{4}{r} + 2\pi r^2$

$SA' = 4\pi r - \frac{4}{r^2} = 0$

$4\pi r = \frac{4}{r^2}$ $\quad r^3 = \frac{1}{\pi}$

$r = \left(\frac{1}{\pi}\right)^{1/3} = \pi^{-1/3} = \frac{1}{\pi^{1/3}}$

$h = \frac{2}{\pi \times r^2} = \frac{2}{\pi \times \pi^{-2/3}} = \frac{2}{\pi^{1/3}}$

$= 2\pi^{-1/3}$

$\boxed{h = 2r}$

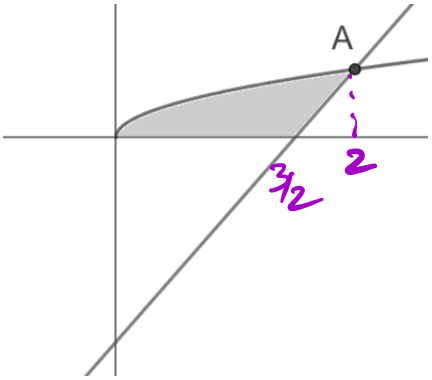
Check for concavity

$SA'' = 4\pi + \frac{8}{r^3}$

$= 4\pi + \frac{8}{\frac{1}{\pi^3}} > 0$ Hence, $r = \frac{1}{\pi^3}$ is the min TP.

Question 33 (4 marks)

The graphs of $y = 2x - 3$ and $y = \sqrt{\frac{x}{2}}$ intersect at point A as shown in the diagram. **4**



Find the area of the shaded region.

$2x-3=\sqrt{\frac{x}{2}}$

$(2x-3)^2=\frac{x}{2}$

$4x^2-12x+9=\frac{x}{2}$

$8x^2-24x+18=x$

$8x^2-25x+18=0$

$x=\frac{-(-25)\pm\sqrt{(25)^2-4\times8\times8}}{2\times8}$

$=2$ or $9/8$

$2x-3=2\times2-3=\sqrt{\frac{2}{2}}=1$

$2(9/8)-3=\sqrt{\frac{9/8}{2}}$ ×

$\therefore x=2$

x-int of line: $x=3/2$

$A=\int_0^2\sqrt{\frac{x}{2}}\,dx-\frac{1}{2}\times\frac{1}{2}\times1$

$=\frac{1}{\sqrt{2}}\left[\frac{x^{3/2}}{3/2}\right]_0^2-\frac{1}{4}=\frac{2}{3\sqrt{2}}\times2^{3/2}-\frac{1}{4}$

$=\frac{13}{12}$

End of the examination

Section I Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.