

Gosford High School

**2011
HIGHER SCHOOL
CERTIFICATE
TRIAL EXAMINATION**

Mathematics

General Instructions

- Reading Time - 5 minutes.
- Working Time - 3 hours.
- Write using a blue or black pen.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (120)

- Attempt Questions 1-10.
- All questions are of equal value.

Question 1 (12 Marks)

Use a SEPARATE writing booklet

Marksa) Evaluate $3e^{3.17}$ correct to 3 significant figures.

1

b) Solve $|2x - 3| \geq 7$.

2

c) Expand and simplify $(2x - 3y)^2 - 5x(x - 2y)$.

2

d) If $f(x) = 2\sin 3x$ find the exact value of $f\left(\frac{\pi}{18}\right)$

2

e) Find the exact solutions of $2x^2 - x - 9 = 0$.

2

f) Factorise $6x^2 - 3xy - 4xz + 2yz$.

2

g) Evaluate $\log_5\left(\frac{1}{25}\right)$.

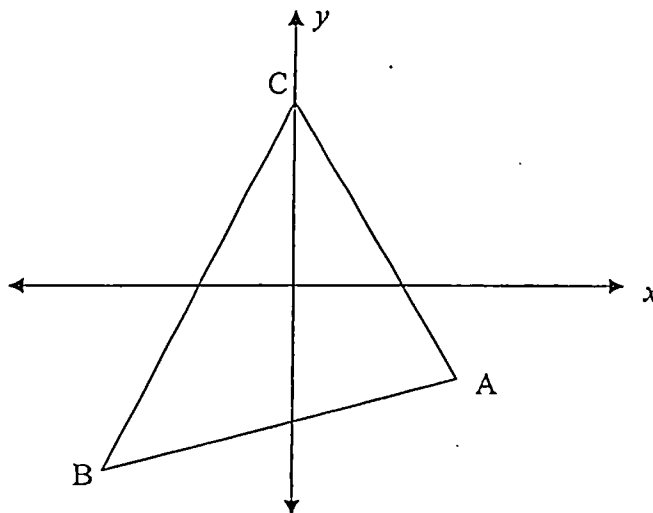
1

Question 2 (12 Marks)

Use a SEPARATE writing booklet

Marks

a)



$A(2,-2)$, $B(-2,-3)$ and $C(0,2)$ are the vertices of a triangle as shown.

- | | | |
|-------|--|---|
| (i) | Find the length of AC. | 1 |
| (ii) | Find the gradient of AC. | 1 |
| (iii) | Find the equation of the line AC in general form. | 2 |
| (iv) | Calculate the perpendicular distance from B to AC and hence the area of triangle ABC. | 3 |
| (v) | Find the coordinates of D such that ABCD is a parallelogram.. | 1 |
| (vi) | What angle does the line AC make with the positive direction of the x axis (answer to the nearest minute). | 1 |

b) Find

- | | | |
|------|-----------------------------------|---|
| (i) | $\int \frac{x^2}{x^3 + 1} dx$ | 1 |
| (ii) | $\int_1^2 \frac{x^3 + 1}{x^2} dx$ | 2 |

Question 3 (12 Marks)

Use a SEPARATE writing booklet

Marks

a) Find :

(i) $\frac{dy}{dx}$ when $y = \frac{1}{3-x}$.

1

(ii) $\frac{d}{dx}(2\tan 3x)$

1

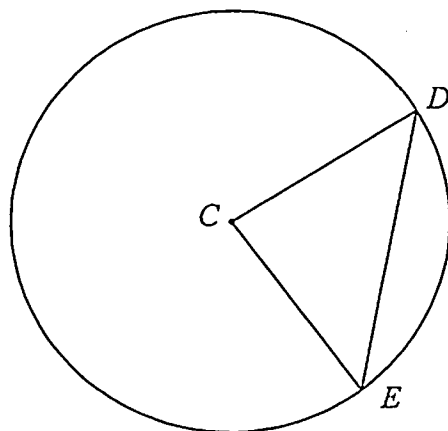
(iii) the derivative of $\frac{e^{2x} + 4}{x^3}$.

2

(iv) $f'(x)$ if $f(x) = (x^3 - 2x) \cdot \ln(x)$.

2

b) A circle has centre C and radius 12 cm. The length of the arc DE is 2π cm.



NOT TO
SCALE

(i) Find $\angle DCE$ (in radians).

1

(ii) Find the area of the minor segment cut off by the chord DE .

2

c) The gradient function of a certain curve is given by $\frac{dy}{dx} = \cos x - 2 \sin 2x$.

3

If the curve passes through the origin find y when $x = \frac{\pi}{2}$.

Question 4 (12 Marks)

Use a SEPARATE writing booklet

Marks

a) For the sequence 100, 95, 90, 85.....

(i) Show the sequence is Arithmetic.

1

(ii) Find the 25th term.

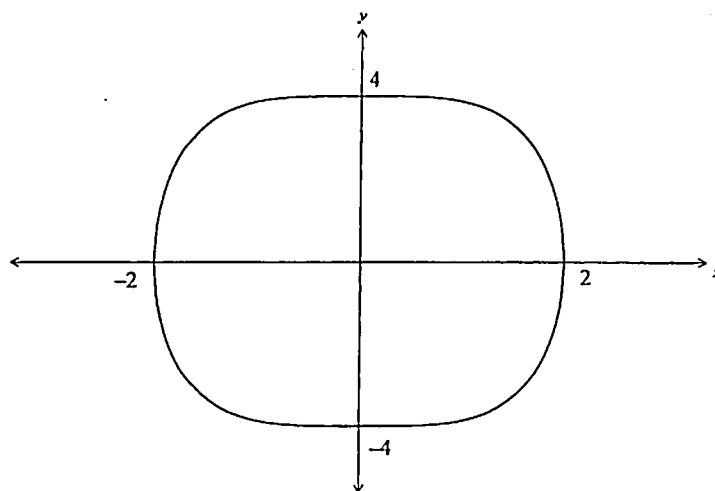
1

(iii) How many terms are needed to give a sum of zero?

2

b) The sketch shows the curve $y^2 + x^4 = 16$.

2



The area enclosed within the curve is rotated about the x axis. Find the volume of the solid of revolution so formed.

c) Find the gradient of the normal to the curve $y = x \ln x$ at the point on the curve where $x = 1$.

2

d) Draw a sketch showing the graph of $y = 2 \sin 3x$ for $0 \leq x \leq 2\pi$

2

e) A die is rolled 3 times. What is the probability that:

(i) No sixes are rolled?

1

(ii) At least 1 six is rolled?

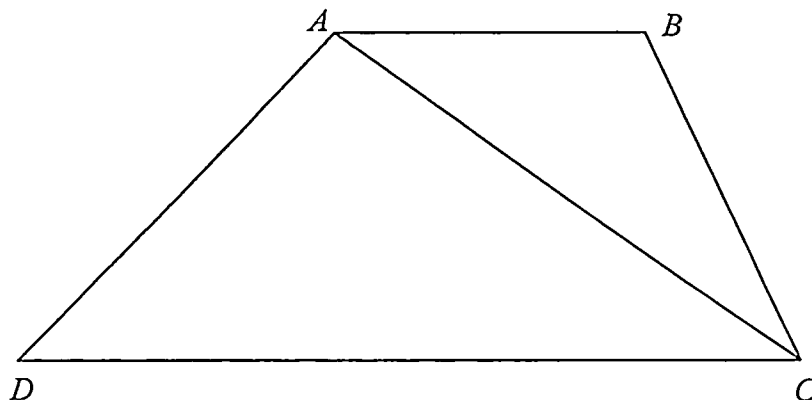
1

Question 5 (12 Marks)

Use a SEPARATE writing booklet

Marks

- a) The figure $ABCD$ is a trapezium. $\angle ABC = \angle DAC$. $AB = 12$ cm, $AD = 20$ cm and $AC = 24$ cm.



- (i) Prove that $\triangle ABC \parallel \triangle ADC$. 2
- (ii) Calculate the length of BC . 2
- b) (i) For the function $f(x) = e^x \ln x$ copy and complete the following table of values. (give your answer correct to 1 decimal place). 2

x	1	2	3	4	5
$f(x)$					

- (ii) Use Simpson's rule with these 5 values to find an approximate value of $\int_1^5 e^x \ln x \, dx$. 2
- c) A particle moves so that its displacement, x metres, from the origin at time, t seconds, is given by:

$$x = t^3 - 6t^2 + 9$$
- (i) Find the initial displacement. 1
- (ii) Find when the particle is at rest. 2
- (iii) Find the distance travelled in the first 4 seconds. 1

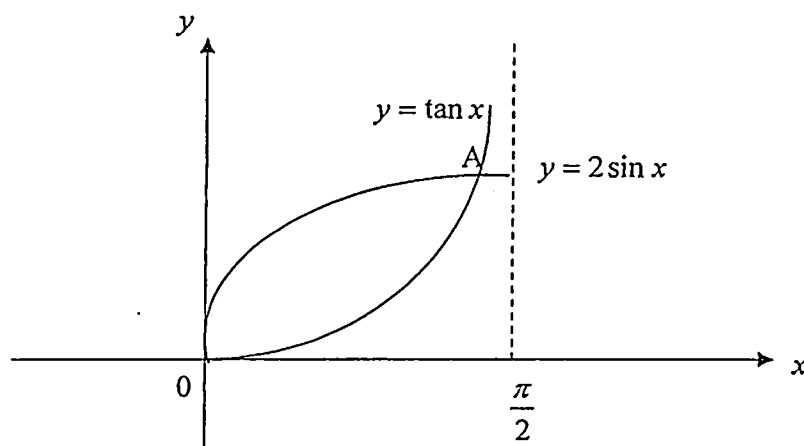
Question 6 (12 Marks)

Use a SEPARATE writing booklet

Marks

- a) Draw a sketch graph of the function $y = x^4 - 8x^3$, for the domain $-1 \leq x \leq 9$, showing any intercepts, turning points and inflections and the end points for the domain.
- b)

6



The diagram shows the curves $y = 2 \sin x$ and $y = \tan x$ for $0 \leq x \leq \frac{\pi}{2}$. A is the point of intersection of the two curves.

- (i) Show by substitution that the coordinates of A are $\left(\frac{\pi}{3}, \sqrt{3}\right)$

1

- (ii) Show that $\frac{d}{dx}(\ln(\cos x)) = -\tan x$

1

- (iii) Hence find the area between the two curves.

2

- c) The area under the curve $y = \frac{1}{x}$ from $x = 1$ to $x = b$ is 1 square unit. What is the value of b ?

2

Question 7 (12 Marks)

Use a SEPARATE writing booklet

Marks

- a) A plane which is initially flying at a speed of 200 m/s cuts its engines. It then decelerates at a rate which is proportional to its speed at that time.

(i.e. $\frac{dS}{dt} = -kS$).

- (i) Show that $S = S_0 e^{-kt}$ is a solution to this equation.

1

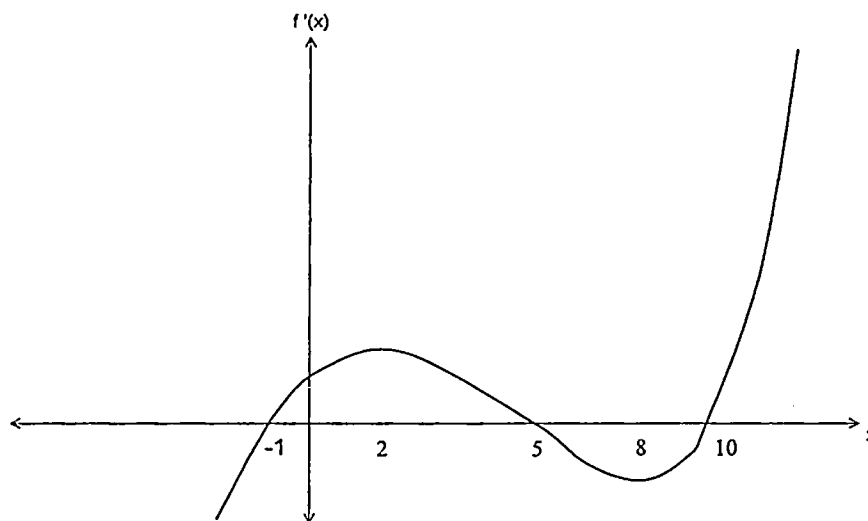
- (ii) After 50 seconds its speed is 120 m/s. Find the values of S_0 and of k .

2

- (iii) Find the speed after 90 seconds.

1

- b) The graph of $y = f'(x)$ is shown below.



- (i) Give the x values for the turning points of $y = f(x)$.

1

- (ii) Draw a sketch of $y = f(x)$.

2

- (iii) Draw a sketch of $y = f''(x)$.

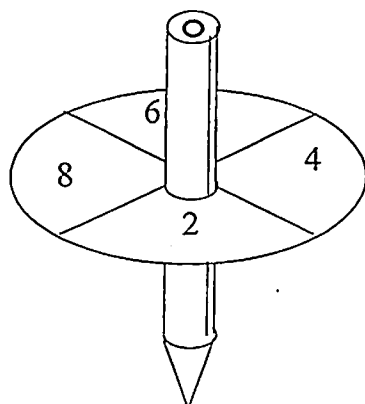
2

Question 7 continues.

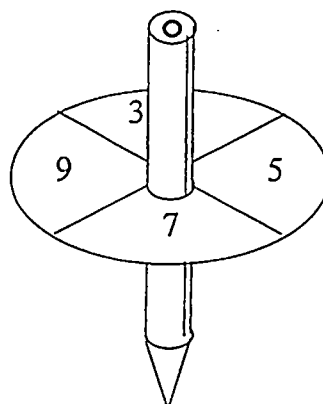
Question 7 continued.

Marks

- c) In a game, on a single turn a player whirls the two spinners shown below, each of which has 4 possible numbers. The score for the turn is obtained by subtracting the score on the second spinner from that on the first.



Spinner 1



Spinner 2

- (i) List the possible outcomes for the scores on a single turn. 2
- (ii) What is the probability of scoring a negative number on a single turn? 1

Question 8 (12 Marks)

Use a SEPARATE writing booklet

Marks

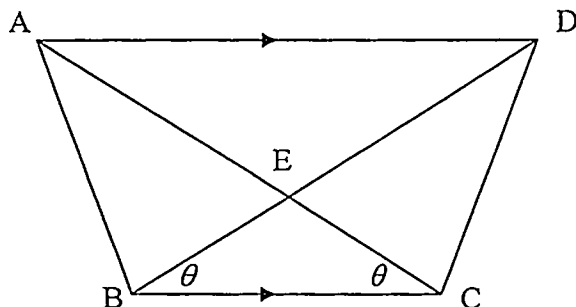
- a) (i) Differentiate $\cos^2 3x$ with respect to x . 2
- (ii) Hence evaluate $\int_{\frac{\pi}{6}}^{\pi} \cos 3x \sin 3x dx$. 3
- b) Sue is raising money for charity by jumping on a Pogo Stick. Her challenge is to jump between two points, A and B, twenty times. On her first attempt she takes 45 jumps. On her second attempt she takes 48 jumps. On her third attempt she takes 51 jumps. She continues this pattern for all 20 attempts.
- (i) How many jumps did she make on her 20th attempt? 2
- (ii) How many jumps did she make altogether? 2
- c) The curve $y = ax^3 + \frac{b}{x^2}$ cuts the x -axis at $x = 1$ and the gradient of the tangent at this point is 2. Find the values of a and b . 3

Question 9 (12 Marks)

Use a SEPARATE writing booklet

Marks

a)



In the diagram AD is parallel to BC and angle DBC equals angle ACB.

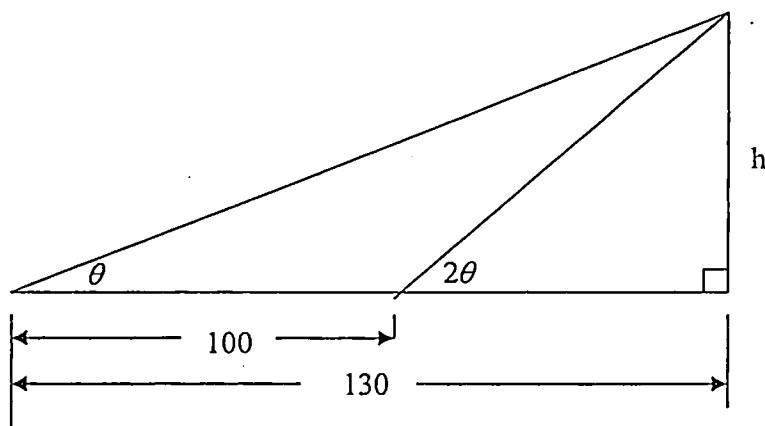
(i) Show that $AE = DE$.

2

(ii) Prove that triangle ABC is congruent to triangle DCB.

3

b)



When at a distance of 130 metres from the foot of a tower its angle of elevation was observed; on walking 100 metres nearer the angle of elevation was doubled.

(i) Find the height, (h), of the tower correct to the nearest centimetre.

2

(ii) Find, correct to the nearest minute, the first angle of elevation, θ .

2

c) Prove that $\sin\theta + 1 + \cos\theta \cot\theta - \operatorname{cosec}\theta = 1$

2

Question 10 (12 Marks)

Use a SEPARATE writing booklet

Marks

- a) A spherical balloon which initially has a radius of 20 cm, is inflated so that its volume increases at a constant rate of $100 \text{ cm}^3 / \text{second}$.

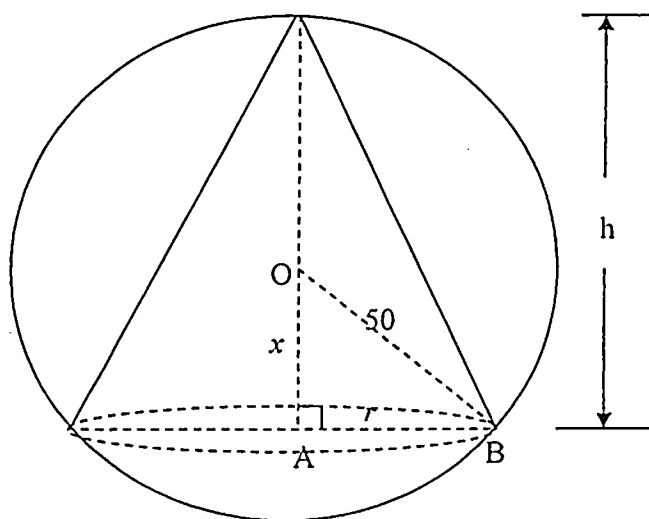
(i) Find an expression for the volume, V , at any time.

2

(ii) Calculate the time required, to the nearest second, for the volume to reach 100 litres ($1000 \text{ cm}^3 = 1 \text{ litre}$).

2

b)



The diagram shows a cone of base radius $r \text{ cm}$ and height $h \text{ cm}$ inscribed in a sphere of radius 50 cm . The centre of the sphere is O and angle $OAB = 90^\circ$. Let $OA = x \text{ cm}$.

(i) Show that $r = \sqrt{2500 - x^2}$

(ii) Hence show that the volume $V \text{ cm}^3$, of the cone is given by:

2

$$V = \frac{\pi}{3} (2500 - x^2)(50 + x).$$

2

(iii) Find the radius of the largest cone which can be inscribed in the sphere.
(Give your answer correct to the nearest mm .)

4

End of Examination

Mathematics 2011 Trial Solutions.

$$\Phi 1) a) 3e^{3.17}$$

$$= 71.4$$

$$b) |2x-3| \geq 7$$

$$2x-3 \leq -7 \text{ or } 7 \leq 2x-3$$

$$2x \leq -4 \text{ or } 10 \leq 2x$$

$$x \leq -2 \text{ or } 5 \leq x$$

$$c) (2x-3y)^2 - 5x(x-2y)$$

$$4x^2 - 12xy + 9y^2 - 5x^2 + 10xy$$

$$9y^2 - x^2 - 2xy$$

$$d) f(x) = 25 \sin 3x$$

$$f'(x) = 6 \cos 3x$$

$$f'\left(\frac{\pi}{18}\right) = 6 \cos\left(\frac{\pi}{6}\right)$$

$$= 6 \cdot \frac{\sqrt{3}}{2}$$

$$= 3\sqrt{3}$$

$$e) 2x^2 - x - 9 = 0$$

$$x = \frac{1 \pm \sqrt{1+72}}{4}$$

$$= \frac{1 \pm \sqrt{73}}{4}$$

$$f) 6x^2 - 3xy - 4xz + 2yz$$

$$= 3x(2x-y) - 2z(2x-y)$$

$$= (2x-y)(3x-2z)$$

$$g) \log_5\left(\frac{1}{5}\right)$$

$$= \log_5 5^{-2}$$

$$= -2 \log_5 5$$

$$= -2$$

$$\Phi 2) A(2,-2), B(-2,3), C(0,2)$$

$$i) AC = \sqrt{(2-0)^2 + (-2-2)^2}$$

$$= \sqrt{4+16}$$

$$= 2\sqrt{5}$$

$$ii) M_{AC} = \frac{2+2}{0-2}$$

$$= -2$$

$$iii) y-2 = -2(x-0)$$

$$y-2 = -2x$$

$$2x+y-2=0$$

$$iv) D = \left| \frac{2x-2+1x-3-2}{\sqrt{2^2+1^2}} \right|$$

$$= \left| \frac{-9}{\sqrt{5}} \right|$$

$$= \frac{9}{\sqrt{5}} \left(\frac{9\sqrt{5}}{5} \right)$$

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2} \times \frac{9}{\sqrt{5}} \times 2\sqrt{5}$$

$$= 9 \text{ Sq units}$$

$$v) D(4, 3)$$

$$vi) \tan \theta = m$$

$$\tan \theta = -2$$

$$\theta = 116^\circ 34'$$

$$b) i) \int \frac{x^2}{x^3+1} dx$$

$$= \frac{1}{3} \ln(x^3+1) + C$$

$$ii) \int_1^2 \frac{x^3+1}{x^2} dx$$

$$= \int_1^2 \frac{x^3}{x^2} + \frac{1}{x^2} dx$$

$$= \int_1^2 x + x^{-2} dx$$

$$= \left[\frac{x^2}{2} - \frac{1}{x} \right]_1^2$$

$$= \left(2 - \frac{1}{2} \right) - \left(\frac{1}{2} - 1 \right)$$

$$= 2$$

$$\Phi 3) a) i) \frac{dy}{dx} \left(\frac{1}{3-x} \right)$$

$$= \frac{1}{(3-x)^2}$$

$$ii) \frac{d}{dx} (2 \tan 3x)$$

$$= 6 \sec^2 3x$$

$$iii) \frac{d}{dx} \left(\frac{e^{2x} + 4}{x^3} \right)$$

$$= \frac{2x^3 e^{2x} - 3x^2 (e^{2x} + 4)}{x^6}$$

$$= \frac{2x^3 e^{2x} - 3x^2 e^{2x} - 12x^2}{x^6}$$

$$= \frac{2x e^{2x} - 3e^{2x} - 12}{x^4}$$

$$iv) f(x) = (x^3 - 2x) \ln x$$

$$f'(x) = \frac{x^3 - 2x}{x} + (3x^2 - 2) \ln x$$

$$= x^2 - 2 + (3x^2 - 2) \ln x$$

$\Rightarrow$ i) $l = r\theta$
 $2\pi = 12\theta$
 $\theta = \frac{\pi}{6}$

ii) $A = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$
 $= \frac{1}{2} \times 12^2 \times \frac{\pi}{6} - \frac{1}{2} \times 12^2 \times \frac{1}{2}$
 $= 12\pi - 36 \text{ cm}^2$
 $(\approx 1.7 \text{ cm}^2)$

$\Rightarrow \frac{dy}{dx} = \cos 2x - 2\sin 2x$
 $y = \sin 2x + \cos 2x + C$
 $0, 4 = 0 \therefore 0 = 0 + 1 + C$
 $\therefore y = \sin 2x + \cos 2x - 1$
 $= \frac{\pi}{2} \therefore y = 1 - 1 - 1$
 $y = -1$

14) a) 100, 95, 90, ...

i) A.P. if
 $95 - 100 = 90 - 95$
 $-5 = -5$

True $\therefore$ A.P.

ii) $T_{25} = 100 + (25-1) \times -5$
 $= 100 - 120$
 $= -20$

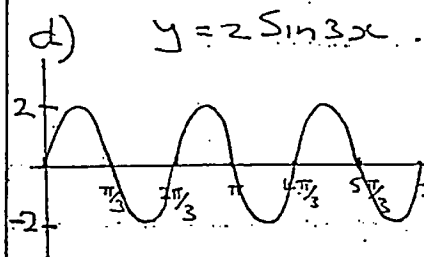
iii) $S_n = \frac{n}{2}(2a + (n-1)d)$
 $0 = \frac{n}{2}(200 + (n-1)(-5))$
 $0 = \frac{n}{2}(200 - 5n + 5)$
 $0 = n(205 - 5n)$
 $n = 0, 41$

$n=0$ not a solution

$\therefore$ 41 terms

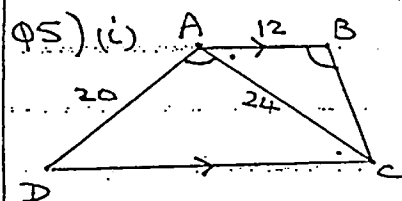
b) $V = \pi \int_{-2}^2 y^2 dx$
 $= 2\pi \int_0^2 (16 - x^4) dx$
 $= 2\pi \left[16x - \frac{x^5}{5} \right]_0^2$
 $= 2\pi \left((32 - \frac{32}{5}) - 0 \right)$
 $= \frac{256\pi}{5} \text{ cubic units}$

c) $y = x \ln x$
 $\frac{dy}{dx} = x \times \frac{1}{x} + \ln x$
 $= 1 + \ln x$
 at $x=1$: $\frac{dy}{dx} = 1 + 0$
 $\therefore$ gradient normal
 $= -1$



e) i) $P(\text{no sixes})$
 $= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$
 $= \frac{125}{216}$

ii) $P(\text{at least 1 six})$
 $= 1 - P(\text{no sixes})$
 $= 1 - \frac{125}{216}$
 $= \frac{91}{216}$



ABCD a trapezium
 $\Rightarrow AB \parallel DC$
 $\triangle ABC \parallel \triangle ADC$
 $\angle ABC = \angle DAC$ given
 $\angle BAC = \angle ACD$
 alternate angles in
 parallel lines
 $\therefore \triangle ABC \parallel \triangle ADC$ (A.A.A)

ii) $\frac{BC}{20} = \frac{12}{24}$
 $BC = 10$

b) i) $f(x) = e^{2x} \ln x$

x	1	2	3	4	5
$f(x)$	0	5.1	22.1	75.7	238.1

ii) $A = \frac{1}{3} \{ 1^5 + 10^5 + 2 \times \text{even} + 4 \times \text{od} \}$
 $= \frac{1}{3} \{ 0 + 238 \cdot 9 + 2 \times 22 \cdot 1 + 4(5 \cdot 1 + 75) \}$
 $= \frac{1}{3} \{ 238 \cdot 9 + 44 \cdot 2 + 323 \cdot 2 \}$
 $= \frac{1}{3} \times 606 \cdot 3$
 $= 202 \cdot 1$

c) $x = t^3 - 6t^2 + 9$
 i) initial when $t=0$
 $x = 9 \text{ m.}$

ii) at rest $v = 0$.

$$v = \frac{dx}{dt} = 3t^2 - 12t$$

$$3t^2 - 12t = 0$$

$$3t(t-4) = 0$$

$$t = 0, 4$$

iii) $t = 4$

$$x = 4^3 - 6 \times 4^2 + 9$$

$$= 64 - 96 + 9$$

$$= -23$$

$\therefore$ distance

travelled

$$= 9 + 23$$

$$= 32 \text{ m.}$$

Q6)

a) $y = x^4 - 8x^3$

$$y' = 4x^3 - 24x^2$$

$$y'' = 12x^2 - 48x$$

S.P. $y' = 0$

$$4x^3 - 24x^2 = 0$$

$$4x^2(x-6) = 0$$

$$x = 0, x = 6$$

$$y = 0, y = -432$$

$$f''(0) = 0$$

$\therefore (0,0)$ possible

horizontal point of inflexion

$$f''(6) > 0$$

$\therefore (6, -432)$ min

t.o.f of inflexion

$$f''(x) = 0$$

$$12x^2 - 48x = 0$$

$$12x(x-4) = 0$$

$$x = 0, x = 4$$

x	-1	0	1
y''	+	0	-

x	3	4	5
y''	-	0	+

change in concavity

$\therefore (0,0), (4, -256)$

are points of inflexion

end points in the

given domain

$(-1, 9) (9, 729)$

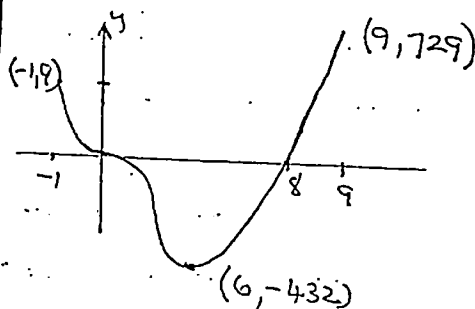
x intercepts. ($y=0$)

$$x^4 - 8x^3 = 0$$

$$x^3(x-8) = 0$$

$\therefore$ x intercepts, 0, 8.

y intercepts, 0.



b) i) $y = \tan x, y = 2\sin x$

at $A(\frac{\pi}{3}, \sqrt{3})$

$$y = \tan x$$

$$y = 2\sin x$$

$$\sqrt{3} = \tan \frac{\pi}{3}$$

$$\sqrt{3} = 2\sin \frac{\pi}{3}$$

$$\sqrt{3} = \sqrt{3}$$

$$\sqrt{3} = 2 \times \frac{\sqrt{3}}{2}$$

true.

$$\sqrt{3} = \sqrt{3}$$

True

as A lies on both curves

it must be a point of

intersection

ii) $\frac{d}{dx} (\ln(\cos x)) = -\tan x$

$$\text{LHS} = \frac{-\sin x}{\cos x}$$

$$= -\tan x$$

$$= \text{RHS}$$

iii) $A = \int_0^{\pi/3} 2\sin x - \tan x \, dx$

$$= \left[-2\cos x + \ln(\cos x) \right]_0^{\pi/3}$$

$$= (-2 \times \frac{1}{2} + \ln \frac{1}{2}) - (-2 + \ln 1)$$

$$= -1 + \ln \frac{1}{2} + 2$$

$$= 1 + \ln 2^{-1}$$

$$= 1 - \ln 2$$

c) $\int_1^b \frac{1}{x} \, dx = 1$

$$[\ln x]_1^b = 1$$

$$\ln b - \ln 1 = 1$$

$$\log_e b = 1$$

$$b = e^1$$

$$\therefore b = e$$

Q7) i) $S = S_0 e^{-kt}$

$$\frac{ds}{dt} = -kS_0 e^{-kt}$$

$$= -kS$$

ii) $t = 0, S = 200$

$$200 = S_0 e^0$$

$$S_0 = 200$$

$$S = 200e^{-kt}$$

$$t=50 \quad S=120$$

$$120 = 200e^{-50k}$$

$$0.6 = e^{-50k}$$

$$\ln(0.6) = -50k$$

$$k = \frac{\ln(0.6)}{50}$$

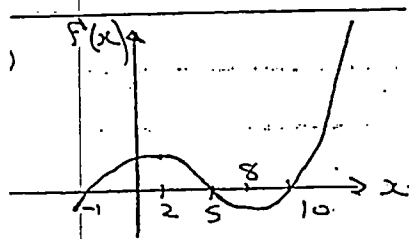
$$k \approx 0.01022$$

$$ii) \quad S = 200e^{-0.01022t}$$

$$t=90$$

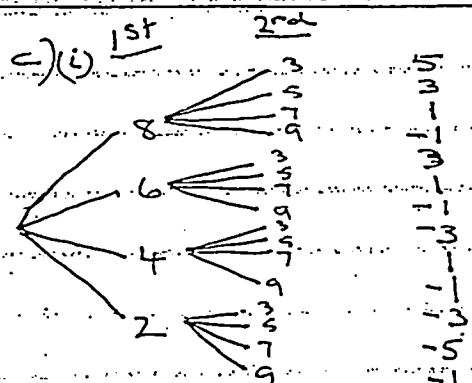
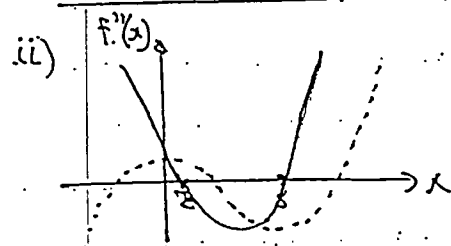
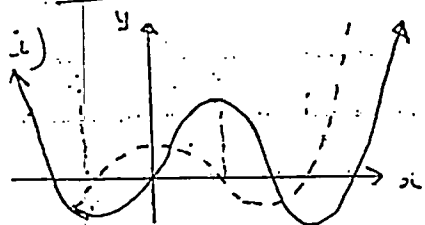
$$S = 200e^{-0.91949}$$

$$S = 79.7 \text{ m/s}$$



i) turning pts. when $f'(x) = 0$.

$$1 < x < 10$$



$$ii) \quad \frac{10}{16}$$

$$= \frac{5}{8}$$

$$Q8) a) i) \quad \frac{d}{dx} (\cos^2 3x)$$

$$= 2 \cos 3x \cdot -3 \sin 3x$$

$$= -6 \sin 3x \cos 3x$$

$$ii) \quad \int_{\pi/6}^{\pi} \cos 3x \sin 3x dx$$

$$= -\frac{1}{6} \int_{\pi/6}^{\pi} -6 \cos 3x \sin 3x dx$$

$$= -\frac{1}{6} [\cos^2 3x]_{\pi/6}^{\pi}$$

$$= -\frac{1}{6} (\cos^2(3\pi) - \cos^2(\pi/2))$$

$$= -\frac{1}{6} (1 - 0)$$

$$= -\frac{1}{6}$$

$$b) \quad 45, 48, 51$$

$$i) \quad T_{20} = 45 + 19 \times 3$$

$$= 45 + 57$$

$$= 102$$

ii)

$$S_n = \frac{n}{2} (a + L)$$

$$= \frac{20}{2} (45 + 102)$$

$$= 10 \times 147$$

$$= 1470$$

$$c) \quad y = ax^3 + \frac{b}{x^2}$$

$$\frac{dy}{dx} = 3ax^2 - \frac{2b}{x^3}$$

Cuts x-axis at $x=1$

$$\text{ie } x=1, y=0$$

$$0 = a + b \quad \text{--- (1)}$$

$$\text{at } x=1 \quad \frac{dy}{dx} = 2$$

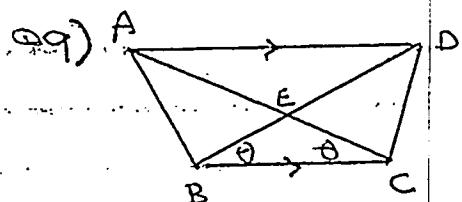
$$2 = 3a - 2b \quad \text{--- (2)}$$

$$(1) \times 2 \quad 0 = 2a + 2b \quad \text{--- (3)}$$

$$(2) + (3) \quad 2 = 5a$$

$$\frac{2}{5} = a$$

$$\text{Sub into (1)} \quad b = -\frac{2}{5}$$



$\angle EBC = \angle ECD$ (given)

$$i) \quad \angle EDA = \angle EBC$$

and $\angle DAE = \angle ECB$

(alternate angles in parallel lines)

$$\therefore \angle EDA = \angle DAE$$

$\therefore \triangle EAD$ is isosceles

- base angles equal

$$\therefore AE = DE$$

i) $\triangle EBC$ is isosceles

given $\angle EBC = \angle ECB$

$$EB = EC$$

from (i) $AE = DE$

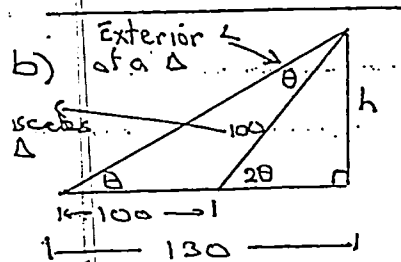
$$\therefore AC = DB$$

$$\angle DBC = \angle ACB$$

(given)

BC is Common

$$\therefore \triangle ABC \equiv \triangle DCB \text{ (SAS)}$$



$$i) h^2 = 100^2 - 130^2$$

$$= 9100$$

$$h = 95.39 \text{ m}$$

$$ii) \tan \theta = \frac{h}{130}$$

$$= \frac{95.39}{130}$$

$$= 36^\circ 16'$$

:-)

$$\sin \theta + \cos \theta \cdot \cot \theta - \cos \theta = 1$$

L.H.S

$$\sin \theta + \cos \theta \cdot \frac{\cos \theta}{\sin \theta} - \cos \theta = \frac{1}{\sin \theta}$$

$$\frac{\sin^2 \theta + \sin \theta \cos \theta + \cos^2 \theta - \sin \theta \cos \theta}{\sin \theta} = 1$$

$$1 + \sin \theta - 1$$

$$= \frac{\sin \theta}{\sin \theta}$$

$$= 1$$

$$= \text{R.H.S}$$

(Q10)

$$i) \frac{dv}{dt} = 100$$

$$v = 100t + c$$

$$\text{at } t=0 \quad v = \frac{4}{3} \times \pi \times 20^3$$

$$= 33510.3 \text{ cm}^3$$

$$\therefore c = 33510.3$$

$$\therefore v = 100t + 33510$$

$$ii) v = 100000$$

$$v = 100t + 33510$$

$$100000 = 100t + 33510$$

$$100t = 66490$$

$$t = 664.9 \text{ sec}$$

$$= 665 \text{ Sec}$$

b) i) by pythagoras

$$r^2 = 50^2 - x^2$$

$$r = \sqrt{2500 - x^2}$$

$$ii) v = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (2500 - x^2)(50 + x)$$

$$v = \frac{\pi}{3} (2500 - x^2)(50 + x)$$

$$\frac{dv}{dx} = \frac{\pi}{3} ((2500 - x^2) + (50 + x)x - 2x^2)$$

$$= \frac{\pi}{3} (2500 - x^2 - 100x - 2x^2)$$

$$= \frac{\pi}{3} (2500 - 100x - 3x^2)$$

$$\frac{dv}{dx} = \frac{\pi}{3} (-100 - 6x)$$

$$\text{max when } \frac{dv}{dx} = 0$$

$$\frac{\pi}{3} (2500 - 100x - 3x^2) = 0$$

$$3x^2 + 100x - 2500 = 0$$

$$x = \frac{-100 \pm \sqrt{10000 + 30000}}{6}$$

$$= \frac{-100 \pm 200}{6}$$

$$= \frac{100}{6} \text{ or } -\frac{300}{6}$$

→ not a solution

$$f''\left(\frac{100}{6}\right) < 0$$

$$\therefore \text{max } x = \frac{100}{6}$$

$$\text{Now } r = \sqrt{2500 - x^2}$$

$$= \sqrt{2500 - \left(\frac{100}{6}\right)^2}$$

$$r = 46.8 \text{ cm}$$