



2018
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A reference sheet is provided
- All necessary working should be shown in Question 11 – 16
- Write your student number and/or name at the top of every page

Total marks – 100

Section I - Pages 2 – 6

10 marks

Attempt Questions 1 - 10

Allow about 15 minutes for this section

Section II - Pages 7 – 14

90 marks

Attempt Questions 11 – 16

Allow about 2 hours 45 minutes for this section

This paper MUST NOT be removed from the examination room

Section I

10 marks

Attempt Questions 1-10.

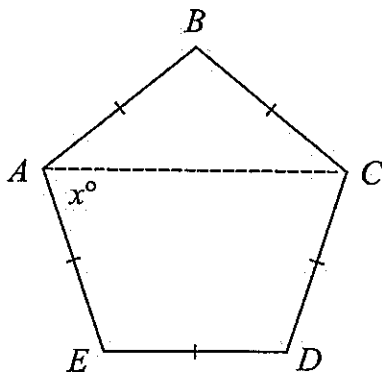
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10.

1 What is 0.00367254 written in scientific notation, correct to 4 significant figures?

- (A) 3.672×10^{-2}
- (B) 3.673×10^{-2}
- (C) 3.673×10^{-3}
- (D) 3.7×10^{-3}

2 $ABCDE$ is a regular pentagon and $\angle CAE = x^\circ$.



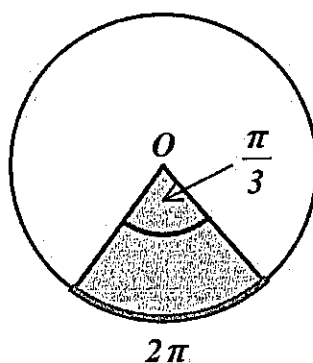
What is the value of x ?

- (A) 36°
- (B) 72°
- (C) 84°
- (D) 108°

3 Which of the following is equal to $\sec^2 x - \tan^2 x - \cos^2 x$?

- (A) $1 - \tan^2 x$
- (B) $-\tan^2 x$
- (C) $\cot^2 x$
- (D) $\sin^2 x$

4 A circle with centre O has a sector with arc length of 2π units and a sector angle of $\frac{\pi}{3}$.



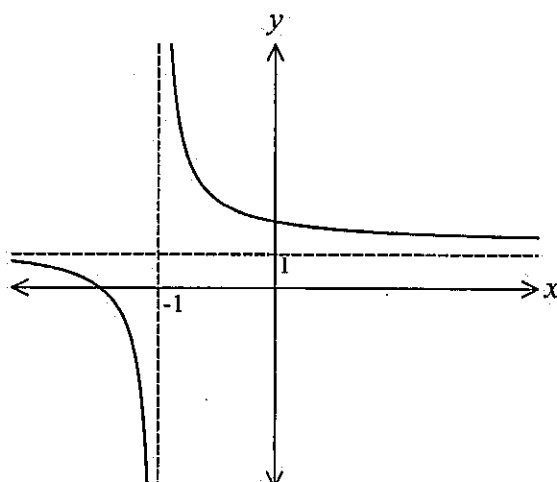
What is the area of the shaded sector?

- (A) 6π
- (B) 12π
- (C) $\frac{\pi^2}{5400}$
- (D) $\frac{\pi^3}{5400}$

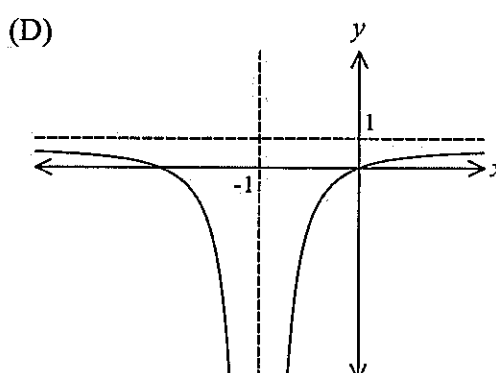
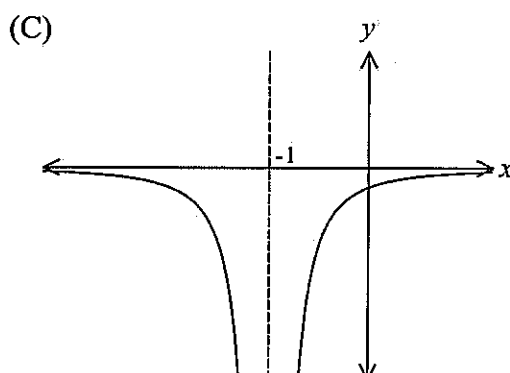
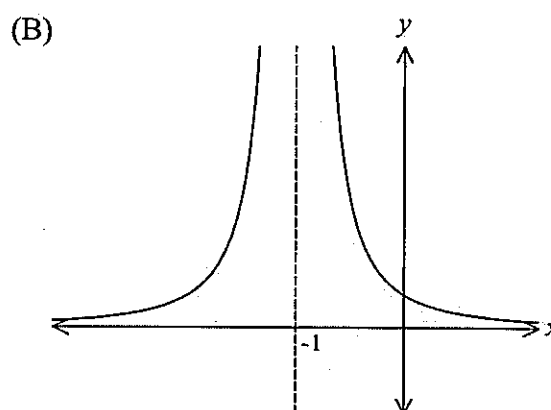
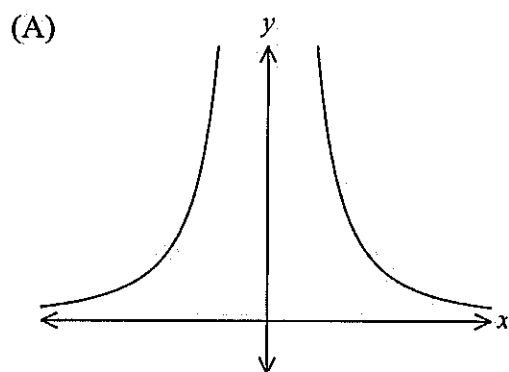
5 Which of the following is the solution to $3^x = 50$?

- (A) $x = 1.70$
- (B) $x = 3.56$
- (C) $x = 3.60$
- (D) $x = 3.91$

- 6 The diagram shows the graph of $y = f(x)$.



Which graph represents $f'(x)$?



- 7 If $a > 0$ and the function $f(x) = ax^3 + bx^2 + cx + d$ is always increasing, which of the following conditions must apply to a , b and c ?

- (A) $b^2 - ac < 0$
- (B) $b^2 - 2ac < 0$
- (C) $b^2 - 3ac < 0$
- (D) $b^2 - 4ac < 0$

- 8 The geometric series $a + ar + ar^2 + ar^3 + \dots$ has limiting sum S .

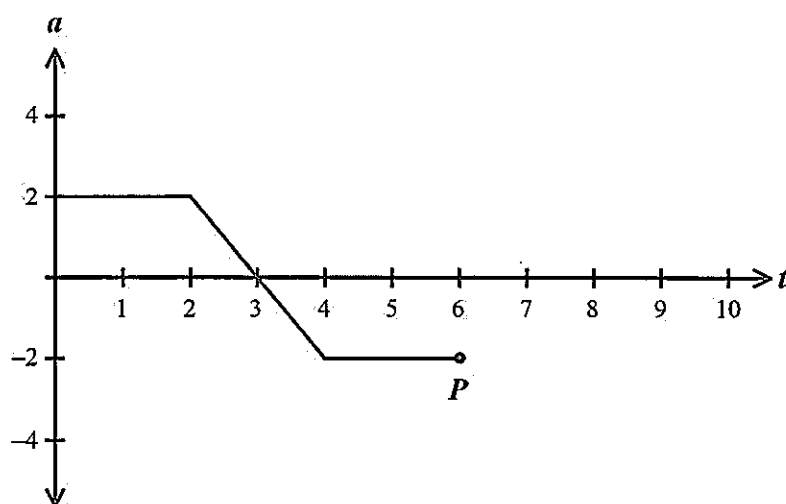
What is the limiting sum of the geometric series $a^2 + a^2r^2 + a^2r^4 + a^2r^6 + \dots$?

- (A) S^2
- (B) $\frac{aS}{1+r}$
- (C) $\frac{S^2}{1+r}$
- (D) $\frac{aS(1-r)}{1+r}$

- 9 Which of the following is a correct expression for $f'(x)$, given $f(x) = [\ln(x^2 + 1)]^3$?

- (A) $\frac{6x}{x^2 + 1}$
- (B) $\frac{8x^3}{(x^2 + 1)^3}$
- (C) $3[\ln(x^2 + 1)]^2$
- (D) $\frac{6x}{x^2 + 1}[\ln(x^2 + 1)]^2$

- 10 A particle is moving in a straight line. The acceleration/time graph is shown below.



If the particle was originally stationary at the origin, which of the following statements best describes the particle at point P ?

- (A) P is stationary at a point on the right of the origin.
- (B) P has positive velocity and negative acceleration.
- (C) P has negative velocity and negative acceleration.
- (D) P has negative velocity and zero acceleration.

Section II

90 marks

Attempt Questions 11–16

Allow about 2 hours and 45 minutes for this section

Answer each question in a new writing booklet.

In Questions 11–16, your responses should include relevant mathematical reasoning and/ or calculations.

Question 11 (15 marks) Start a new writing booklet.

(a) Solve $\frac{2x}{3} - 1 = \frac{x}{2}$. 2

(b) Evaluate $\lim_{m \rightarrow 4} \left(\frac{2m^2 - 9m + 4}{m^2 - 16} \right)$. 2

(c) If $(2\sqrt{2} - \sqrt{6})^2 = a - b\sqrt{3}$, find the values of a and b . 2

(d) Differentiate $\frac{3x + 2}{2x^3 - 5x^2}$. 2

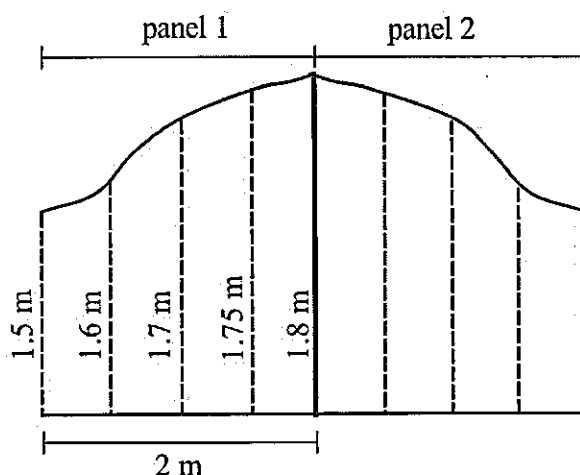
(e) Solve the inequality $|2x - 3| \leq 3$. 2

(f) Evaluate $\int_0^1 \frac{1}{2x+1} dx$ 3

(g) Find the equation of the tangent to the curve $y = 2(\tan x - 1)$ at the point $\left(\frac{\pi}{4}, 0\right)$. 2

Question 12 (15 marks) Start a new writing booklet.

- (a) A gate is being installed for the drive-way of a home. 3
 The gate consists of two steel panels that are exactly the same.
 Each panel is 2 m in length and is divided into 4 equal intervals.



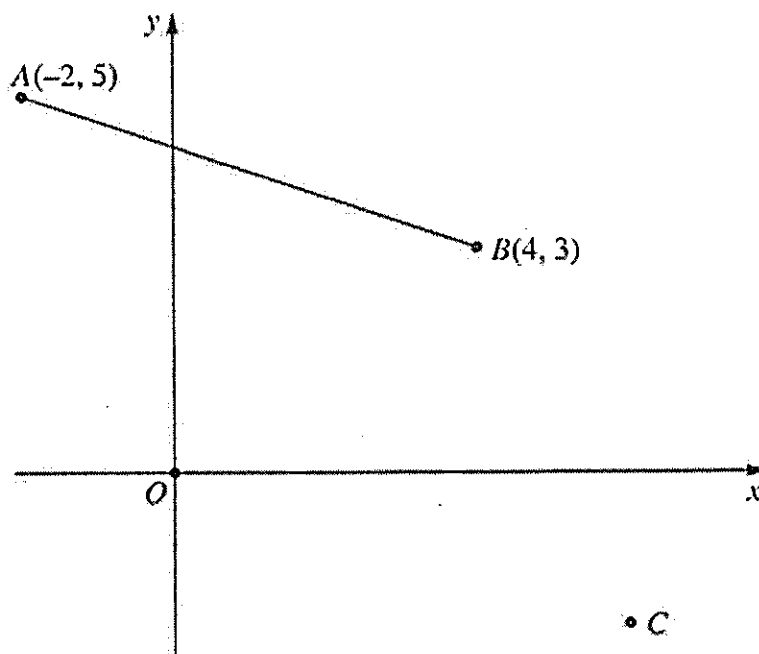
Using Simpson's rule with 5 vertical lengths, calculate the total area of the gate.

- (b) Find the domain and range of $y = \sqrt{x^2 - 1}$ 3
- (c) Solve $8\sin^2 x = \operatorname{cosec} x$ for $0 \leq x \leq 2\pi$ 2
- (d) The roots of the equation $2x^2 - 3x + 8 = 0$ are α and β .
- (i) Find the value of $\alpha + \beta$. 1
- (ii) Find the value of $\alpha\beta$. 1
- (iii) Find the value of $\alpha^2 + \beta^2$. 2
- (e) The region bounded by the curve $y = e^x$ and the x axis between $x = 0$ and $x = \log_e 3$ is rotated through one complete revolution about the x axis. 3

Find, in simplest exact form, the volume of the solid formed.

Question 13 (15 marks) Start a new writing booklet.

- (a) The diagram shows the points $A(-2, 5)$, $B(4, 3)$ and $O(0, 0)$. The point C is the fourth vertex of the parallelogram $OABC$.

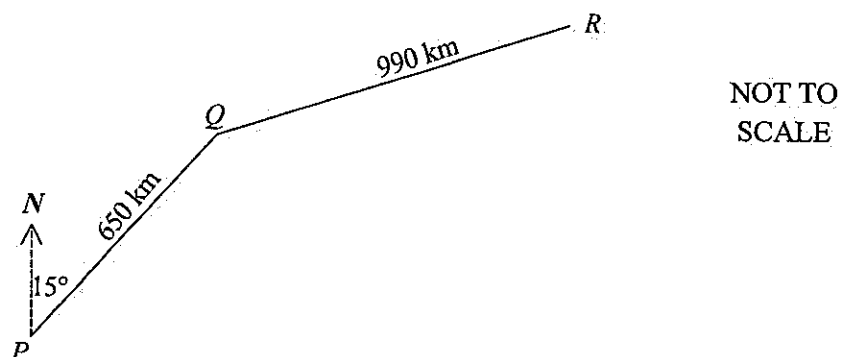


- | | | |
|-------|--|---|
| (i) | Show that the equation of AB is $x + 3y - 13 = 0$. | 2 |
| (ii) | Show that the length of AB is $2\sqrt{10}$. | 1 |
| (iii) | Calculate the perpendicular distance from O to AB . | 2 |
| (iv) | Calculate the area of parallelogram $OABC$. | 1 |
| (v) | Find the coordinates of C . | 1 |
| | | |
| (b) | Consider the curve $y = -x^3 + 3x + 2$. | |
| (i) | Find the stationary points and determine their nature. | 3 |
| (ii) | Find any point(s) of inflexion. | 2 |
| | | |
| (c) | Shade the region in the Cartesian plane which simultaneously satisfies the inequalities $y \geq 3^x$ and $y > 4 - x^2$. | 3 |

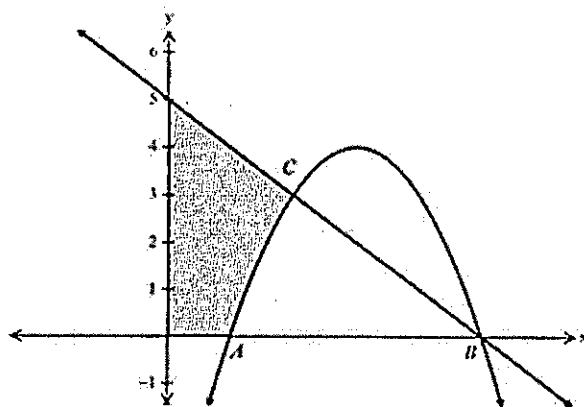
Question 14 (15 marks) Start a new writing booklet.

- (a) An aeroplane flies directly from town P to town Q . The distance from P to Q is 650 kilometres. The bearing of Q from P is 015° .

At town Q , the aeroplane turns onto a bearing of 040° and heads to town R which is 990 kilometres from town Q .



- (i) Show that $\angle PQR = 155^\circ$ 2
- (ii) Calculate the distance from town P to town R , correct to the nearest kilometre. 2
- (b) The parabola $y = -x^2 + 6x - 5$ meets the x axis at points $A(1,0)$ and $B(5,0)$. 3
The line $y = -x + 5$ meets the parabola at points B and $C(2,3)$.

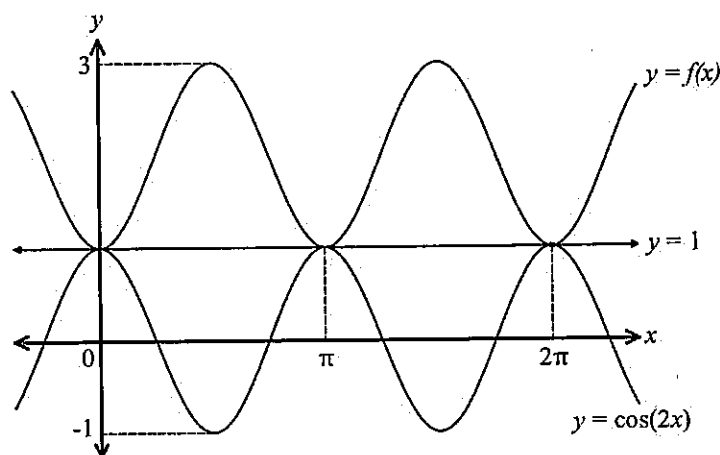


Find the area of the shaded region.

Question 14 continues on page 11

- (c) The diagram below shows the graph of $y = \cos(2x)$ and $y = f(x)$ from $x = 0$ to $x = 2\pi$. 2

The graph of $y = f(x)$ is a reflection of $y = \cos(2x)$ across the line $y = 1$.



Find the equation for the graph of $y = f(x)$.

- (d) Consider the parabola $y = x^2 - 8x + 4$.
- Find the coordinates of the vertex. 2
 - Find the coordinates of the focus. 1

- (e) In March 1958, 24 koalas were introduced on Kangaroo Island. By March 2018, the number of koalas had risen to 5000.

Assume that the number N of koalas is increasing exponentially and satisfies the equation $N = 24e^{kt}$, where k is a constant and t is measured in years from March 1958.

- Show that $k = 0.0890$, correct to 4 significant figures. 2
- Predict the number of koalas that will be present on Kangaroo Island in March 2024. 1

End of Question 14

Question 15 (15 marks) Start a new writing booklet.

- (a) The point $P(x, y)$ moves such that the gradient of AP is twice the gradient of BP , where the points A and B are $(0, -5)$ and $(-2, 3)$ respectively. 3

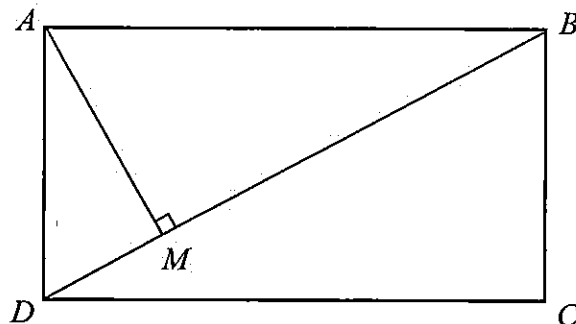
Find the equation of the locus of P .

- (b) A series is given by 2

$$\log_{10} e + \log_{10} e^2 + \log_{10} e^3 + \cdots + \log_{10} e^n$$

Show that this is an arithmetic series.

- (c) $ABCD$ is a rectangle with $AB = 12$ cm and $AD = 9$ cm. AM is perpendicular to BD .



- (i) Find the length of BD . 1
- (ii) Prove that $\triangle ABM$ is similar to $\triangle DBA$. 2
- (iii) Hence find the length of BM . 2

Question 15 continues on page 13

- (d) At the start of the month, Katrina opens a bank account and deposits \$300 into the account.

At the start of each subsequent month, Katrina makes a deposit which is 1.5% more than the previous deposit.

At the end of every month, the bank pays Katrina interest at a rate of 3% per annum on the balance of the account.

- (i) Show that the balance of the account at the end of the second month is 2

$$\$300(1.0025)^2 + \$300(1.015)(1.0025)$$

- (ii) Show that the balance of the account at the end of the n^{th} month is given by 2

$$\$300(1.0025)^n \left(\frac{\left(\frac{1.015}{1.0025} \right)^n - 1}{\left(\frac{1.015}{1.0025} \right) - 1} \right)$$

- (iii) Calculate the balance of the account at the end of the 60th month, correct to the nearest dollar. 1

End of Question 15

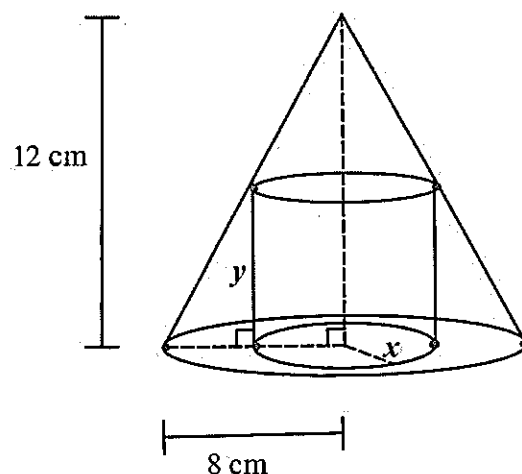
Question 16 (15 marks) Start a new writing booklet.

- (a) A particle travelling in a straight line is initially at rest.
Its acceleration as a function of time t (seconds), is given by $a = 4 \sin 2t$.
- (i) Show that the velocity of the particle is given by $v = 2 - 2 \cos(2t)$ 2
- (ii) Sketch the graph of the velocity as a function of time, for $0 \leq t \leq \pi$ 2
- (iii) Explain why the particle never changes direction. 1
- (iv) Find the total distance travelled in the first π seconds. 1

- (b) (i) Find $\frac{d}{dx} [x \tan x + \log(\cos x)]$. 2

- (ii) Hence find $\int_0^{\frac{\pi}{4}} \left(x \sec^2 x - \frac{\sin x}{\cos x} \right) dx$. 3

- (c) The diagram below shows a cylinder of radius x cm and height y cm, inscribed in a cone. The cone has a radius of 8 cm and its height is 12 cm.



- (i) Show that $y = \frac{3}{2}(8 - x)$. 1
- (ii) Show that the volume of the cylinder is given by $V = \frac{3}{2}\pi x^2(8 - x)$ cm³. 1
- (iii) Find the value of x for which the volume is a maximum. 2

End of paper

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- | | | |
|------|------|-------|
| 1. C | 5. B | 9. D |
| 2. B | 6. C | 10. A |
| 3. D | 7. C | |
| 4. A | 8. B | |

Question 11

$$\begin{aligned} \text{a) } \frac{2x}{3} - 1 &= \frac{x}{2} \\ 4x - 6 &= 3x \\ x &= 6 \end{aligned}$$

$$\begin{aligned} \text{b) } \lim_{m \rightarrow 4} \left(\frac{2m^2 - 9m + 4}{m^2 - 16} \right) &= \lim_{m \rightarrow 4} \frac{(2m-1)(m-4)}{(m+4)(m-4)} \\ &= \frac{2(4)-1}{4+4} \\ &= \frac{7}{8} \end{aligned}$$

$$\begin{aligned} \text{c) } (2\sqrt{2} - \sqrt{6})^2 &= 8 - 4\sqrt{12} + 6 \\ &= 14 - 8\sqrt{3} \end{aligned}$$

$$\therefore a = 14, b = 8$$

$$\begin{aligned} \text{d) } \frac{d}{dx} \left(\frac{3x+2}{2x^3-5x^2} \right) &= \frac{(2x^3-5x^2) \times 3 - (3x+2)(6x^2-10x)}{(2x^3-5x^2)^2} \\ &= \frac{6x^3 - 15x^2 - 18x^3 + 30x^2 - 12x^2 + 20x}{(2x^3-5x^2)^2} \\ &= \frac{20x + 3x^2 - 12x^3}{(2x^3-5x^2)^2} \end{aligned}$$

$$\text{e) } |2x-3| \leq 3$$

$$\begin{aligned} 2x-3 &\leq 3 & \text{or} & & 2x-3 &\geq -3 \\ 2x &\leq 6 & & & 2x &\geq 0 \\ x &\leq 3 & & & x &\geq 0 \end{aligned}$$

$$\therefore 0 \leq x \leq 3$$

$$\begin{aligned} \text{f) } \int_0^1 \frac{1}{2x+1} dx &= \frac{1}{2} \int_0^1 \frac{2}{2x+1} dx \\ &= \frac{1}{2} [\ln(2x+1)]_0^1 \\ &= \frac{1}{2} [\ln 3 - \ln 1] \\ &= \frac{1}{2} \ln 3 \end{aligned}$$

$$\text{g) } y = 2\tan x - 2$$

$$\begin{aligned} y' &= 2\sec^2 x \\ m_r &= 2\sec^2 \frac{\pi}{4} \\ &= 4 \end{aligned}$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= 4(x - \frac{\pi}{4}) \\ y &= 4x - \pi \end{aligned}$$

CBDA | BCCB | DA

Question 12

$$a) A \approx \frac{h}{3} [y_0 + y_n + 4(y_1 + y_3) + 2(y_2)] \times 2$$

$$A \approx \frac{0.5}{3} [1.5 + 1.8 + 4(1.6 + 1.75) + 2(1.7)] \times 2 \\ = 6.7 \text{ m}^2$$

$$b) y = \sqrt{x^2 - 1}$$

$$\text{Domain: } x^2 - 1 \geq 0 \\ x \leq -1 \text{ or } x \geq 1$$

$$\text{Range: } y \geq 0$$

$$c) 8 \sin^3 x = \operatorname{cosec} x, \quad 0 \leq x \leq 2\pi$$

$$8 \sin^3 x = 1$$

$$\sin^3 x = \frac{1}{8}$$

$$\sin x = \frac{1}{2}$$

$$\frac{*S/A*}{T/C}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$d) 2x^2 - 3x + 8 = 0$$

$$(i) \alpha + \beta = \frac{-b}{a} \\ = \frac{3}{2}$$

$$(ii) \alpha\beta = \frac{c}{a} \\ = 4$$

$$(iii) (\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2 \\ \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \\ = \left(\frac{3}{2}\right)^2 - 2 \times 4 \\ = -\frac{23}{4}$$

$$e) V = \pi \int_0^{\ln 3} (e^x)^2 dx$$

$$= \pi \int_0^{\ln 3} e^{2x} dx$$

$$= \frac{\pi}{2} [e^{2x}]_0^{\ln 3}$$

$$= \frac{\pi}{2} [e^{2\ln 3} - e^0]$$

$$= \frac{\pi}{2} [e^{\ln 9} - e^0]$$

$$= \frac{\pi}{2} [9 - 1]$$

$$= 4\pi \text{ units}^3$$

Question 13

$$a) (i) \frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-5}{x+2} = \frac{3-5}{4+2}$$

$$6(y-5) = -2(x+2)$$

$$6y - 30 = -2x - 4$$

$$2x + 6y - 26 = 0$$

$$x + 3y - 13 = 0$$

$$(ii) d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$d = \sqrt{(4+2)^2 + (3-5)^2}$$

$$= \sqrt{36+4}$$

$$= \sqrt{40}$$

$$= 2\sqrt{10}$$

$$(iii) d_{\perp} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$d_{\perp} = \frac{|1 \times 0 + 3 \times 0 - 13|}{\sqrt{1^2 + 3^2}}$$

$$= \frac{13}{\sqrt{10}}$$

$$(iv) \text{Area} = b \times h$$

$$= 2\sqrt{10} \times \frac{13}{\sqrt{10}}$$

$$= 26 \text{ units}^2$$

$$(v) C(6, -2)$$

$$b) y = -x^3 + 3x + 2$$

$$(i) y' = -3x^2 + 3$$

Stat. pts when $y' = 0$

$$-3x^2 + 3 = 0$$

$$3x^2 = 3$$

$$x^2 = 1$$

$$x = \pm 1$$

$$\text{When } x=1, y = -1 + 3 + 2 = 4$$

$$\text{When } x=-1, y = 1 - 3 + 2 = 0$$

$$y'' = -6x$$

$$\text{When } x=1, y'' = -6$$

$$\text{When } x=-1, y'' = 6$$

$\therefore$ Max turning point at $(1, 4)$, min turning point at $(-1, 0)$

(ii) Possible points of inflexion when $y'' = 0$

$$-6x = 0$$

$$x = 0$$

Check concavity

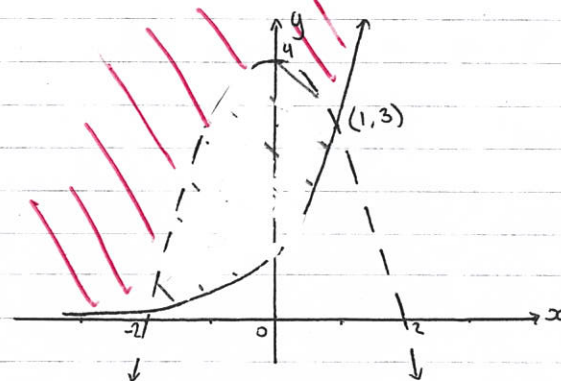
$$x \quad -0.5 \quad 0 \quad 0.5$$

$$y'' \quad > 0 \quad 0 \quad < 0$$

$$\text{When } x=0, y = 0 + 0 + 2 = 2$$

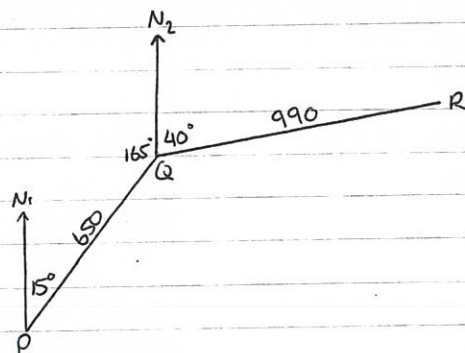
$\therefore$ point of inflexion at $(0, 2)$

c)



Question 14

a)



- (i) $\angle PQN_2 = 165^\circ$ (co-interior $\angle$'s in || lines)
 $\angle RQN_2 = 40^\circ$ (bearing of R from Q)
 $\angle PQR = 360 - 165 - 40$ ($\angle$'s in a revolution)
 $= 155^\circ$

(ii) $PR^2 = 650^2 + 990^2 - 2 \times 650 \times 990 \times \cos 155^\circ$
 $= 2569018.122 \dots$
 $PR = 1602.8156 \dots$
 $\approx 1603 \text{ km}$

b) Area = $\frac{2}{2} (3+5) - \int_3^5 -x^2 + 6x - 5 \, dx$

$$= 8 - \left[-\frac{x^3}{3} + 3x^2 - 5x \right]_3^5$$

$$= 8 - \left[-\frac{8}{3} + 12 - 10 - \left(-\frac{1}{3} + 3 - 5 \right) \right]$$

$$= \frac{19}{3} \text{ units}^2$$

c) $y = 2 - \cos 2x$

d) $y = x^2 - 8x + 4$

(i) $y = x^2 - 8x + 16 - 12$
 $y + 12 = (x - 4)^2$

Vertex $(4, -12)$

(ii) $4a = 1$
 $a = \frac{1}{4}$

Focus $(4, -11\frac{3}{4})$

e) (i) $N = 24e^{kt}$

When $t = 60$, $N = 5000$

$$5000 = 24e^{60k}$$

$$e^{60k} = \frac{5000}{24}$$

$$60k = \ln\left(\frac{5000}{24}\right)$$

$$k = \frac{\ln\left(\frac{5000}{24}\right)}{60}$$

$$k = 0.0889856 \dots \quad 0.08899 *$$

$$= 0.0890 \text{ to 4 sig. figs.} \quad \leftarrow$$

(ii) $N = 24e^{0.0890 \times 66}$
 $= 8536$

or if using complete k value,

$$N = 24e^{66k}$$

$$= 8528$$

* Various answers according to rounded response part e) i;

Question 15

a) $A(0, -5), B(-2, 3), P(x, y)$

$$m_{AP} = 2 \times m_{BP}$$

$$\frac{y+5}{x-0} = 2 \times \frac{y-3}{x+2}$$

$$(y+5)(x+2) = 2(y-3)(x)$$

$$xy + 2y + 5x + 10 = 2xy - 6x$$

$$11x + 2y - xy + 10 = 0$$

b) $\log_{10} e^2 - \log_{10} e = 2 \log_{10} e - \log_{10} e$
 $= \log_{10} e$

$$\log_{10} e^3 - \log_{10} e^2 = 3 \log_{10} e - 2 \log_{10} e$$

 $= \log_{10} e$

$$\therefore T_3 - T_2 = T_2 - T_1$$

$\therefore$ it is an arithmetic series with $d = \log_{10} e$

c) (i) $BD^2 = 12^2 + 9^2$
 $= 225$

$$BD = 15 \text{ cm}$$

(ii) Let $\angle ABM = \alpha$

$$\angle BAM = 90 - \alpha \text{ (L sum of } \triangle BAM)$$

$$\angle DAM = \alpha \text{ (adjacent complementary L's)}$$

$$\angle AMB = \angle DAB = 90^\circ$$

$$\angle ABM = \angle DBA = \alpha \text{ (as above)}$$

$$\therefore \triangle ABM \parallel \triangle DBA \text{ (equiangular)}$$

(iii) $\frac{BM}{AB} = \frac{AB}{BD}$

$$\frac{BM}{12} = \frac{12}{15}$$

$$BM = \frac{12^2}{15}$$

 $= 9.6 \text{ cm}$

d) (i) $r = 3\% \text{ p.a.}$

$$= 0.0025 \text{ per month}$$

$$A_1 = 300 \times 1.0025$$

$$A_2 = (A_1 + 300 \times 1.015) \times 1.0025$$

 $= 300 \times 1.0025^2 + 300 \times 1.015 \times 1.0025$

(ii) $A_3 = (A_2 + 300 \times 1.015^2) \times 1.0025$
 $= (300 \times 1.0025^2 + 300 \times 1.015 \times 1.0025 + 300 \times 1.015^2) \times 1.0025$
 $= 300 \times 1.0025^3 + 300 \times 1.015 \times 1.0025^2 + 300 \times 1.015^2 \times 1.0025$
 $= 300 (1.0025^3 + 1.015 \times 1.0025^2 + 1.015^2 \times 1.0025)$

$$A_n = 300 (1.0025^n + 1.015 \times 1.0025^{n-1} + 1.015^2 \times 1.0025^{n-2} + \dots + 1.015^{n-1} \times 1.0025)$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1.0025^n \left[\left(\frac{1.015}{1.0025} \right)^n - 1 \right]}{\left(\frac{1.015}{1.0025} - 1 \right)}$$

$$\therefore A_n = 300 (1.0025)^n \left[\frac{\left(\frac{1.015}{1.0025} \right)^n - 1}{\left(\frac{1.015}{1.0025} \right) - 1} \right]$$

$$(iii) A_{60} = 300 \times 1.0025^{60} \times \left[\frac{\left(\frac{1.015}{1.0025} \right)^{60} - 1}{\left(\frac{1.015}{1.0025} \right) - 1} \right]$$

$$= \$30\,835.37$$

$$\approx \$30\,835$$

Question 1b

$$a) (i) \frac{dv}{dt} = 4 \sin 2t$$

$$v = \int 4 \sin 2t \, dt$$

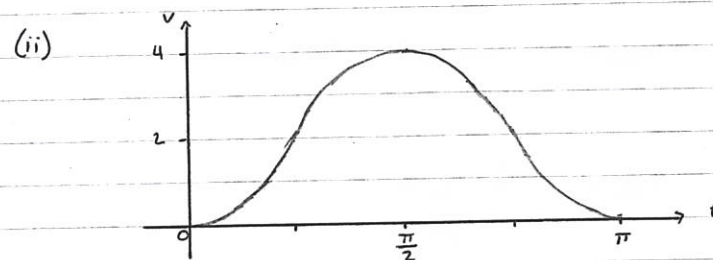
$$v = -2 \cos 2t + c$$

$$\text{When } t=0, v=0$$

$$0 = -2 \cos 0 + c$$

$$c = 2$$

$$\therefore v = 2 - 2 \cos 2t$$



(iii) Velocity is never negative

(iv) Distance = area under curve

$$= (4 \times \pi) \div 2$$

$$= 2\pi \text{ units}$$

$$b) (i) \frac{d}{dx} [x \tan x + \log (\cos x)]$$

$$= \tan x \times 1 + x \times \sec^2 x - \frac{\sin x}{\cos x}$$

$$= \tan x + x \sec^2 x - \tan x$$

$$= x \sec^2 x$$

$$(ii) \int_0^{\frac{\pi}{4}} x \sec^2 x - \frac{\sin x}{\cos x} dx$$

$$= \left[x \tan x + \log(\cos x) + \log(\cos x) \right]_0^{\frac{\pi}{4}}$$

$$= \left[x \tan x + 2 \log(\cos x) \right]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4} \times \tan \frac{\pi}{4} + 2 \log(\cos \frac{\pi}{4}) - [0 + 2 \log(\cos 0)]$$

$$= \frac{\pi}{4} + 2 \log\left(\frac{1}{\sqrt{2}}\right) - 2 \log 1$$

$$= \frac{\pi}{4} + 2 \log(2^{-\frac{1}{2}})$$

$$= \frac{\pi}{4} - \log 2$$

c) (i) Using similar triangles,

$$\frac{y}{12} = \frac{8-x}{8}$$

$$y = \frac{12(8-x)}{8}$$

$$y = \frac{3}{2}(8-x)$$

$$(ii) V = \pi r^2 h$$

$$V = \pi x^2 y$$

$$= \pi x^2 \times \frac{3}{2}(8-x)$$

$$= \frac{3}{2} \pi x^2 (8-x)$$

$$(iii) V(x) = 12\pi x^2 - \frac{3}{2}\pi x^3$$

$$V'(x) = 24\pi x - \frac{9}{2}\pi x^2$$

Max or min when $V'(x) = 0$

$$0 = 24\pi x - \frac{9}{2}\pi x^2$$

$$0 = \frac{3}{2}\pi x (16 - 3x)$$

$$x = 0, \frac{16}{3}$$

But $x > 0$

Check concavity

$$V''(x) = 24\pi - 9\pi x$$

$$V''\left(\frac{16}{3}\right) = 24\pi - 9\pi\left(\frac{16}{3}\right)$$

$$= -24\pi$$

$$< 0$$

$\therefore$ max value when $x = \frac{16}{3}$